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Video Lectures on
Convective Heat Transfer

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Lecture 16
Similarity solution for flow and heat transfer with transpiration at walls

So good morning all of you we are continuing our derivations for similarity solution two different kinds of external boundary layer problems so we had looked at Falkner Scan solutions and the corresponding heat transfer solution associated to Falkner Skanvelocity similarity solution in the last class and we had taken some special cases for that where m = zero indicates flat plate and m = 1 indicates stagnation flow and in fact the stagnation flow is very useful because if you take for example the case of flow past a circular cylinder.

And if you are looking at the stagnation region you can convert the you can convert the coordinates from whatever you have here to the cylindrical coordinate and you can derive an expression for Nusselt number in the stagnation region of a circular cylinder that is what we had seen last day okay so apart from that in fact the most general generic case which we should have done is to consider a case of a boundary layer where you have transpiration at the walls.

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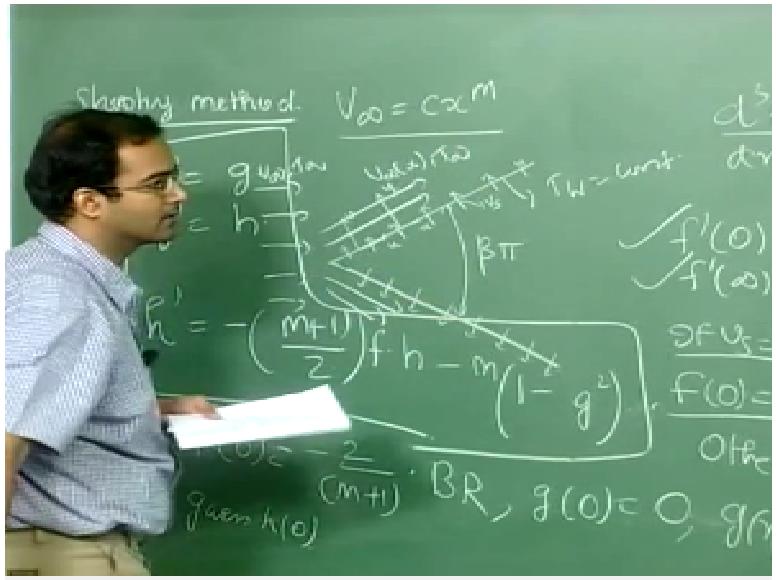

Okay so the Falkner Scan solutions that we did consider no slip at the wall and no transpiration okay so in most of the cases where you employ what is called as boundary layer control okay so you want to control the boundary layer because it is sometimes detrimental sometimes it is beneficial so accordingly we may want to control it to your requirement so most of the times you may have to remove the reduce the boundary layer or if you have a separation you want to reduce the separation so all these things can be done if you actually not make it impervious but slightly porous okay.

So that means you allow some kind of vertical velocity which could penetrate through the walls of this edge okay so this is basically your wedge angle b η pie okay and your coordinate system is this is your u velocity V velocity and this is your X and y alright the limiting case of M = zero leads to b η = zero for which you have the flat plate case where you have transpiration from the walls of the plate all right.

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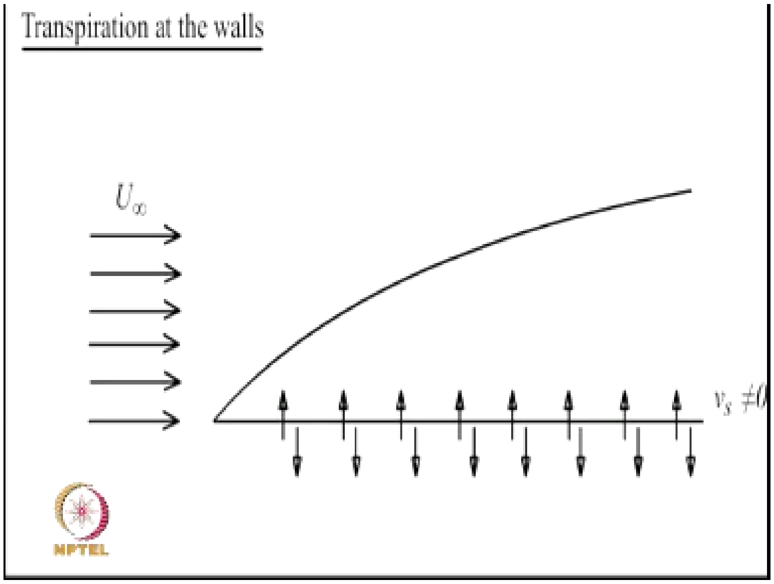

