Indian institute of technology madras
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National Programme on technology enhanced learning
Video lectures on
Convective heat transfer

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Lecture 23
Duhamel's method for varying surface temperature

Good morning all of you so any clarifications on integral methods. I think I have been going a little bit slow in the last two three classes so takes enough time for you to understand what I had been doing so last class we had looked into the approximate solution for flow with pressure gradient Duhamel's method and applied that to heat transfer problem and finally we also looked into an approximation to the Duhamel's solution okay the ordinary differential equation can be simplified and directly integrated by what is called as the volts approximation okay so there he has his plotted H of K as a function of K and found that is a level a very linear curve and he has given the approximate profile for the linear curve.

And that is what if you use it becomes much straightforward to integrate the equation to get the a momentum thickness okay so once you get the momentum thickness from there you can get your other thicknesses like displacement thickness and boundary layer thickness which are required to calculate the flow average flow properties like skin friction coefficient and the same way you can solve the heat transfer problem and in the heat transfer problem you will get an ordinary differential equation for ζ which is the ratio of your thermal boundary layer thickness to momentum boundary layer thickness once you have the expression for momentum boundary layer thickness so therefore you can calculate your average integral heat transfer quantities like heat transfer coefficient and therefore we can get the expressions for nusselt number okay.

So this is the standard procedure for all the solutions whether it is similarity solution or integral solution this is the standard procedure in the integral solution you guess a profile you know you approximate the velocity profile and temperature profile and from there you calculate your boundary layer nests and thermal momentum and the thermal boundary layer thickness okay whereas in the case of similarity solution you have to solve the ordinary differential equation numerically by some shooting technique or whatever and get the curvature at the w and from there you can get the other properties such as skin friction coefficient and also you can get your boundary layer thickness and other thicknesses and for heat transfer.

You can also get the slope of the temperature profile at the w and from there you can calculate your expression for heat transfer coefficient and Aselton okay so what we will do today so we had so far looked at application of integral techniques to for flows without pressure gradient that is a similar to the Blasius similarity solution we approximated that with the integral method and also we extended that to flows including the pressure gradient terms that is like the Falkner Skan similarity solution we can apply the integral methods for a similar problem for a wedge with different badge angles and we can derive approximate solutions for this okay we had also looked at the case of circular cylinder okay.

Which is which is a limiting case is a stagnation flow and also the heat transfer for flow past a circular cylinder now we will look at a case where you can consider a simpler case or maybe even the wedge but we will take a simpler case that is a flow past a flat plate however the boundary condition here need not be a uniform w temperature or uniform heat flux okay so most of the real cases you will find that the temperatures are actually varying along the surface okay so if you want to consider non-uniform temperature or non-uniform heat flux so how do we solve such kind of problems okay.

We do not have a straightforward similarity solution to such kind of problems but it is quite likely that we can develop a similarity solution for a particular case.

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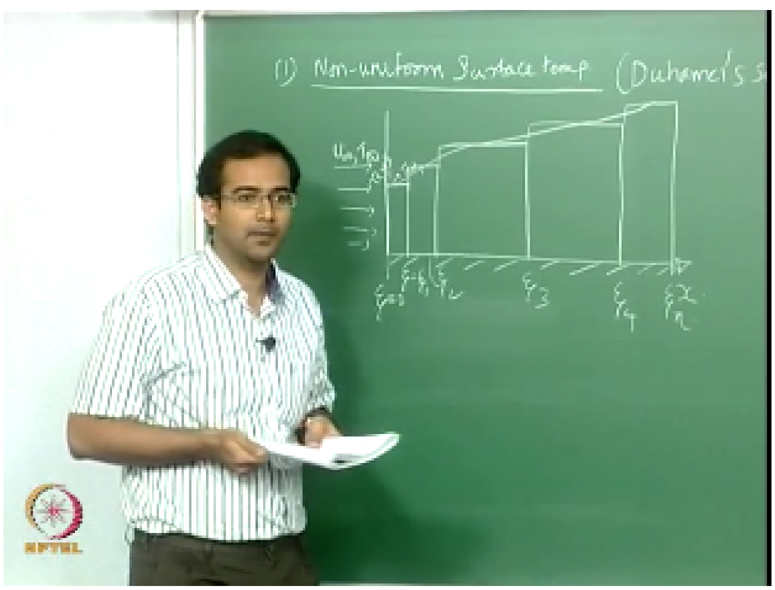

Where you are variation is predetermined that is if you have a flat plate and your variation of the w temperature is something like a power-law X^{M} variation okay where M is some real number and so this is a power law kind of a profile so for this you can develop a similarity solution also okay we can also very easily solve this by integral methods which we will see now and it need not be a variation like this it can be any kind of variation okay that is the advantage of the approximate methods okay the approximate method give you a lot of flexibility in approaching problems with different kinds of boundary conditions and also things like where the boundary condition there is an unheated starting length okay.

