**Indian Institute of Technology Madras**
**NPTEL**

**National Programme on Technology Enhanced Learning**
**Video Lectures on**
**Convective Heat Transfer**

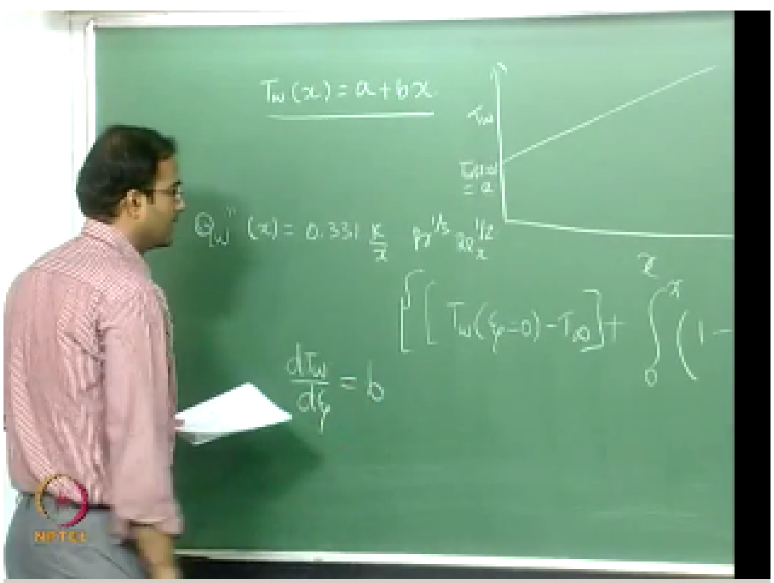
**Dr. Arvind Pattamatta**

**Department of Mechanical Engineering**
**Indian Institute of Technology Madras**

# Lecture 24
# Laminar External heat transfer with non uniform surface temperature

So today we will take up a problem on the variable w temperature case and we will look at how to get the expression for the w heat flux variation as well as the heat transfer coefficient so let us take the example of a linear surface temperature variation that is of the form.

(Refer Slide Time: 00:34)

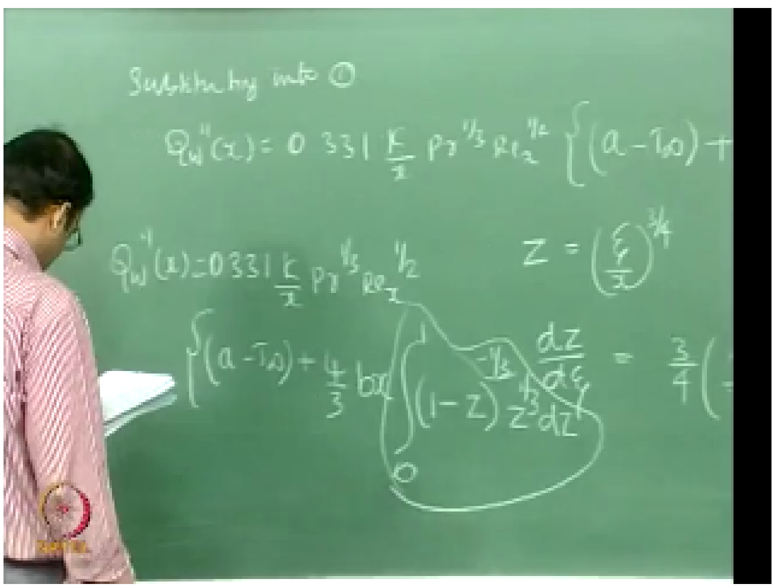



T w (x) = a + BX okay so if you plot the surface temperature as a function of X so at X is = 0 that will be = constant an and from there it varies linearly okay so this is the value T w at X is = 0 which is = a so if you go back to what we derived yesterday with the Duhamel superposition integral if you have a continuous variation of the w temperature so we have derived the expression for the w heat flux this was 0.33 1 x K / X P R 1 by 3 and you have T w at $\theta$ = 0 - T $\infty$ + 0 to X 1 - $\theta$ by X the whole power 3 by 4 - 1 / 3 DT 1 by D Z x okay ,so this is our expression you know now if for the limiting case where you do not have any slope the slope is 0 so this is Steve all - T $\infty$.

