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Video Lectures on

Convective Heat Transfer

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Lecture 18

Approximate (Integral) methods for

Laminar external flow and heat transfer

Good afternoon, anyway good that at least a few have turned up otherwise I would have to reschedule the entire class anyway so why is there one more small correction to the previous class that we that I just want to bring forward I think one is one person who observed it and he was correct the mean temperature.

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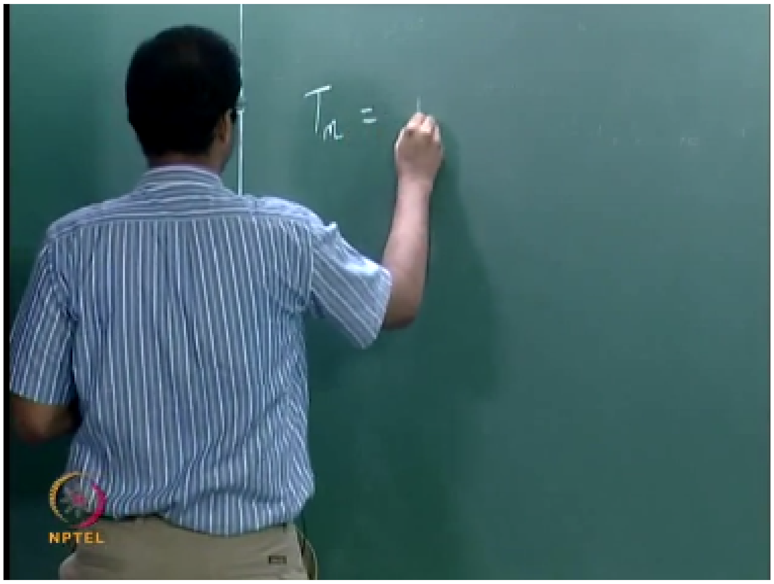

For the high speed flows I had told that it should be 0.5 _ ∞ let us see in - I think it should just reverse the sign it should be T adiabatic wall _ T wall where C = to 0.2 so I think I had given this as C into T wall _ T adiabatic someone has correctly pointed out that since the adiabatic wall temperature is greater than the wall temperature. So this has to be positive so that the mean temperature now is increasing rather than it should not decrease because now with the use of adiabatic profiles for example if you had plotted Y versus T which we had done this is your T ∞ from here the profile should start something like this and this is your T wall if you had applied a isothermal boundary condition.

Without viscous dissipation it would have come something like this okay so therefore you see the average temperature has to now go up okay earlier the average temperature was between T 1 and T ∞ okay now the actual temperature will be like this something like it will go up inside so therefore the average temperature has to account for this increase.

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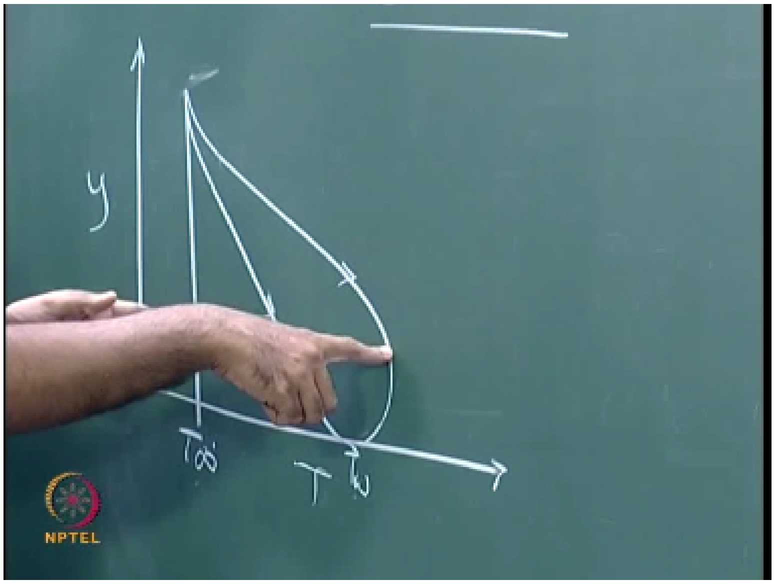

In the therefore this has to be positive here if this was T wall - so generally your adiabatic wall temperature will be greater than your wall temperature. okay so this will be positive and therefore this will increase your T mean temperature otherwise it will decrease the T mean temperature which is not physically correct okay so this please incorporate this particular correction.

And now we will start a new topic it is the same solution to external boundary layer flows but we will use what is called as an approximate method of solution okay so far we have been doing exact solution so-called exact solutions where we had derived the similarity send equations for the different configurations and we had solved it but we cannot solve it exactly we have to solve it using some numerical method but nevertheless the solution is the solution is for an equation which is actually an exact equation okay now we will look at techniques where we will approximate the solution by means of some profiles rather than trying to solve by a numerical technique.

That the way that we were doing till now so we do not know the profiles how they look so but we got that as a solution of the similarity equation now we will approximate the profiles as something depending on the order of accuracy now you can use a linear profile or quadratic or cubic or fourth order whatever polynomial so we will substitute that into the governing equation once you know the profiles we can determine the expression for boundary layer thickness the thermal boundary layer thickness and therefore the heat flux nusselt number and so on so this kind of technique is also called the integral method.

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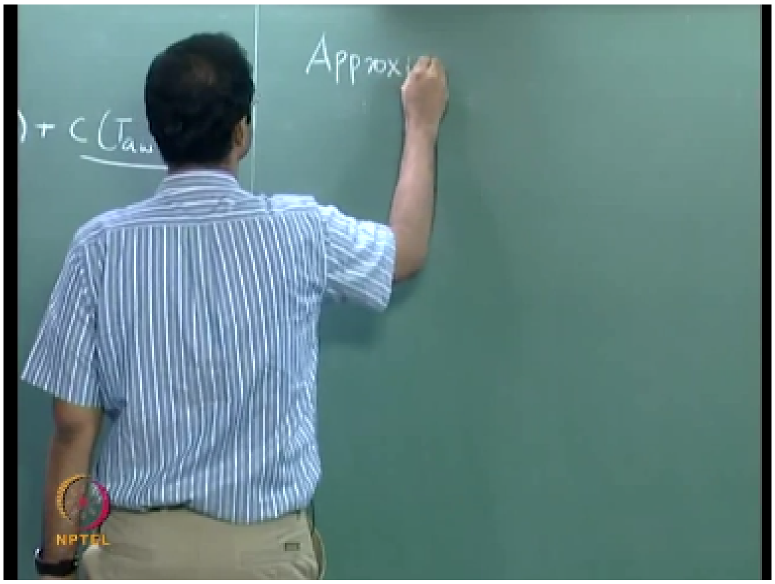

Many people also refer this as the approximate okay so the starting point of this is to start with essentially an integral form of why this is called an integral method is you operate with the integral form of the governing equation form of boundary layer momentum and okay so why do we use the integral form is of course the integral form is more helpful when you are approximating some solution putting it inside and integrating.

