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Video lectures on convictive heat transfer

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Lecture 20
Integral method flows with pressure gradient
(von Karman –Pohlhausen method)

So in today is class we will look at some more approximate solutions but this time we will take into account the pressure gradient term know in the last couple of classes we have ignored the pressure gradient we have done the approximate solution for flat plate and we have assumed they say linear profile and we saw with the linear profile you get some kind of number which is in terms of the boundary layer thickness and the skin friction coefficient there little bit off from the exact solution if you assume a cubic profile it is much closer okay.

So it I mean it is something to do with the right boundary conditions that you are satisfying so you start with the wall boundary conditions which are to be satisfied and then go to the free stream boundary condition so if you satisfy those necessary boundary conditions definitely your solution accuracy will improve and today what we will do is will extend this basic approximate methods to also non Blazes cases and we will start with a very general solution.

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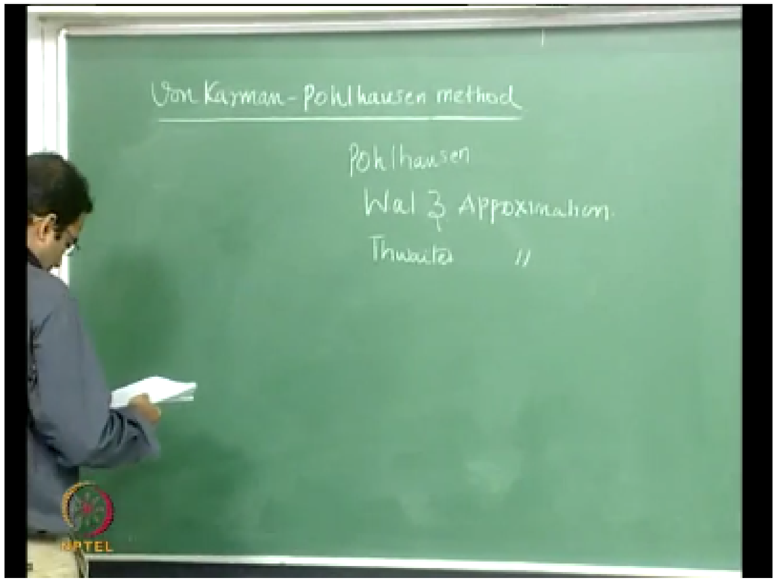

And this solution is called as Von Karman Pohlhausen method okay so this is also an extension of the boundary layer integral method but this is applied to any general case now you can think that this is a Falkner Skan case instead of a similarity solution we are doing this by new by the approximate methods so this method was first introduced by Pohlhausen so that is why it was named after him and in fact not only by that not only by that this name is given as Pohlhaus Method.

But also Von Karman was involved because it was Von Karman who has who had given the idea to Pohlhausen and von Karman was a student of prandtl and he gave the suggestion to Pohlhausen that we can extend these approximate solutions to a class of problems including the pressure gradient so therefore this method was named after both these pioneers and called as Von Karman pohlhausen method.

And this is a complicated method in terms of solution so later on it was simplified considerably / a person called waltz and this came to be known as the walls approximation okay the waltz introduced a further approximation to the integral solution of Von Karman pohlhausen some in some books this is also referred to as Thwaites approximation okay so both mean the same thing the same kind of approximation they have introduced okay.

So the basic assumption in this case before we go into the Von Karman Pohlhausen let us rewrite the boundary, boundary layer integral equation for a generic case.

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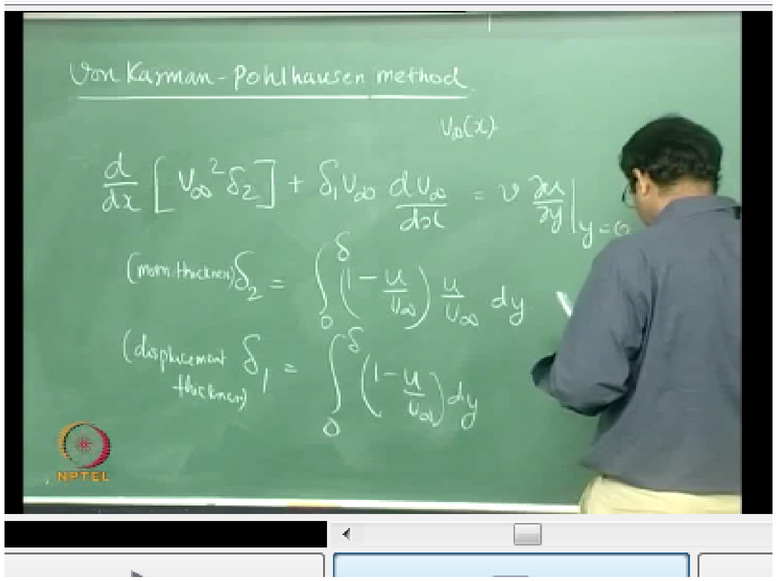

So D / DX okay so this was the generic momentum integral that we had derived where what is your Δ this is your momentum integral okay so one over you can write this is one over u ∞ R I will take the U ∞ inside so 0 to Δ 1 - U / U ∞ into U / U ∞ DY ok and Δ 1 is your so this is your momentum thickness or momentum integral this is your displacement thickness 0 to Δ 1 U / U ∞ DY ok .

So in the case of Flash is solution we ignored this term and then we simply had only the momentum integral and then we substituted the approximate profile and integrate it will doubt in this case we have to also see the term which includes the displacement thickness okay so now the next step what we are going to do is to make a guess for the approximate velocity profile.

