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Video lectures on convictive heat transfer

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Lecture 21
Integral method with pressure gradient: Heat transfer

Good morning all of you so yesterday we were looking at application of the approximate method to close the pressure gradient and so this method is also called as the Fond Carmen poll Hassan technique .

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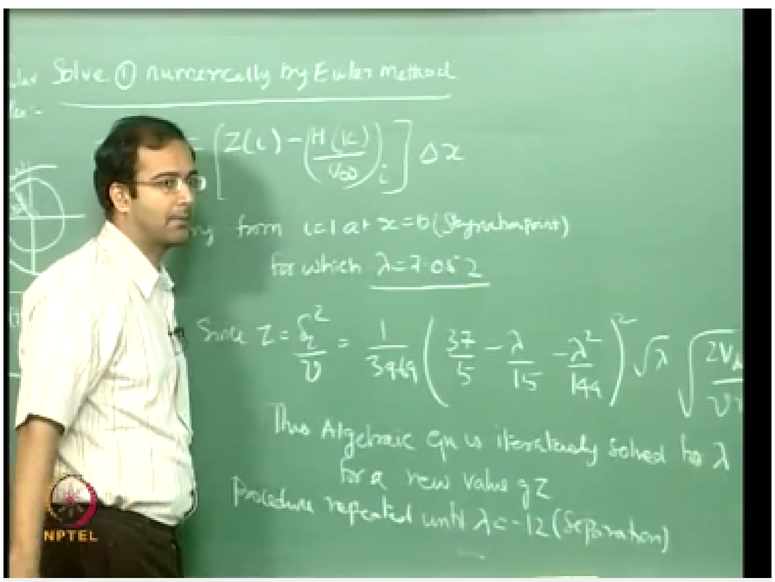

Which was originally conceived by Von Karman and he suggested that we can use higher order polynomials and we can substitute that x the momentum integral equation and try to find solution for flows including pressure gradient. So this is what the procedure that we did and finally the equation that we obtained from the momentum integral after making all those substitutions for aquatic polynomial so this is the final expression and so this cannot be solved just like that analytically because it is a non linear odie because you u ∞ is a function of X okay.

So therefore we have to use some numerical method like the Euler method that we had seen that for shooting technique also so we can just discretise this particular ordinary differential equation for many number of points starting from the stagnation point so suppose you want to apply this to the flow past a circular cylinder okay so for this we know the variation of the free stream velocity so suppose you are the free stream which is approaching the cylinder is u ∞ V∞ .

So we can express the velocity variation of the free stream velocity variation past the circular cylinder as u 2 u ∞ sin x / r 0 or maybe i can use some capital R to indicate the radius okay so this is the velocity profile obtained from the invisible flow analysis and so now we directly integrate this particular odie by the Euler method and so we start from the stagnation point so that is where the i = 1corresponds to I = 123456.So if you want to look at the coordinate system in terms of X okay.

So that corresponds to X = 0 which is nothing but the stagnation point okay so we have also determined the value of λ is a function of X as I told you and λ at x = 0 comes out to be 7.052 because we have seen that u ∞ o fx at x = 0 becomes 0 so in order for this particular odie to be finite we have to force H (K) to be 0 at x = 0 from which we arrived that the root for λ which is acceptable root is basically 7.052 okay . So now we know the value of λ therefore we have the expression for Z in terms of λ for a given flow conditions like your V ∞ μ and r not or capital R so in fact if I use capital are here to let me use this here so this is my cylinder radius okay so this is fixed for the particular geometry and the flow condition okay so I can calculate my Z at the stagnation point based on the value of λ so that from there I can start integrating it.

So now i can find the value of z that the second point okay that is at i = 2 and once i do that again i come back to this expression okay to this algebraic expression for which i can iteratively find the value of λ corresponding to the μ value of Z ok so like that i can do keep doing this procedure multiple times ok so for all values of Z I have to keep finding the corresponding value of λ and then using that to calculate the other parameters like H ok because all of them are functions of λ okay.

So if I keep doing this integration till I reach a particular point where I will find that the λ comes to - 12 ok that indicates that the flow is separating at that particular point and that is where I stop the integration okay so with this method I can actually track how the boundary layer thickness because this is nothing but Δ 2 is what the momentum thickness okay.