It out directly okay so therefore this is more convenient for the approximate solution to work with an integral form and I am NOT going to now derive the integral form because if I remember correctly I had already derived a particular version of the integral form if you may recall recollect your earlier notes for the flat plate lass a solution we try to find the expression for boundary layer thickness Δ.

So as something like √ of Nu X by u ∞ right so this was derived by integrating the momentum equation from 0 to Δ and from there we had solved an OD to get the expression for Δ. Okay so in fact at the time I told that I am not going to derive it again because we will be using that as a starting point so the point where we left off so we integrated the X momentum equations I am just going to write.

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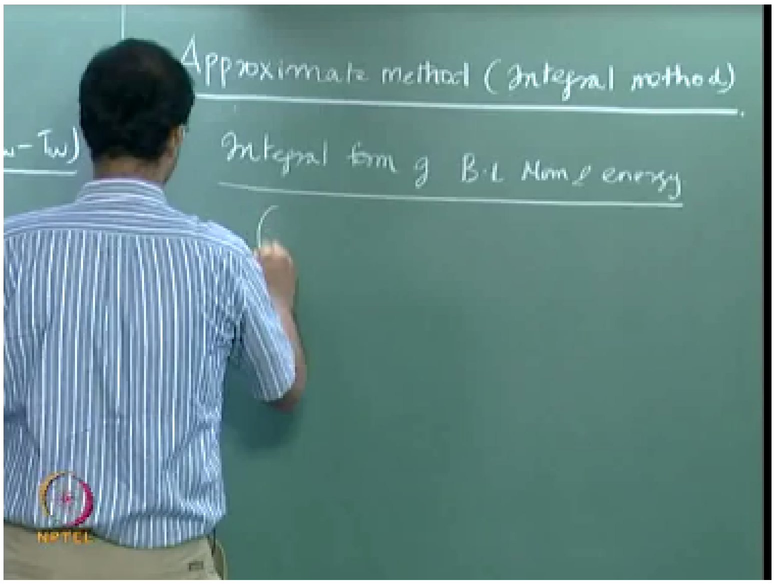

It just to show you how it looked so this is d u by DY, DY is = to new do you my D between the limits = to 0 and okay so we just integrate the boundary layer momentum equation for a flat plate okay we could also include your pressure gradient term so that will be additionally + u ∞ D u ∞ by DX and integrated between 0 to Y DY it is nothing but Δ okay so how we got this because originally this term was nothing but _ 1 by Rho DP by DX and we saw in the boundary layer that the pressure gradient along Y is negligible.

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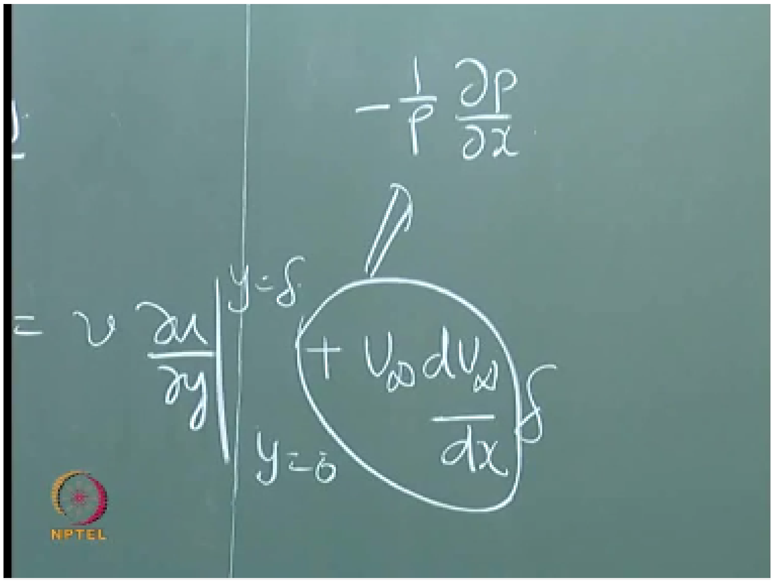

Therefore we can apply this equation to the invest potential flow outside and we can directly find that this is = to u ∞ D u ∞ by DX okay and when you when you integrate it across the boundary layer so that will be this will be constant okay for a particular Y location so you can directly say that is multiplied by Δ okay so if you have a pressure gradient that term also enters into this final expression and after we broke this term we integrated by parts and then we use the continuity equation to make some approximations and finally we arrived at this particular form 0 to Δ u D u by DX especially the left hand side - you ∞ 0 to Δ V you by D X D Y this is = to nu D u DY 0 to Δ + okay.

So I think we arrived at this particular form for the entire left hand side term after we use the continuity and we integrated it out and from here we are going to start now okay so this is nothing but the momentum integral equation but still not the final form okay so I am just going to rearrange some of the terms in fact what I am going to do here is I can once again integrate by parts okay so I can do this as D of u ∞ u by DX _ u into V u ∞ by DX again I can just re express we express that way and I am going to multiply throughout by a negative sign and if I just rewrite this a little bit so this will be D by DX 0 to ∞ 0 to Δ u ∞ _ u so u ∞ into u is basically this term _ this can be neatly written as directly.

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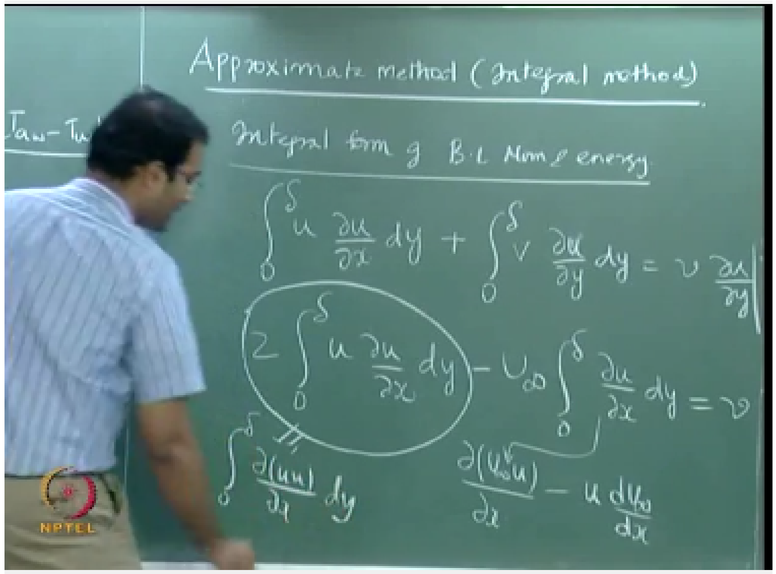

0 to Δ D into u by DX right D u square is nothing but 2 UD u DX okay so I can just write this as d by DX so I can take D by DX out so 0 to Δ applying throughout by negative sign okay so I can write this as so this is already negative of negative is become positive so u ∞ into u _ u ok so that is this particular term okay + the other term which I which I have written this is d u ∞ by DX which I can take common from here as well as from the right hand side term okay so this can be written as D u ∞ by DX integral 0 to Δ okay so here I might I have a negative sign here and I multiply it with a negative sign so this should become positive okay so this should be a positive again I multiplied.