So we will consider the similarity solution for that today and in fact this will be the most generic case and you can also apply the case that we derived before where you say there is no vertical velocity and that gives you the solution we derived in the last class okay so for this particular case we will look at only the this flat plate but it doesn't matter we will apply the general Falkner Skan solution applicable to many configurations so the similarity equation that we derived in the last class is still valid.

Because the boundary condition is not going to change the similarity equation right so what we did was simply substitute for u u velocity V velocity and also all the gradients derivatives everything in terms of the similarity variables so that is still going to remain the same and therefore the similarity solution that similarity equation that we derived similarity equation for Falkner Skan solution is still going to be valid .So I am going to write it down.

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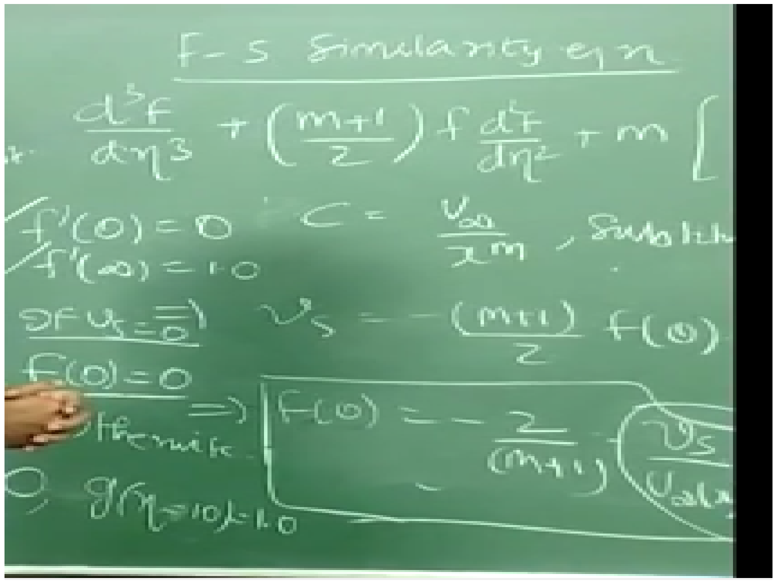

Okay so let us call this as equation number one this is your Falkner scan similarity equation okay so the equation is the same the only thing that you need to modify is what the boundary condition okay so earlier we put the boundary condition that since u = 0 .

So you are DF by D H at H = 0 so this should be = 0 right and at H going to ∞ this should be = 1 and apart from this what was the other boundary condition that we gave the Falkner Skan case we just talked to only about the flow right now f = 0 okay so what does that indicated V = 0 so this corresponding to the boundary condition V = 0 right okay so in fact if you had look at the expression for the velocity V maybe I can just write it down for your benefit we go back to yeah.

So that that is basically V by u ∞ is = half √of μ by u ∞ into X into H DF by D H - F so this was the expression for V so at the at the wall we have already mentioned that DF by DT = 0 so in order to satisfy the condition no-slip be equal to 0 this also has to be 0 right so this corresponds to the fact that we = 0 now in this case we cannot do that because you have some small velocity transpiration velocity okay which has to be specifically accounted for in the boundary condition so therefore the solution for the equation has to be different because the equation although it is the same the boundary condition is now different okay so this is how you are including the effect of transpiration into the Falkner Scan solution so far this let me let us look at what is the appropriate boundary condition.

So now let us write down the expression for V will let us derive it from first principles because now you are u ∞ is a function of X ok so let us derive from first principles and see how it looks so this is nothing but D by DX the stream function in terms of the similarity variable can be written as u ∞ μ X into f of η right so all of you agree so we had derived this for the Blasius case so the same thing is valid only thing now your u ∞ is a function of X okay so we are looking at solution to problems where u ∞ varies as CX power M okay.