So I can track how the momentum thickness is progressively growing with X and from there I can calculate all the other thicknesses because I know Δ 1 / Δ i have Δ 2 / Δ i can calculate my displacement thickness i can calculate my boundary layer thickness everything is a function of λ okay and consequently i can also determine the point where the flow is separating okay so the exact angle where the flow detaches about 75 degrees okay.

So this is the rigorous way of doing it but nevertheless it is a very good useful tool now otherwise when you do experimental studies and flow visualization that is one thing but if you do not have access to those experiments theoretically you can use this technique approximate technique to determine at least to the extent where the flow separation takes place okay or otherwise we have to do an expensive numerical simulation of the entire navier-stokes equation which is too rigorous okay this is a less rigorous procedure but nevertheless it is useful till the separation point beyond the separation point the boundary layer theory is not valid okay .

So this is was what we have seen yesterday know a very useful technique which was proposed by Von Karman and demonstrated by paulasin now this is for flow let us extend this technique to heat transfer problem okay you can consider the same adverse pressure gradient our favorite favorable pressure gradient flows let us introduce the approximate solution for temperature okay.

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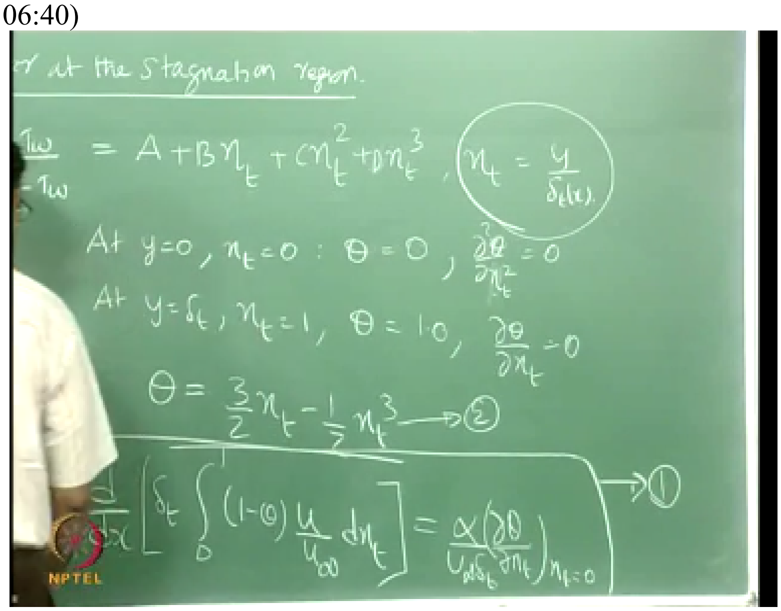

So heat transfer now what we are going to do when we are looking at the Nusselt number correlation the same way for the similarity solution we derived only for the stagnation region okay the same way we are going to focus as far as the heat transfer is concerned yes yeah i just gave are you in 3 D(x) so the bracket with one the before that and this should be a + sign you're right yeah I think please correct this you're right okay so this Δ X should be multiplying only this and then this is Z i + 1 – Z i so it becomes positive on the other side yeah so okay.

So if you are looking at heat transfer we will focus our heat transfer problem to region which is close to the stagnation region the same way we did the similarity solution Falkner Skan solution for the case of m = 1was nothing but the stagnation point solution and that we had converted to cylindrical system and that gives us the Nusselt number for the stagnation region for a circular cylinder the same way with the approximate method we are going to do the heat transfer solution for the stagnation region .

So for the same problem let us assume that the free stream or the ambient temperature is T ∞ and the walls of the cylinder is maintained as at an isothermal condition where your t wall is constant ok now this gives rise to the heat transfer problem for as far as the temperature profile is concerned ok Von Karman or pools and did not specify anything about how rigorous the profile should be as far as the flow is concerned you have to take a quadratic polynomial but with respect to temperature profile we can still use the third order polynomial that we were using before for blushes solution .

So let us define our θ t - t wall / t∞ - t wall so that it scales between 0 and 1 and 0 at the wall and 1 at the free stream so this can also be assumed to be a function of some η T which is a non dimensional coordinate corresponding to the thermal boundary layer ok so let us assume a cubic polynomial we have four coefficients to be determined and here η T is nothing but y / Δ T okay.