So this becomes = your flat plate with a uniform temperature expression okay so of course if you have continuous variation but also intermediate temperature jumps okay so there you have to introduce an additional $\delta$ T x H you know for those temperature jumps in between okay so apart from that this is the expression now we will substitute for the given temperature profile whatever we require so for

example in order to evaluate this Duhamel integral we need to first get the slope of the w temperature profile and since this is a linear profile it is very simple so in this case DT w by D Z will be for this particular profile B right so we will substitute this x the equation.

(Refer Slide Time: 03:20)



Let us call this as equation 1 so this is T wet Z = 0 is nothing but a okay so right at X is = 0 this is nothing but a so I'm just substituting for T wet Z = 0 as a - T $\infty$ + integral 0 to X 1 - Z / X to the power 3 by 4 the whole power - 1 / 3 DT w by D Z is nothing. But B so this will be B x D Z okay so now all we need to do is evaluate this integral because everything else is known T $\infty$ is given for this problem whatever value it is known as is a constant B is a constant so we need to find this particular integral and this particular integral whatever slope that you get and put it here is of the following form which we will reduce it to so we can assume a variable Z which is = Z / $X^{3}$ / 4 okay.

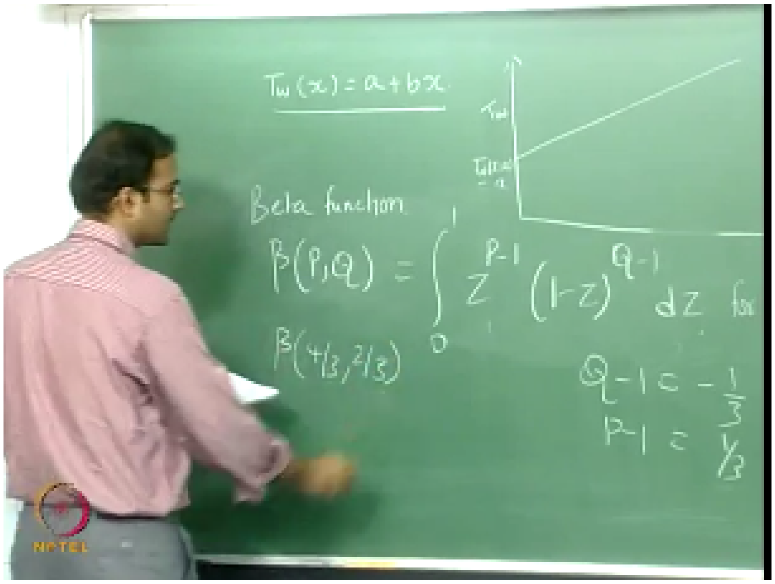
So I am just going to transform the variables again so I am saying that $\theta$ by $X^{3}$ / 4 is = some other variable Z so therefore DZ by D Z should be = 3 / 4 Z /$X^{3}$ by 4 - 1 which is - 1 / 4 x with respect to D Z Φ so X is a constant so it will be 1 by X right so this is my derivative okay so therefore now I can substitute for D Z in this integral in terms of DZ okay so this will give me my DZ is = 3 by 4 x X power this is $X^{1/4}$ - 1 so this will be $X^{-3}$ /4 okay x $\zeta^{-1}$ by 4 x D Z alright it is just I am just doing some algebraic manipulation there so therefore now I can substitute for D Z I can transform these variables from D Z 2 DZ okay so I can substitute for Z / $X^{T2}$ / 4 s Z and D Z.