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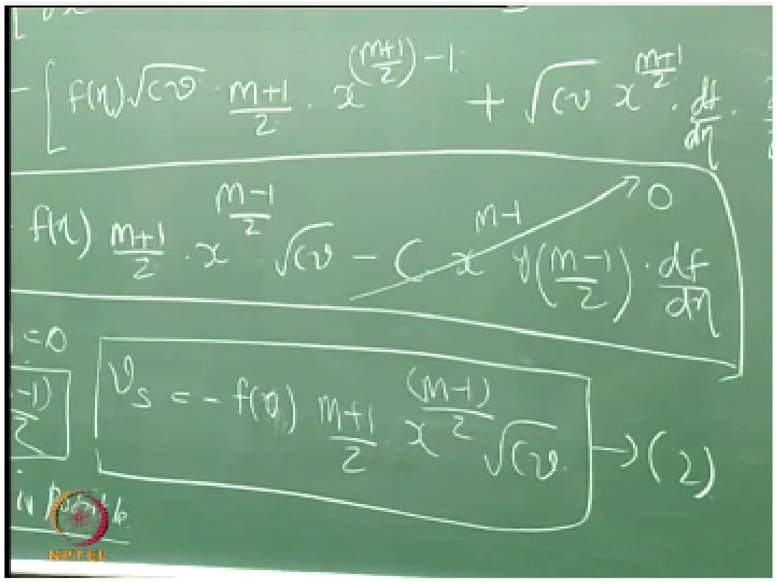

So if you now differentiate it let me also do that once myself - D by DX of we can take f out differentiate by + you have square root of μ ∞ UX x DF / D H in fact this will be B total derivative here DF by D H into the H by d x okay so you can substitute for u ∞ in terms of CX power M so this will be M + 1 inside the √okay so this also can be written as C nu X power M + 1 right so now how will you differentiate it so you have - f of H so d by DX of this √of C nu this is a constant so this is M + 1 by 2 right into X to the power M + 1 by 2 - 1 all right so then + you have √of C nu X power M + 1 so you can also write this as √of C nu into X power M + 1 by 2 into DF by D H into D H by DX.

So now again we have to differentiate H with respect to X and we know our η is nothing but Y √of u ∞ by nu X okay now again you ∞ you have to write in terms of X so this will be Y √of C by new so X X power M - 1/2 okay 2x M - 1 by 2 so when I say D H by DX so this will be M - 1 by 2 into y √of C by μ into X power M - 1 by 2 - 1 okay, so this is what I get for D H by DX.

Which I can substitute here okay so I can I can just write it touch D H by DX in the next step I am going to substitute and simplify so this will be - F of H M + 1 by 2 into X + X power M - 1 by 2 into √ C nu + - C into so what I am going to do is now my D H by DX has √ C by μ square root of nu and nu cancels √ C into √ C is C.

So therefore I get a C and X power M - 1 by 2 - 1 x X power n + 1 by 2 okay so what does it give X power M - 1 okay so therefore I will have X power M - 1 into I have M - 1 by 2 into y so I have Y into M - 1 by 2 into DF by D H okay so finally this is what I get for V velocity in fact you need to know this when we derive the Falkner Skan solution I think I omitted these steps because I want you to do that yourself now I have done that you can substitute then for u and V into the boundary layer equations and finally use cancel of all the common terms you will end up with this equation .

Okay I am doing the same thing now because we are more interested in the V velocity at the wall therefore I am doing this specifically now so now at the wall you are y = zero right so what happens is the entire term disappears okay so therefore at y = zero let us call this as let us call the vertical velocity for transpiration as some V small s okay so therefore this will be = V small s which will be - F of 0 H = 0 M + 1 by 2 X power M - 1 by 2 √of C mu ok so this is the expression for the transpiration velocity at the wall now what can you say what can you say about the variation of the transpiration velocity with respect to X now the transpiration velocity need not be a constant okay.

So you see from here it says that it can vary with X okay but to find a similarity solution this particular boundary condition gives us a constraint for the variation of V s so can you guess what should be the variation of V s how it should vary and it vary in area arbitrary manner say some x power n suppose it varies as X power M let us say okay so what will be the expression for f of 0 we have something to the power dependence on X right so once you have dependence on X is that a similarity solution so it will not be a similarity solution right so it will not lead to if you have a boundary condition in terms of F.

