

**Indian institute of technology madras**  
**NPTEL**  
**National Programme on Technology Enhanced Learning**  
**Video lectures on**  
**Convective Heat Transfer**  
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**Lecture 19**  
**Integral method of laminar external thermal**  
**Boundary layer over isothermal surface**

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Approximation (integral method)

$$\frac{d}{dx} \left[ U_{\infty}^2 \int_0^{\delta} \left(1 - \frac{u}{U_{\infty}}\right) \frac{u}{U_{\infty}} dy \right] + U_{\infty} \frac{dU_{\infty}}{dx} \int_0^{\delta} \left(1 - \frac{u}{U_{\infty}}\right) dy = \nu \frac{\partial T}{\partial y} \Big|_{y=0}$$

MOM thickness
displacement thickness

$$\frac{d}{dx} \int_0^{\delta} (T_0 - T) u dy = \alpha \frac{\partial T}{\partial y} \Big|_{y=0}$$

NPTEL

Hello everyone today we will look at the approximate methods of solution so far we have been looking at triggered solutions / using similarity methods and solving the resulting similarity equation / the odd / the shooting technique and other numerical methods now we will introduce a approximate way of a getting solution to different configurations may be flow without pressure gradient like the flat plate case are including the pressure gradient for all of these cases we can get approximate solutions / making suitable approximations to the velocity and temperature profile so in the last class we had derived the integral form of the momentum and the energy equations okay so the generic form including the pressure gradient and finally.

We expressed the momentum integral equation in this particular form so where this includes momentum thickness which is nothing but this is the momentum integral sitting inside this and

the displacement thickness which is which is the displacement integral here okay so if you consider flat plate case where your free stream velocity does not change with the position you can neglect this complete term right here and therefore this is a more familiar form which you

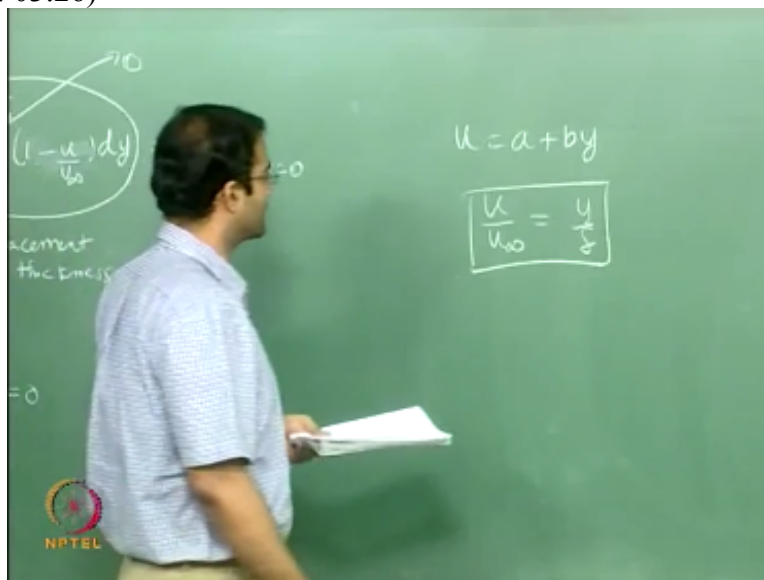
have been using in your earlier heat transfer class to solve with approximate methods so you will have only the momentum thickness so this is the rate of change of momentum thickness.

Along the axial direction will balance the shear stress okay so this is wall shear stress basically so this is basically the momentum integral equation very very important equation and why we are using the integral form here is that we guess some approximations for velocity in terms of some polynomials may be linear or quadratic cubic fourth order whatever we will substitute that into this and we will integrate it out over the entire boundary layer thickness.

And we will find an expression for calculating the boundary layer thickness  $\delta$  from here the same way we can also integrate the energy equation across the boundary layer and this is the resulting energy integral equation that you get and once again we can make an approximation guess a value of  $s$  a profile for temperature whichever profile that you want to guess substitute here integrate it out and then you get a resulting equation in terms of the thermal boundary layer thickness  $\delta_T$ .

So with that we will get an expression for  $\delta_T$  therefore with that we can use that to calculate the wall shear stress skin friction coefficient heat transfer coefficient and your result number okay so this is how our approach will be okay and yesterday we did this for the flow problem we.

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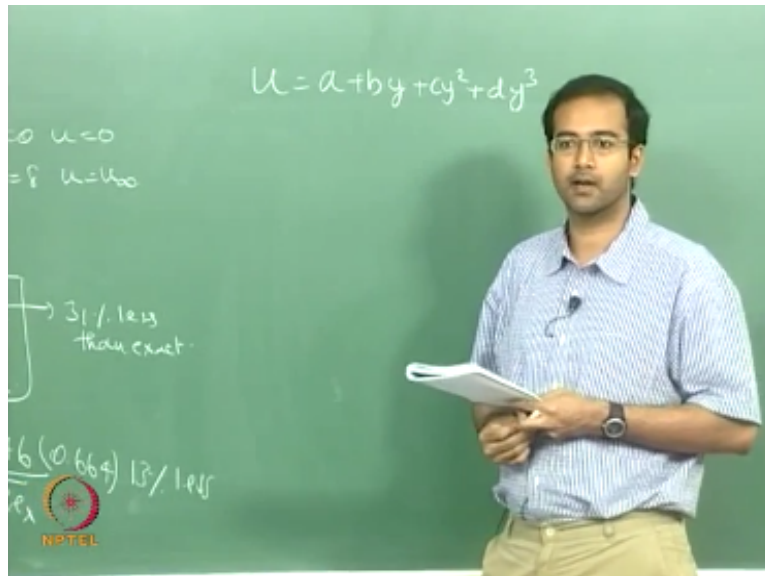
Guessed initially a linear profile okay so a linear profile of the form  $U = a + B Y$  and finally when we plug in the profile comes out very simply as  $u / u_{\infty} = Y / \delta$  okay so we were putting boundary conditions to satisfy this particular profile and determine the constants  $a$  and  $B$  so that is at  $y = 0$   $u = 0$  and at  $y = \delta$   $u = \infty$  okay and there and when we when we use the linear profile so that means we have assumed we assume that now the variation in the boundary layer is completely like this okay which is totally unlike the solution what Prandtl has got.