With a negative so that should become negative and this one is already negative goes to the left hand side term that becomes positive so this becomes u ∞ _ u DY okay on the right hand side now if I integrate this from 0 to Δ so at this at y = to Δ D u by DY should vanish okay so this becomes _ nu D u by DY with a _ sign so this becomes positive this becomes μ D u by DY at y = to 0. Okay so I have trouble skipped a couple of steps but I think you can understand straight away how that comes out you want me to explain again or is it okay. So till here you know how it has come out okay from the previous derivation.

So from here I am just rewriting this first M as Du by DX and this I am integrating by parts again okay so I can write this as u ∞ u D by DX _ UD u ∞ by DX so therefore so this term and this term can be combined I can take D by DX out of the integral so 0 to Δ u ∞ _ u ok so this is u ∞ into u this is you so I take small u common u ∞ _ u DY and the other two terms so here D u ∞ by DX is common to this in this okay so and I can write this as instead of writing this as

Δ I can write this as 0 to Δ DY ok so that is u ∞ that is basically this u ∞ into DY _ there is again this u DY okay so which is what I am writing here together on the right hand side term this is 0 to Δ so at Δ D u by DY is 0 right so therefore this is _ D u by DY at y = to 0 already a multiplying throat by _ so this becomes new d u by DY at y = to 0 okay so this is my final expression in fact I can also rewrite in a more familiar form so I can write this as d by DX if I can take multiply and divide by u ∞ square for this particular term so this will be u ∞ square into I will just denote this as something like Δ 2 + in this case I can multiply by multiply and divided by u ∞ okay so I will call as some Δ 1 into D u ∞ by D X on the right hand side.

I have so where my Δ 1 will be so multiplied and divided by u ∞ so this will be u ∞ 0 to Δ so this is u ∞ by u ∞ which is 1 _ u by u ∞ into okay so , I am not going to write exactly like this I am just rewriting slightly in a different manner just a minute so multiplying and dividing it so that Δ 1 into u ∞ is here and Δ 1 is defined in this particular fashion.

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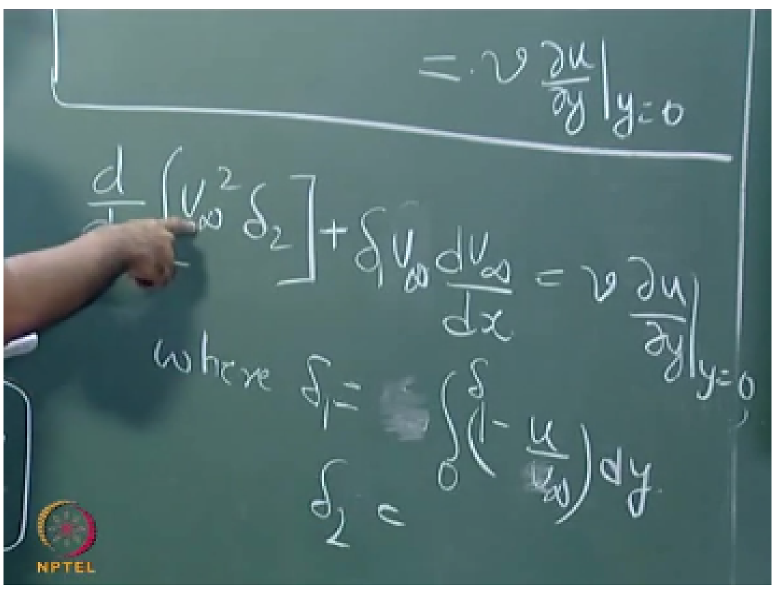

Okay and Δ 2 so μ multiplying and dividing by u ∞ Square and divided by u ∞ square this will be 0 to Δ 1 _ u by u ∞ into u by u ∞ DY okay so this is also another nice form that you find in many of the textbooks nevertheless all the three whether it is this form or this form this form they are all the momentum integral equations or integral momentum equation okay I personally prefer looking at this form this is almost the final form and this gives you a lot of information because here this Δ one which I have defined here there is a particular name to it this is also called the displacement thickness and Δ two here is called the momentum thickness okay so and this is okay I hope you have a good understanding of what is displacement thickness and momentum thickness okay from your incompressible flows I am not going to go into an explanation for that now it is only so this these are not really measurable.

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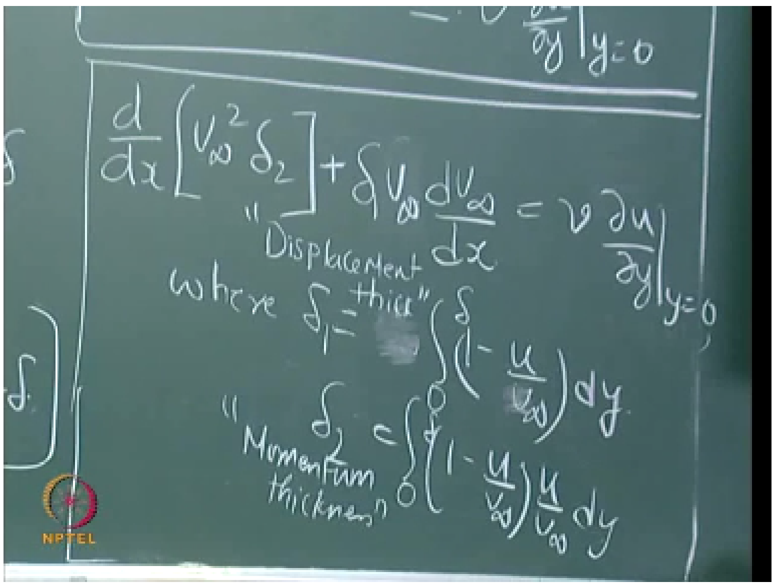

Okay this is what you have to understand unlike the boundary layer thickness. Which you can measure at as a location these are conceptual values which give you a sense if you suppose replace the boundary layer with a potential flow profile so how μ CH the wall has to be pushed up okay so are displaced in order to satisfy continuity or satisfy conservation of momentum okay so that is this location so these are all not measurable okay these are some things you are conceptualizing and usually these are μ CH smaller than your actual boundary layer thickness okay so this is your momentum integral equation.

So you are writing this in terms of the momentum integral or your displacement integral and so on for the case of flat plate without the pressure gradient the second term will be 0 so you do not have anything in terms of the displacement thickness so for flat-plate.