So there is no similarity solution if you have an unheated startlingly so these kind of problems with the variation in the boundary temperature and the heat flux can be easily solved using the approximate solution okay so today we will look at extending the simpler approximate solutions to a case where you have more complicated boundary conditions so the first case we are going to do is non uniform surface temperature okay so here you can consider a flat plate where your temperature is varying in an arbitrary manner okay and of course you have your free stream velocity and free stream temperature and we will look at a particular technique called the Duhamel superposition method for solving this problem okay ,so the specified condition is something like this you can you can maintain locations where you want to specify a particular oil temperature for example.

Let us call this as ζ at the location $\zeta = 0$ you start with some specified temperature okay that could be something like tu T w one so let me draw how the w temperature profile will look so this is my plot of Tw of course as a function of my position so at this particular location $\zeta = 0$ I will have for example a piecewise constant value of w temperature which is T w 1 okay so this is existing till a value of $\zeta = \zeta 1$ okay so like this I will have multiple piecewise constant values of surface temperature okay so this is a simpler case to begin with of course your actual variation need not be piecewise constants.

Now it can be a gradual variation smooth continuous variation ok so like this we can look at temperatures which are success successively increasing the w temperatures which are successfully increasing in a piecewise constant manner okay so like that you can go up to some value of finally ζ n alright so this is how your surface temperature is now plotted as increasingly you know a piecewise constant which is increasing okay so this is like an approximation to a profile which is like this suppose your w temperature profile was this them the basic approximation for this is to assume piecewise constant you break this continuous curve into piecewise constant okay and you have an increasing trend in the w temperature.

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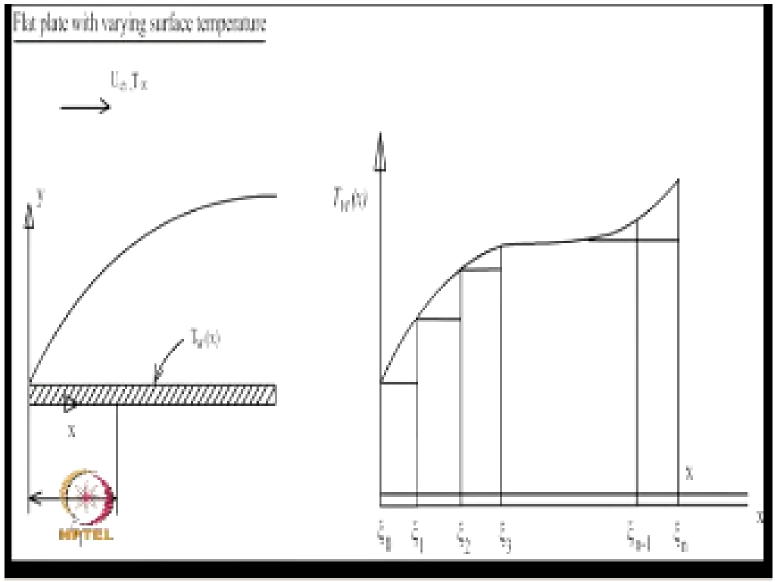

Okay so far this how do we approach the problem okay thankfully the equation that we are solving the energy equation is linear once you know the corresponding velocity profiles so the velocity profiles are not going to get affected as long as your properties are not affected by the temperature okay so in that case your velocity is decoupled from the temperature and you can plug in the particular value of velocity at a location into the energy equation and your energy equation becomes quasi linear and therefore for any linear partial differential equation if you have varying boundary conditions okay you can break the solution into multiple solutions and superpose the solutions for each boundary conditions linearly okay.

So the resulting solution is a linear combination or linear superposition of multiple solutions each corresponding to a different boundary condition okay so if you solve by method of separation of variables you will know that for example the heat conduction problem okay.

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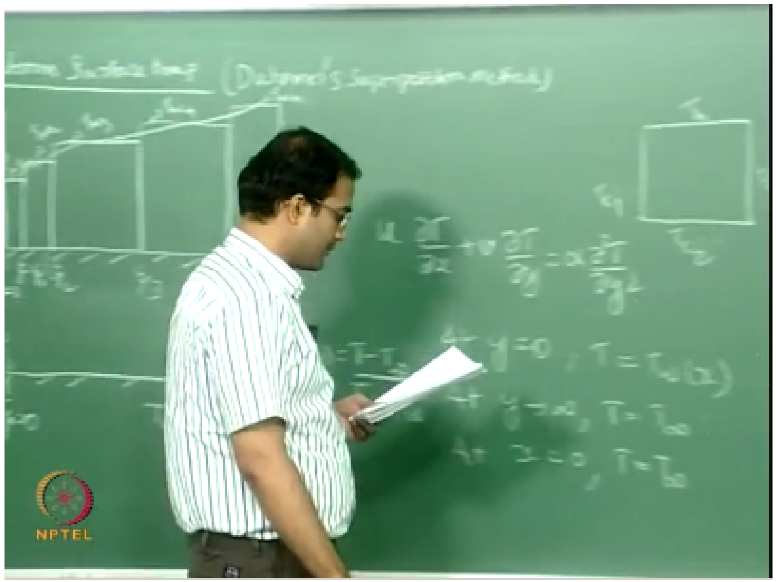