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$$\frac{\delta}{x} = \frac{3.47}{\sqrt{Re_x}} \rightarrow 31\% \text{ less than exact.}$$
$$c_{fx} = \frac{0.576}{\sqrt{Re_x}} (0.664) \text{ 13\% less}$$

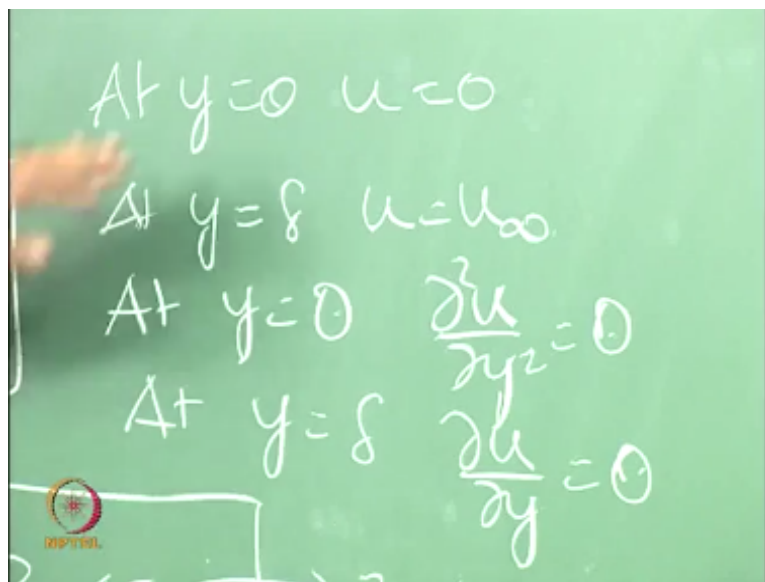
Okay this is your  $\delta$  and this varies from 0 to  $u_\infty$  okay but even in spite of this big assumption when you substitute this into the momentum integral and determine an expression for  $\delta$  we got the expression for  $\delta/x$  to be  $3.47 / \sqrt{Re_x}$  okay compare this to the Blasius solution where  $\delta/x$  is  $5 / \sqrt{Re_x}$  okay still this is a reasonable way to get started to get an order of magnitude of the boundary layer thickness okay so this is about 31% less than the exact solution and so was your local variation of skin friction coefficient which came out to  $0.576 / \sqrt{Re_x}$  Reynolds number the actual value was 0.664 okay so this is about 13% > the exact solution okay so then what we did we improved the profile assumption from a linear profile to a cubic profile.

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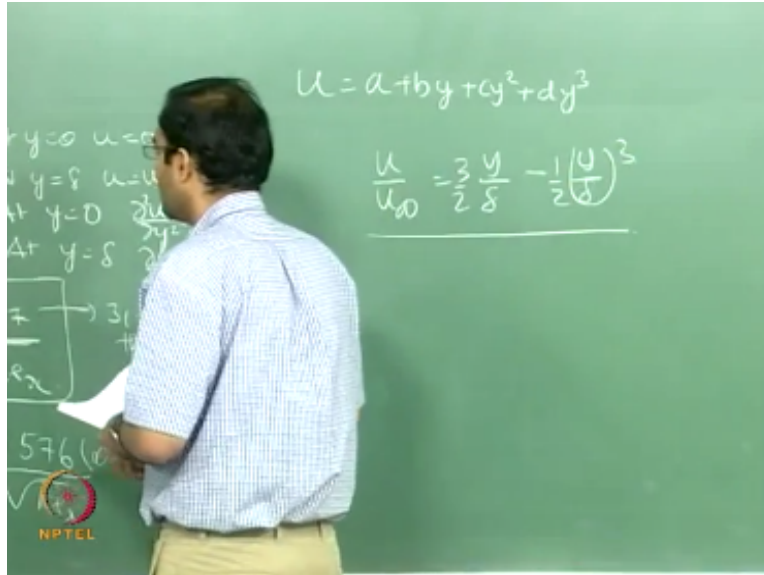
Okay so we assumed a profile something like  $a + B y + C y^2 + d y^3$  so now that is a higher order polynomial we need to satisfy more boundary conditions to determine these additional constants and apart from these two boundary conditions they say these are the most important boundary conditions in terms of the order of relevance okay so you need to first satisfy no slip at the wall and the condition at  $y = \delta$ .

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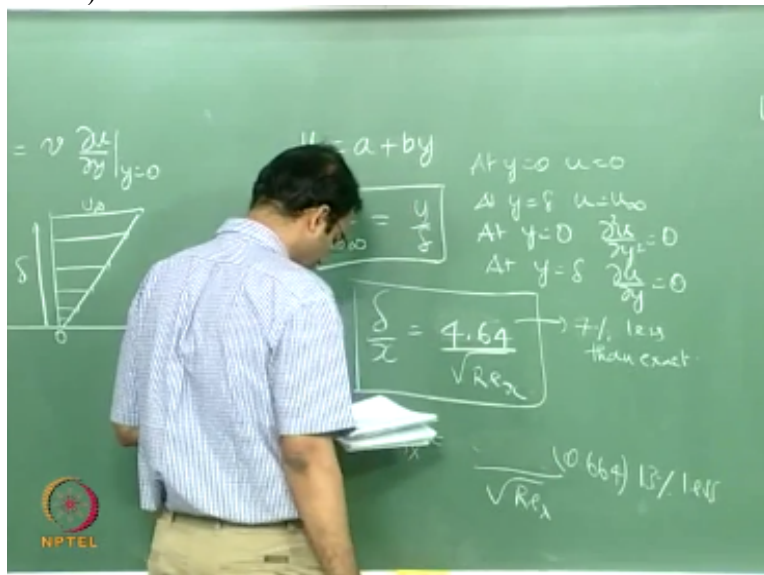
To  $\delta$  and apart from that we have also used that at  $y = 0$   $\frac{du}{dy} = 0$  which comes from the momentum equation at the wall okay also at  $y = \delta$  the condition  $\frac{du}{dy} = 0$  has to be satisfied okay so these four conditions have to be satisfied in this particular order and with that you will get a profile which is  $u / u_{\infty} = \frac{y}{\delta} - \frac{y^3}{3/2 \delta^3} - \frac{1}{2} \frac{y}{\delta}$ .

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Right this was what we saw yesterday and if you plug in this velocity profile now instead of the linear profile into the momentum integral integrate it out across the boundary layer thickness we found that the resulting boundary layer thickness is actually improving a great deal you know from 3.47 here we finally reach a value which is 4 point anybody remember 4.64 okay so which is 7%.

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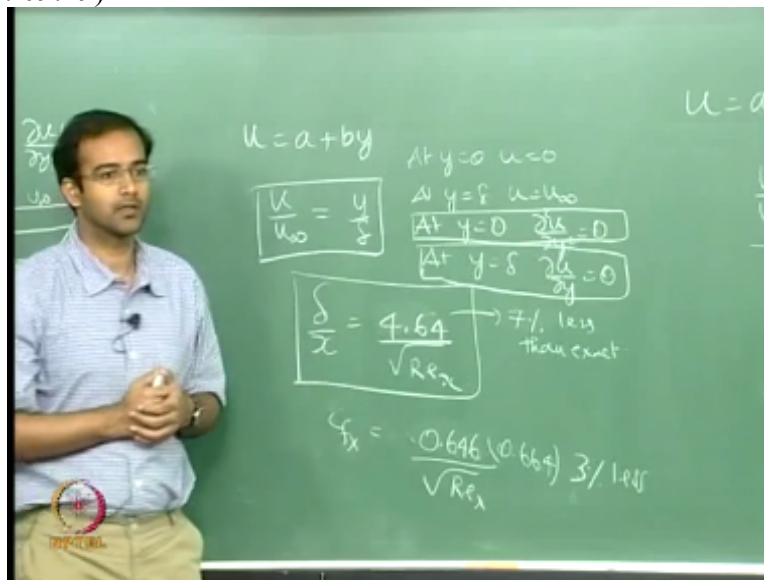
Less than the exact but it is a very close very close to the exact solution okay the same way with the skin friction coefficient if you calculate this wall shear stress and therefore the non-dimensional skin friction coefficient this value comes out to be point 646 okay which is also very close 3% less and the actual okay therefore you can see the higher order polynomial is definitely better than using a linear polynomial and.