Now in this case you have included this term because your u ∞ is going to be a function of X so this includes any kind of general case including the Falkner Skan which problem now in order to first substitute you know we have to make a guess value so what, what Von Karman Pohlhausen did okay in fact Pohlhausen profile.

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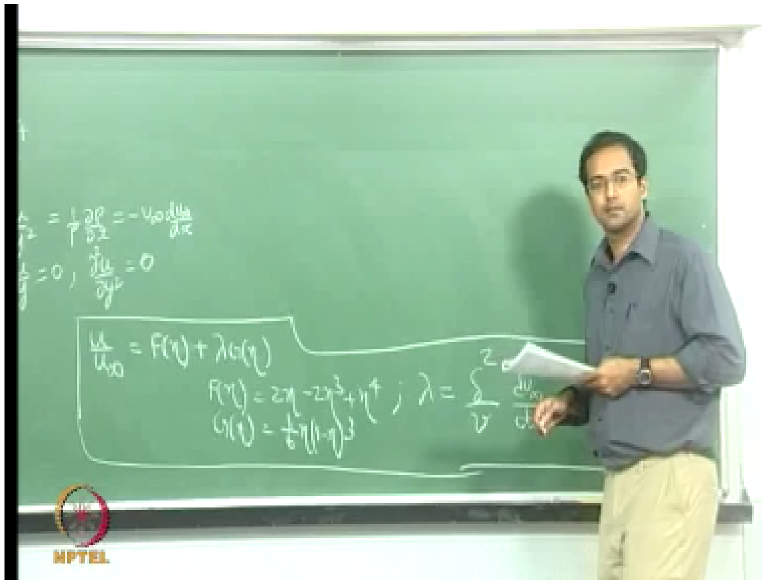

Was something like this he used a fourth order kwatick polynomial A + B H + C H ^{2} + D H ^{3}+ C H^4 to power four okay so where your H was defined basically yes Y upon Δ okay so he assumed a fourth order polynomial okay not a linear not the quadratic neither cubic okay so why he did it in fact so in order to satisfy what we call this important boundary conditions and when you include also the pressure gradient another boundary condition has to be satisfied okay.

We will see those following boundary conditions which have to be applied to find these coefficients so at Y = 0 so what is the boundary condition U = 0 that is the first boundary condition and at Y = Δ your U should be equal to your ensure two boundary conditions we have so we need three more okay so at y = 0 what else okay so D^2 U / DY^2 now in the case of Blazes solution we made this directly a 0 from the momentum equation okay so if you write the momentum equation at the wall so at the wall the momentum terms the inertial terms are 0 okay so you are friction term exactly balances your pressure gradient term unlike in the blushes clays case where this is 0 and therefore this is 0 okay.

So you can write your new D^2 U / DY^2 as 1 / P DP / DX so this is your condition and the wall and also we know that outside the boundary layer if you go out if you write the momentum equation there you do not have any frictional term and you do not have any V velocity you have only the velocity in the X direction okay that will be u ∞ D U ∞ / DX should be exactly = 1 - 1 / P DP / DX okay set Y going to ∞ this is at Y = 0 and we also have seen that in the boundary layer approximations your DP / DY = 0 therefore P is approximately the same anywhere therefore we can replace this term 1 / P DP / DX with U ∞ so this will be - U ∞ U ∞ / DX this is a very important boundary condition ok.

Now we have three so we need two more so now at y = Δ what is the other boundary condition this has to be anyway 0 because the profile velocity equals U ∞ and there is no friction there so outside the boundary layer so it has to satisfy the 0 gradient condition ok and finally the last condition will be so in terms of the order you see we started with the wall this is you equal to 0 and then so we do not know what is the gradient at the wall but we have applied a second order derivative condition at y = Δ you know = ∞ .

We know the first order derivative should be 0 and again the curvature also so this is, this is how all the five conditions are put and if you substitute these five conditions into the assumed profile the fourth order polynomial finally you will get the following profile so which is of the form u / u ∞ is = capital F of H + λ G of H now where your f of H is nothing but 2H - 2 H^3 the η^4 okay.

G of H is = 1 / 6 and finally your λ is nothing but a non-dimensional parameter which is the ratio of the pressure force to the viscous force it is written as ∇^2 / μ into D U ∞ / DX which can also be written as - L 2 / μ so I can write my D U ∞ / DX as 1 / P U ∞ into DP / DX so this will become μ into DP / DX okay so is it clear so I leave this as a exercise in fact I will give you as a part of the assignment where you can substitute these conditions and you can check if you will be finally reaching this particular profile okay.

Now the thing is this value this λ carries some meaning here okay so typically if you are looking at adverse pressure gradient flows so where your DP / DX is positive.

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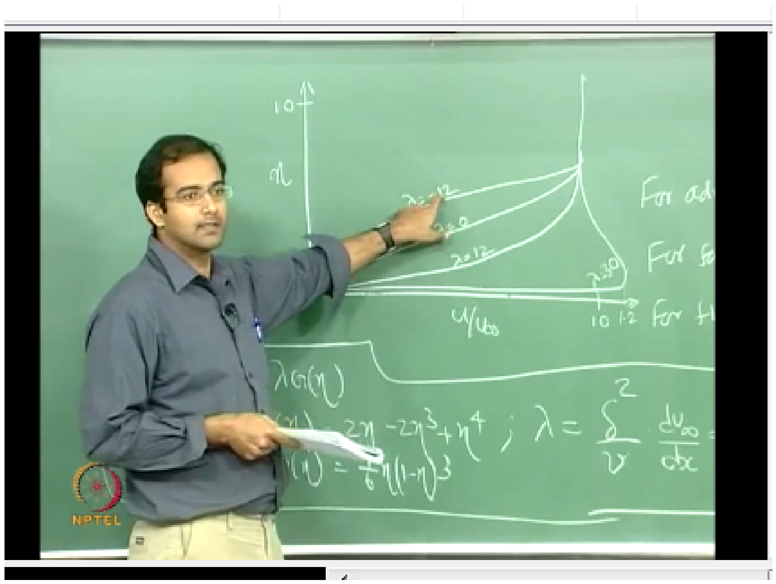