I can substitute in terms of DZ in fact I can also write this in terms of Z ok now that is Z by $X^3$ by 4 so $Z^1$ by 3 I can put it because this is already so my DZ will be D Z 4 by 3 here $X^3$ by 4 $Z^1$ /4 okay so θ by X so I can write this as 1 by 4 $X^{-3}$ by 4 I can take 1 by 4 X yeah so I can write this as Z by X the whole power 1 by 4 this one - yeah right yeah right I can write it like that because this is anyway $3^{43}$ / 4 I can write it as X / $X^1$ /4 okay so now this is nothing but θ by X to the power 1 by 4 is nothing but $Z^1$ / 3 okay so this I can rewrite as that to the power this entire thing has that to the power one-third okay so what I am doing is I am transforming all my variables from Z plane to Z plane okay.

So where ever I have Z that also how to include that so therefore if I substitute x this expression 0.33 1 a/ X P $R^1$ /3 Rd X to the power so you have a - T ∞ + so I haven't substituting for D Z so 4 by 3 is constant 4 by 3 also B is a constant I can take out and also inside the integral this is integrated with respect to now DZ therefore X also can be taken out of the integral now 0 to this is originally 0 to X here so I can transform this to 0 to one okay because that Z = X this becomes Z = one so therefore this will be the upper integral will be one upper limit of integration so this will be 1 - Z 1 - Z sorry 1 - Z - 1 by 3 and you have this $Z^1$ /3 here so that will also come outside and by 3 x DZ okay just check 1 - Z to the power - 1 by 3 x that to the power 1 /3 DZ okay so now I have anyway transformed that to this integral right here now how should I integrate so I will just give you the formula.

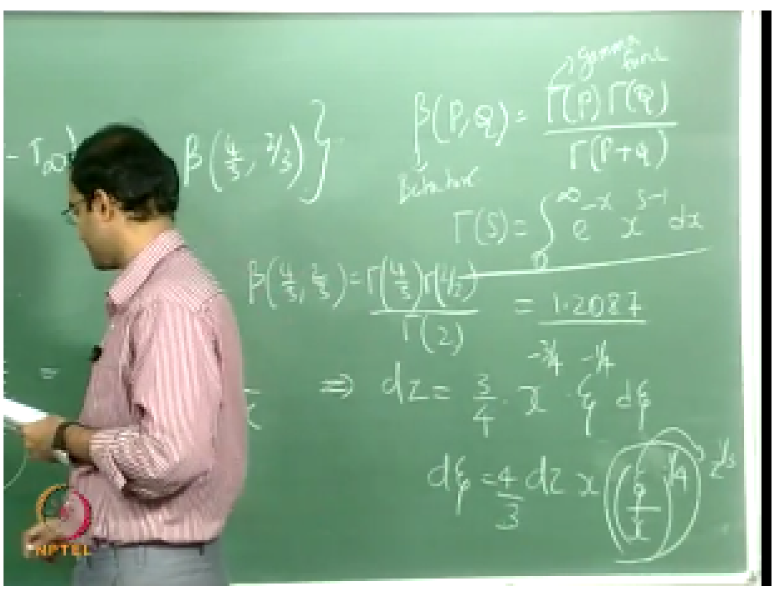
Now this is of the form of what is called as a β function okay the β function can be expressed so generally the problems with the Duhamel integral will be of the form of a β function once you transform the variables from Z to DZ okay so you will be ending up with a function something like this and you can express this β function.

(Refer Slide Time: 10:46)




In terms of constants P and Q 0 to 1 $Z^{P}$ - 1 1 - Z Q - 1 valid okay this is valid for positive values of P and Q > ∞ okay so this is how your β function is defined okay, now if you compare this with this expression you can see 1 - $Z^{Q}$ so therefore Q - 1 is = - 1 by 3 and P - 1 is = 1 by 3. Right therefore Q is = 1 1 - 1 / 3 2 / 3 and P = 1 + 1 / 3 4 / 3 okay so therefore this P and Q will be nothing but 4 /3 , 2 / 3 okay, so therefore this entire term can be written in terms of β function.