Which is a function of X so it does not give a similarity solution so therefore in order to make sure your f is a constant so only a constant value of f can lead to a similarity solution it should not vary with X or Y okay so for that case so your vs has to vary as X power M - 1 that is the only possible way correct only then your f of f at y H can be equal to 0 can be a constant okay so from this it clearly demonstrates that you can have a similarity solution to the Falkner Skan problem with transpiration only if your velocity varies as X power n - 1 by 2 so only then the similarity solution is possible okay only then your F of not is a constant for a given value of M then your F of not will become a constant okay.

So now putting this condition we can write in terms of the boundary condition for f of f at H = zero let us call this as equation number two.

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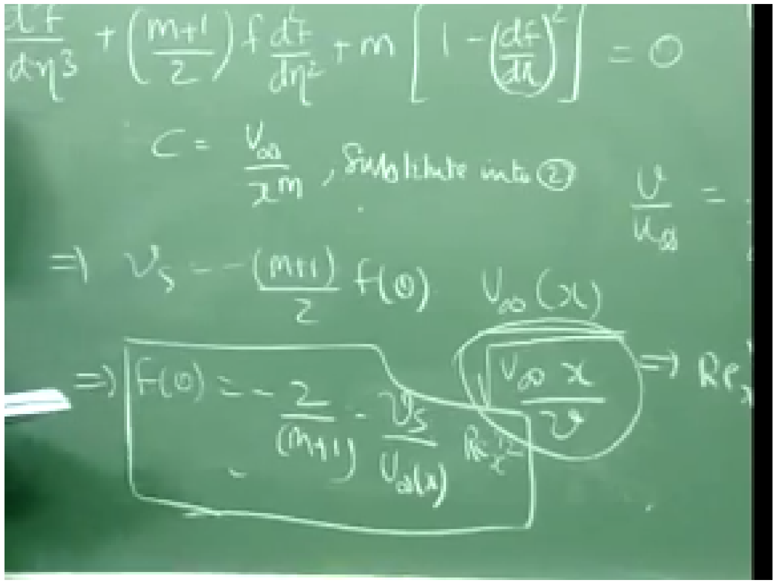

So therefore from two you can in fact you can also expand one step further you can write your C in terms of u ∞ by X power M so let you can write C is = because that is your condition here for the free stream and you can substitute into two so that will give you V s as - M + 1 by 2 into F of 0 into so if you multiply and divide by u ∞ I think we should be getting something like u ∞ of X by √of u ∞ into X by nu okay you please check this is correct u ∞ by X power M.

So this is basically √of u ∞ that is correct yeah X power m the thing that you give you square X power - half yeah right so that should lead to this particular form right so this is nothing but √of u ∞ by square root of x by nu ok so if you substitute this into c there you will be ending up so why am doing the same this rewriting in terms of Reynolds number that is all okay so now from here you will get your F of 0 as - 2 by M + 1 okay into vs by u ∞ of X now this is nothing but what Reynolds number to the power half so that will go to the numerator so this is your boundary condition now okay so you have put a condition on.

The variation of your transpiration velocity with your exit has to vary this way for a given value of n okay so when do you say that you have a similarity solution so you have of course reduced your partial differential equation in terms of the similarity variable H you do not have any X&Y here okay also the boundary condition should not have any terms in terms of x and y okay now when you write down the boundary condition for F of 0 now you see it is a function of some X okay, if you want to make this independent of X and make it a constant it has you have to make sure that your transpiration velocity varies exactly by the same factor okay.

So that is this that this is the constraint that is required in order to get a similarity solution for this particular boundary condition so on so that as this has to satisfy this particular constraint okay for a given value of M your V has to vary according to along X as this particular relation okay only then you get the similarity solution because once you rewrite now what you are saying is this entire term is now what constant okay if your V as varies that way your F not has to be constant so this entire term has to be some ratio which is constant or fixed okay.