The method of separation of variables will work if you have for example homogenous boundary conditions in three directions and one non homogenous boundary condition okay for example in conduction case you will maintain this at some high temperature and the remaining three sites or maybe you can assume that or you can non-dimensionalize the temperature. In such a way that the non-dimensional temperature is zero okay, for this you can solve the conduction equation 2d conduction equation by means of separation of variables okay so there will be an eigenvalue problem in this direction basically where you have two homogenous boundary conditions in the other direction.

You apply the remaining homogenous boundary condition and finally whatever non homogenous boundary condition is there to get the final constants okay but if you have non homogenous boundary conditions in all the four sites so how do you solve this problem you can still solve it by separation of variables but you need heigen value problems okay for eigenvalue problems you need homeowner homogenous boundary condition in a particular direction so to do that if suppose these were nonzero ok this was some something like TC 1 TC 2 and T C 3 which were not 0 okay, so you can break this into problem where you have Th here and you can put TC 1 here TC 3 and 0 okay, or what you can if you want to make it completely homogeneous.

You can break this as three zeros here + you can make the other three as zeros and you can make this as TC 1 + of course you can make these three as zeros and this is your TC 3 + this is your TC 2 and the other three zeros so you can you can apply you know Eigen value problems in different directions you know in this case you have Y Direction y direction you have X direction here in this case you have the X direction okay so you create four equivalent eigenvalue problems and you get solutions for each of these case okay, with one non-homogeneous boundary condition and finally you superpose all these four solutions and that will give you the solution for this problem okay.

This can happen only if the partial differential equation is linear and conduction equation is linear right so the same way if you look at the convective heat transfer the energy equation governing convective heat transfer that is also linear so therefore if you have a combination of multiple boundary conditions like this you can superpose solutions where you have for example you have multiple w temperatures you can solve one problem where you have T w one throughout okay and the solution to that is known + you have another problem where from ζ = ζ 1 to the end you have T w1 - T 1 - or t12 - T w one okay that is the δ δ t w so that that is that is applied to the entire plate and again you know.

The solution for that and from ζ = ζ to till the end so that is you have t w 3 - T w - so like that you keep on applying successive δ and you apply that as your boundary condition and you solve the problem now if you if you do that you have individual solutions where you have an unheated starting link and then the rest of the plate where you have a uniform temperature okay so like that you break up the problem into multiple boundary conditions and then you get the solutions you already have the solution so you add them you superpose all the solutions together and that will give you the solution for this problem okay, that is why it is called as a superposition method okay.

So this is also called the Duhamel method so let me so let me indicate this T w - this is T w through T w 4 and so on so this is your T w and n - one okay or okay let me call this as P w okay so the solution for these is anyway you have to solve the energy equation boundary conditions are now at y =0 you do not have a constant value of temperature but T = t1 which is a function of X okay and the other boundary conditions are the same Y going to ∞ T is = T ∞ and at X is = zero okay now following some analogy similar to the conduction problem we're also the partial differential equation is linear and you can convert the problem into equivalent for equivalent problems with convenient boundary conditions.

So you can do the same way here also and you can super force superpose the solution so therefore the solution for this problem will be something like you can take one problem like this where you have your free stream temperature T ∞ okay and this is your starting from your ζ = zero entire plate you maintained at temperature t1 right so this is your first problem and already you have the solution uniformly heated plate right from without any unheated startlingly okay so the if you non dimensionalize the temperature your solution to be found out will be in the form Φ for the case θ = zero and it is a function of x and y will be t - T ∞ by T w one - T ∞ this is the way.