(Refer Slide Time: 12:40)

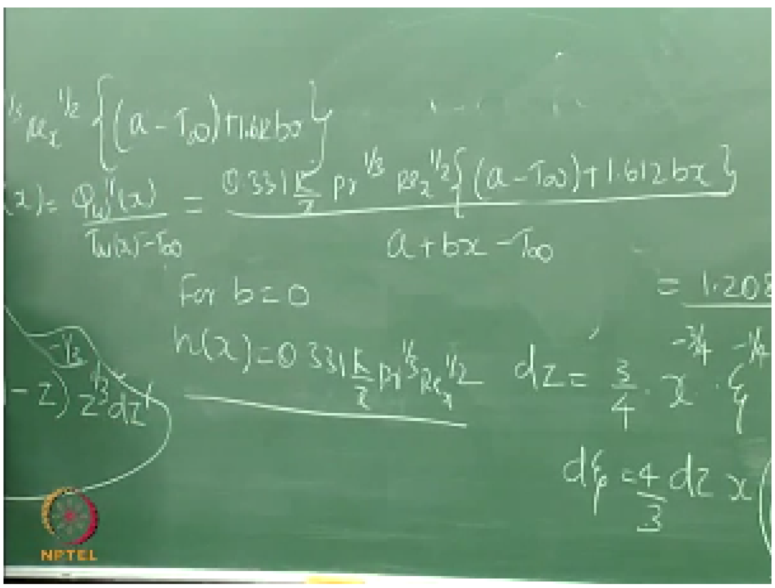



Okay so everything here as it is a - T ∞ + 4 by 3 VX x β function okay this entire integral is β function the parameters are 4 / 3, 2 / 3 okay so the integral is replaced by the β. And why do we now need to write this in terms of β function because we can the β functions are tabulated okay in fact very specifically the β function itself can be expressed as a function of another function called the $\gamma$ function okay so on $\gamma$ function tables are quite common you know they are tabulated for different values of the function.

So therefore we will express the $\beta$ function in terms of $\gamma$ function as follows this is your $\gamma$ function $\beta$ function we are generally your $\gamma$ function $\gamma$ of say some variable s is represented as e power - X $X^{S}$ - 1 DX so this is your $\gamma$ function basically ok and this has been tabulated you know you can do this integral numerically also for different values of s but this have been tabulated $\gamma$ function charts are there so you can look up for the values that we have here so therefore $\beta$ of 4 / 3 , 2 / 3 will be $\gamma$ of 4 / 3 x $\gamma$ of 2 / 3 / $\gamma$ of what 6 /3 that is 2 okay so if you plug in from the $\gamma$ function tables which are available online.

You can Google $\gamma$ function charts and you will find those nice charts for different values of this you know for P and Q so if you plug in you will get this $\beta$ of 4 by 3 , 2 by 3 as 1 .2 0 8 7 okay so this is the resulting expression for $\beta$ 4 by 3 , 2 by 3 so if you substitute that value x this so this is 1 .2 0 8 7 x 4 / 3.