This ratio is called the blowing ratio okay you can give maybe some other notation to that I do not want to give anything I just want to describe V s by u ∞ into re X basically is nothing but your blowing ratio which means that is something like ratio of V by X power M - 1 by 2 which is nothing but some constant okay so that blowing ratio is specified for a particular problem you specify that you're blowing ratios say some value something like 0.5 or - point 4 depending on whether you blow or whether you are sucking the flow and based on that for a given value of M there is a constant value of f(0) which gives you a similarity solution okay so you solve the same similarity equation.

Okay your other boundary conditions remain the same so what is your other boundary condition you are f dash 0 = 0 okay and your f dash at ∞ equal to 1 so these two boundary conditions are same the only boundary condition which is now slightly different is this and this is also not so different earlier this was 0 okay so for the case where m = 0 okay so this for this particular case vs will can vary only as X power - half okay so in that limiting case you can you can get F of 0 as 0 okay you can probably just check so when m = 0 vs varies as X power - half okay.

So yeah and if you don't have any transpiration velocity of course this will be zero so therefore if your vs is = zero so this gives your F of zero as exactly zero otherwise this is your corresponding expression so you know what is the blowing ratio depending on that you calculate the constant value of f (0) okay so the same procedure that you use to solve the other similarity equation the shooting method exactly the same way you do it okay only thing is now you have a value of f of zero which is some constant value instead of zero okay so that is the only difference okay if you want if you want me to write down the summarize the shooting method equations probably I will do that again.

So now you say your F ′ = some G G ′ = H and therefore what will be the corresponding equation this will be H ′ is = - of M + 1 by 2 into F x h + - M into 1 - what is DF by D H G G ^{2} = okay so this is your this is your set of Odie's first-order Odie's that you have reduced from the given third order Odie and the corresponding boundary condition so you are earlier it was F of 0 = 0.

Now F of 0 is = - 2 by M + 1 into blowing ratio okay so this entire thing let me use the notation V R it could be some constant value by either positive or negative okay so therefore for a given value of M this is some constant what is the other boundary condition G of 0 will be 0 because V of 0 is f ′ okay so the other boundary condition is what G at large value of H say H going to 10 = 10 for example this will be = 1 okay so you don't know the value of H so once again you can guess the value of H of 0 okay such that finally this condition is satisfied right so you know the value of F of 0 G of 0.

You do not know value of H of 0 so you guess h of 0 I iteratively I have given you the algorithm Newton's algorithm such that you better and better guesses for H so that it satisfies this particular equation ultimately okay so the same exactly whatever the Newton's procedure and shooting technique that you use to solve it is perfectly applicable only that your F of zero is not zero anymore it's just a constant value that is all and for if you write a program it does not matter for the program only you have to give some constant value there okay so you do not have great effort.

In so this is the most generic solution so you have this Falkner Scan solution you have generic boundary conditions for the case variant blowing ratio is 0 this becomes no transpiration case and for different values of M you get different configurations ok so let me just summarize and write down the corresponding values of the curvature at the wall that is f ″ that is basically nothing but H of 0 for different values of blowing ratio ok for a flat plate case .Since I have so many parameters now have to fix some of the parameters.

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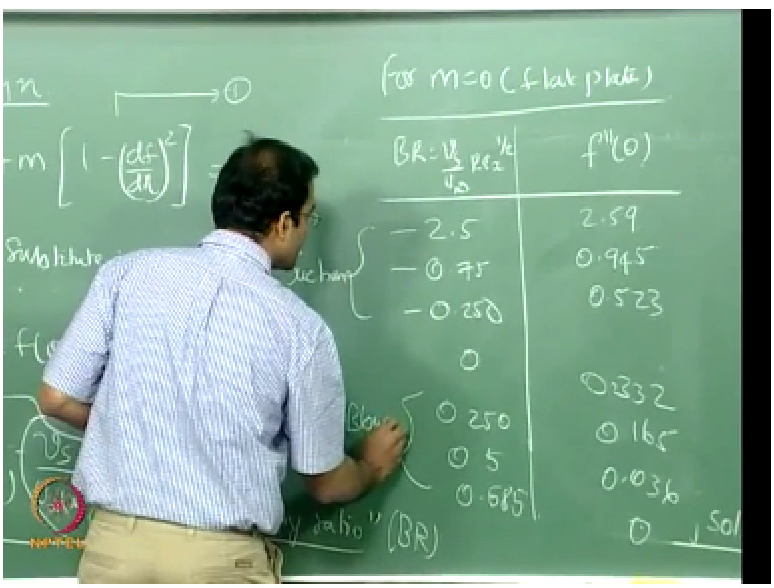