So here this is not a similarity variable as I said as far as the approximate solution is concerned your η T does not denote any similar it is just your non dimensional you when you when you substitute in terms of Y and you get the profile you will have terms like Y /Δ so in order to make it compact you denote your Y / Δ as a non dimensional η but it does not mean that this is a similarity variable ok so of course also coincidentally for the similarity solution this turns out to be this similarity variable ok but here it does not have anything to do with similarity.

So now we have to propose the boundary conditions to satisfy this particular temperature profile so what are the corresponding boundary conditions at y = 0 okay this is basically η T = 0 right what should be the value of θ should be 0 and at y = Δ T corresponding to η T =1 ok θ T should be equal to 1 okay and even at y= 0 now if you go and look at the higher order boundary condition from the energy equation again = to α d^2 t /dy^2 of course we have neglected the viscous dissipation term so near the wall we can neglect the inertial terms so therefore we can also say that $d^2t / dy^2=0$ or $d^2 \theta / dy^2 =0$ okay in terms of η also the same thing will apply in terms of η T same condition applies okay now we need one more boundary condition to close this problem .

So what will be the fourth condition zero set y = to Δt and H η T so your D θ / D η T should be = 0 so now you have the required boundary conditions you can get the profile for θ and what will be the profile we have already done the same thing for Blasius solution we have assumed a cubic profile same boundary conditions okay.

So what was the profile θ= 3 / 2 η t - 1 / 2 η T^3okay so as far as the temperature is concerned so it does not matter you know as I mean it only depends on the flow problem the flow problem is different for the pressure gradient and without pressure gradient and as far as the temperature profile is concerned so this is the temperature profile and the energy integral equation also remains the same okay so you have to substitute this x the energy integral do you remember the energy integral equation for without the viscous dissipation term .so let me erase this part.

So I hope you can recollect the energy integral D / DX okay so what should be the term inside integral 0 to Δ T now i am going to transform the variables from y 2 η 0 to 1 1 -θ u / u ∞ x D η T okay so this will be d η by so you have basically dy there okay so dy / D η / dy okay so basically that will be where Δ T that will come out outside and on the right-hand side you have your α DT / or D θ / dH T at η t = 0 and you have also from the transformation of variables you get u ∞ x ΔT okay .

So this is your energy integral expression let me call this as now equation let this call let us call the profile as equation number 2 and the energy integral expression as number one okay so all of you are familiar with this okay so you have your one x so if you want me to expand again from the basic equation so d / DX of 0 to ΔT 1 - θ u / u ∞ dy okay, so this I am writing in terms of η 0to 1 1 - θ u / u ∞ x dy / D H T x D T and dy / d H T is nothing but Δ T okay.

So that is how this factor Δ T comes okay so this is my left hand side and on the right hand side i have = α d t or d θ / dy at y = to 0 okay this I can again write as α dθ / d η η t H = 0 x D H T / dy DT / D Y is 1 /Δ T so that is why I have the factor Δ T here okay and there should be so yeah so I am multiplying and dividing by u ∞ here so that u ∞should be taken to the right hand side ok this is originally 1 - θ x u dy so i am multiplying and dividing by u ∞ and this u ∞ is going to be so this is the final form in terms of non dimensional θ and non dimensional coordinate η T okay.

So I can now substitute my temperature profile and the velocity profile that I obtained from the quadratic polynomial into this particular equation one.

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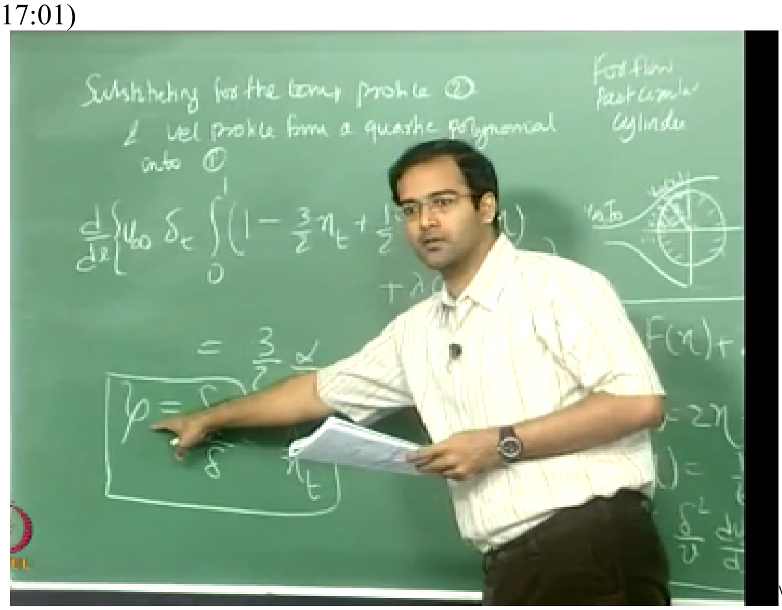