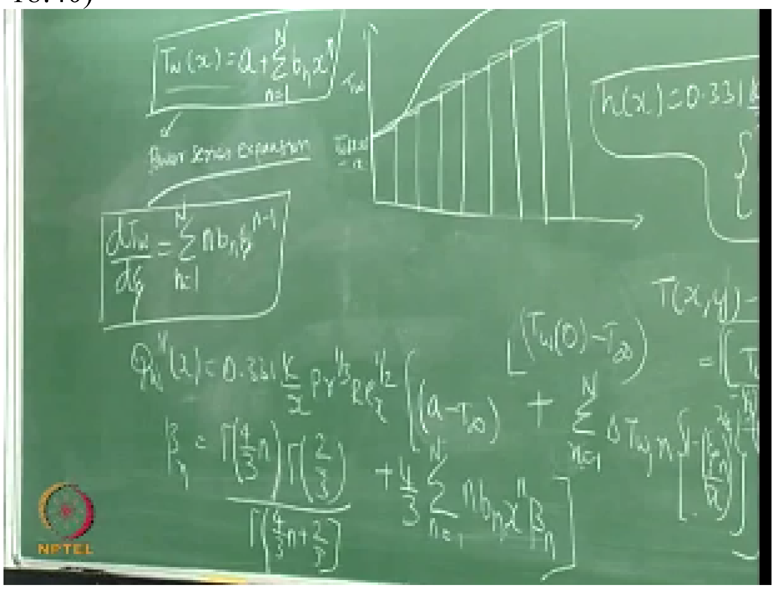
(Refer Slide Time: 15:30)




Which comes out as 1 .6 1 2 okay so now we have a complete expression which gives you the variation of heat flux with respect to X okay provided you know your constant A and B and you know the free stream temperature $T_\infty$ all right so now therefore from this the heat transfer coefficient can be defined okay so H of X can be defined as $Q''$ X /T w of X - $T_\infty$ okay now T w of X - $T_\infty$ can be written as a + B X - $T_\infty$ so therefore you have this entire 0.33 one a by X B R to the power one-third 6 1 2 BX the entire thing / T w - $T_\infty$ which is nothing but a + BX - $T_\infty$ okay so therefore for the given constants you can now determine the local variation in the heat transfer coefficient and for the limiting case.

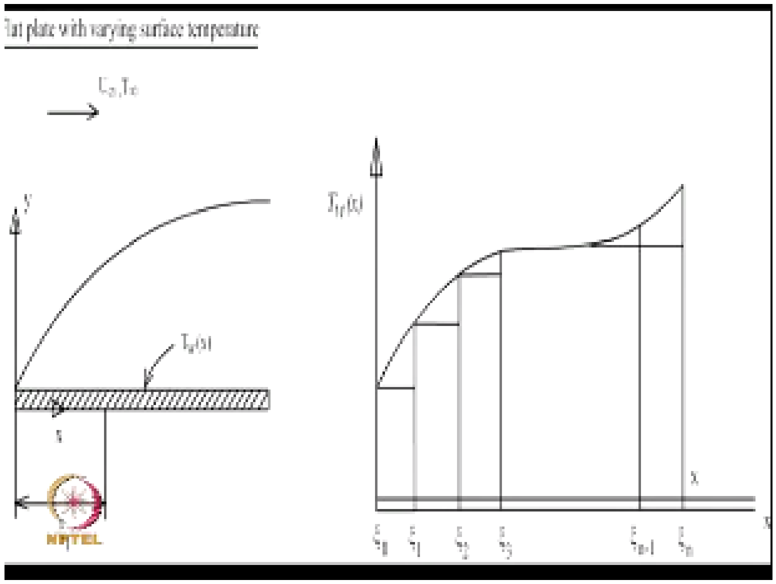
Where your $B = 0$ $B = 0$ gives me a uniform w temperature okay so for that case how does it reduce this becomes $a - T_\infty$ here the denominator this cancels with $a - T_\infty$ here B is anyway 0 okay so then you will what will be the expression the constant w temperature that is 0.33 1 K by X $PR^{1/3}$ re $X^{1/2}$ okay so this is your constant w temperature boundary condition so for the limiting case where $B = 0$ you retrieve your constant w temperature heat transfer coefficient okay so it is a very straightforward method as such you know so what it finally means that so you can also solve this by dividing.

(Refer Slide Time: 18:40)



These x piecewise constants like we did yesterday you know you can assume that you can represent this by piecewise constant like.