so for flows with adverse pressure gradient typically what should be a condition for λ your DP / DX is positive then this has to be negative okay for favorable pressure breaking so their DP / DX will be negative so λ should be greater than 0 and for flat plate case λ should be = 0 okay so in fact if you put $\lambda = 0$ this profile will become u / u ∞ is capital f of λ so which is nothing but the flat plate profile that you get if you assume the quadratic polynomial okay.

So that is the check now so this is how you get it now you can also plot the value of u / u ∞ as a function of H okay so H along the y axis so η varies from 0 & 1 and same thing with u / u ∞ okay so now what I am going to do is I am going to plot this for different values of λ now theoretically speaking I do not know what is the extent of λ okay λ could range from - ∞ to + ∞ but if I plot for λ some value like 30 what is observed is so this is this is like 1.2 .

So this is for $\lambda = 30$ so the value of U / u ∞ crosses 1 somewhere which is non-physical basically with this kind of this is an approximate profile so you don't know where it the profile will become physical or non-physical but if you use very large values of λ like 30 this becomes non-physical okay so now if you go to smaller values like for example $\lambda = 12$ now this becomes well-behaved it goes smoothly to 1 and then λ of 0 this is your flashes profile and λ of say -12 so this is your λ - 12 $\lambda = 0$ λ equal to 12 okay.

So now you could have it gone to various negative values but we are just plotting till here now the reason why we are stopping at - 12 is that you can check from the profile you can check the condition for separation okay so the condition for separation is what D U / DY at Y = 0 = 0 so if you put that condition and check the value of λ comes out to be - 12 that is the λ 4 criteria for separation so that is why we are stopping it 4 - 12 so B if you go below - 12 so the boundary layer would have separated and you cannot find solutions from the boundary layer equations.

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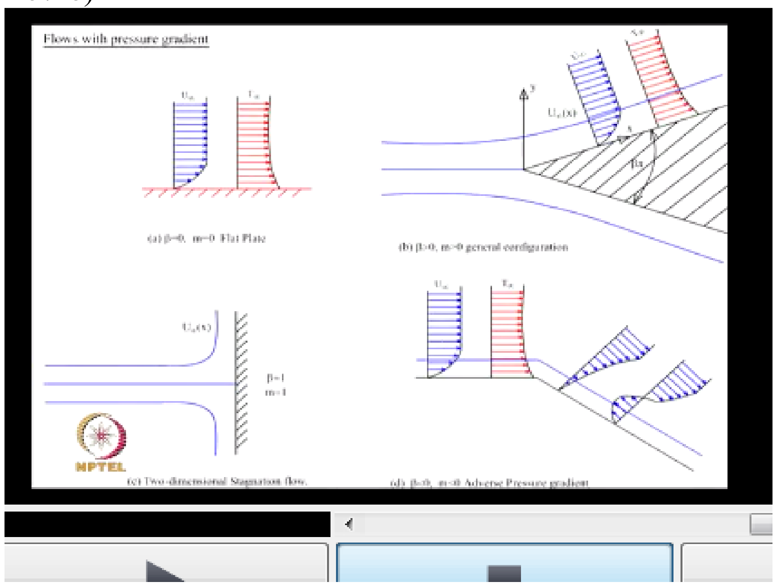

Okay so that is the lower cutoff and the upper cutoff is not thirty because it becomes non-physical so we are stopping from the value of 12 so the λ values that we are usually interested will range from - 12 to 12 okay - 12 that is the extreme case of separation and even when you have very favorable pressure gradients you should not have such large pressure gradients such that the profile becomes non-physical okay .

So this is the condition under which you know you are doing the solution so a condition of separation that is this you will check that now okay.

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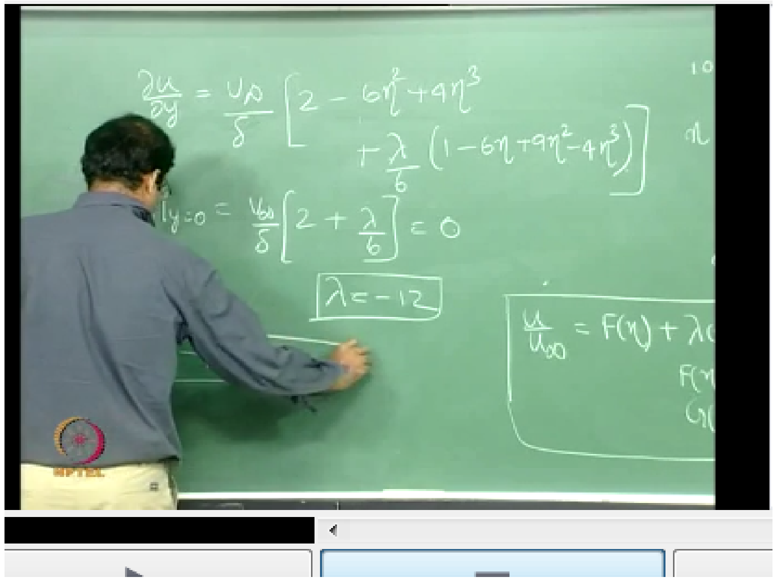

So let us check D U / DY so from this profile that will be what U ∞ into so this I can write as D U / D H into D H / D Y okay so my η is nothing but Y / Δ so D H / DY will be 1 / Δ so that will be this x so you have to differentiate F which will be 2 - 6 H^2 + 4 H^3 + λ into G H okay if you differentiate that you will get λ / 6 1 - 6 H + 9 H^2 - 4 meter ^{3} okay so now D U / DY at Y = 0 nothing but D U / DY at H = 0 here which is C U ∞ / Δ into all these terms get knocked off 2 + λ / 6 .