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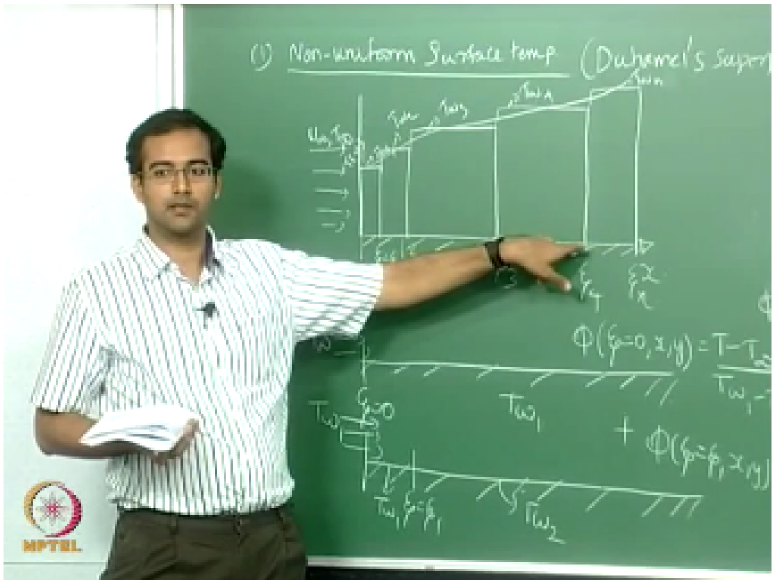

I am going to non-dimensionalize okay, so and how does the corresponding w temperature the non-dimensional temperature profile look at the w if I if I plot this fee at ζ = zero X , y = zero so I am I am plotting the temperature at the w so this will become T w one okay and throughout it is T w one okay so therefore it will be just one everywhere alright so this is one case now I can break it up into multiple problems okay my second problem will be what so now I have solved one problem where everywhere it is T w 1 but now you can see from ζ = ζ 1 2 ζ 2 it is T w 2 so now what is the next problem that I have to solve ok so to do that what should i what should I do now I should not solve for any heat transfer problem till ζ 1.

Because I already have solved it here okay, so I should maintain what is what is something called as the unheated static link to ζ 1 right and what should be the free stream velocity that I should take so that there is not be any heat transfer here now this is already at T 1 1 okay. So I have to choose a free stream velocity such that there will not be any heat transfer so that this remains unheated so what is that free stream velocity that I have to take I cannot take t ∞ if I take T ∞ the T ∞ is different from T 1 1 t12 - t1 one way it should be t1 one okay so if I take a free stream which is at T w one so this is also a t-ball one so therefore they will not be any heat transfer till here now at this point onwards this will be a t1 to till ζ two okay.

So now I can introduce another non-dimensional P corresponding to ζ = ζ 1 non-words okay now how do I non-dimensionalize this t - T w 1 by t1 2 - T w 1 exactly okay so if you draw the non-dimensional profile at the w. Let us the sex how does it look now initially till ζ 1 there will not be any heat transfer so from here it will start and this will be 1 throughout so this will be a T w - throughout okay so that means here this portion is now T w 1 this entire portion is T w - okay so now you are solving for T 1 2 - T w 1 that is the difference that you are solving ok so that means now you have solved for T 1 1 here now here you have already solved for T 1 1 + T 1 2 - T 1 1 so that is basically s T w - now same way you have to break this into problems depending on.

The number of piecewise constant that you have all right so now if you extend this to well just give you a representation for the third one so this + this + okay.

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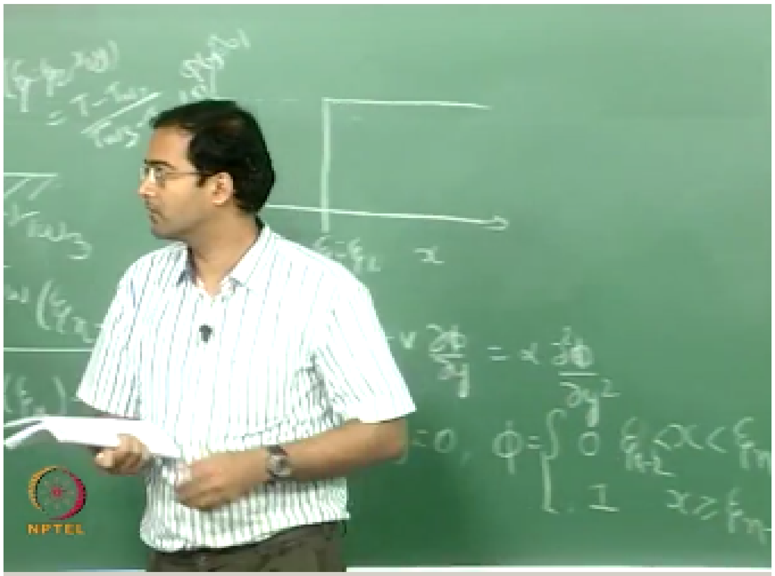

So till θ equals from $\zeta = 0$ to $\zeta = \zeta\ 1$ you have T w 1 okay now from $\zeta =$ up to $\zeta\ 2$ you have T w to now you have to maintain T w 3 throughout okay so to do this again we take assume a free stream velocity where you are maintaining everywhere has now T w to and your free stream not everywhere I am sorry from $\zeta = 0$ to ζ equal to $\zeta\ 2$ as T 1/2 and your free stream velocity is know T 1/2 from $\zeta\ 2$ onwards it will be T w 3 so you maintain everywhere as T w 3 and you define your fee as beta = $\zeta\ 2$, X , Y which is = P - T w 2 by t 1 3 - T w - so if you plot again C at y = 0 as a function of X okay.