(Refer Slide Time: 18:53)




This you right you can divide this x piecewise constants and instead of the integration that we did here we will replace this by discrete summation okay so if you do that you can also get somewhat similar expression but that will be in a slightly discrete form so in that case you will have something like point 3 3 1 x K by x re X power 1/2 P R power one-third so this will be e 1 0 - T ∞ so when you differentiate it the original profile you have a fee so that will be - K DV / dy at y = 0 this will be nothing but the heat transfer coefficient H okay.

So that H you have already substituted as 0.33 1k by x re X power 1/2 PR power 1 by third x 1 - Z by X the whole power - 3 by 2 GJ so what is that can you go back and revise the expression for H of X 0.33 1k by x re $X^{1/2}$ P R power 1 third 1 - Z by X to the power what 3 by 4 the whole power $^{--1/3}$ okay so this we had to substitute x the expression for Q all of X which was so you had the original expression for T w - T ∞ that is - T ∞ was T w 0 - T ∞ x fee of 0 , X , Y + summation of n = 1 to capital n δ T w and fee of X , Y so this was what we saw a study this was for the local variation in temperature.

We have super post the solutions where you have a uniform temperature okay and one by one so then the incremental temperatures so all of that when we superpose so first you have this is the basic solution that you have + the incremental solutions which is basically this okay so now when we want to calculate the heat flux so we had to say - K x D Φ / dy at y = 0 which was nothing but the heat transfer coefficient H so this we had to substitute x this expression so therefore this is taken out as common you have T w 0 - T ∞ as one of the terms and the second term will be the rest of the terms will be in n = 1 to capital n number of discreet intervals you have δ T w and okay x 1 - Z by $X^{(3)}$ / 4 full power – 1/ 3 okay.

So this is what you will have if you have a discrete variation if you had a continuous variation you will replace this by an integral okay integral over D Z okay now you have a discrete variation therefore you just substitute for δ T as it is and of course the heat transfer coefficient for the first location where Z = 0 that is no unheated starting link so therefore you don't have this term for that for the subsequent boundaries and conditions you maintain at Z = Z 1 Z = Z 2 so there you have unheated starting link.

So there you have to substitute the values of so this will be corresponding value of Z θ n okay so by doing this also you can calculate your local wall flux variation instead of using the do Hamill's integral method you can just linearly superpose this is the superposition method right so you can divide this continuous curve x small discreet intervals where you have no variation of temperature like this you can substitute that x this discreetly and you can also estimate the w heat flux okay, so both will be more or less the same they are continuous is the more accurate because you are taking x account the slope accurately okay so this is to just give an example okay how you take a problem.

Where suppose you have a w temperature variation like this and you can use either the Duhamel integral or a simple superposition technique and you can calculate your local heat transfer coefficient and your w heat flux okay so any questions on this okay now for more complex profiles you know you it's more likely that most of the ball temperature variation cannot be approximated just by a straight line it will be more complicated profile so for a more complicated w temperature variation what is common practice is that we can approximate the w temperature variation as something like power law series so instead of using an A + B X relationship we can write this as a + summation n = 1 to N B and X okay.