The condition for separation is this should be 0 therefore λ will be - 12 this is, this is why we are stopping there okay so we will restrict our values of λ to - between - 12 and 12 for most of the cases that we are interested okay so now so we have estimated the approximate profile so this is how pearl house ended he guest assumed aquatic profile and then he let us look at the values of λ because this is the separation case and the other one goes non-physical beyond that and then this profile is now substituted into the momentum integral equation okay.

So if you if you now substitute this so first we calculate the momentum and the displacement thicknesses from the profile.

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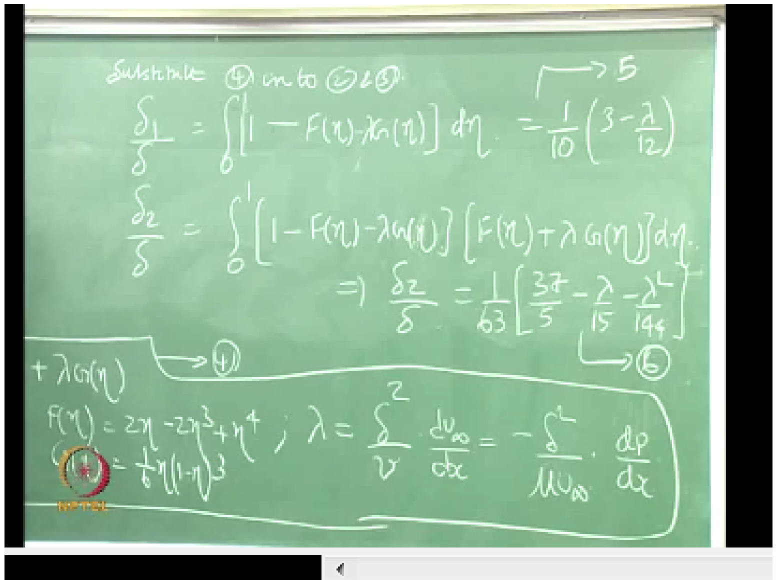

So first if I substitute okay let me call this profile as let me call this as number 1 okay this is number 2, 3 and this is profile is number 4 substitute 4 into 2 and 3 and calculate your δ1 and δ2 so we will get Δ 1 / Δ so what I am going to do I am transforming my variable Y in terms of H okay so this will be DY / D H into D H so DY / D η is nothing but Δ okay so therefore I can bring the Δ in the left hand side so this will be Δ 1 / Δ will be the limits will become now 0 to 1 into 1 - u / u ∞ which will be 1 - this profile right here which is F of H - G of λ into G of H.

Okay now if you substitute this profile and you integrate it out you will find that this comes out to be 1 / 10 into T - λ / 12 okay let this call let us call this as number 4 okay this is number 5 alright so similarly if you substitute into the momentum thickness and integrated this is 0 to 1 , 1 - F of - G λ your B η sorry into F of H + λ into G of H D H so if you integrate it out this gives Δ 2 / Δ as 1 / 63 37 / 5 - λ / 15 - λ^{2} / 144 so this I will call as equation number 6.

So what I am doing this I am just calculating estimating my an expression for the momentum thickness and the displacement thickness in terms of λ okay because now λ is a function of X strictly speaking if you look at a particular problem and if you define your X like this your ∞ is a function of X your λ is also a function of X so because your boundary layer thickness changes with X and so is your displacement thickness and momentum thickness so everything is a function of λ which is internal for internal function of X okay.

So for different way of positions as you keep marching from the stagnation point say somewhere so the value of λ keeps changing and accordingly the value of these different thicknesses will change okay so once we have estimated this we will now substitute this into the momentum integral one so before that we are going to slightly rewrite the momentum integral such that we will eliminate terms which are appearing in terms of Δ okay.

We will reconstruct the momentum integral so what we will do is we will.

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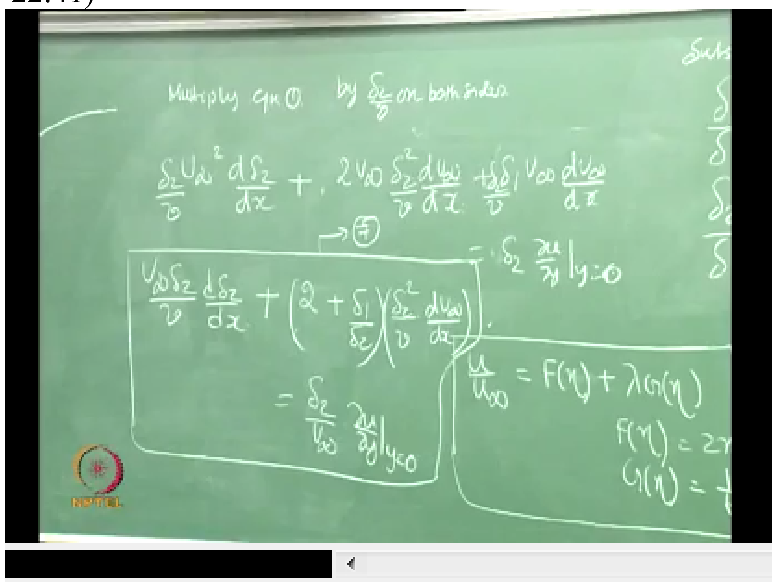