You can also see that it is important to satisfy all these four boundary conditions if you want to get a reasonably accurate solution okay so especially at the wall you have to also satisfy the curvature = 0 and at  $y = \delta$   $D u / D Y$  has to be 0 shear stress has to be 0 at the interface so therefore these are the important things now the question you may ask is whether if you go to fourth order or fifth order polynomial if you get can we get.

A more closer solution it need not be ok because these are the four most important boundary conditions to be satisfied and if you are introducing additional constants and polynomial so you have to introduce higher order derivatives to be 0 okay nothing more than that for example if you introduce an additional constant you have to say  $d \sqrt{u} / dy \sqrt{u} = 0$  and further than  $d^3 u / dy^3$  at  $y = 0$ .

0 and so on okay so they are relatively less important and therefore you will find you will not get a significant improvement beyond this in fact using quadratic polynomial will in fact even deteriorate the solution okay that is that is also because of the fact in quadratic case you use this.

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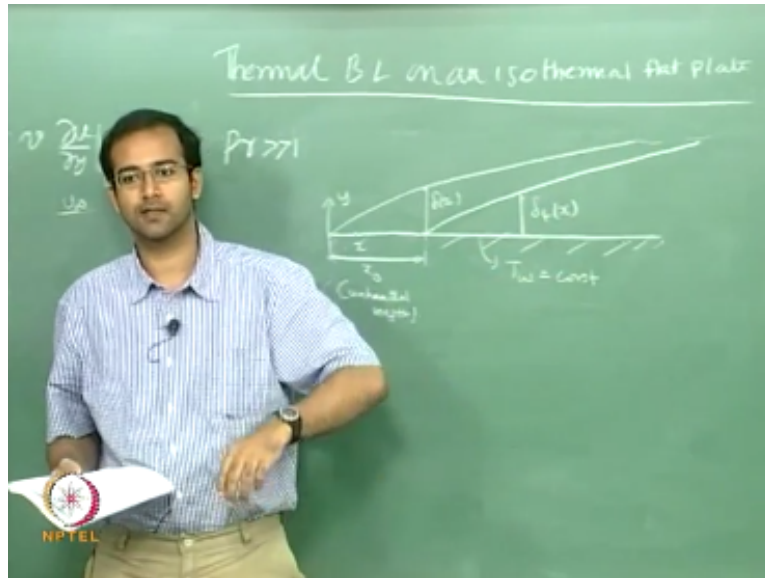


Boundary condition at  $y = \delta$  okay rather than using this boundary condition because you have only up to quadratic term you cannot take derivative up to the second order.

You have to take up the first order of plate so therefore you do this before applying this and this is a very important boundary condition okay so to be satisfied so which is not applied in the second-order polynomial case and therefore the second order polynomial will not be as accurate may be I in my opinion I think if you do the calculations it may come out to be somewhat even worse than the linear polynomial okay and definitely not when you anyway close.

To the third order polynomial okay so this this is what we did yesterday with the flow solution so today we will look at such kind of approximations for a temperature for a flat plate case okay so so now we will look at the thermal boundary layer on an isothermal flat plate so apart from the.

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Development of the velocity boundary layer now you also maintain a temperature fixed wall temperature which is uniform and this is your coordinate  $x$  and  $y$  so your velocity boundary layer grows and your boundary layer thickness keeps increasing with the axial location now when you apply this  $T$  d  $T$  wall is constant it it need not mean that you are heating all the way from the leading edge okay so you can for as a more generic case you can actually not heat the initial portion.

Okay so this could be what is called the unheated starting length okay and you can start the heating after some certain distance so that distance will be  $X_0$  okay so this  $X_0$  is your unheated length okay in order to get a generic solution you know for the case where your  $X_0 = 0$  the plate is completely heated right from the beginning okay now in this case the boundary the velocity boundary layer may grow all the way from the leading edge but the thermal boundary.

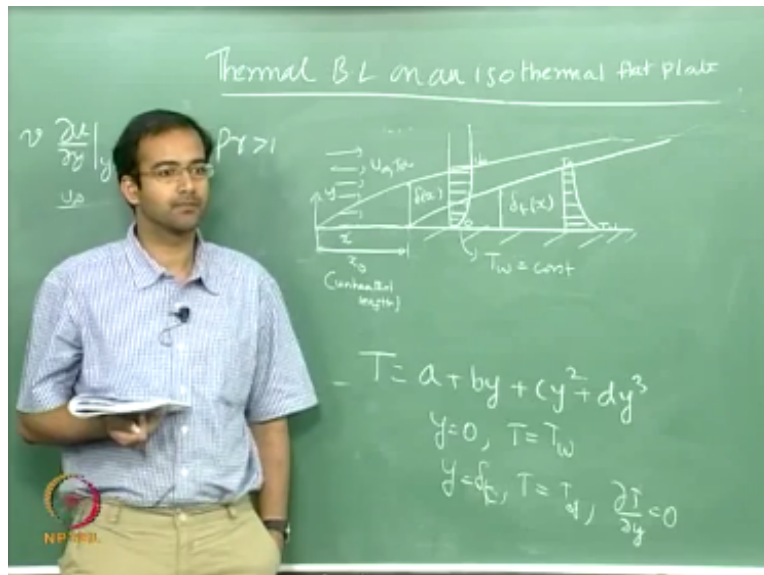
Layer will not grow unless it sees the wall temperature okay so therefore it will start from here and it will grow okay so this is your  $\delta$   $T$  and function of  $X$  okay so depending on the case whether your Prandtl number is  $> 1$  or approximately 1 or much  $< 1$  once it starts growing so this boundary layer may over take the momentum boundary layer thickness okay so if your Prandtl number is much  $> 1$  so in that case this will be lower than this okay.

The momentum boundary layer thickness will be growing at a much faster pace so this will not be able to catch up okay now for the case when your Prandtl number is much  $< 1$  so then

you will find that this will rapidly overtake and finally somewhere here your thermal boundary layer thickness will be greater than exceeding the momentum boundary layer thickness beyond a certain position okay if your Prandtl number is = one so they will be both growing at the same rate however this is already shifted here .

So once again this will never be able to meet meet up to the growth of the momentum boundary layer okay so these are certain things that you should know very clearly for sure which for which Prandtl number case how the rates of the growth of the either boundary layers appear okay so right now we will consider a case where your Prandtl number is > one okay we can also use the integral method for the other case where your Prandtl number is much < one but we will start with this particular case so let us now also assume directly a third degree polynomial for temperature something.

