Indian institute of Technology Madras

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Video Lecture on

Convective Heat transfer

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Lecture 22

Heat transfer across a circular cylinder:

Walz Approximation

Good morning all of you, yesterday we were looking x the von Karman poll house in solution for heat transfer problem I just want to make a small correction probably when we looked x the solution for Z so we had the λ a value for the stagnation region as 7.0 5/2 and the resulting expression for Z so we have to serve hard.

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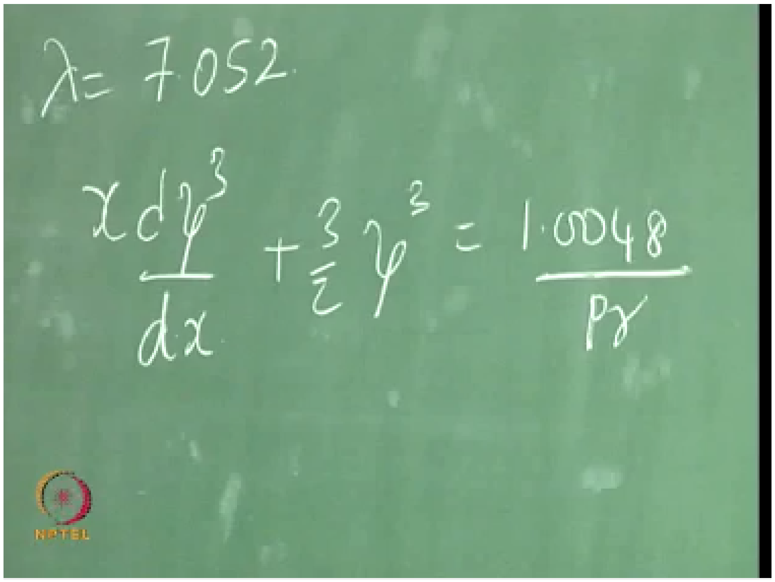

To solve this OD and the solution as a combination of the homogeneous and the non homogenous parts came out 0.669 9 / Pr okay so you can look at the non-homogeneous part. Is actually based basically a constant so that is why we know the particular integral has to be a constant okay if it is a function of X in any form then we have to assume that particular form and you have to calculate the particular integral and coming to this particular equation now this is for the case where we have flow past a circular cylinder you have the heated circular cylinder.

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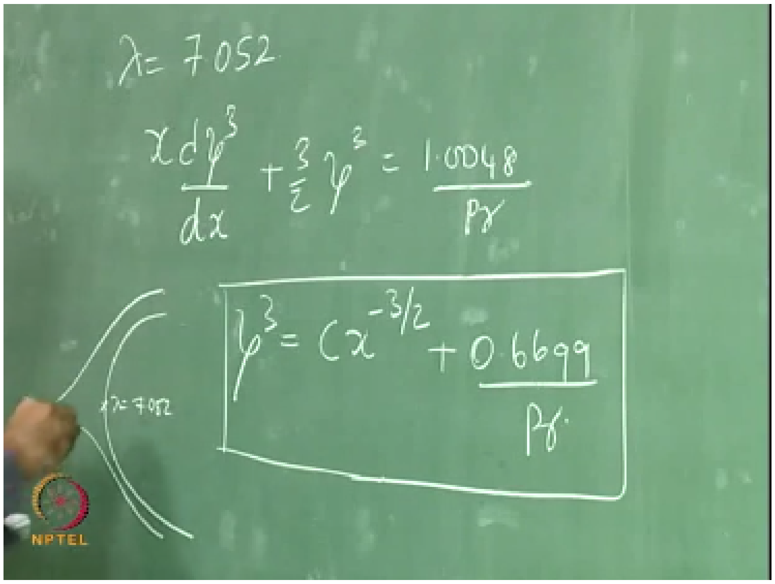

Okay and this is the stagnation value of λ which corresponds to seven point 0 five two so now at X is = to 0 this is where you are starting your X okay so you should be careful that the boundary layer thickness is actually not 0 okay because the stagnation flow comes ahead and then it basically bifurcates like this so you have a certain value of boundary layer thickness and the same thing holds true for your thermal boundary layer thickness also.