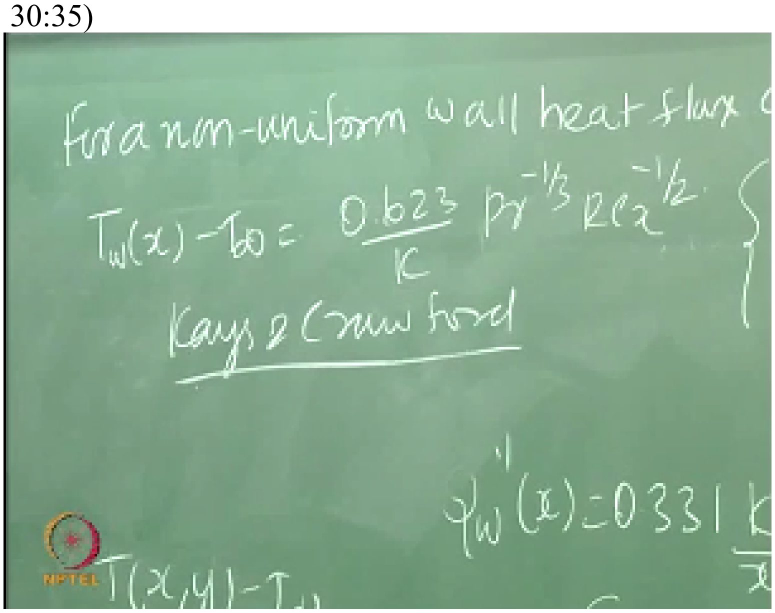
So this is the power series expansion so which is which is K which can be used to approximate more complicated nature of profiles you know if you have a profile something like this so you can you can use power series expansion you can fit the coefficients to know you can do a regression fit the coefficients such that you can approximate this curve with the power series expansion okay so this is a better way to represent this than using a straight line right so you can substitute this now x this expression here so you had a term here DT w by D Z so now you have to calculate DT w / D Z for this so what will be the expression for DT w by D Z summation n = 1 to n you have n B and X power n - 1 or Z power and - 1 okay.

So this has to be now substituted x the expression where we had DT by DT w by D Z okay and if you do that the resulting expression for the heat flux comes out to be everything is the same only you have the summation term 3 3 1k /X so everything up to here is the same except you have the summation term and everything inside the summation term goes there so n = 1 to n you have n be n so now so this will be $X^{n-1}$ x there will be an X which will come out of the transformation okay so that will be giving you $X^{n}$ okay x the other the β function will be there as it is so you will have β now this β function also will become a function of n okay so where this β function we can be written as γ function of 4 by 3 originally it was 4 by 3 now it becomes 4 by 3 n x γ function of 2 by 3 / γ function of 4 by 3 n + 2 by 3 okay so now depending on the value of n that you use ok so the value of β will change and then you have to sum them over all the values of n.

So suppose you are using five terms you have to sum them for each value of n and for all the 5 times you have to sum them together so then that that will give you the variation if you have a more complicated profile you know you can approximate that with the power series expansion and you can use this expression to calculate the local heat flux variation okay now the question is given local variation in the temperature profile we can use this to calculate the local heat flux but what about the other way suppose your w boundary condition is a locally varying w heat flux okay so how do you calculate the local w temperature as well as the heat transfer coefficient okay so that is also a little bit more complicated derivation.

I am not going to do that I will just only give you the final solution for the for the w temperature variations.

(Refer Slide Time: 30:35)




Okay for a non-uniform w flux so for this case you can calculate the w temperature variation can express it as follows .6 to three by K P R $^{-1}$ / 3 re X $^{-1/2}$ you have 0 to X 1 - Z by X $^{3}$ / 4 the whole raised to the power - 2/3 x Q okay so this is the local variation in the w flux whatever you have so that can be substituted x this and you can get the corresponding w temperature variation okay so this is this given in your textbook case in Crawford okay he has also not derived it.

But I think there is a reference to some literature where they have done it and they have shown that shown this kind of an expression anyway so this is beyond the scope of your this thing but you should understand that you can do either of these in when using the approximate solutions given non-uniform w temperature how do you calculate the variation in the w heat flux or given a non-uniform w heat flux how can you calculate the variation in the w temperature so both are possible by using the approximate methods so I think with that we will kind of wrap up the external laminar external flows external boundary layer flows.

So we have covered quite a bit you know we have got almost covered whatever possible similarity solutions under external boundary layers and also the approximate methods we know whatever external laminar similarity solutions they have a complimentary integral methods also integral solutions also like we have seen if you have the Falkner Skan solutions for which problem you have similar foreign Carman pool house and solution when you use the integral methods okay of course like the blusher solution there you have you can approximate some velocity profile and very easily find out the expression for say nacelle local nusselt number okay.