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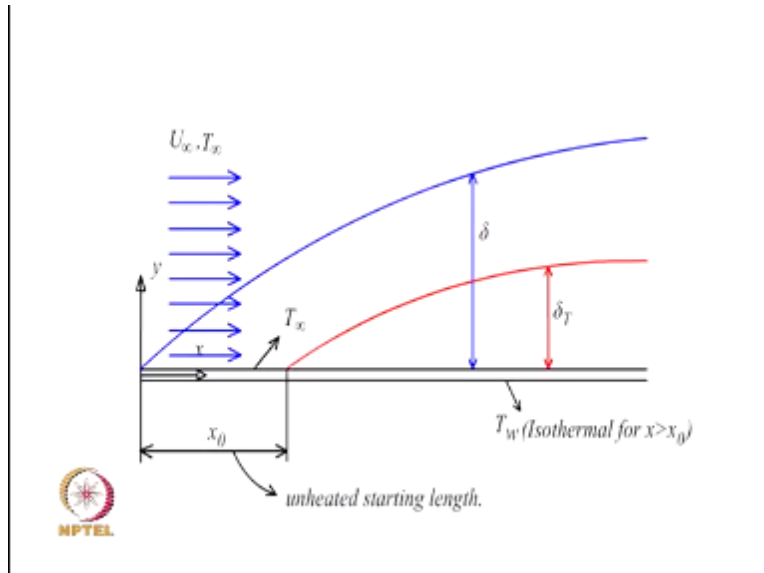
This sort now you have to tell me which are the boundary conditions that can be applied to satisfy this okay  $y = 0$   $T = T_w$  okay this is still dimensional okay I am not going to non-dimensional okay and then  $y = \delta$   $T = T_\infty$  okay so  $T = T_\infty$  because if you look at suppose you are assuming that your  $T_w$  is > your  $T_\infty$  so this is your  $T_\infty$  this is your  $T_w$  okay so similarly  $T_\infty$   $T_\infty$  the same way with the velocity boundary layer also so this is your  $u_\infty$  this is 0 so we need two

more boundary conditions okay so in  $y = \delta$   $\frac{\partial T}{\partial y} = 0$  so you can see that once this reaches  $T_\infty$  after that the gradient will be 0 right after the thermal boundary layer so at the interface you have to satisfy this interface continuity condition continuity in temperature and slope should be continuous okay and then we need one more heat flux at the wall.



But we do not know this is the case where we have constant wall temperature yeah the same way if you look at the momentum the same way that you did for momentum if you look at the energy equation if you do not include the viscous dissipation term so at wall these are 0 therefore  $D \sqrt{t} / dy \sqrt{}$  has to be 0.

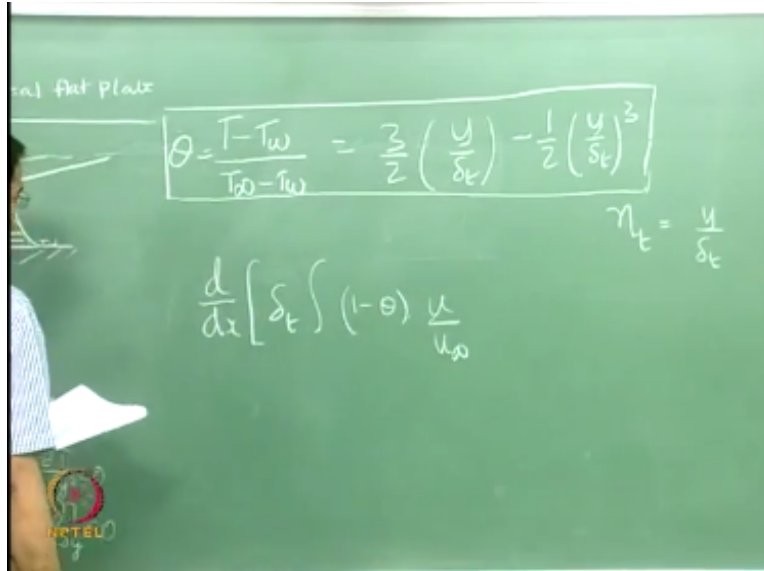
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Okay you so you can you can see that you have four boundary conditions in 4 most important boundary conditions and if you substitute them into the profile you should be getting a profile something like  $t - t_w / t_\infty - T_w$  will be  $3/2 (y / \delta t) - 1/2 (Y / \delta T)$  the whole cube now we can define this as my non-dimensional temperature  $\theta$  which I also defined in the similarity solution so this is the resulting non-dimensional temperature profile.

Just like I have a non dimensional velocity profile exactly the same expression only in terms rather than boundary layer thickness here you have the thermal boundary layer thickness  $\delta t$  okay the same cubic approximation so the next step will be to substitute this into the energy integral and do the same procedure that we did for the momentum integral just a little bit I am going to rewrite the energy integral in terms of non-dimensional temperature  $\theta$  ok so right now this is my T.

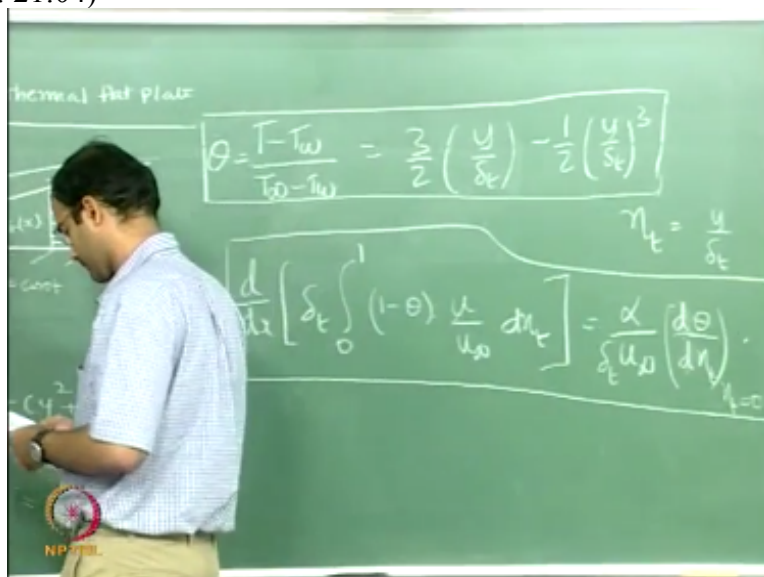
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$\infty - T$  I can express this in terms of  $\theta$  as  $d / DX 1 - \theta$  also I can multiply and divide /  $u \infty$  so  $u / u \infty$  and I can transform my variables from  $Y$  space to  $h$  space where I am going to define a variable  $H$  subscript  $t$  as  $y / \delta T$  okay so this is not the similarity variable okay it is just some notation that I am giving here okay coincidentally this is also for the similarity solution the similarity variable exactly related as  $Y / \delta T$ .

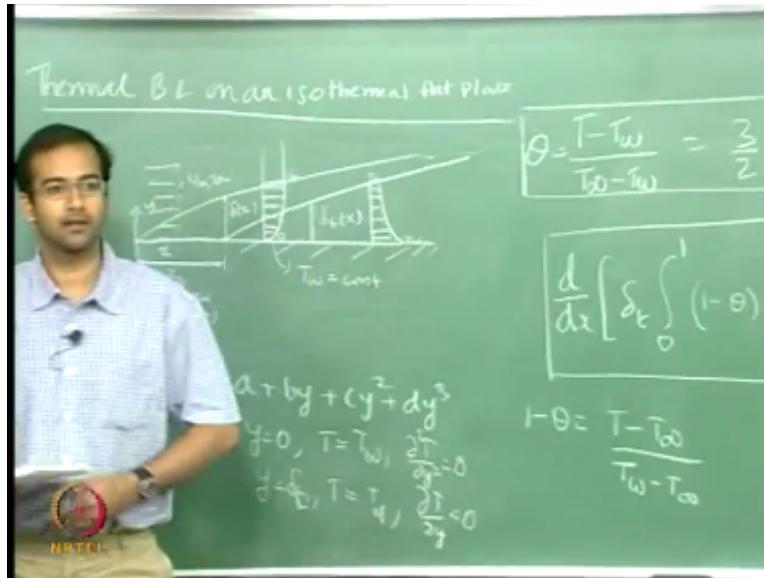
So I am just transforming in terms of  $h$  so this will be  $dy / D h$  into  $D h$  so  $dy / D h$  is nothing but your  $\delta T$  which will come out and the limits of the integral  $0$  to  $\delta T$  will now become  $0$  to  $1$  so you have  $D h T$  all right on the right hand side of course you multiplied and divided /  $u \infty$  so that I will take it to the denominator on the right hand side this will become  $\alpha / u \infty$  and  $DT / dy$  I will write in terms of  $\theta$  and.