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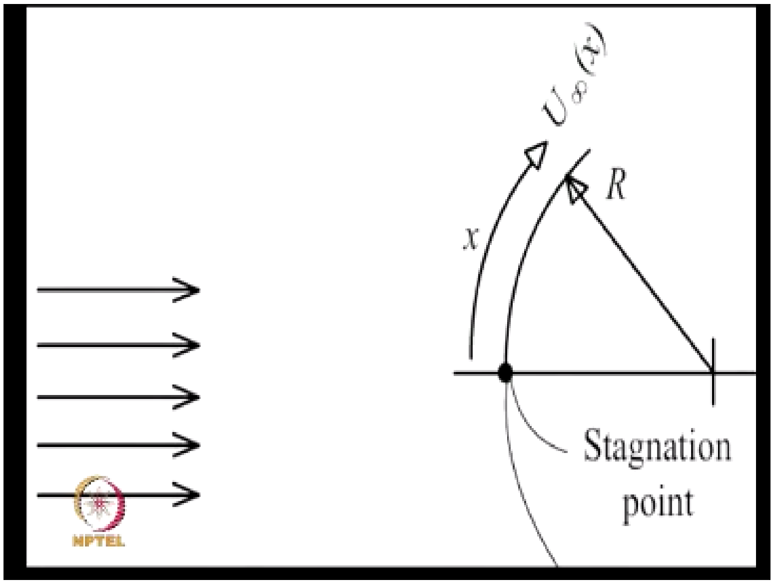

You so you have to be careful then when we stated that at X is = to 0 we came to the conclusion that the constant is 0 essentially because of the fact as x2 tends towards 0 the solution will go it to ∞ okay because this is a negative power so in order to make this value of Δ T that is the thermal boundary layer thickness finite at X is = to 0 this constant has to go to 0 okay so this is why we put this constant as a problem I did not explain it very carefully so you have to be careful that at the stagnation region both the thermal boundary layer thickness and the momentum boundary layer thickness are both non0 okay and therefore in order to may give a finite value of Δ T.

So your constant has to be 0 that is why the final solution was cube is = to 0.66 99 / and from this we have calculated the expression for a self okay so this is I just want to clarify before we proceed further today we will introduce a smaller approximation to the entire approximate method and this is also called as.

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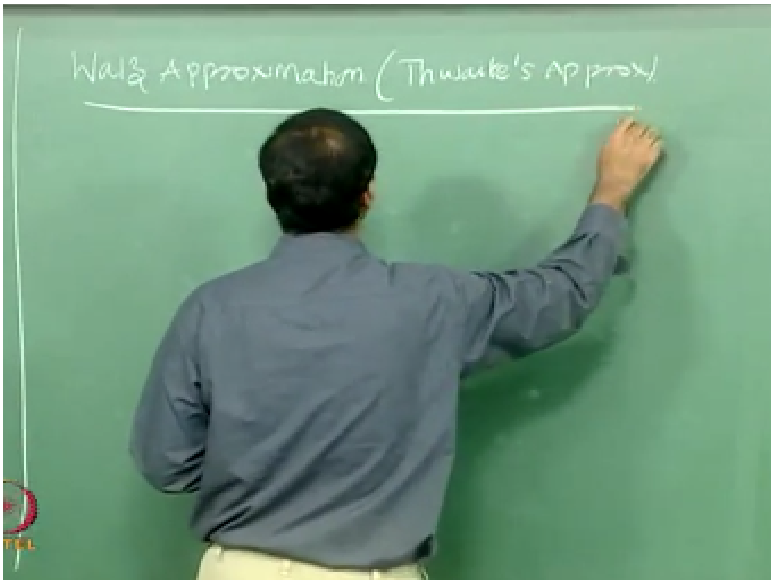

Was approximation some people also referred this to a straights approximation I am not sure why this is attributed to two people I do not know the historical reason but some books some of the more recent books may refer this to all is approximation in some of the earlier books refer this to a straights approximation and what essentially happened is this person walls he pointed out that we had defined this function H of K right so a complex function which was function of F 1 F 2 K and so on so he said and we were thinking that we can construct a lookup table where we will change the values of λ for different values of λ.

We will plot F 1 F 2 and therefore we can plot H so he said when he did this plotting we found this H of K was actually a linear function of K okay so and he got a curve fit of a straight line which was 0.47 _ 6 K so he just simply plotted H of K as a function of K at k = to 0 this was like 0.47 and then it was decreasing like this.