Multiply equation 1 / Δ to buy new on both sides and rewrite so this term can be split as you can write this as U ∞^2 D Δ 2 / DX + you can write this as 2 U ∞ into Δ 2 L U ∞ / DX + you have Δ 1 U ∞ be U ∞ / DX is = so if you multiply so Δ to my new Δ to buy new so this will become simply Δ to okay now if you rearrange this so what I am going to do I am dividing throughout / u ∞ okay so the first term here will be u ∞ Δ to / µ into D Δ to / DX + I can combine these two terms because D u ∞ / DX is common to both.

Okay the second and third terms and I can write this as 2 + Δ 1 / Δ 2 into Δ 2^2 / µ into D U ∞ / DX so I can combine these two terms in this way so the first term will be twice so I am dividing / U ∞ so twice Δ 2^2 / µ into D U ∞ / DX the second term the third term here will be so basically Δ 2 cancer so this is Δ 1 Δ 2 / µ into D U ∞ / DX okay it is just a little bit of rewriting on the right hand side I have Δ 2 / U ∞ into me you / okay now this is my modified slightly modified form of the momentum integral equation which I will number as number 7 okay.

Now the reason why we are rewriting this is now these are in terms of Δ 1 / Δ 2 okay and Δ 1 / Δ 2 will eliminate basically Δ okay and even terms like Δ 2^2 into D u ∞ / DX is actually can be written in terms of λ and this particular expression so all of them will in will not finally involve an expression for Δ okay so now I am going to introduce certain terms here just to simplify the equation I am going to say.

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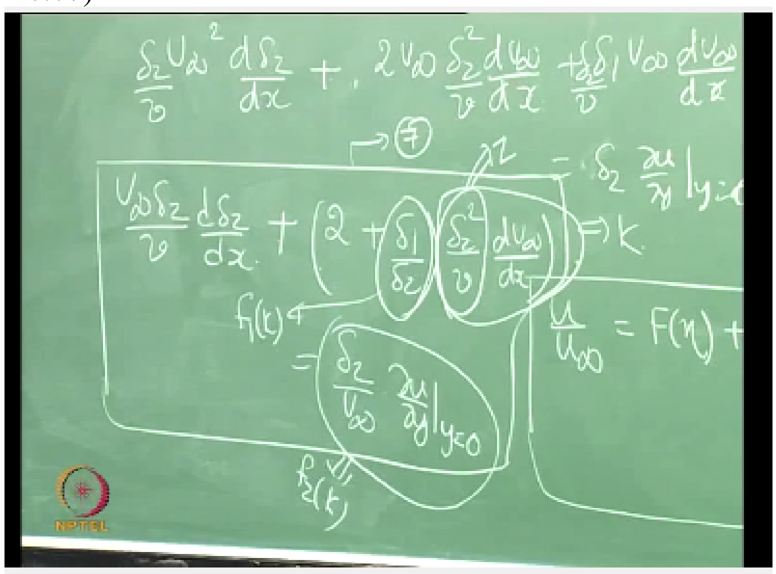

That $\Delta 2^2 / \mu$ I will call this as some parameter Z and this entire factor here as a parameter K okay so and since K is a function of λ because now K is a function of Δ - 2 D ∞ / DX which is nothing but a function of λ okay so K is a function of λ and I will I will express this ratio Δ 1 / Δ 2 as some function of K which is also a function of λ basically indirectly okay if I write f 1 as a function of K which is nothing but a function of λ and the term on the right hand side I will call this as another function f 2 which is a function of K which is nothing but a function of λ so therefore.

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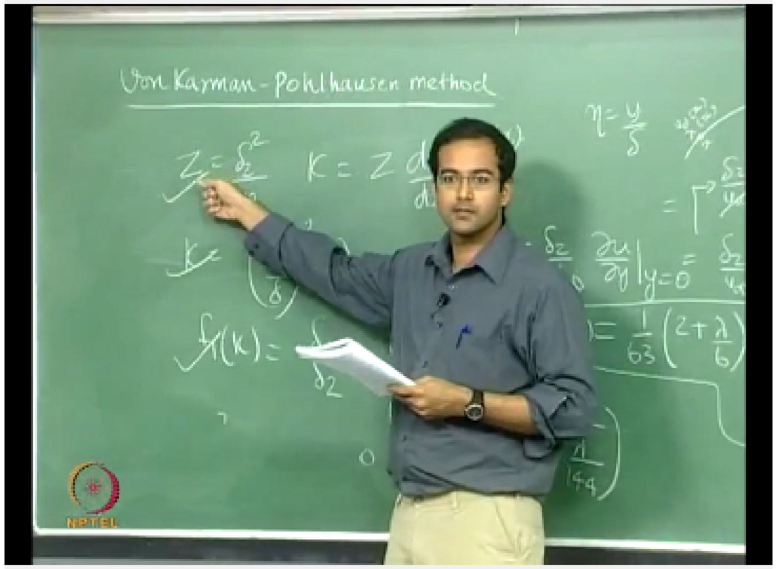

My Z is = ∇ 2^2 / μ and my K is nothing but Z into D u ∞ / DX and my K is = now I can rewrite this K now this K is Z into D U ∞ / DX I can rewrite this in terms of Δ 2 / Δ the whole 2 into λ because this D U ∞ / DX I can write this as λ into μ / Δ 2 okay so already this Z is Δ 2^2 / μ cancel so you have Δ 2 / Δ the whole 2 into λ so therefore I can express my K in terms of λ directly okay so once again you see now the K is in terms of Δ 2 / Δ okay .