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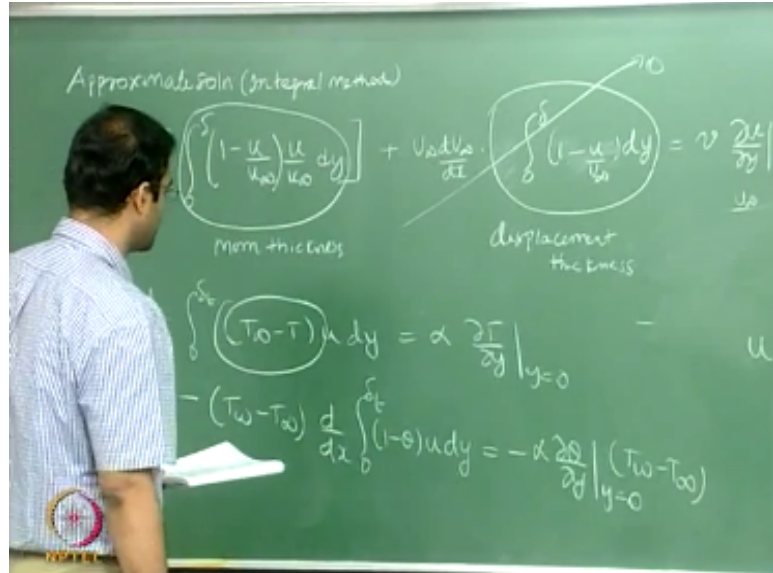
H so this I can substitute in terms of  $\theta$  so this will be  $D\theta / Dh$  into  $Dh / dy$  okay so  $D\theta / Dh$  okay so here I have to be yeah I think it is fine so a  $h = 0$  this is  $h = 0$  into  $Dh / dy$  which is again  $1 / \delta T$  I will keep the  $\delta T$  here ok so this is the non-dimensional form of the energy integral let me call this is number 1ok everybody is clear ok so so now this is 1 so  $1 - \theta$  will be what  $t - T$ .

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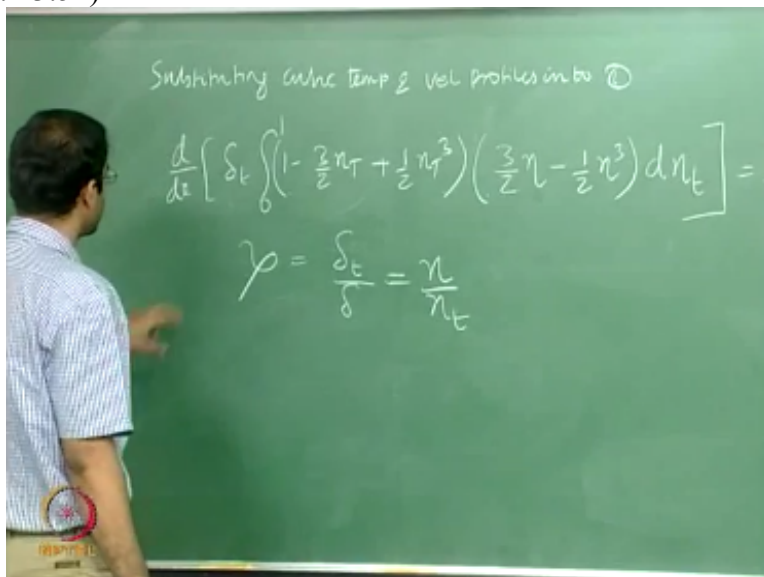
$\infty / T_w / T_\infty - T_\infty$  is that right so I can I can just substitute for  $T_\infty - T$  okay, so that is  $1 - \theta$  and on this side  $DT / dy$  should be  $- D\theta / dy$  ok so  $\delta T$  cancels on both sides  $d w$  is a constant okay so is that right see okay so pre  $\infty - T$  now I can write it as  $\delta T (1 - \theta)$  into  $T_\infty - T$  okay so so  $T_w$  and  $T_\infty$  they are constants so  $\delta T (1 - \theta) D / dx$  from  $0$  to  $\delta_t$  into  $1 - \theta$   $u dy$  ok on this side I have  $DT / dy$  is nothing but  $- D\theta / dy$  so this is  $- \alpha D\theta / dy$  at  $y = 0$  into you have  $T_w - T_\infty$  okay so I think now this cancels alright so now you now the rest of the things is I am.

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Multiplying dividing  $u / u_{\infty}$  and this is  $u / u_{\infty}$  here and the other  $u_{\infty}$  I am taking to the denominator here ok and I am transforming from  $Y$  to  $T$  that is all ok any any doubts I hope everything is clear so if you have any doubts please let me know I am going a little bit fast assuming that you could follow all right so with this is the final expression because we have in terms of  $\theta$  the profile and also  $u / u_{\infty}$  so therefore we can directly substitute that into this expression so if you do that so now let me erase this so substituting Cubic temperature and cubic.

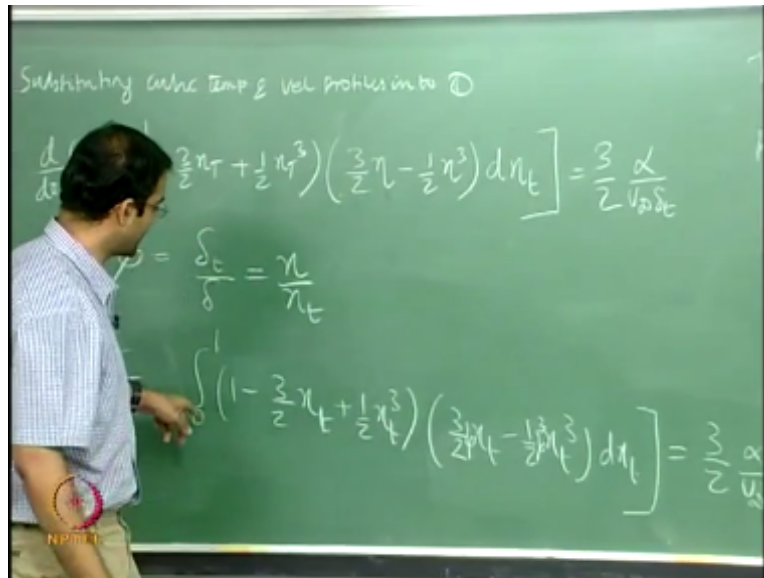
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Velocity profiles into one so  $d / DX \delta T$  0 to 1 so my cubic temperature profile is now  $1 - \theta$  so that is  $1 - 3/2 \eta - \frac{1}{2} \eta^3$  ok so I can write it as  $1 - 3/2 \eta T$  and - of - is  $p + \frac{1}{2} \eta D Q$  okay this is my temperature profile  $1 - \theta$  and what is my velocity profile  $3/2$  let me also define the same way I defined  $h T$  let me define  $H$  where  $h$  equal to  $Y Y / \delta$  okay so my velocity profile

can be written as  $\frac{3}{2} h - \frac{1}{2} h^3$  ok so into  $Dh T$  so this is integrated over  $Dh$  ok this is my left hand side on the right hand side yeah on the right hand side now what is  $D\theta / Dh T$  ok so this is my  $\theta / Dh T$  at  $h = 0$   $\frac{3}{2}$  that is it okay.