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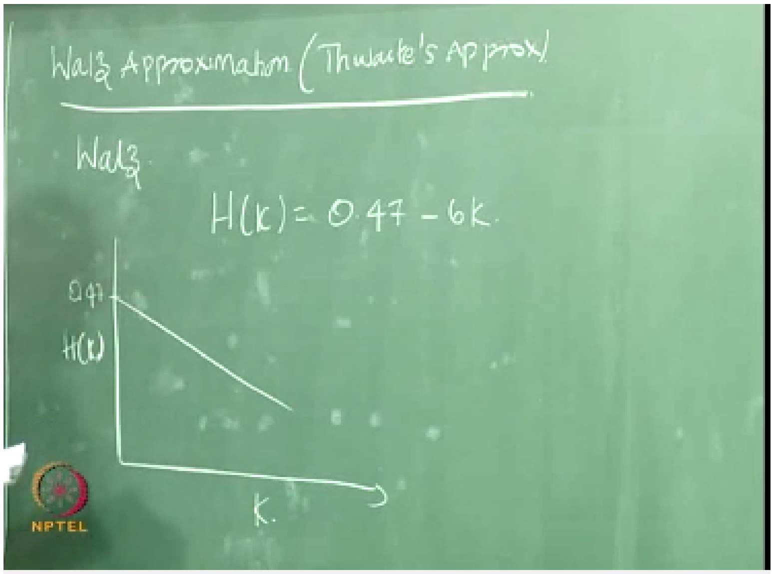

Okay so this was a very good news because now since this is a linear relationship we can simply directly use this to calculate Z okay so we know the expression for the momentum reduced momentum integral equation which was DZ / DX is = to H of K / u ∞ so this was the expression that we derived after we substituted those velocity approximate velocity profiles finally where what is your z del 2 square / μ so this is nothing but a parameter which involves the momentum thickness okay so this has to be solved in order to get new values of Z and then we found from there you iteratively solve for new values of λ and keep doing this as you keep marching from one region to the other region till it separates.

Okay now this was because we did not know a very good functional relationship for H of K so now when Wald's plotted H of K as a function of K we found this is nothing but you can fit it very nicely with a linear line and therefore the linear approximation was derived now we will substitute this x this expression so this will be 0.47 _ 6 K / u ∞ okay so this is called the walls approximation the rest of the thing is it is just how do we now simplify this equation now directly we have DZ / DX is = to this we can integrate it out so one more step we can write this as u ∞ DZ / DX is = to 0.47.

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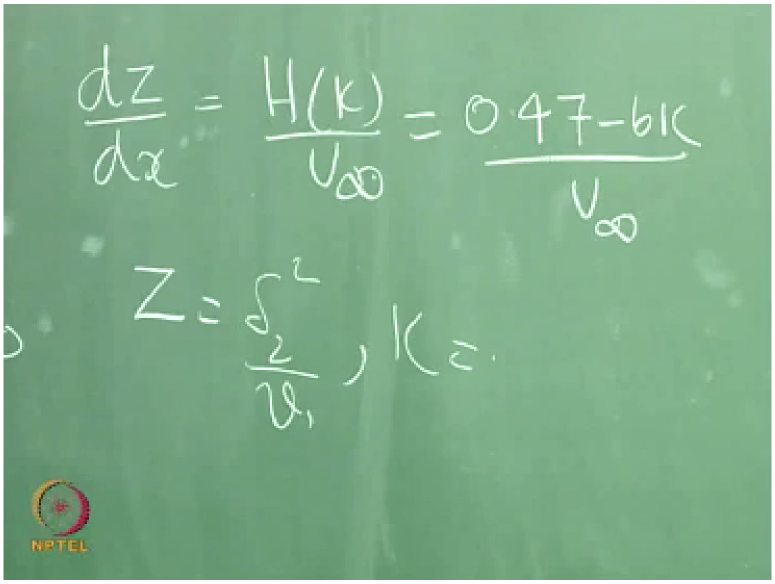

Anybody remember what K is Z x D u ∞ / DX this is coming essentially from the pressure gradient parameter okay Z x D u ∞ / DX right so therefore this is a function of K is nothing but basically a function of Z okay so it is wiser to put this in terms of Z D u ∞ / DX now you see we have an equation directly for Z okay which we can integrate it out we can also rearrange this as u ∞ power 6 Δ 2 square / μ so I am going to substitute my Z as Δ 2 square / ν so this can be rearranged in this manner Oh point 4. 7 u ∞ power 5 okay so now you can expand and check so this will be the same so you have ν is a constant which can be taken out so you have D / DX 1 / μ so we can take Δ 2 square.

And u ∞ so this will be 6 x u ∞ power 5 Du ∞ / DX + I have u ∞ power 6 okay 2 Δ 2 x little T Δ 2 / DX is = to 0.47 ∞ power 5 okay so I have six and now my Z is nothing but ν square Δ 2 square / ν I have this so I can divide throughout / u ∞ power 5 so this will give me Δ 2 square / ν leave ∞ this is 6 leave ∞ / DX + u ∞ - Δ - Lee Δ 2 / DX is = to 0.47 so this is U ∞ so this is six set D u ∞ / DX and I have u ∞ yeah so this DZ / DX is nothing but 2 Δ 2 x D Δ 2 / DX that is = to 1 so therefore you can you know all this can be combined x one neat way of clubbing together this particular term.