And F 1 of K which I have introduced here is nothing but Δ 1 / Δ 2 okay now we have got expressions for Δ 1 / Δ 2 / Δ so I am just going to take the ratio of these two okay so this comes out as 63 into 3 - λ / 12 / this is 15 - λ 2 / 144 okay so this is just the ratio of Δ 1 / Δ 2 okay now my F2 of K is nothing but Δ 2 / u ∞ into DU / DY at Y = 0 this is my RHS term so even this can be written so D U / DY at Y = 0 we have already determined that was coming out as 2 + λ / 6 if you remember okay .

So this is this is nothing but Δ 2 / U ∞ into D u / D H okay into so in fact this is the D U / D H into at H = 0 into D H / D Y okay so DU / we have already the polynomial for you if you put that you will get 1 / 6 2 3 2 + λ / 6 into 37 / 5 - λ / 15 - λ 2 / 144 so this will be the expression for F 2 of K okay so this D U / D H = 0 we have already derived that is two + λ / six okay we have we have differentiated the profile in the previous step you can check that we have differentiated the profile and at y = 0 that council comes out as U ∞ / Δ into two + λ / 6 and U ∞ U ∞ there cancels so this becomes ratio of Δ two / Δ okay so this which is nothing but this expression right here 1 / 63 okay.

If you are having problem I will just maybe expand it so this will be Δ 2 / U ∞ into U ∞ / Δ into 2 + λ / 6 okay so this entire term right here is this okay this is from the previous step when we calculated D U / D H equal to 0 into data / device this entire expression okay this is Multiplied so U ∞ this cancels now this is 2 + λ / 6 into D Δ 2 / Δ so we have already derived an expression for Δ 2 / Δ in terms of λ so this gives you the final expression for so now we have expressed everything in terms of λ okay so that is a function of $\Delta 2^2$ and Δ 2 is a function of λ K is a function of λ f 1 is a function of λ f 2 is a function of λ okay.

So now that we have everything so we can substitute these expressions into the momentum integral equation of the form 7 and probably as you might have guessed by now we cannot solve this equation by hand okay so this is a tedious expression so we will once again how to use the numerical method to solve it so I will just cast it in the final form so I can I can express I can write my Δ - D Δ 2 / DX in terms of Z ok so I have Z is $\Delta 2^2$ / μ therefore DZ / DX will be twice Δ 2 into D Δ 2 / DX ok so this will be half of what DZ / DX okay.

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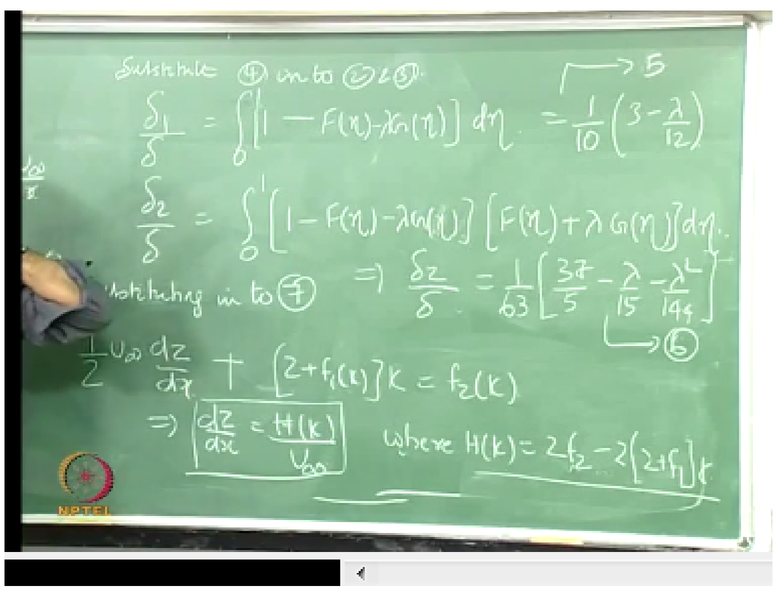

So substituting into equation number seven I can write this as half of U ∞ into DZ / DX okay so I am writing this in terms of Z here and the other terms we cannot do anything they are just there as it is algebraic terms so this will be 2 + F1 okay 2 + F1 of K multiplied by K this entire terms K on the right hand side you have F2 of K okay now K is a function of λ so therefore this can be written as DZ / DX is = H of K / U ∞ where my H of K is nothing but what 2 F 2 - 2 times 2 + F 1 into K okay.

So this is the final ordinary differential equation that I have to solve okay in order to get the value of Z and now Z is nothing but Δ 2 which is nothing but the momentum thickness okay so once you get your momentum thickness from this expression you know for a particular value of Δ 2 you can calculate the corresponding value of λ and therefore you can calculate your Δ 1 and also your Δ okay so this is how this is how it goes you know you have to solve this OD for different values of Z and you know now Z can be expressed in terms of λ okay .

So for that particular value of Z you have to solve for λ and then you calculate the other expressions for displacement thickness and the boundary layer thickness so this is how we have to numerically solve this equation so to just give an example in fact if you if you just happen to see this particular equation this H of K can be predetermined before it can be solved and kept as a table because it is purely a function of K which is a function of λ so for different values of λ between - 12 and 12 you can get all these functions right.