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So  $\frac{3}{2}$  into  $\alpha / \delta T \times \infty$  okay so this is therefore the final expression after we substitute now we have to simply integrate it out before we integrate it we have to write  $h$  in terms of  $h T$  because  $\eta$  and  $\eta T$  are both connected okay so therefore what we are going to do is we are going to introduce a variable another non-dimensional variable I will call this as  $Z h$  which is nothing but the ratio of the thermal boundary layer thickness to the momentum boundary layer thickness.

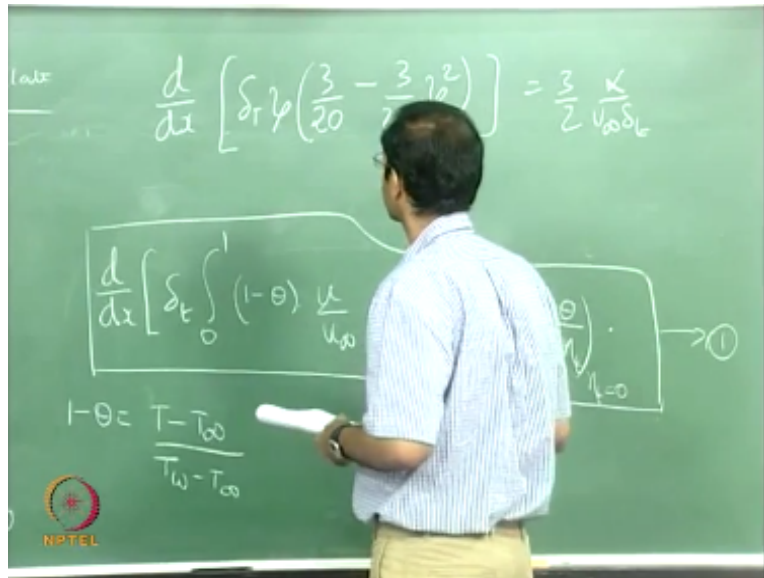
So since this is the ratio of this and this this has to be a function of prandtl number okay because this is the ratio of the two boundary layer thicknesses so therefore since I define my  $h$  as  $Y / \delta$  and  $h T$  as  $Y / \delta T$  you can also express this as the ratio of  $h T$  and  $H$  okay so this will be nothing but  $h / h T$  okay so now therefore I can substitute  $h$  as the  $Z h$  into  $\eta T$  okay so this can be substituted into this and we can write this as  $d / DX$ .

So I can write this as  $\delta T$  and this  $Z h$  can be taken out of the integral because it is fixed for at a particular  $Y$  location okay so we need not now integrate it but whereas in this case  $h$  is a function of  $Y$  okay so that therefore that has to be integrated across the boundary layer now  $Z h$  is fixed value at a given  $X$  location so therefore it can be taken out outside the integral so this will be  $\delta T Z h$  and inside the integral you have  $0$  to  $1 - \frac{3}{2} h T$ .

Plus  $1 / 2 h T$  cube into  $\frac{3}{2}$  this is again  $h T - 1 / 2 H T$  cube okay so I am just substituting as  $Z$  into  $\eta T$  so okay so this should actually be written as  $y-a$  so this is  $Z h$  here this

should be  $Z h^3$  okay so I just keep it one more step okay so on this side you can r in the terms as it is so now you can integrate integrate it from 0 to one so integrate it with respect to  $H T$  okay so if you do that I am NOT now going to do step / step but I will so this.

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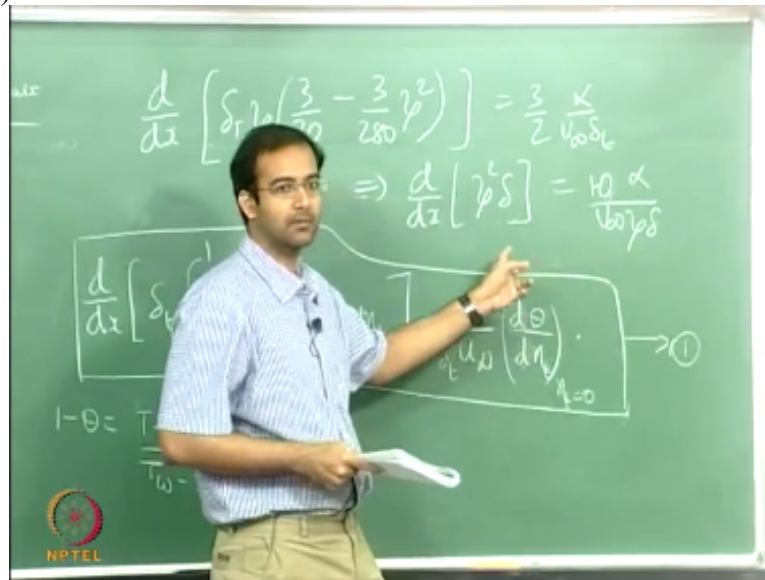


Should come out to be  $D / DX$  and  $\delta T$  if  $Z h$   $Z h$  if you take common this should become  $3 / 20 - 3 / 20$  okay so this  $Z h$  cubed term we can write it as  $Z h$  common out and  $Z h^2$  inside on this side you have  $3 / 20 - 3 / 20$  okay so if you integrate it you can do that yourself so each of these terms you have to multiply and integrate it okay so you have terms here  $\eta$  your  $\eta^2$  here at  $T$  power 4 again again  $\eta^3$   $\eta^4$  okay so you have to integrate each of them between 0 to 1 so finally the resultant.

Solution will be in  $D$  will be independent of  $t$  and this is what you will get okay so from here we can make an approximation now already I said this is for a case where prattle number is  $> 1$  if you make the approximation so so far we have not introduced any approximation on the Prandtl number okay so if you now bring in the approximation that prattle number is  $> 1$  here if parental number is  $> 1$  what what should be  $\delta T$  with.