So till your $\zeta = \zeta\ 2$ there is no heat transfer from here it becomes 1 okay so this is how you get different solutions okay now you have actually solved here for the remainder tea w three - T w two for this particular region okay so already you had solved for T w 2 - T 1 1 and also for T 1 1 - T ∞ so finally that will be the T w 3 - T ∞ okay so then you can superpose all these solutions together yes so you have to maintain this at T w to throw out because you have already solved for this till here you are not interested in this region okay so you maintain this entire region at T w 2 and then this will be from T w 3 so T here also you have already solved here ok so you don't need any solution the solution is already there that is this solution till this region you already have solution from this so you do not have to solve for anything here.

So you maintain a unheated starting length forcibly by maintaining the free stream temperature and this the same the same way here ok so you already have solved from here to here so you maintain a same temperature = the free stream temperature so that there is no heated length okay so therefore you define your Φ in generic terms as t - T w ζ n - 1 by T w ζ n - T w ζ n - 1 this is your generic formula for defining your non dimensional free okay so where for example if n = 1 so that is basically that that is your ζ 0 that is this first case okay n = 2 that is your second case n = 3 will be this particular case okay.

So this is a generic formula how you are defining your non-dimensional temperature okay and the non-dimensional form of the energy equation will be you D Φ by DX + at y = 0 now what will be the value of fee now till your X < 0 or maybe you can say if you want to write it in generic terms θ n - 1 2 ζ n okay so this should be what 0 and for X < ya so for X greater than or = ζ n it should be 1 okay and you just check that it has to be easy to eat AM ETA n - 2 - 1 is that right okay so when you start with of course something negative it means it is you can say free stream okay free stream so there it will be 0 and then when you start with n = 1 so up to for this case so up to n 0 basically you do not have any heat transfer from here.

Onwards you have greater than that you have your fee = 1 okay so this is your generic solution okay and this is your generic way of non dimensionalizing fee now therefore the solution.

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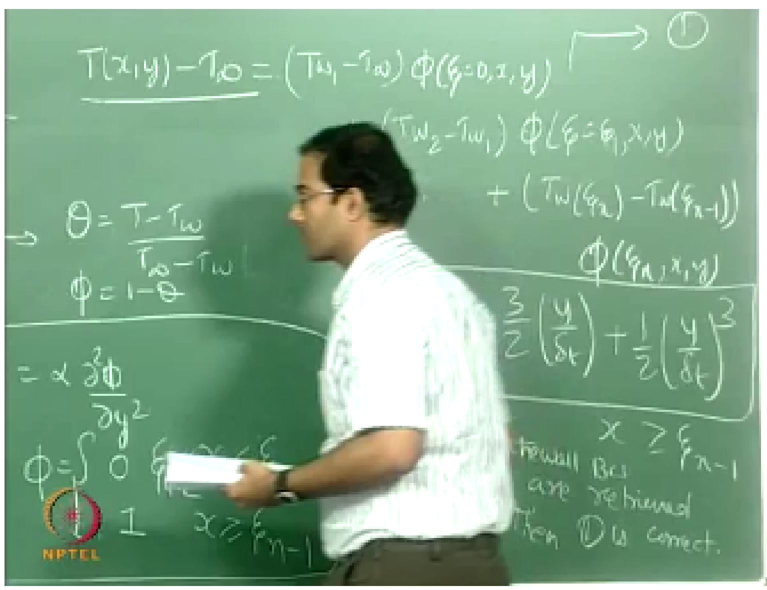

To the problem for temperature t - t ∞ so this is what we finally want to find out so now we have broken this into sub problems and for each already we have the solution okay we have the unheated starting length problem okay so we can just combine all these for the case where your ζ =equal to 0 how do we express t - T ∞ t - T ∞ is fees n = 0 times T all 1 - T ∞ okay so this is T 1 1 - T ∞ into P ζ = 0 X , Y okay + the next problem will be t 1 t - t 1 1 which is t 1 2 - t 1 1 into p ζ = ζ 1 x , y + all the way till t w ETA n - T w ETA n - 1 pi ETA n X , Y so that is your final this thing okay.