You can calculate now this is a function of what F 2 F 1 and K okay so you can get expressions for K F 2 F 1 everything as a function of λ put it in a nice table and therefore you can determine H of K directly okay so now this can be arranged as a table now every time that you solve for new value of Z you put the corresponding value of H of K corresponding to that value of λ okay.

And then get the new value of Z and from there you return my new Δ and therefore you determine the new value of λ so from the lookup table you can keep on taking the correct for current value of H of K okay so in fact all this had been tabulated by Holstring Bohlen and if you happen to look at the book by boundary layer theory by schlickking so he has mentioned he has tabulated all these values know F 1 F 2 K H of K as a function of λ ok this has been table in fact you yourself tend to that as a very nice exercise you know this is not difficult.

You take values of λ between - 2l and 12 calculate all these parameters we just create a table and once you do that every time you solve for new value of Z and you calculate new value of λ you have to simply take the value of H corresponding to that λ that is it okay so like that you keep marching so this is also a marching problem you start from some point maybe the stagnation point and from there you keep marching for different values of X so for each value of x you get a new value of Z you new value of Δ 2 Δ 1 Δ like that you keep marching till separation okay.

So how do you know whether it is separated so each value of Z you now calculate the new value of λ so λ comes out to be - 12 that means that at that point the flow has separated okay so this can be applied to any kind of pressure gradients you know adverse or favorable for favorable it is restricted to a maximum value of $\lambda = 12$ okay for adverse pressure gradient it is restricted up to the point of separation that is up to $\lambda = -21$.

So using this let us quickly apply this to a small case where it is flow past a circular cylinder and we will see how we can use this algorithm to calculate all the parameters okay. so I am I am just going to use this final expression here.

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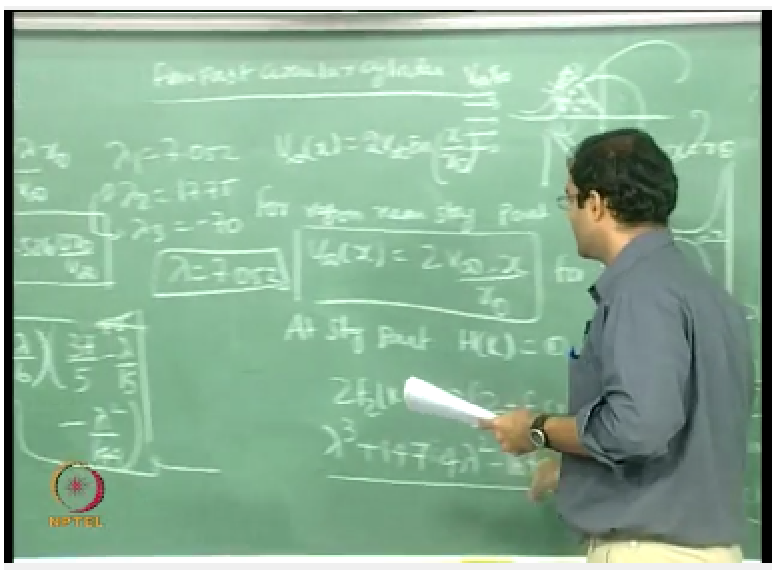

So flow past circular cylinder so this is your cylinder with radius let us call R or not and θ therefore the coordinate that we are talking about X this is the is basically the angular coordinate the space swept by the sector that is nothing but R θ okay and when it is approaching the cylinder the flow has a velocity a constant velocity V ∞ T ∞ now once it comes encounters the stagnation and starts flowing past this then this becomes the free stream velocity which is a function of X okay.

So for this particular problem if you are looking at a region close to the stagnation point how are we going to find the solution to the stagnation point problem so once again in the similarity solution we did this okay we took the stagnation flow for the Falkner Skan solution and we have transformed that to a cylindrical coordinate system and that, that gave us the solution for the stagnation region skin friction coefficient as well as the nasal number.

So here you similar way we will assume a profile from the invades potential flow which is nothing but X Y or not okay so at X is = 0 so this becomes 0 and at X is = R naught / 2 so this becomes exactly = twice of U ∞ okay so for this particular case we are going to look at for the region close to stagnation point so I can replace my sign X / R 0 as X / R 0 you know for small values of X / R 0 so therefore u ∞ of X in that particular region will be 2 V ∞ into X / R 0.

This is valid only close to the stagnation region where your X / R naught is very small so the condition is X / R 0 is quite small now the velocity distribution we have already determined and we have also determined the final expression for DZ / DX now if you if you are smart enough and it quickly observe now you are U ∞ now here is a function of X and at stagnation point your U ∞ is 0 okay so for this expression to be finite okay if U ∞ goes to 0 that means DZ / DX goes to 0 at stagnation point to avoid that H of K should go to 0 okay.

So therefore at the stagnation point the condition is for the circular cylinder case H of K has to be 0 so we already have the expression for H of K which is nothing but this has to be 0 that means 2 F2 of K - 2 into 2 + F 1 of K = 0 so if you substitute for F 1 F 2 and K in terms of λ you will get a nice cubic polynomial into a cubic algebraic equation in terms of λ this will be 147 point 4 λ^2 so this is the resulting algebraic expression that you will get .