So substituting for the temperature profile 2 and velocity profile from a quadratic polynomial into 1 so I have d / DX okay i am taking my u ∞ to the left hand side I have Δ T 0 to 1 1 - 3 /2 η t - + η T^3 and my velocity profile u / u ∞ will be so what will be the velocity profile u by what was that we had derived yesterday one / two where is this coming from yeah so yeah so if you substitute everything in the compact notation it was F(η) + λ G(η)right so where your F(η) of was what 2 η - so this was what probably your 2 η - 2 $η^3$ + $η^4$ your g(η) sorry should be 1 / 6 η x $(1-η)^3$ ok and your λ is nothing.

But non dimensional pressure gradient parameter which can be written as $δ^2$/ μ d u ∞ / DX in terms of DP DX your d u ∞ x DX can be written as - $δ^2$/ u∞ μ x DP / DX so far flows with adverse pressure gradient your λ will be negative with favorable pressure gradient λ will be positive so this was the velocity profile that we derived yesterday assuming a quadratic polynomial and satisfying those five boundary conditions okay so your λ here is the non-dimensional parameter which is giving us something like a ratio of your pressure gradient to or pressure forces to the viscous forces okay so this is one measure of how adverse pressure gradient how adverse the pressure gradient you maintained its okay and what is the criteria for separation is given by λ so λ = - 12 indicates the point of separation okay.

Now from this we can substitute for the velocity profile so this will be F of we will put it in terms of f (η) + λ G (η)okay d η T is the variable with which you are integrating and thats it okay so this is your left hand side basically on the right hand side this will be = α x u u ∞ I have taken to the left hand side you have d θ / d η T so from this profile what is d θ / D η T at η T = 0 3 / 2 and you have already α / Δ T okay so this will be your equation now we can introduce my Z Zη as which is nothing g but Δ T / Δ and from the way that we have introduced Y / Δ as my H and H T is = to y / Δ T so this is nothing but the ratio of H/ H T okay so i am just introducing a parameter called zη now if I for a given Prandtl number you know that this Δ t / Δ is a function of parental number.

So this becomes fixed for a given Prandtl number and also if you have starting if you do not have any unheated static link that means you maintain an isothermal temperature condition right from the beginning ok so this Z η is going to be a constant because both the thermal boundary layer and the momentum boundary layer will grow simultaneously and for a given Prandtl number that ratio of the boundary layer thicknesses has to be a constant so therefore this Z η will not be a function of X anymore it will be constant if you heat the play heat the cylinder right from the beginning okay so right now we are considering a case which is isotherm throughout okay we do not have any unneeded starting length .

Okay so in that case you can take your Z η as a constant you can write your Δ T in terms of Z η and Δ and also you can substitute for H in terms of Z η and Δ H T okay so now you can integrate this no now since this is a function of H T and you are integrating across H T so this will result final expression will be independent of H T it will contain only terms with respect to Z η and finally Δ okay so that comes out to be Zη   x Δ.