So whatever is possible in fact you can also use the approximate solution for example a flow with transpiration you have boundary where you have porous boundary with suction or blowing so we have derived the similarity solution for that in fact we have we have identified the condition for the variation of the suction profile velocity profile so that you can get a similarity solution okay so the same way we can derive an approximate by using approximate method you can we can derive expressions for the local skin friction coefficient as well as the nusselt number variation with transpiration so every similarity solution has a counterpart.

In the approximate methods and more than that you can also derive some special cases such as the unheated starting length which you cannot derive by similarity solutions and also cases such as these where you have non-uniform w temperature variation of any of any given profile which you can approximate as a power series expansion or a non uniform w heat flux okay so for these kind of boundary conditions you know it will the similarity solutions are not possible or it becomes very rigorous so their approximate solutions are much easier okay so this is in a nutshell giving you an idea what we covered so from the next week onwards.

We will look at internal laminar internal boundary layer flows okay so they are actually strictly speaking the boundary layer concept doesn't have a meaning that the way that laminar external flows has okay the strict definition of boundary layer flow does not hold for internal flows okay because once you have a fully developed flow both the boundary layers merge and everywhere you have viscous effects there is no place where you can use potential flow and approximate that with the potential theory and somewhere you can solve with the solving the full navier-stokes okay so therefore we have to resort to a complete solution of the navier stokes equations in some cases.

In some cases we can make approximation to the velocity gradients or the temperature gradients okay so there we can obtain exact solutions of reduced form of the navier-stokes equations okay so that is also very important and interesting because most of your practical problems in heat transfer okay although there are many external flow problems you will find most of the heat exchangers they are encountered you will be encountering internal flows there and in those cases you should understand the approximations that you can make and how you can get the expressions for local variation in the nusselt number and in a fully developed case how the nusselt number variation there is no variation in the nusselt number and so on okay.

So that we will cover in the next 9 to 10 classes starting from next weeks in about three weeks I think we should be able to cover the laminar internal flows and then from the following we converts that is about the fourth week of March professor Koehler will start turbulent flows so that will be for about seven lectures or so and or seven or eight lectures and maybe natural convection for another seven or eight lectures you.

**Laminar External heat transfer with non uniform surface temperature**

**End of Lecture 24**

**Next: Laminar internal Forced convection -Fundamentals**


**Online Video Editing / Post Production**

M. Karthikeyan
M.V. Ramachandran

P.Baskar

Camera
G.Ramesh
K. Athaullah

K.R. Mahendrababu
K. Vidhya
S. Pradeepa
Soju Francis
S.Subash
Selvam
Sridharan

Studio Assistants
Linuselvan
Krishnakumar
A.Saravanan

**Additional Post –Production**

Kannan Krishnamurty & Team

Animations
Dvijavanthi

**NPTEL Web & Faculty Assistance Team**

Allen Jacob Dinesh
Ashok Kumar
Banu. P
Deepa Venkatraman
Dinesh Babu. K .M
Karthikeyan .A

Lavanya . K
Manikandan. A
Manikandasivam. G
Nandakumar. L
Prasanna Kumar.G
Pradeep Valan. G
Rekha. C
Salomi. J
Santosh Kumar Singh.P
Saravanakumar .P
Saravanakumar. R
Satishkumar.S
Senthilmurugan. K
Shobana. S
Sivakumar. S
Soundhar Raja Pandain.R
Suman Dominic.J
Udayakumar. C
Vijaya. K.R
Vijayalakshmi
Vinolin Antony Joans
Adiministrative Assistant
K.S Janakiraman
Prinicipal Project Officer
Usha Nagarajan

**Video Producers**
K.R.Ravindranath
Kannan Krishnamurty

**IIT MADRAS PRODUCTION**
Funded by

Department of Higher Education
Ministry of Human Resource Development
Government of India

Www. Nptel,iitm.ac.in