If you substitute for F 1 F 2 so if you solve this algebraic equation you will get 3 roots because it is a cubic polynomial you will get the value of λ one of the roots will be 7.052 the second root will be 17.75 third root will be - 70 okay since our region of interest is – 2 , 12 to 12 will ignore these two roots we will take only the root which gives you seven point 0 five two therefore the value of seven λ = 7.052 corresponds to the stagnation point okay.

This is the value of λ stagnation point please remember that at the stagnation point also there is a certain value of the boundary layer thickness boundary layer thickness is not 0 because here the flow comes like this and it divides like this if you remember the 2D stagnation flow for the

Falkner Skan which problem where your M = 1 okay the flow comes like these hundred bifurcates okay and here the boundary layer thickness is not 0 okay.

At the stagnation point okay I am corresponding to that you have a finite value of λ which is seven point 0 five two okay so this value of λ corresponds to a particular value of Δ in fact okay so you can calculate the value of boundary layer thickness for this particular value of λ now how do we do that how do we calculate the value of Δ so we already have expressions for λ which is nothing but ∇ 2 / μ into μ u ∞ / DX you remember this is how we defined our λ okay.

So once you have your λ you can directly calculate your Δ from here okay because you know for this particular problem what is the free stream velocity gradient okay so therefore your Δ 2 will be new λ by which will be nothing but new λ or this problem it will be 2 V ∞ R 0 okay so this is your profile alright so therefore your Δ 2 will be nothing but if you substitute the value of λ 7.052 in the stagnation region your Δ 2 will be 3.526 into V R 0 / u ∞ okay.
So if you multiply and divided by R 0 you can write this in terms of the Reynolds number now U ∞ R 0 / μ so this becomes the Reynolds number so you can get an expression for Δ2 to Δ / R naught in terms of the Reynolds number okay so this is how you have you get the expression for Δ and now once you get it this is valid only for the stagnation point or the region which is very close to the stagnation point.

Now if you are interested in calculating the flow past the cylinder from the stagnation point all the way where it separates okay so it may separate somewhere here for example maybe at 75 degrees okay so till separation you can use this technique to mask our word and calculate the value of boundary layer thickness the displacement thickness momentum thickness also you can calculate the separation point okay.

The location where exactly it separates okay for that you have to use the complete velocity profile okay so use the full velocity profile and you have to solve this ODI numerical so you start from the stagnation point at the stagnation point you know the value of Δ from this expression okay so you know the corresponding value of Δ 2 and therefore you know the value of Z okay.

So at the stagnation point you know the value of λ you know the value of Δ so you know you can calculate the value of Δ 2 okay therefore you can calculate the value of Z so that is their initial condition ok you start from there you march from the stagnation point you keep solving this OD / Euler's method you can get the value of Z at each of those locations for example you can discretized this into 100 points for example okay I am not putting 100 points I am just giving a rough estimation.

So for each of these points you can calculate Z and on the right hand side this is the value of Z for the previous point okay so H of K corresponding to the previous point so every time you calculate Z now you can you have to calculate the new value of λ okay so for that we can use the expression that is = $\Delta 2^2$ / μ and okay so let me erase this okay.

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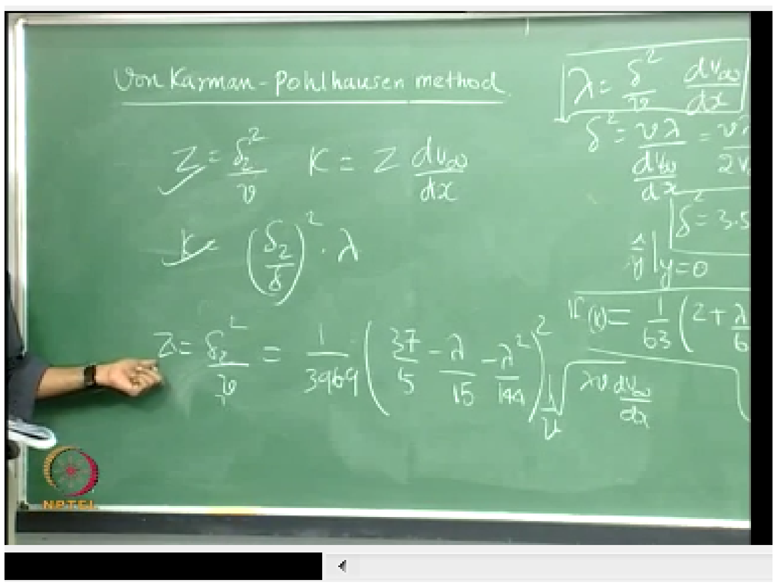

So Δ to 2 I am substituting from here okay so this is nothing but this into Δ 2 and Δ 2 is nothing but this particular expression right here okay so I am combining this expression where I can calculate Δ and put it into this I can calculate my Δ to and from there I can determine my relation between λ and Z so for a given problem this is all fixed okay now the value of ∇ λ is to be determined okay.

So for a given value of Z that you solve so this equation is an algebraic equation now this is not so straightforward to solve by hand you have to solve it iteratively again you can use some numerical method like Newton's method or bisection method okay solve this iteratively for the latest value of λ at that value of Z so now this will become from this value of λ you are already prepared the lookup table you can calculate the latest value of H and now you can go to the next point get the value of Z.

And again keep doing it so till you reach a point where λ reaches - 12 okay at that point you stop okay so this is how you determine where exactly the separation happens and also the corresponding values of all the three thicknesses boundary layer displacement and momentum okay so with that I think we will stop here and tomorrow will continue some this point to the heat transfer problem.

Integral method for flows with pressure gradient
(Von karman –Pohlhausen method)
End of Lecture 20

Next: Integral method with pressure gradient:
Heat transfer

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