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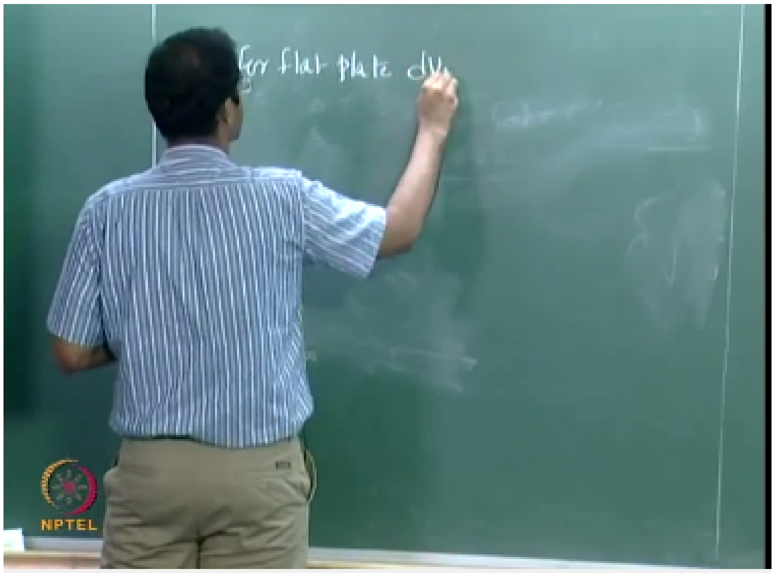

Your d u ∞ by DX is = to 0 and this will lead to the form you can write u ∞ square d by DX 0 to Δ okay so this is the integral momentum equation for flat plate let me call this is number one.

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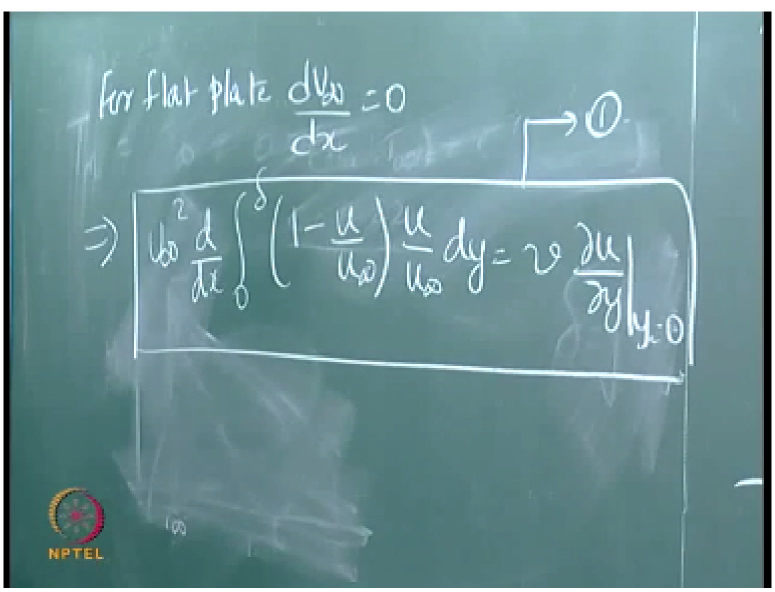

Okay this is the one firm that we will be using to substitute all our approximate profiles now same way we will derive the integral form of boundary layer energy equation okay we start from the conventional energy boundary layer equation and then we integrate it out across the boundary layer thickness and we arrive at a similar integral energy equation okay so we start with.

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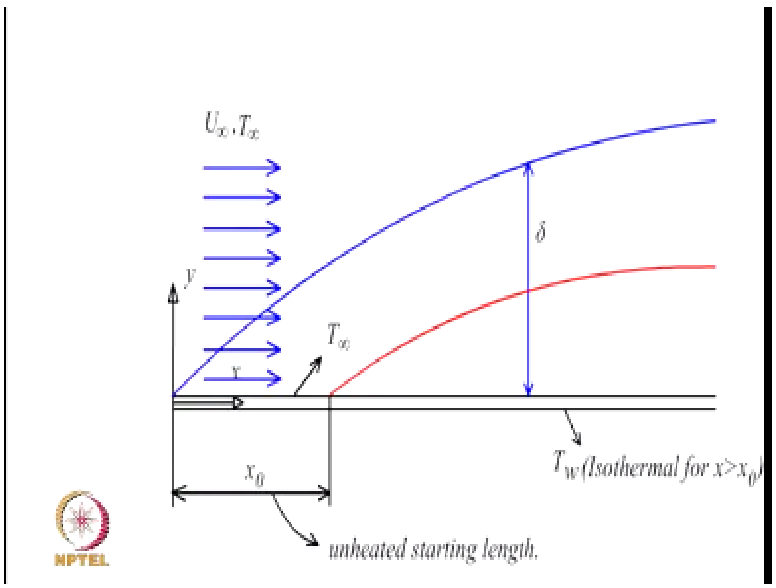

Our you standard energy equation we do not assume any viscous dissipation term here for flat plate case without viscous dissipation this is your equation at y = 0 it could be either wall temperature is fixed or your prescribed heat flux it is constant and at Y going to ∞ is = to t ∞ at X = to 0 also the same thing follows so what we are simply doing is integrating again now when we are integrating now we cannot integrate it till the boundary layer thickness because now we have to integrate it till the characteristic thickness.

For the energy which is the thermal boundary layer thickness okay so we integrate it till Δ t of course you know we are not making any assumption whether Δ T is greater than Δ less than Δ whatever extent it may be we are just integrating till the edge of the thermal boundary layer and so we will try to eliminate V so that we write everything in terms of U so we will see that the second term can be integrated by parts so this can be written as.

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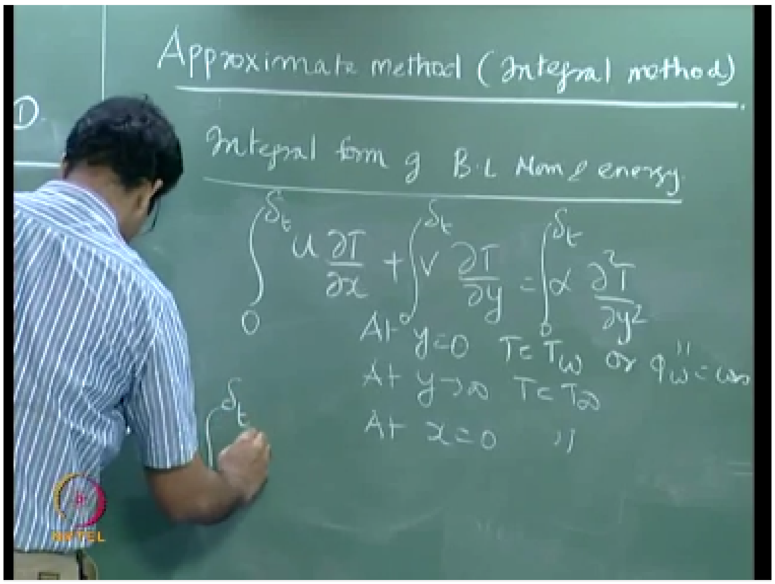

0 to Δ T okay I can write this as d by DY of VT integrate between 0 and Δ T set that is nothing but V into t between the limits 0 and Δ T okay _ integral 0 to Δ T into T DV by DY on the other side you have α Δ T by ∇ Y between the limits 0 and Δ T and a Δ T DT by DY is 0 okay it has to satisfy continuity in slope at the edge of the boundary layer so therefore so this will become _ α t by DY at y = to 0 okay so this is coming because that X is = to Y going to ∞ t = to t ∞ therefore DT by DY has to be 0 okay.