So T w ETA n - T w ETA n - 1 that is the last this thing okay so you keep adding up all these solutions such a way that finally if you add up you get the solution to the problem where you have variation like this in the boundary condition okay so already we have the solution two feet fee is what t-- you can you can look at this t - T ∞ by T 1 - T ∞ so what is it suppose if you assume a cubic temperature profile so what is the how can we write fee suppose you assume cubic temperature profile you non dimensionalize it y1 - correct your what you have said is correct but y1 - us what you are saying is correct but it should be 1 - why it should be one - signs have changed no how did we define the non-dimensional temperature θ.

When we were fitting a cubic profile t - T w by T ∞ - T one so my fee is what so that should be 1 - θ right so therefore what it should be 1 - 3 by 2 y by δ T + 1 by 2 y by δ T the whole cube correct so this is the profile satisfying the condition X < = ζ n - 1 right so where so this is the corresponding w temperature so uniform w temperature correct corresponding to that this is the profile in the boundary layer thermal boundary layer now I already know the solution so only thing I have to know linearly combine all the solutions that's all and how do you check that how do you know that this is the correct solution correct for way of superposing the solutions how do you verify.

That the simplest verification is getting the boundary condition itself so you apply this at y = 0 what it will be now until so this profile will be 1 everywhere okay so this will be T while 1 - T ∞ ok suppose you want to check that at between ζ = ζ 1 2 ζ 2 that this gives me D w 2 - T ∞ does it give that is the check read so this is T w 2 - T 1 1 and this is 0 before and it is one between ζ = 1 ζ 1 and ζ 2 and the other things are all 0 okay, the T 1 1 T 1 1 cancels so this is T 1 2 - T ∞ so this retrieves my boundary condition ok this is a good check that therefore this is the correct solution because your boundary condition also is a part of the solution right so this is the therefore how you have to write it and if you check if the w boundary condition are retrieved then let me call this as equation number 1 ok then one is correct okay.

So or one is represented correctly okay, so now that's it once we write the solution like this is valid if you have a piecewise constant variation in the actual case the variation should be continuous so instead of having a summation of piecewise constants we can replace that by an integral which represents a continuous summation okay so that is what we are going to do now so I would like to rather than writing P w ζ - the T w ζ and n – 1.

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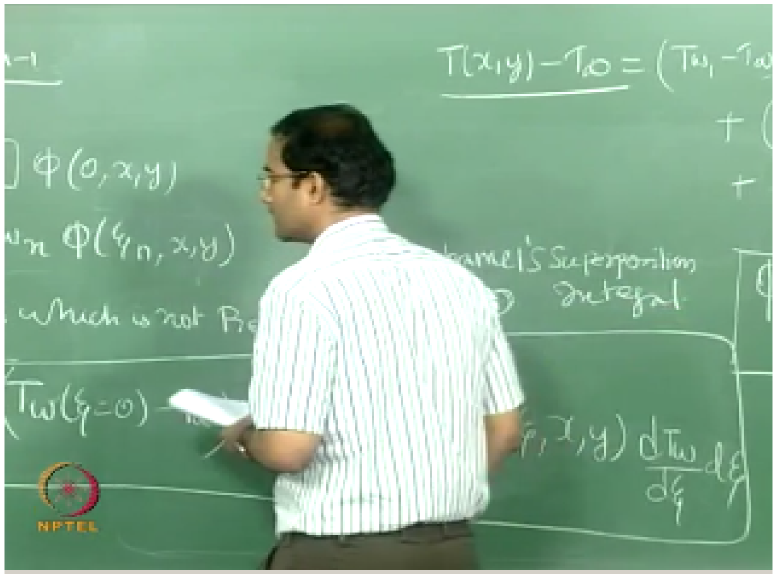

I am going to introduce δ T w and which is = P w ζ n - T was ζ n - 1 this is my representation so therefore equation 1 can be written as P X , Y - T ∞ is = T w one - T ∞ into fee zero , X , Y. So this first set notation here is corresponds to ζ okay so ζ = zero + I can write the rest of summation n = 1/2 totally N δ T w n p ZT n , X , Y so the rest of them I am just summing over those n number of discreet intervals okay so that I can put under a big summation like this is that right so if the temperature variation is not a piecewise constant but actually is something like this which you have approximated as a piecewise constant okay so now I am going to convert this discrete summation into a continuous integral okay that is all I need to do so for w temperature variation which is not piecewise constant but continuous so that is either linearly increasing or not linear it is continuously increasing or decreasing you can rewrite this discrete summation as T w ζ equal to zero - T ∞ into p zero , X , y + integral 0 to X P X ζ X Y now how do.