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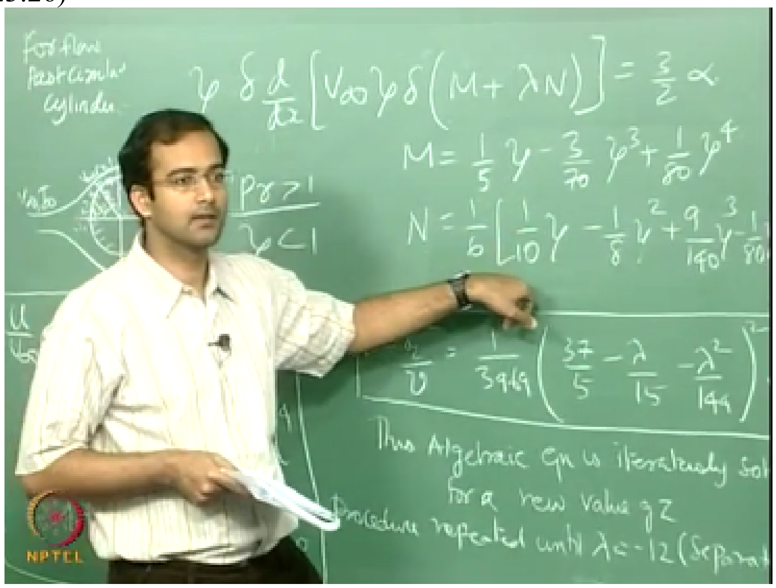

So i am writing I am taking my Δ T on the right hand side here so and I can I can write this as Zη times Δ x D / DX   x u∞   x Δ T and again I can write it as a DA   x Δ and if integrate this entire expression out in terms of H T. So I will be getting expression like this 3 / 2   α where my M now he what okay now I am substituting here for H whatever function i have all this in terms of Ht + Z Zη okay now if I integrate it over H T now what should be the resulting expression should be a function of only Z η okay so therefore M will be coming out as 1 / 5 zη- three x 70 zη q + 1 x 80 zη ^{4} and my n will be 1 / 6okay so it is not that difficult you know it is just one step integration which I am not doing it here you can just substitute for all this for f of H in terms of H and Zη multiply these two and then you integrate it out so you will get one bunch of expressions in terms of Zη.

The other which is contained within the parameter λ okay so I am separating these two and writing it out okay it is clear till here now if you consider a particular case where your parental number is <1 so now what kind of approximation does it mean on Zη okay so thermal boundary layer thickness is much smaller than your velocity boundary layer okay so therefore this corresponds to the fact at my Zη >1 so therefore all the higher order terms of Zη so the second power and higher can will be very small and therefore can be neglected okay so all these terms can be neglected only you can the first power terms okay with that we can constantly simplify this expression okay.

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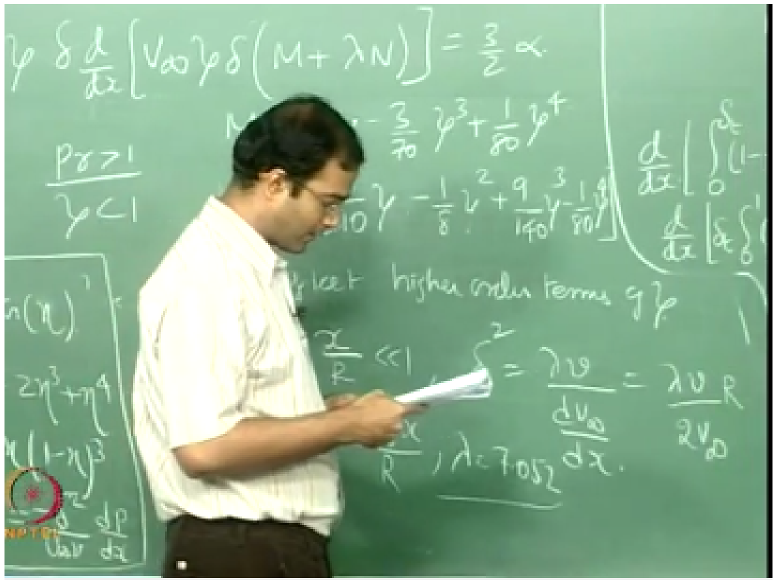

So therefore neglect higher order terms of $Z\eta$ and also we are interested in the region which is close to the stagnation region as I said we want to get an expression for Nusselt number in the stagnation region so this is for the case where my x / r is much lesser than10 k is quite much lesser than one okay so for this case we have also derived the expression for Δ square now you have to tell me what will be the expression for Δ square here if make the approximation that is close to the stagnation region ok so my Δ square comes from the expression of definition of λ if you remember okay, this is λ you in / d u ∞ xDX and for small values of x / r my $U\infty$ of X will be 2 V ∞ xx / r okay so this will be λ nu x2 V ∞ or okay, so this I can substitute in first so I have a Δ here.