So now this particular again at y = to 0 V Is 0 so this will be valid number only when you look at y = to Δ T so where once again your P will become t ∞ there okay so a Δ T this will become ∞ okay so this is at T is it fine so now what we can do is once again we can integrate this by parts you can write this as 0 to Δ T into D by DX of UT DY _ 0 to Δ T DU by DX DY okay so therefore you can combine this and this term right here so you can take T common this is d u by DX + DV by DY which is nothing but the continuity equation alright therefore I can write this.

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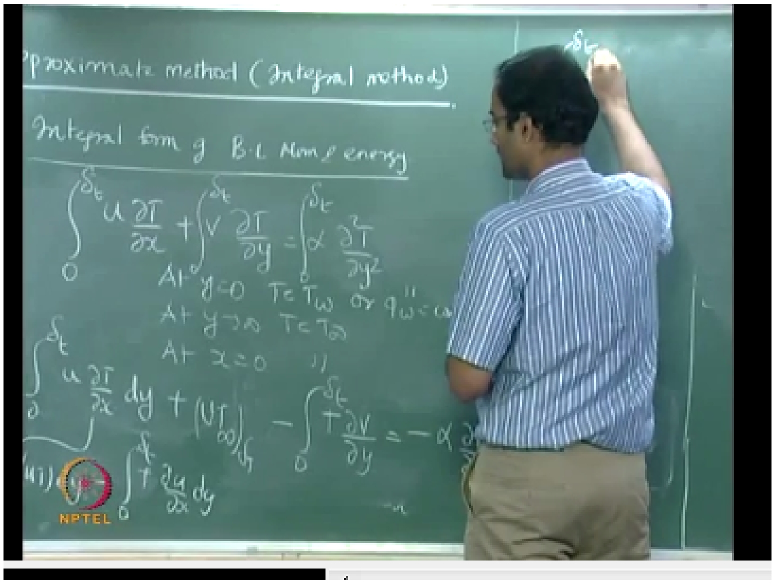

As 0 to Δ T D by DX of UT DY + V T ∞ - I can take a 0 to Δ T common d u by DX + DV by DY this is = to _ α DT by DY at y = to 0 so and this also satisfies continuity so this goes to okay and also from continuity you know that my V = to _ 0 to integrate from 0 to Δ T the continuity equation I can write my V velocity in terms of u velocity so just the continuity of integrating okay so I can substitute I know I can eliminate V from this equation so if I write this in terms of u velocities this will be 0 to Δ T I can take D by DX out D by DX u t DY _ I have T ∞ into integral 0 to Δ T be u by DX _ α DT DY y = to 0.

Okay so I just want to combine these two terms I can write this as now D by DX okay 0 to Δ T by _ sign throughout this will be T ∞ _ T so U is common for both I can write this as u DY this is = to α DT by DY at y = to 0 okay so this is my final form this is V energy integral equation okay so I can combine these two terms because T ∞ is anyway constant I can take this they can write this is D by DX of T ∞ u so u is common in both the cases in combines okay so now we have both of them so I am going to define my Θ in fact I can write in terms of Θ but let me do it later so just let me write down the energy equation.

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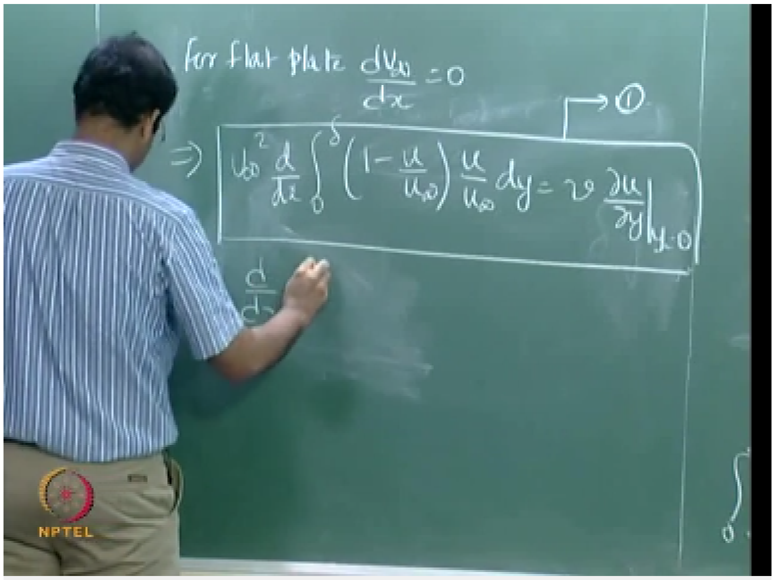

Energy integral 0 to Δ T ∞ _ t DY should be = to α okay so this is your equation number two okay so the first step is that we derived the integral form of momentum and energy and we have put it in this particular pattern and next what we are going to do in the approximate solution we are going to make an approximate assumption for the velocity and temperature has some polynomial okay so it can be any order polynomial and we have to see which boundary conditions that the profiles have to satisfy to calculate the coefficient of these polynomials so this is the next step so now let us take the case where we have a linear velocity profile.

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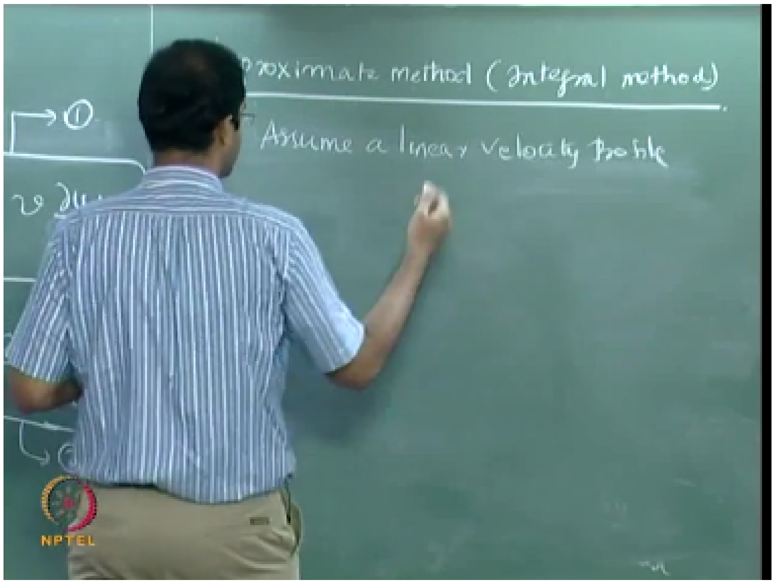

You know that is the most basic profile that you can start with so for the laminar flow let us assume a linear velocity profile so I am just saying that my you would be something like a + B Y okay this is the linear form of the velocity profile that I am assuming so in order to get the constants a and B I have to satisfy boundary conditions okay so the most you have to start with the most basic boundary conditions before.