I write this in an integral δ T so I am going to write this as DT w by D ζ which is the dummy variable D ζ okay so all I am doing is I am converting this discrete summation into a continuous integral so if I have a finite slope and the slope is continuously varying with the location okay so all I need to do is get the slope of the w temperature variation with respect to ETA ζ and that will be used here okay so this will this is also called as we do Mac do hammers superposition integral okay let us call this as number two this is referred to as we do Hamill's superposition integral okay so the general super superposition which we discussed above is a general method and usually it is applied where you have this discrete piecewise constant kind of approximation.

If you don't make that but directly you put the slope of the w temperature variation as a continuous variation if you do that then the resulting expression is called the Duhamel integral method okay so now once you know the local variation of the temperature now we can calculate all the other things like the w heat flux for example so now for example in this case we had varied the w temperature and now for the varying w temperature case we want to calculate what is the corresponding variation in the w flux the w flux also varies right so it is not a constant anymore the w flux also keeps varying with the fixed location so we can calculate an X right.

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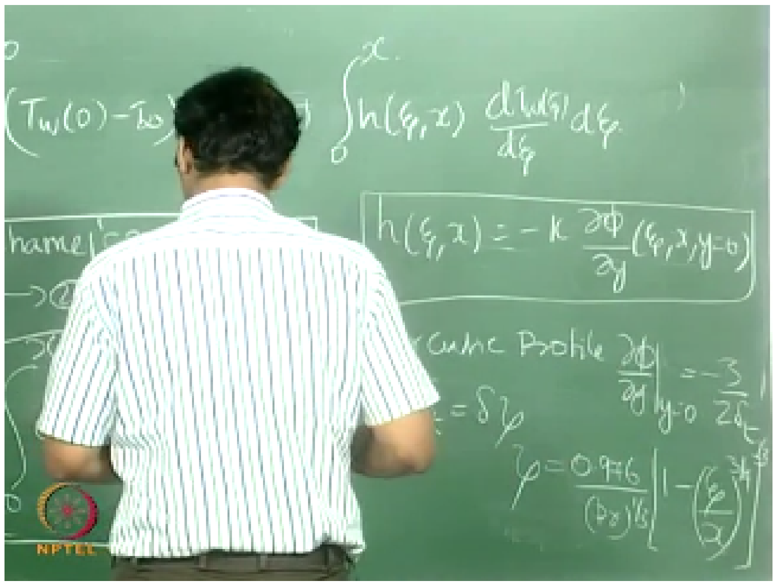

And expression for so the local fault flux can be estimated as this is a function of X of course and that will be written as T w zero - P ∞ into H zero , x + integral 0 to X its data X into DT w by D ζ 2 D ζ where your H of ζ , X is nothing. But - K D Φ by D Y ζ X , y = 0 right my this is my definition of heat transfer coefficient - - K DT by dy by T w - T ∞ okay so fee is nothing but your t - T ∞ by T 1 - T ∞ so this is your heat transfer coefficient so if you differentiate this with respect to Y at y = 0 you can replace your fee as directly now that - KD DV by dy is directly H okay now so how do you calculate the local heat transfer coefficient so already if we know for cubic profile my DV by dy at y = 0 what is the value - 3 by 2 δ T okay so also I know the δ T okay so how do I know that because δ T is = δ into ζ okay and I have got an expression earlier which we derived for the flat place flat plate case with unheated starting lengths okay.

So that comes out as 0.9 76 by Prandtl number to the power one third one - so the unheated starting line there was assumed as X not in any generic case where you know the location of the unheated starting length you can replace the text not by the Z here the whole power three by four entire thing raised to the power one-third right so this was what we derived for flat plate with unheated startlingly so this has to be substituted into this expression to calculate the heat transfer coefficient and do you remember how that comes out you remember the expression for heat transfer coefficient that we derived earlier.

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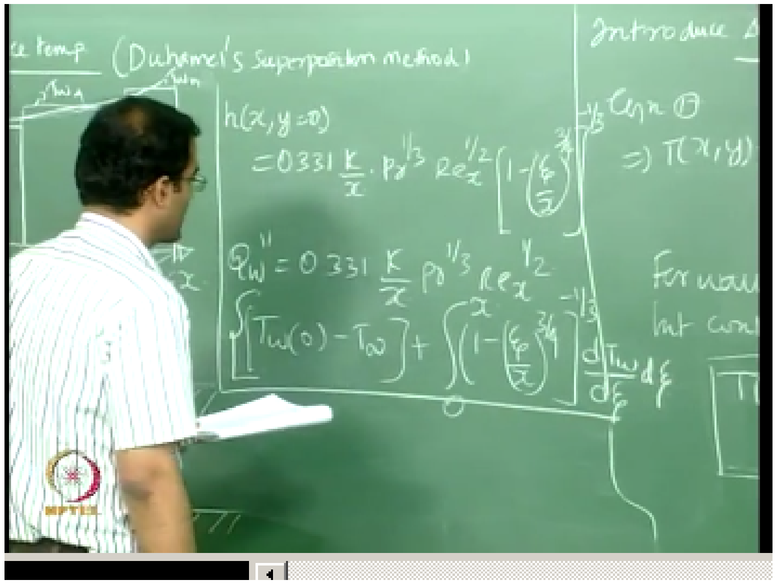