So I can substitute my expression for Δ from this x this expression and also the value of λ for stagnation point okay we have already determined the value of λ which is what 7.05 too so we can substitute the value of λ here as well as the Δ from here so and therefore the resulting expression will be so let me call this as number three okay so substituting all these approximations now in 23.

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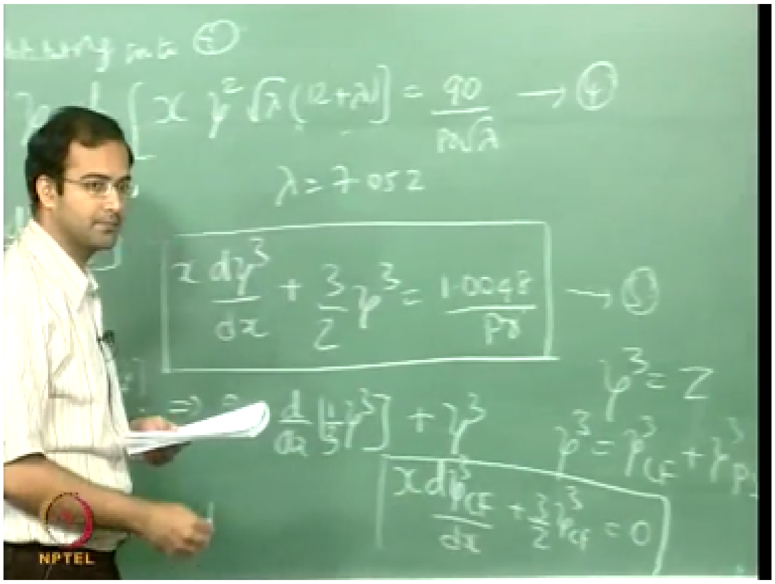

Now you should be getting Z η D / DX okay have already substituted for Δ okay Δ in terms of so this is a know the value of λ also so I will substituted directly x Δ and this will be x times z η √ λ x 12 + λ which will be = to 90 x Prandtl √ λ okay so I have substituted for Δ from here so everything will be in terms of λ and of course.

You are z η you have neglected all the higher order terms okay all these terms have been neglected so this will be λ here this will be θ Z η square only the z η^2 term will be there okay so this will be 1 / phi z η + 1 x 60 in 2 z η x λ okay so if you take the z η term out so that will be z η square here and you substitute for Δ in terms of whatever expression that we have so everything will be reduced to this particular form which is called let us call this is for now for λ =to 7.05 to this will be getting even more simplified and you can expand this differential equation as X D θ cube/ DX + 3 / 2 Z η cube 1.00 48 my frontal number okay so this is my final expression the OD in terms of Z η okay so I so all I am substituting is for λ so this entire thing is a constant and it comes out and of course this is also in terms of λ.

So now I can separate this d / DX I can take this inside so this will be λ cube x if I if I write this in terms of λ cube ok suppose I ignore all the constant I have Z η x z t^2 ok so I can write this as okay, so Z η D / DX I can take θ cube so this is 2 Z η x θ^2 Z η x okay + I can write this as Z η q x d DX d/ DX of X will be one okay so this is how I am expanding it now once again this term right here Z η cube θ can be written as 2 x d / DX of Z η cube okay x 1 / 3 okay so I have +Z η cube so basically I am dividing throughout / 2 / 3 and therefore I have this final expression okay.

So just comes through a couple of steps of algebra which is not that difficult okay so therefore this is the final OD and you know now how to solve this we can this is a non-homogeneous OD but it is linear so it is not a problem we can directly give a get an analytical solution for this you have to the solution will be one will be for a homogeneous OD which is nothing but the complementary function the other including the non homogenous part separately so that will be the particular integral okay so we can combine both the solutions together let us do that now you have to tell me what is the solution to the homogenous part of the Saudi all of you take couple of minutes.