We go to satisfying the more higher-order boundary conditions okay the most important boundary condition start from the wall at y = to 0 to e = to 0 okay and after this what should be the second most important boundary condition so once you have given at y = 0 you have to give something at y = Δ ok u = ∞. These are the most basic boundary conditions which are which are which have to be satisfied i cannot directly say d u by DY at y = to Δ is 0 has to be satisfied then it does not tell what value it should reach at y = to Δ ok just only say this slope is 0 that is it.

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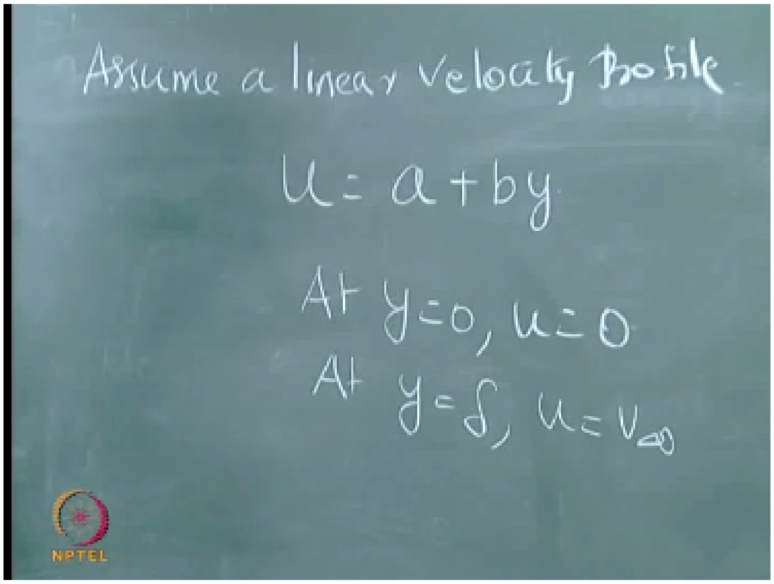

So if you substitute can you tell me what will be the profile that I get so if you directly substitute this y = to 0 = to 0 array will be 0 at y = to Δ you will become u ∞ therefore B will be u ∞ by Δ ok so therefore your u will be = U ∞ Y by Δ so u by u ∞ will be = to Y by Δ so this is my linear velocity profile so if I if I plot this profile for a flat plate.

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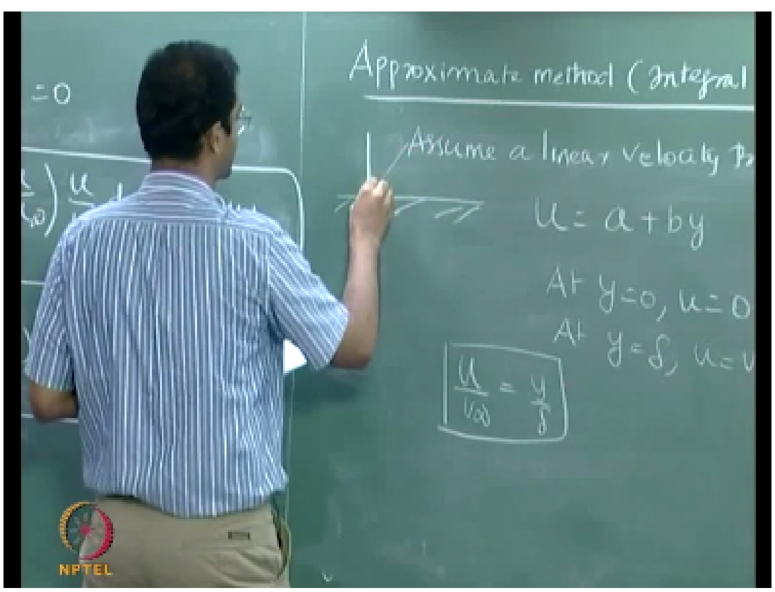

At any location it will show only something like this a very linear variation from 0 to u ∞ but y = to 0 U will be 0 and varies linearly at y = to Δ u will become u ∞ so this will be Δ something like a coed flow profile okay.

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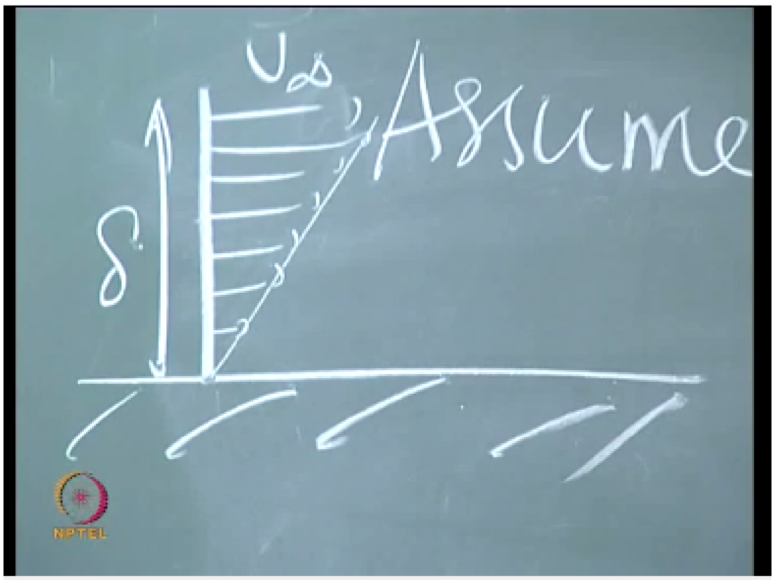

So this is not actually the real profile right your real profile is for satisfying and a different relationship which we have seen from the Blasius solution and definitely the shape does not look like this okay that is a slope there is a curvature terms are all that which we are not accounting for in this particular ok the slope is constant and there is no curvature so therefore this may not be a very good profile but still this is an approximation okay so why we are approximating it is once we may give this kind of an approximate profile we can substitute for velocity in this momentum integral and we can directly calculate the boundary layer thickness expression for Δ can be determined so what I am going to do now.

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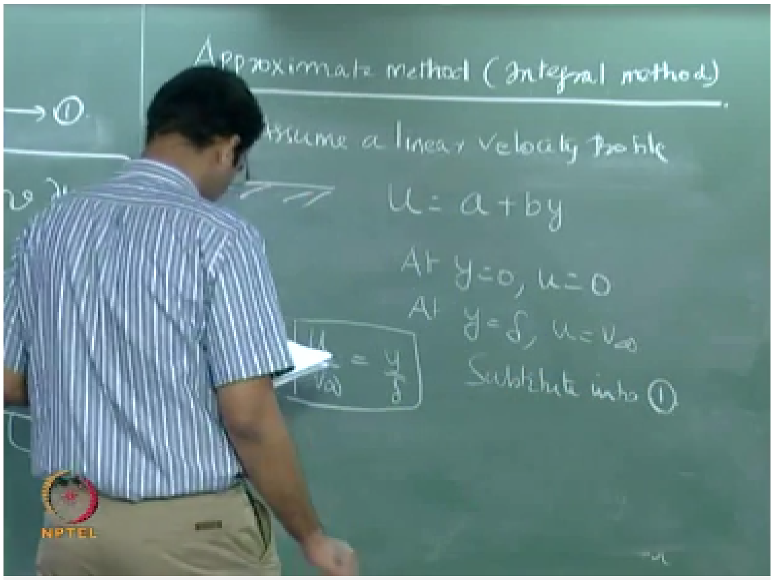

Substitute this profile into one so if I substitute into one that I will get u ∞ square D by DX 0 to Δ into 1 _ so u by u ∞ now will be Y by Δ into y by Δ into DY on the right hand side I have new D u by DY at y = to 0 so D u by DY now will be what u ∞ by Δ okay from the linear profile so therefore so anyway that is a constant slope it does not matter whether it is at y = to 0 or so if this is just linear profile so the slope is constant so this will be μ into u ∞ by okay so very simple profile and the equation also looks simple now I am going to introduce a non-dimensional variable H which is = to Y by Δ.