And this will be nothing but the expanded version of this okay so now with this we can we have an expression directly in terms of Δ 2 which we can integrate it out okay so if you if you integrate it so integrating so we can say Δ 2 square is = to 0.47 new / u ∞ power 6 of X okay and so I am going to integrate the right-hand side which will be 0 to X u ∞ to the power 5 of some Z DZ okay so this is my final expression that I now get so I simply integrate both sides.

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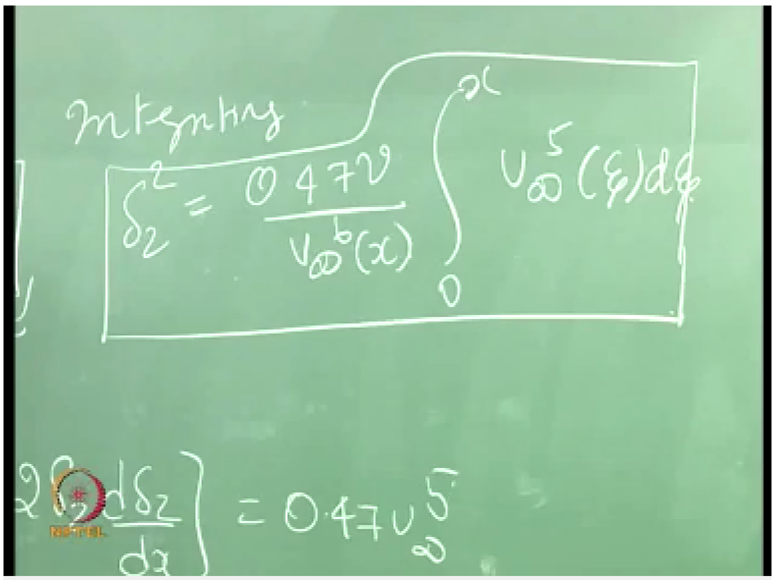

Okay and just I write this in terms of Δ 2 square so therefore I will call this as number 1 so your earlier expression for solving for Z in fact that is solving for Δ 2 was doing numerically you have to solve this OD numerically now once you introduce this linear slope approximation ok constant slope approximation x this and you directly integrate it out you get a simpler integral okay if your function is known u ∞ you substitute here you directly integrate it out and you can get an expression for Δ two right away so there is no numerical work involved in this okay although this is an approximation you know nevertheless.

It is a very good approximation in fact I can give one homework where you can check this that this linear approximation is valid okay you can calculate H for different values of K plot and check for yourself all right so this is the thing and here we have used the fact of course then you integrate it out they have a constant here constant of integration okay so now what basically X is = to 0 when you are integrating this with respect to X is = to 0 you are you ∞ that you are using so this entire term has to be 0 because I have the stagnation point your free stream velocity is 0 so therefore this entire term will be 0 therefore the constant will be 0 okay so we have used that that at X is = to 0 to R u ∞ over 6 Δ 2 square / ν = to 0 okay so it is not that your boundary layer thickness is 0 but your u ∞ is 0 which leads to this particular conclusion that the constant

Is 0 therefore this is the final expression okay now we will apply this again for the cylinder problem earlier we had I had given you the algorithm to solve for the cylinder case in a more rigorous way solving the OD in numerically okay so first we will look at the cylinder problem near the stagnation region so if you substitute the profile of for u ∞ or u ∞ is basically two V 0 V ∞ R 0 / sorry x / r 0 so if you substitute this and you integrate it out you get Δ 2 square is = to 0.23 5 x μ x R 0 ÷ / V ∞ X power 6 okay so this is basically 0.47 ÷ / 2 and you have V ∞ X power 6 ok that is U ∞ power 6 actually everything is power 6.

But inside the integral also when you substitute for u ∞ to the power 5 the extra terms cancel out so you have Z power 5 DZ is what is left out inside okay so you are basically substituting for u ∞ here u ∞ power 6 here u ∞ power 5 so you have to you in to V ∞ / r-not which is the same ok so then you will he'll be left with the factor that this is V ∞ x V ∞ ÷ / R 0 here you will have X power 6 here you will have C power 5 so this you can integrate out directly so this will be point so if you integrate this.