You can assume some Z cube is= to something like p or Z something and you can say that yours IQ solution will be cube complementary function as I Q particular integral okay so this is the solution to the homogeneous part of the OD and you can do it and tell me what will be the homogenous part solution okay so that is basically it has to satisfy x d SI q SI f / DX + 3 x 2CF cube = to 0 so this is the solution to the homogenous part yes so What should be the complementary function a zero Y solve this /separation of variables you have forgotten all your foodie basics you can now if you are confused looking at this Z cube you can take this Is some Z or something like that now the Z η cube so now then it becomes the standard form of no D which you can separate / variables okay x power -3 / 2 okay.

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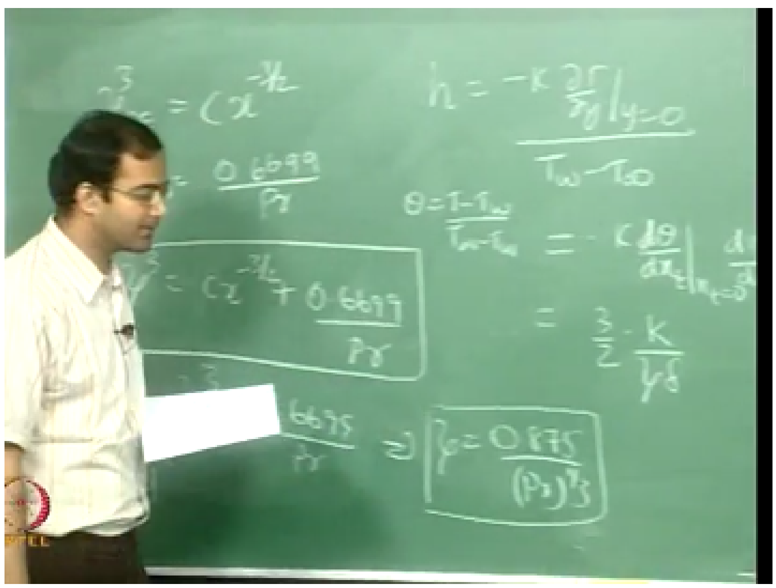

So some constant that is some constant x power - 3 / 2right so now how do you find the particular integral so you say that this is some Z complementary function + Z particular integral ok and you substitute that x the original equation which is this okay so from there you already know the solution for the complementary function okay so then you subs you calculate what is your particular integral solution from there okay if you do that you will get that your QP I will be a constant basically which will be 0 point 6699 / PR okay so all you have to do is substitute for this + some P I particular integral x this OD five okay and you get directly an expression for the particular integral.

Which will be constant finally it will be just point 66 so you please go back and revise if you forgot how to solve anon-homogeneous Saudi okay non homogeneous linear ordinary differential this is the most basic you know so from this therefore your final solution will be Z η cube will be C X power - 3 / 2 + 0.6 699 / PR so this is your final solution now if you had a unheated starting length then you could have substituted that X at X is = to X not my Z η = to zero but in your case for the present case we will assume that it is heated right from the beginning therefore at x is= to zero θ = to zero okay.

Therefore this constant has to be 0 okay so in our case directly the solution will be 0.6 699 / PR okay or this will be 0 point 875 / PR to the power one third so once you found out Z η now the entire problem is almost nearly solved because to get the heat transfer coefficient H is - K DT / DY at y= to 0 / t wall - T ∞okay now for this particular temperature profile can you tell me how this will reduce I know my temperature profile for θ which is nothing but 3 / 2 H t- 1 / 2 H TQ so this can be written as - K in terms of θ this will be left defined as t – Tw / T ∞ - t1 okay so DT/ d Y will be D θ / DY x T∞ - T wall okay so – of that will be t wall - T ∞ so this will be K x D θ / DY so once again I can transform the variable as d θ / D H T at it at = to0 x D H T / DY okay so my expression here will be basically d DH D θ / DT at H T = to zero will be nothing but 3 / 2 so this will b e 3 / 2 x K and what is DET / DY 1 / Δ T and since my z η is = to Δ t /Δ I can write my Δ T as Z η x Δ okay.

So finally since we have found the expression for Z η it's wiser to put everything in terms of Z η right so on therefore - alt number can be defined as H X / K or in the case of cylinder my characteristic length will be the diameter or the radius so I will choose the diameter as my characteristic length so this will be H in 2d not this is the diameter of the cylinder okay so this will be nothing but so 3 / 2 x D 0 divided /Z η x Δ so I can substitute my expression for Z η x this okay and I can directly simplify to get – L number expression.