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Okay do not confuse this with the similarity variable of course, I am using the same variable here because coincidentally in the similarity solution also H was the similarity variable which is which was related to Y in Δ exactly like this but here I am just using H to represent any non dimensional variable so if I do that I can transform Y in terms of the non-dimensional H this will become u ∞ square d by DX now integrated from 0 to Δ will become 0 to 1 in terms of H okay so 1 _ H into H into this will be DY by D H into D H ok DY by D H will be Δ so basically I can I can put this Δ here into D H okay this will be = to on the right hand side new Infiniti by Δ ok now this Δ has to be with inside D by DX because Δ is a function of your position along the flat plate.

So if you do the integral maybe you can take a couple of minutes and do it but since we do not have time I am just going to give you the final solution so this will come out to be 1 by 6 okay so the integral 0 to 1 _ H into H D H should come out as 1 by 6 okay so in that case this will become I cannot cancel u ∞ on both sides and this will be Δ will come to this side so Δ D Δ by DX will be = to 6 new by ok so now this is a very simple OD which I can solve by separation of variables straight away ok so this will be Δ square will be = to 12 new by u ∞ X + some constant ok this is the solution for Δ.

Of course we can find the constant by applying the boundary condition that at X is = to 0 Δ = to 0 right the boundary layer thickness is 0 at the leading edge of the plate and therefore your constant will be 0 okay so your expression for Δ now reduces to the form Δ is = to √ of 12 µ X by u ∞ which is nothing but 3.47 √ of Nu X by or Δ by X can be written as 3.47 I can divide by X on both sides and this will be u ∞ X by nu which will be nothing but √ of Reynolds number will be √ of 10 a number now you can compare this with the similarity solution from which we calculated the expression for Δ do you recollect what is the expression there that was 5 okay so how did we do that we calculated for different values.

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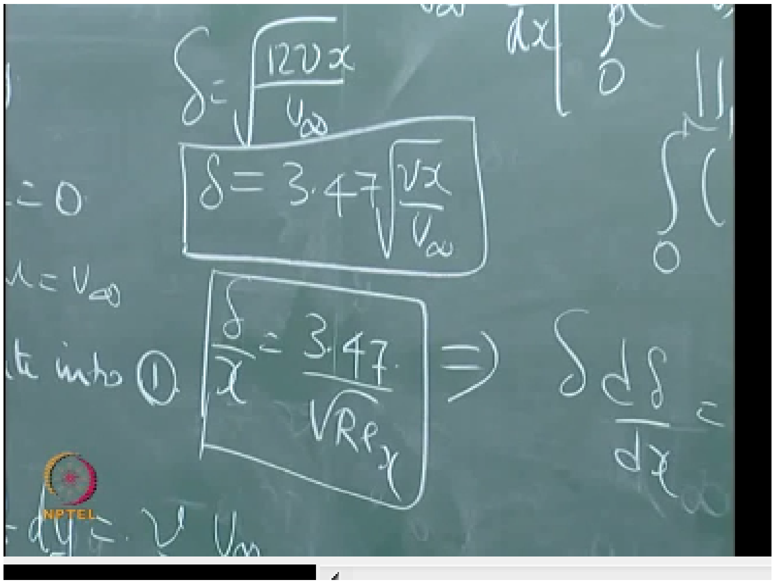

Of b η F of f of 0 F prime of 0 and also at all other values of b η we have tabulated and we found out the corresponding value of H aware you are u by u ∞ or your F prime was 0.99 corresponding to s prime 0.99 we found the data was = to exactly 5 and that that is nothing but the value of Δ okay so this is how we have got it approximately the actual solution is Δ by X is = to 5 by √ of re X so the since we are using an approximate profile this is the variation you know so this is about 31% less than the exact solution.

Okay so there is an underestimation that we get by using the linear profile and we can go ahead and calculate the wall shear stress local variation which is μ Du by DY at y = to 0 okay so in this case d u by DY at y = to 0 is nothing but for the linear profile u ∞ by Δ okay so this is new u ∞ by Δ so we have already got the expression for Δ which we'll substitute here and if you do that you will get 0.288 u ∞ √ of when you X okay so now if you define non-dimensional local skin friction coefficient as towel by Rho ∞ square so this will come out as 0.576 by √ of announce number okay so and if you remember the exact solution was giving 0.664 okay so this is about 13% less than exact solution.

Okay so if you also go ahead and integrate the local skin friction coefficient over the plate and calculate the average skin friction coefficient for the entire plate that is your CFL is = to 1 by L 0 to L C of X DX okay so we will end up with the expression 1.15 2 by √ of re l okay so finally the bottom line is this you can you can get an approximate estimate of all these quantities like boundary layer thickness your local shear stress in friction coefficient everything using some approximate profiles and this is μ CH easier as you can see than doing a rigorous solution to the similarity equations okay and in fact if you use a better profile you will be amazed to find that the agreement will be even better okay so just to give an example if you go from a lenient profile to a cubic profile okay which I will just show you and stop there okay so if you assume a cubic velocity profile U is = to a + B y + C Y square + DY cube.

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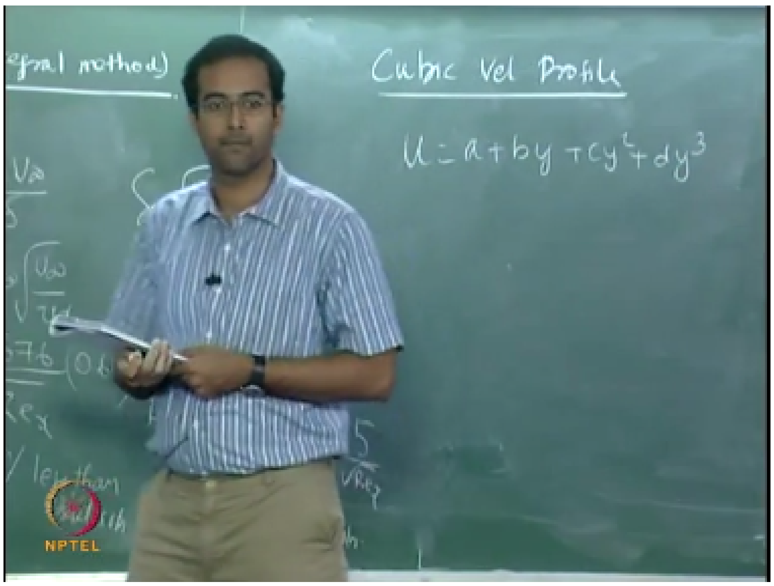

Okay so now you have one, two, three, four coefficients and we have to satisfy therefore four boundary conditions so what are the possible boundary conditions y = to 0 u = to 0 and at y = to Δ u ∞ so these are the important boundary conditions and what is the next important boundary condition at y = to 0.