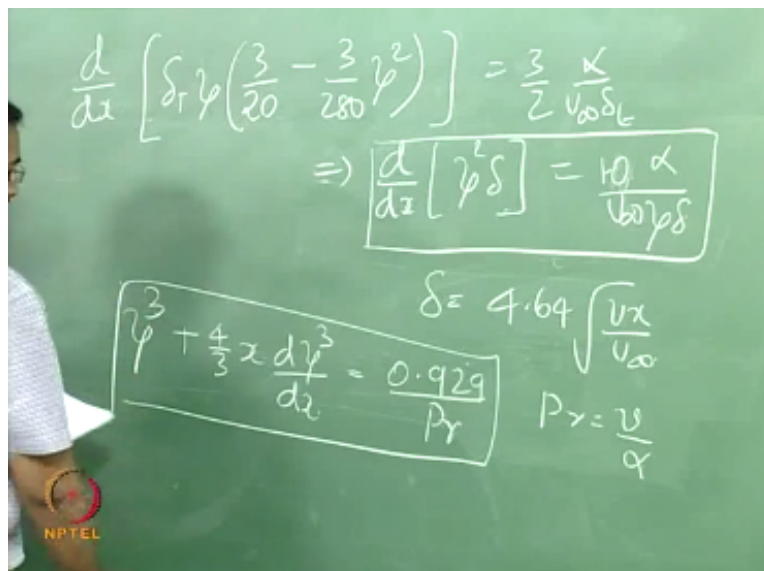
Respect to  $\delta$  less therefore  $Y$  or  $Z$  should be less than 1 okay so for prattle number much  $> 1$  our  $zh$  should be much  $< 1$  ok so if you bring in that approximation you can neglect the higher order terms in terms of  $zh$  okay they are very very small so therefore if you do that if you in only the most important term most significant term will be the first term so this will be  $3 / 20$  into  $zh$   $\delta t$  once again I can write  $\delta t$  s  $zh$  into  $\delta$  so therefore this has become  $\theta \sqrt{\delta}$  on this side I am taking  $3 / 20$  and factoring it out this will become 10 okay so this is this cancels this becomes  $10 \alpha / u_{\infty}$  and once again  $\delta T$  I can write in terms of  $\delta$ .

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And  $\delta$  so finally I am reducing the entire equation to an equation only in terms of  $\delta$  and  $Z$  ok so I know now the expression for  $\delta$  coming from the solution to the momentum integral equation which I can substitute into this and now I will have an ordinary differential equation in terms of the unknown  $Z$  okay so I think with that I will be able to calculate also my thermal boundary layer thickness so this is my equation so if I substitute for the cubic velocity profile  $\delta$  was  $4.64 \sqrt{u x / u_\infty}$  okay so if I substitute this value of  $\delta$  into this and if I expand this particular OD so this will be  $\theta$  cube.

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Plus  $4/3 \times D \delta^3 / D X$  this should be  $= 0.929 / \text{Prandtl number}$  where Prandtl number is  $= \nu / \alpha$  this is what you will finally get as the ordinary differential equation so I am

just expanding this particular term right here okay so what I am doing is I am taking this  $\delta$  here okay I am saying this as  $1/3$  into  $D/DX \delta^3$  cube ok if you want me to explain this I will say so this can be written as  $1/3$  or I can say this is  $d/DX 1/3 \delta^3$   $\theta^3 \delta^3 = 10$ .

$\alpha/u \infty$  into  $\delta$  okay I think yeah should be right so now I can expand this into two terms so one is  $Zh^3$  into  $D/DX 1/3 \delta^3$  so  $\delta$  you can substitute and you can differentiate it and the other term will be  $1/3 \delta^3$  into  $D Z h^3 / D X$  okay and if you substitute for  $\delta$  so you already have  $\delta$  you can multiply the entire left-hand side /  $\delta^4$  which have the expression for boundary layer thickness and it will simplify.

On the right hand side you will have  $\alpha/u$  because you have an expression for new okay here which will come and you will exactly get a factor of  $\nu/\alpha$  in the denominator which is nothing but the Prandtl number so couple of steps which you can do yourself you know and I am just simplifying that once you simplify you will get this is your final order differential equation okay so this is very straightforward to solve okay this is a non-homogeneous.

Body so you can find two solutions one is your complementary function which is the solution to your homogeneous OD and a particular integral assuming that some particular solution exists you substitute so you will get those two solutions which you can add and that. (Refer Slide Time: 35:49)

Handwritten mathematical derivation on a green chalkboard:

$$\frac{d}{dz} \left[ \delta^3 \left( \frac{3}{20} - \frac{3}{280} \delta^2 \right) \right] = \frac{3}{2} \frac{\nu}{u_{\infty} \delta^2}$$

$$\Rightarrow \frac{d}{dz} \left[ \delta^3 \right] = \frac{10 \nu}{u_{\infty} \delta^2}$$

$$\frac{d}{dz} \left[ \frac{1}{3} \delta^3 \right] = \frac{10 \nu}{u_{\infty} \delta^2}$$

$$\delta^3 + \frac{4}{3} x \frac{d\delta^3}{dz} = \frac{0.929}{Pr}$$

$$\delta^3 = Cx^{-3/4} + \frac{0.929}{Pr} \rightarrow PI$$

CF

$$\delta = 4.64 \sqrt{\frac{\nu x}{u_{\infty}}}$$

$$Pr = \frac{\nu}{\alpha}$$

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Solution will be finally  $Zh^3$  should be  $CX$  power some constant  $C X$  power  $-3/4$  plus  $0.929/Pr$  so this is your complementary function and this will be your particular integral so this is your solution for  $Zh$  okay so once you determine the solution now we have to

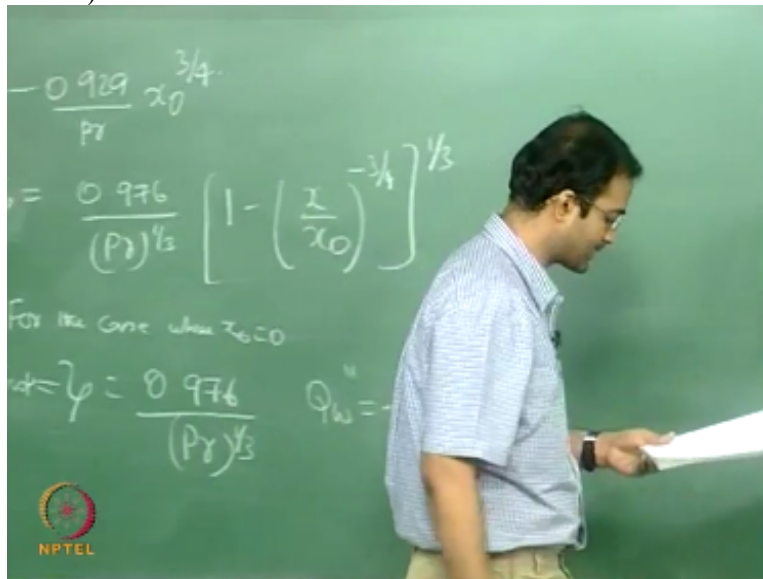


determine this constant so therefore we should give a boundary condition for  $Z_h$  okay so that is we know for the generic case that  $X$  is  $X = X_0$  or  $Z_h$  should be 0 right.

So if you put that condition at  $X = X_0$  is  $X_0$  okay so what what will be the constant the constant will can you substitute and let me know what the constant comes out to be  $0.976 / Pr$  and this will be  $X$  not power  $3/4$  which will go to the other side this will become  $X$  not power  $3/4$  okay so therefore the final solution for  $Z_h$  can be expressed as  $0.976 / Pr$  to the power  $1/3$   $1 - X / X_0$  to the power  $3/4$ .

$T$  whole raised to  $1/3$  okay so I am substituting for  $C$  into this into this expression right here and I will get the final expression for  $Z_h$  okay so now this is done therefore we we now know the thermal boundary layer thickness expression because it has nothing but  $\delta T / \delta$  and  $\delta$  is already known so we can find the expression for  $\delta T$  and to derive a specific case okay for the case where you do not have the unheated length that is you start maintaining a wall temperature condition right from the beginning at  $X = 0$  okay so far for.