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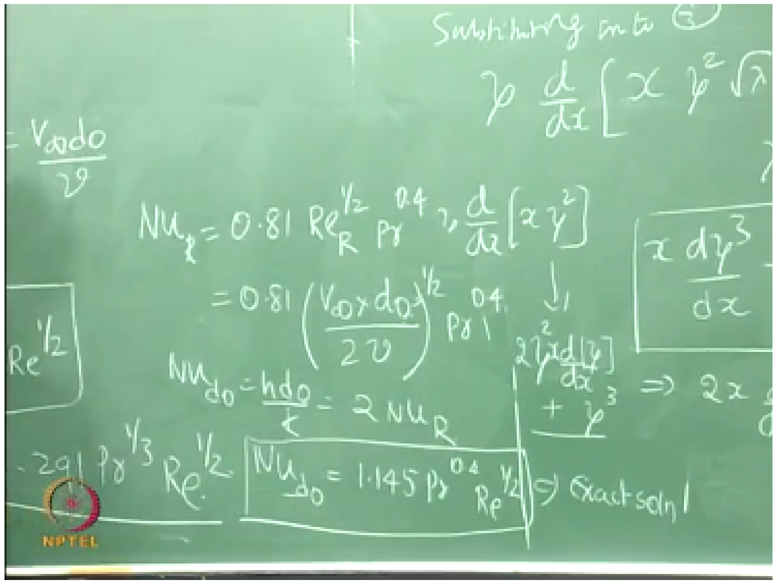

So my expression for H would turn out to be 0.6 4 5 in 2 K / r x PR 1 / third re power half okay so this is already in terms of PR and my Δ I know the expression for Δ okay so which is nothing but square root of nu λ R / 2 p ∞ correct. So I am substituting everything x this and finally i can write in terms of parental number and Reynolds number where my Reynolds number here is defined as V ∞ x D not buy μ Saudi not is two times are okay so then you can finally get the expression for Nusselt number which is HT not / k so which comes out to be one point two nine one x prandtl number power one third re or half okay so this is my expression for Nusselt number in the stagnation region okay in terms of Reynolds number which is defined this way and the Prandtl number now compare this with the similarity solution for the similarity solution you also you derived the expression for Nusselt number that was in terms of the radius okay.

If you remember that was n u r +OH point 81 times re based on the radius here Oh point for please go and go back and recollect from the Falkner Skan solution we have determined the expression from the similarity solution for the cylinder case this was defined based on the radius so if you convert this in terms of diameter okay that is you know it is a straight forward conversion so this can be written as point 8 1 V ∞ x D 0 x 2 x μ over half Prandtl number the power point for and you can define your assault number based on diameter has HD not / k which is nothing but twice of n UR okay so if you put this so if you multiply by 2 times so this will be nothing but you are not and on this side you will have a factor of 2 x 0.81 divided by square root of two okay.

So this will give you my nu D 0 has 1.145until number 0.4 re based on your diameter over half so this is your exact solution right from the similarity from the Falkner Skan similarity solution so you can compare that with the approximate expression so this is 1.2 91 almost 1.3 this is 1.14 so closed and the dependence on Reynolds number is the same the Prandtl number dependence is slightly different this is giving 0. 33 whereas that is to the power 0.4ok so from the similarity solution you can see the agreement is pretty close not that exactly the same but very close to the similarity solution so with that we will stop here today.

So tomorrow let us make another assumption so that we can simplify the kar man pole house and solution itself ok and with that we will get the same expression for Nusselt number and we will see how close it is to any questions ok so as these approximate solutions are therefore no very useful you do not have to solve the Odeon numerically ok so only get some profile and substitute and finally you find the resulting expressions correlations for Nusselt number specially the integral quantities like Nusselt number are pretty much similar to the similarity solution okay.

Although the profiles themselves may be different for example if you assume a linear temperature profile it is completely off from the actual temperature profile okay but the integral quantities like skin friction coefficient and the Nusselt number you get a reasonably good agreement with the approximate solution okay.

Integral method with pressure gradient:
Heat transfer

End of Lecture 21

Next: Heat transfer across a circular cylinder:
Walz approximation

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