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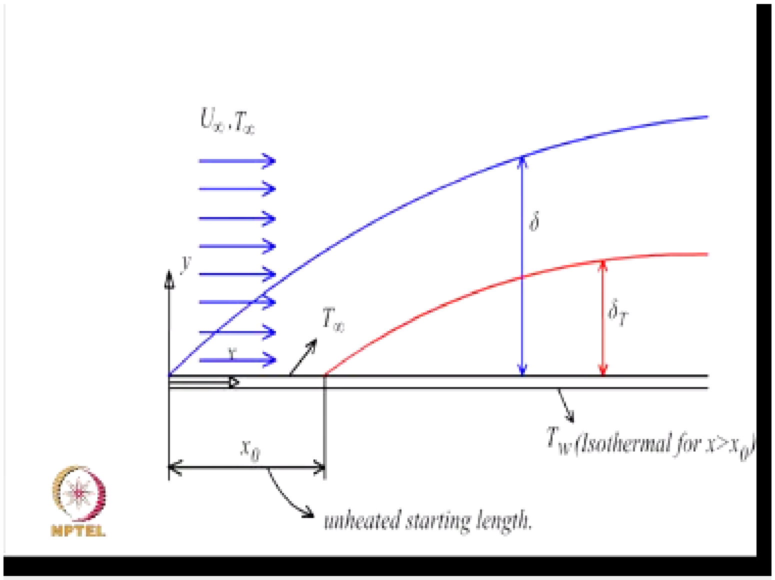

You u by way so if you if you look at the momentum equation okay so you D you by DX + B D u by DY is = to µ v square u by DY square at the wall okay both are 0 so therefore d square u by DY square has to be this is the next important saying the order of importance i am writing it okay and then finally what is the last boundary condition that you have to give cubic.

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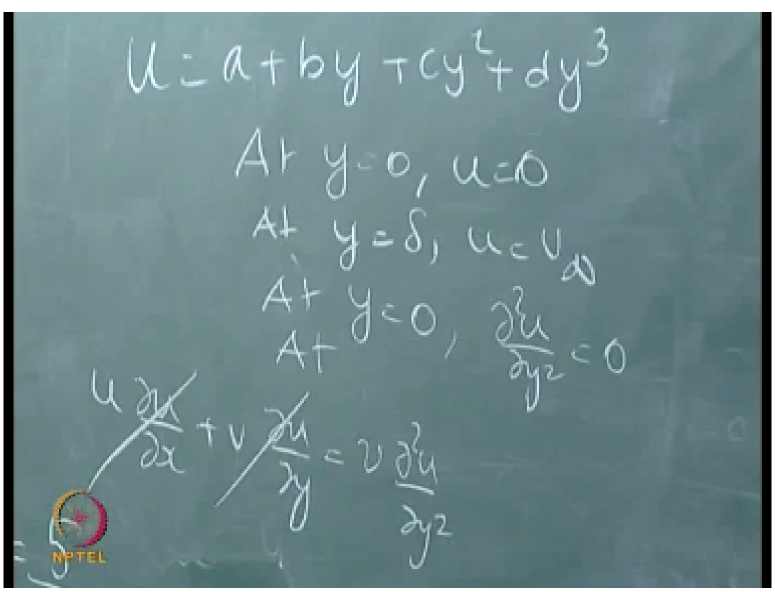

In terms of the order of importance okay so these are most important and then at y = to 0 this is to be given and then y = to what Δ? Okay so these are the four important boundary conditions so if i substitute these boundary conditions into the profile and you calculate all the coefficients finally i will be able to arrive at an expression which is like this u by u ∞ will be 3 by 2 y by Δ _ 1 by 2 y by Δ whole cube okay so this is the cubic velocity profile that you will get I am sure most of you are taken the heat transfer course have done this also in hand once heat and mass transfer incompressible fluid flows also okay.

So cubic profile is like the accurate profile that you can do okay so once you get this and if you substitute the same way that we did the linear profile into the momentum equation so let us see what happens to the final expressions the final expression for Δ that comes out with the cubic profile will be 4.6 for that is a correct Δ by X will be 4.6 4 by √ of re X so you can see remarkable improvement okay so this is like 7% less than the exact solution and when you look at the skin friction coefficient so this value comes up as 0.6 for 6 by √ of Rx this is like almost there 3% + okay so this is 0.66 4.6 for 6 almost there.

So you can see that with a cubic profile you get a very good approximation very accurate solutions ok rather than going for a very rigorous similarity solution so nowadays I think many of the people they are not so interested in the similarity solution because you have numerical techniques where you can directly solve the governing equations okay and or you can also resort to solutions like approximate methods with the integral equation where you can make use of some approximate polynomials and also get the solution so that that is why these are more popular methods okay which are also workable in a very short time but you have to be cautious that does not mean that if you keep increasing the polynomial.

l now this accuracy will become better and better okay so for example if you go from linear to quadratic and from quadratic to cubic you will find that it does not progressively increase in terms of accuracy okay in fact quadratic in fact spoils the solution a little bit it is mainly because in the case of quadratic boundary conditions we give this boundary condition this boundary condition and finally this boundary we do not give this particular boundary condition okay so because we have only up to here.

We do not give up to this boundary condition because of that you will find there is a degeneration in the accuracy the quadratic case again with the cubic it satisfies all the fundamental boundary conditions and the accuracy goes up and if you use a fourth order polynomial again you may have to give something like y = to Δ D square u by DY square = to 0 you have to introduce higher derivatives basically and they may not be very important so therefore you may not get on the error may not come down all the time so that is why maximum you go to cubic and we can stop there okay so in the next class tomorrow we will make an approximation for the temperature profile the same way that we did and we will substitute into the energy equation and calculate the thermal boundary layer thickness and therefore the heat transfer coefficient.

Okay so we will see these aspects in the next class so quickly we will move from flat plate so flat plate I am sure many of you have familiar with the heat transfer course so we are not going to spend much time we will do this analysis for flows with pressure gradient similarity flows that is that dealt with in the other courses or know where you have done similarity solution for adverse pressure gradient flows or pressure gradient flows I am sure something must have been done in incompressible flows the slides method and Carmen pole house and method okay so maybe we will cover those aspects in the future next four or five lectures of you.

Approximate (Integral) methods for

Laminar external flow and heat transfer

End of Lecture 18

Next; Internal methods for laminar external thermal

Boundary layer over isothermal surface

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