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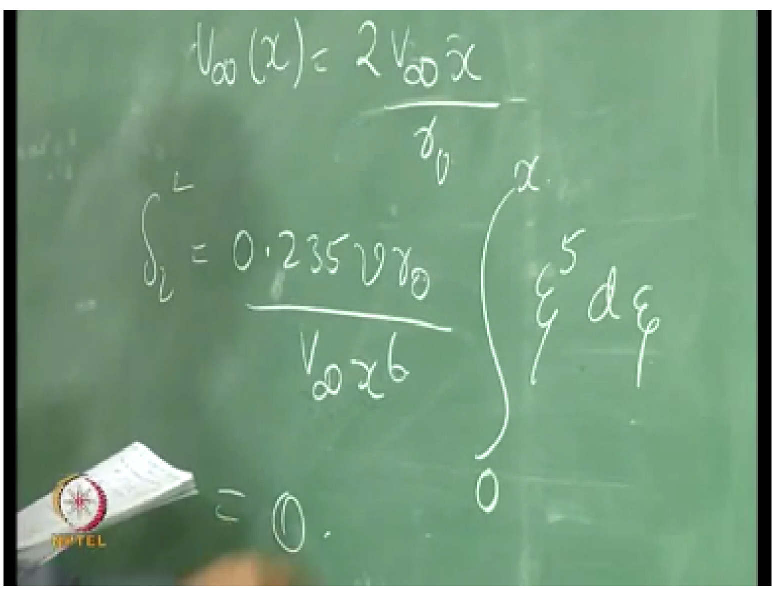

Will be nothing but X power 6 / 6 okay so 0.235 ÷ / 6 will be point 0 3191. X power 6 X power 6 cancels so you have knew R 0 / v ∞ okay so therefore there you go you directly have got an expression for the momentum thickness without solving the ODE rigorously you know near the stagnation point directly you find it okay and you can also calculate the value of λ for doing that we need to estimate the boundary layer thickness which is estimated so how do you estimate the boundary layer thickness Δ square there is an expression because Δ is a function of λ okay what is the original expression how did we define Δ λ ν / D u ∞ / DX right so we defined this parameter λ like this okay so Δ Square D u ∞ / DX / ν is that right okay so therefore for this particular case we can substitute D u ∞ / DX which is nothing but a constant - V ∞ / r 0 and that comes out to be λ / 2 x ν R 0 / V ∞ ok so now we know λ Δ as a function of λ ok we have an expression for Δ 2 so we can calculate the ratio of Δ 2 / Δ the whole square so you let me know what this expression will be so if you take the ratio of these two VR 0 / V ∞ will be cancelled out okay this will be point 0.391 x 2 / λ.

Which is 0.078 - what we are trying to do is we are trying to solve for λ ok so Δ is a function of λ and we have we have an expression for now Δ 2 / Δ the whole square which is now a function of λ we also have derived Δ 2 / Δ square we have expressed this as another function of λ do you remember that originally that is 1 / 63 square 37 / 5 _ λ / 15 or you can say that this is = to √of this and then you can you can write like this λ square / 144 okay all right so now we have a expression algebraic equation in terms of λ since Δ two / Δ is originally this in terms of λ okay now we have another expression for this particular problem.

Which is like this so we can equate both and you can find out λ if you do that in fact this has to be also solved iteratively so λ will come out as 7.25 okay now you can compare this with the λ that we got earlier that was 7. 05 right so it is slightly off but nevertheless it is very close okay so I think this is now μ CH easier to solve than the earlier case what we will do is now go ahead and complete the heat transfer solution so for the heat transfer problem whatever we have derived we have to assume the cubic polynomial and then we have substituted x the energy integral and we have finally calculated the expression for Z. Which is the ratio of Δ T / Δ s point this is there is Z cube this is .69 / frontal number right so the same expression is valid because as far as the energy integral is concerned.

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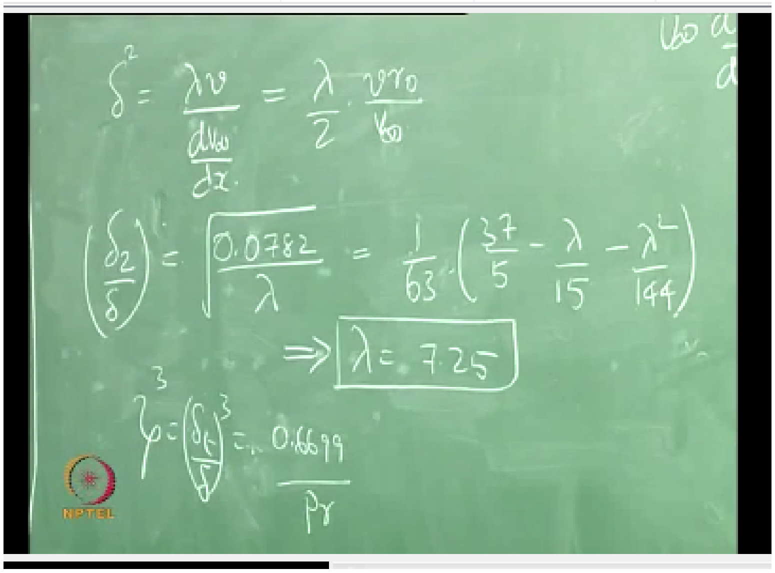

All you are doing is you are substituting the velocity profile approximate velocity profile approximate temperature profile and finally you have expression in terms of Z which you are getting the solution for okay now when you are getting the solution you are substituted a value of λ there okay the original expression was something like this.