After we substitute this and also for δ δ we can assume a cubic velocity profile and we got an expression in terms of Reynolds number right. So that in this we substitute into this expression for H we get a final expression you recollect that 0.33 1k by X into Prandtl number to the power watt 1 / ^{1} /2x 1 - ζ by X $^{(3)}$/ 4 the entire thing power - 1 / 3 this is what we had derived all right so therefore you substitute this value of H into this expression now H is a function of ζ okay so that ζ will tell you what is the unheated starting length for that particular problem so when you are breaking this problem into multiple boundary conditions for each configuration you have on one heated starting link so it starts initially at $\zeta = 0$ then ζ 1 then ζ 2 like that you have location of the unheated starting length keep shifting.

So that value of ζ has to be used in calculating the local heat transfer coefficient and that goes into this expression right here okay, so that will give us the value of local w heat flux which is 0.33 1 k by X so this is a constant term which will come out and the rest of the terms that is T w zero - T ∞. This is your term and H the other terms we have taken out as constant now for the very first case H of 0 there is no unheated starting length therefore this will be 1 + you have integral 0 to X in this case you have the unheated start starting length so there you put 1 - θ by X - 1 by 3 DT w by D ζ easy all right so this is your final expression for calculating the local variation in the heat flux all right.

So what we'll do is tomorrow we will apply this to a problem I will just do for a simple case where you have a linear variation what happens if you have a linear variation then we will calculate the heat transfer coefficient and the local heat flux w heat flux okay now what happens now if you if you have a continuous variation like this and apart from that if you also have local jumps something like this.

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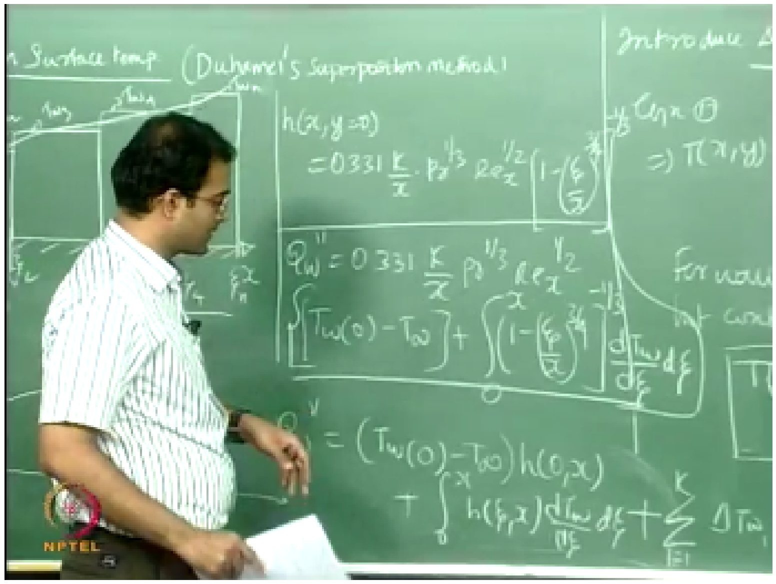

So you have a continuous variation and suddenly you have a jump here and again you have a continuous variation. So it is what piecewise continuous okay, so in such a case you know the location of these jumps maybe at ζ 1 liter tour so the same expression + you have to account for these jumps local jumps local discontinuities okay so then your expression for Q well double prime will be H of zero , X + whatever you had before + what should you do so this is the same as this correct now additionally you have this local discontinuity so what you should do to account for that so once again that becomes locally discrete so then you have to make this again discrete this becomes H into δ T for continuous variation we replace the discrete representation with the continuous thing now once you have a discrete jump or once again.

You will have to replace this integral with the summation okay so summation suppose you have say K number of jumps okay I am going to go from I = 1 to K δ T w into H that is it okay. So this will take care of both the continuous variation wherever and wherever you have discrete jumps at these locations you have you have to write as δ the and sum them ok so this will give you the local variation of heat flux so with this we will stop tomorrow we'll work out solve an example for assuming a linear variation in the w temperature and we will see how to calculate you.

Duhamel's method for varying surface temperature

End of Lecture 23
Next: Laminar External heat transfer with
Non uniform surface temperature

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