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The case where  $X_0 \neq 0$  okay so this is  $X_0 / X_0$  to the power  $3/4$  so this will go to 0 and therefore your  $Z_h$  will be simply  $0.976 / Pr$  raised to the power  $1/3$  okay also for the case where there is no unheated length in its heated right from the beginning for a given Prandtl number the ratio of  $\delta T / \delta$  will be constant because Prandtl number is a constant okay so therefore the ratio of  $\delta T / \delta$  will be constant.

Therefore you can see from this expression for a given Prandtl number the  $Z_h$  will be actually a constant so if you do not have an unheated length if you have an unheated length you can see  $Z_h$  is also function of the position correct make sense rate because if you if you

heat it right from the beginning both the boundary layers grow according to the Prandtl number and at any cross section at any section axial location the ratio of the boundary.

Layer thicknesses will be such that it is a constant whereas if you maintain a starting length heated length and  $u$  heated length then you can see at different locations the ratios will become different okay so that is what is coming out of these expressions so for the case where you do not have unheated length this is your expression so with this we can calculate the heat flux -  $K \frac{DT}{dy} \Big|_{y=0}$  so  $\frac{DT}{dy} \Big|_{y=0}$  can be written as of course.

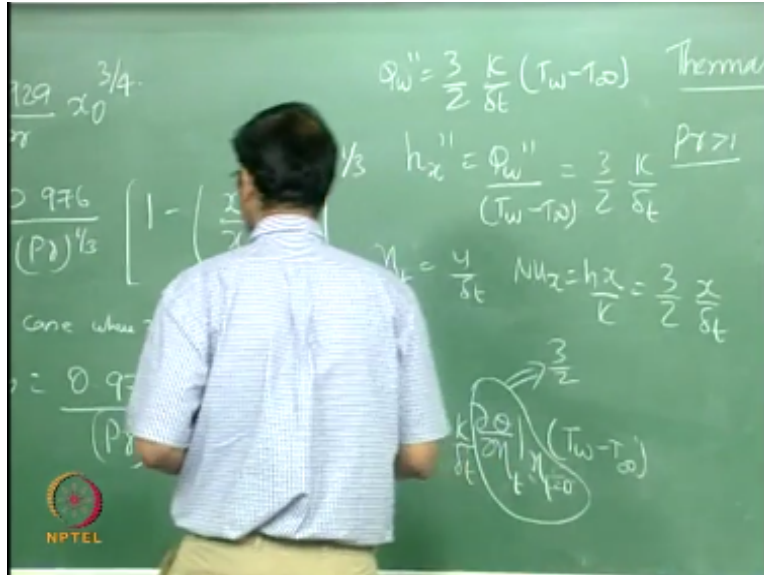
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$$q_w'' = -k \frac{\partial T}{\partial y} \Big|_{y=0} = k \frac{\partial \theta}{\partial y} \Big|_{y=0} (T_\infty - T_w)$$

You are if you remember your definition of  $\theta = T - T_{wall} / T_\infty$  okay so you can you can write this as  $D \theta / dy$  at  $y = 0$  and multiplied by  $T_\infty - T_{wall}$  okay so this will be - of course so - I am going to write this in terms of  $T_{wall}$  - okay again you can make a transformation so  $D \theta / D H$  into  $D H / D Y$  okay so you can run this is  $H T$  it at = .

So in  $D H / D Y$  so what is  $D H / D Y$  your  $H T$  is  $= Y / \delta T$  so  $1 / \delta T$  so that will be  $1 / \delta T$  in the denominator here okay and  $D \theta / D H H T$  at  $H = 0$  this is nothing but  $3/2$  okay so therefore this can be written as  $3/2 k / \delta T$  into  $T_\infty - T_{wall}$  okay so from this everything else follows you can define your local heat transfer.

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Coefficient as  $q_w'' = \frac{3}{2} \frac{k}{\delta} (T_w - T_\infty)$  which will be  $\frac{3}{2} \frac{k}{\delta} (T_w - T_\infty)$  and therefore you can define your nusselt number locally as  $h_x = \frac{q_w''}{(T_w - T_\infty)}$  okay which will be  $\frac{3}{2} \frac{k}{\delta} \frac{x}{x}$  okay, so finally you can substitute the expression for  $\delta$  and  $\delta$  so already you know that  $\delta / x$  is  $= 4.76 / \sqrt{\text{Reynolds number}}$  and this is nothing but  $\delta T / \delta$  so finally for  $\delta$  T if you substitute this should come out as  $0.33 \text{ Re } x^{-1/2} \text{ Pr}^{1/3}$  to the power half Prandtl number to the power  $1/3$  okay so I think this should come out and in terms of the unheated starting lengths this will also add come as a factor  $1 - x/x_0$  to the.

Power -  $3/4$  the power -  $1/3$  so this will be the expression for the local variation of the nusselt number okay so far then for the heated length for the heated case right from the beginning it is not  $= 0$  so a new  $x$  will become  $0.33 \text{ Re } x^{-1/2} \text{ Pr}^{1/3}$  and mantle number four was unworthy now you compare this with the polich and solution similarity solution that was what  $0.332$  so it is very very close okay so here I am skipping couple of steps you

Can just do that yourself okay because you have this is your  $\delta t / \delta$  you can write this  $\delta T$  as  $Zh$  into  $\delta$  and  $\delta$  you already have the expression you can just substitute that into this and finally you will get a very neat expression in terms of Reynolds number and prandtl number okay so for the case where you are heating right from the beginning this is your polasancase since case now you remember when we did the similarity solution.

We cannot make the assumption that you have an unheated length and then get the similarity solution okay the similarity solution was derived based on the assumption that the wall temperature is uniformly applied through the entire length of the plate otherwise we cannot derive the similarity solution so therefore the approximate method the integral method gives us a choice to also impose this condition of unheated length okay.

See not only it makes our derivation simpler relatively compared to the similarity solution but also it gives some flexibility in terms of applying variable boundary conditions for

example we will see later on down the line in a few classes three to four classes that we can solve use the similarities use the approximate solution method to take a case where the temperature is varying along the wall not just something like this you know it could be.

Linearly varying it could be some piecewise constant variation whatever may be that can be handled with the approximate solution okay which which is which is quite difficult in which cannot be done in fact with the similarity solution so therefore although this also looks quite tedious but this can be worked out with the hand and unlike the similarity solution which needs programming to solve the ordinary differential equation okay.

So relatively in terms of the total effort this is much better than doing complete similarity procedure and finally you also get a very accurate result in terms of the heat transfer so we will stop here today and I just want to also tell you we can we can also do a similar approximate solution for Prandtl number less than one which I am not going to do but we can make an approximation there since for prattle number much lesser than one your your thermal boundary layer thickness is going to be far  $>$  your momentum boundary layer thickness so therefore you can make a uniform velocity approximation rather than.

Substituting a profile okay so the velocities boundary layer is so small that most of the thermal boundary layer will be looking at a uniform velocity okay so that is a much simpler case to deal with and I will give you that as a homework assignment so in the next class on Tuesday we will look at flows with pressure gradients where we can apply the approximate solution you.

**Integral method for laminar external thermal  
Boundary layer over isothermal surface  
End of lecture 19**

**Next: Integral method for flows with pressure gradient  
(Von karman-pohlhausen method)  
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