So your expression was Z D / DX at Z Square √of λ 12 + is = to 90 up till here everything is the same okay so this is coming from substituting x the energy integral right for the velocity profile and the temperature profile and simplifying a little bit we are where we are neglecting all the higher-order terms of Z and only Z square comes finally so now from here we should use the particular value of λ.

Which we got from the walls approximation okay so that is basically 7.25 instead of 7.05 which we substituted earlier if you do that earlier you got your Z as 0.75 875 / PR to the power 1 / 3 now with this you will be getting Z as 0.864 / PR power 1 / 3 so only the constant will change a little bit.

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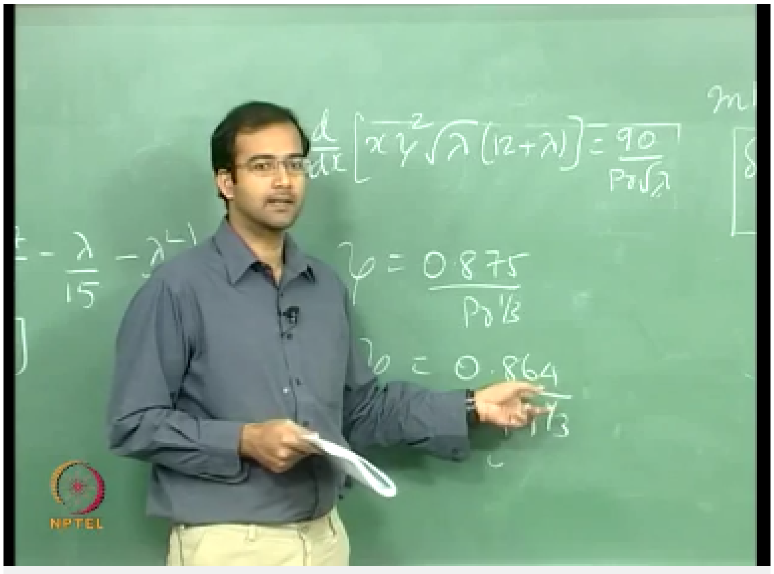

Because now your λ is slightly different if you substitute it your solution for Z will be just slightly differing / constant it will be 0.864 / PR power 1 / 3 so that is it so once your Z is determined. You can calculate your heat transfer coefficient and your nusselt number okay so you know that the heat transfer coefficient is 3 / 2 K / Z Δ okay so the expression for Δ is the same for both the walls approximation and the original von Karman pool house and solution so only the value of Z is differing slightly if you substitute this you will be getting an expression for H which is like point six for Phi K / r- 0 x prandtle number or 1/3 re power half okay now if you define your nusselt number H x D 0 / K okay so this will be what 1.29 prantle number power one-third re power half okay where you are is nothing.

But V ∞ D 0 by new so compare this with the expression that we obtained yesterday you remember that expression when we used λ 7.05 what was the expression that we got one point once you know that that is the exact solution 1.291 correct okay so this is 1.291 so finally in terms of nusselt number there is any hardly any difference okay whether you use the walls approximation or not so that is I see this is to just give a good idea that walls approximation is pretty good especially in terms of the heat transfer calculation you get a very good agreement with your poll how since solution as well as your exact solution is what 1.145 and that has prantle number 0.4 independent.

So there is some difference between the exact similarity solution and the approximate solution but using the waits approximate walls absorb approximation is much more judicious you know you get faster solutions than solving the OD now the same problem where you are looking at circular cylinder can be done by using walls approximation right okay so the same way you take the profile the complete profile u ∞ of x is = to 2 V ∞_{θ} X / r-0 where your X is = R 0 $_{\Theta}$.

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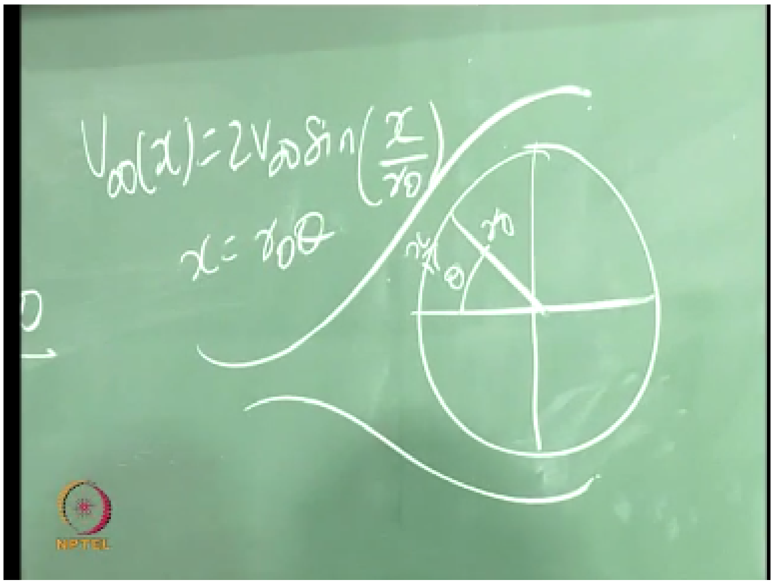

You right you take your full profiles which is coming from the potential flow for describing the free stream velocity for a circular cylinder and that can be substituted x this okay maybe expression number 1 right here so that u ∞ can be substituted now you can integrate it now you have to be careful when you integrate it you are integrating sign power 5 X / are 0 there okay so in the stagnation region you have made the approximation that for small values of x / r 0 $_{\theta}$ x / r 0 is = to x / or not but if you are looking at integrating for the entire flow from the stagnation point still the separation point you have to use this profile as it is okay and when you put this profile you have $_{\theta}$ power v term here $_{\theta}$ power sixth term.

So if you are finding it difficult to integrate you can use some numerical techniques such as you know trapezoidal rule for integration simple trapezoidal rule or even simple rectangular rule will work so you have to start from some point right here at this point you know the value of Δ to that that is what you have determined okay you also know the value of λ and everything so you can start from that point 0 to some point next okay so you can keep on integrating that way okay till any X that you want till you go to separation okay for each point that you go so now you calculate the new value of Δ to square and therefore you can calculate the new value of λ like this because you know the ratio of Δ two / Δ correct for each value of x you know the new value of Δ 2 / Δ you also know the original form original relationship you can equate those two and solve for λ so we will give you solutions for λ basically okay so this you keep doing till you hit separation point.

Where your λ becomes = to _ 12 so now this is a little bit more easier you can do this on an excel sheet you do not have to program it so you can simply do numerical integration and you can solve this equation also you know you can find roots of this in an excel sheet itself so that will give you again the boundary layer thickness and the momentum thickness and also the separation point so everything can be obtained from the walls approximation okay then this will be nearly as good as doing it / the solution numerical solution to the OD.

Okay so in fact I will give the next assignment which will be assignment 3which will include all the integral solution problems I will also give ask you to do this for scepter cylinder and see for yourself how you can basically find the separation point and get expression for boundary layer thickness momentum thickness okay so the same thing can be done for any wedge problem you know the Falkner Scan solutions where you had assumed some kind of a wedge flow and a velocity profile of this particular sort.

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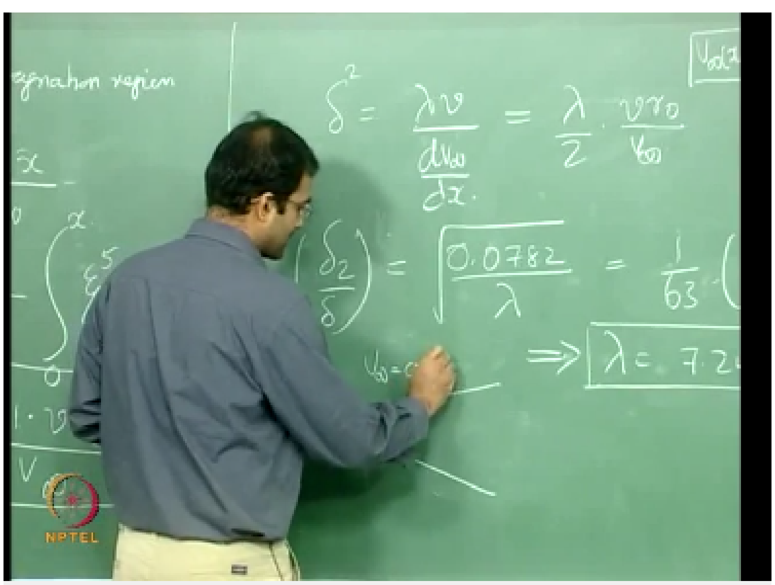

Now you can apply the walls approximation you can substitute for u ∞ here you can integrate it out okay and finally you can calculate the value of λ and from there of course you are boundary layer thickness and again you can use that x the heat transfer problem where you substitute the expression for Δ and the particular value of λ you calculated you can derive the expression for nusselt number. So you can do this for any value of M okay like the way you did your similarity solution you can do this and you can compare the result with the similarity solution you will find the agreement once again is pretty good.

Okay so any problem with pressure gradient and we handled with the walls approximation okay so probably we will I will stop here but I will probably will have some kind of a small discussion so if you have any doubts because I will start I have one more last topic left under integral method which is non-uniform temperature boundary condition so far we have assumed and the entire plate is heated with the uniform temperature or uniform heat flux in most of the cases that is not true okay the profile will be varying.

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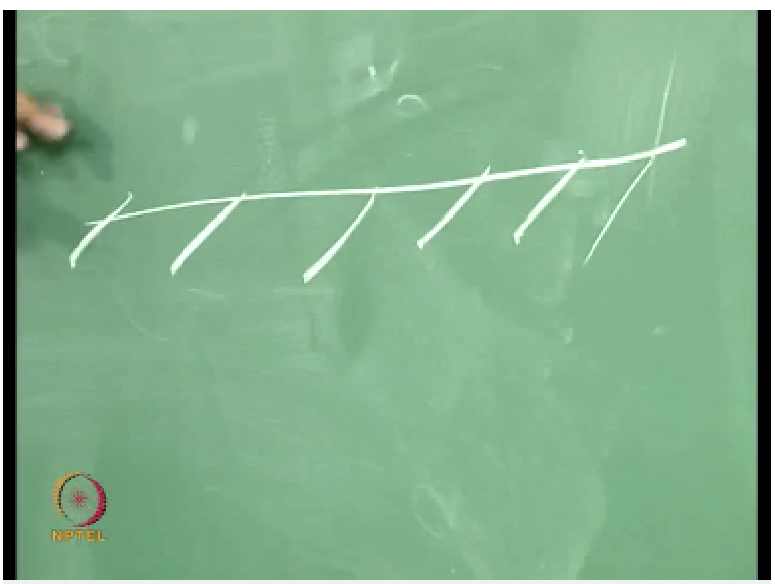

So what happens if you have a variable temperature profile okay so we cannot do with the similarity solution this kind of a problem of course you can use similarity solution if it varies only linearly but if it is any other variation you have to do integral solution so we will see for a flat plate case what is the technique to how to introduce a non uniform temperature boundary condition how do we solve this okay so that will be the last topic I will take that about a couple of hours next week before your quiz 1 okay so quiz 1 will have all the topics till your integral solutions.

So pretty much training any doubts that you have in this I hope you understood the walls approximation because you will be now using this to solve this flow past circular cylinder okay so if you have any questions you please ask me is it is it clear you have you understood huh this is just a dummy variable yeah this is just a dummy variable you can if you if you are having say CX power M this will be C Z power M so that will be just a dummy variable and it indicates that whatever upper limit till where you are interested to integrate.

So that will be used x the dummy variable okay so earlier X indicated some arbitrary location now when you are integrating it you have to use the dummy variable because you should not confuse this will with the limit of integration okay so many a times we use the same thing you know but we know that we should use the upper limit but to differentiate it clearly strictly speaking you always have to use a dummy variable okay so you are clear with all of all of this like how to calculate your Δ - numerically and all that you have to numerically integrate it by representing rule you can start from say the stagnation point ok assume this is a trapezoid between these two so you can if you are integrating to the first point it is just only one step if you are integrating somewhere till somewhere here you have to assume any trapezoids and sum them all together so the area under each trapezoid sum them all the trapezoids together will use it will give you the area under the integral basically so that is that is the approximation that we are so at each location so you calculate the value of Δ - and so like this.

You will have an expression for Δ - right here right now we have completely there is no dependence on X here ok so there when you are putting a sign profile there it will have a dependence on particular value of x so for that particular value of x you will have a particular constant okay for Δ - okay so and then you now this is so this is the expression for Δ - this expression for Δ will be there this is again a function of the position because you are now differentiating sign profile okay so there will be a X dependence so for each position the value of Δ 2 and Δ will be different and that you take the ratio.

So you will be getting an expression in terms of λ okay only this constant will keep changing for each location and then you can solve this find roots of this okay get the new value of λ and you know that whether we are the region is know as and when the λ keeps going negative and increasingly negative you know that you are approaching close to the separation point and then you keep doing this integral till we are at a particular point where λ becomes _ 12 and then you know that you have hit the separation okay so all this can be done in a excel sheet you know you do not have to write any program for that separate.

Heat transfer across a circular cylinder:

Walz approximation

End of Lecture 22

Next; Duhamel's methods for varying surface temperature

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