I am saying for m = 0 that is for flat plate okay now this is a flow problem so I do not have to fix any other number like prantle number or anything like that so for this particular case I have to check what is my F ″ of zero for different values of blowing ratio which is nothing but vs by u ∞ re X power half okay if I tabulate it in fact I have given you one problem in the tutorial second tutorial to do this I have asked you to vary the blowing ratios to different values and check what is the corresponding value of F ″ at 0 because why do you need F ″ 0 because this is directly related to the skin friction coefficient or wall shear stress okay.

Similarly you are the η ′ of 0 is directly related to a heat transfer coefficient so these are finally what you require in order to calculate your heat transfer coefficient and skin friction okay for the value of - 2.5 so these - values actually indicate suction right the negative velocities and zero indicates that okay now you tell me what should be the value for blowing ratio of 0.332 which we have already derived from lays solution so for 0.25 zero then you have 0.5 and 0.85 okay this is like two point five nine.

So you can see the curvature increases with suction and actually decreases with blowing point nine four five point five two three point one six five zero point zero three six and finally it becomes zero okay so what do you think is happening here it becomes zero for a blowing ratio of 0.585 sub flow is separating there okay at the separation your d u by dy is exactly zero where f ″ is also exactly zero there okay so once it separates there is no point in going be beyond a blowing ratio of 0.5 eight five that is why it is limited to that value so boundary layer theory is not applied applicable to that okay.

So this is all blowing and the above one section in fact the curvature you can see increases because you are trying to if you even have a separation if you suck it the separation gets minimized and you will have a more of a boundary layer flow okay so we will now quickly go to the heat transfer problem what do you think will happen to the heat transfer similarity solution how about the similarity equation for heat transfer will it change from what we derived for Falkner Scan solution okay.

.Let me also write the fact that the skin friction coefficient is nothing but twice of f ″ 0/ √of number okay so for the case of M = 0 this becomes 0.664 by √of Reynolds number ratio derived for the flat rate case .

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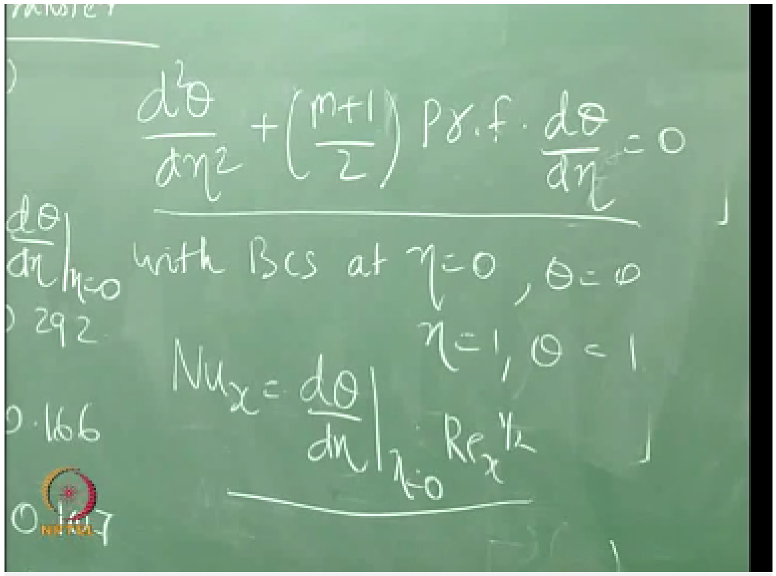

So coming to the heat transfer problem okay was this covered under incompressible flows transpiration how about the heat transfer advanced heat transfer and mass turns okay whereas to derive the similarity solution PTL solutions yeah integral solutions are anyway what we are going to do from maybe tomorrow onwards okay we will try to complete the similarity solution in the next one or two classes coming to the heat transfer so what should be the equation for similarity equation for heat transfer exactly identical to what we derived right no change and boundary conditions also know do not change to assume that you have a constant wall Temperature .Okay and you ∞ to ∞ and once it comes here this becomes view ∞ of X and still T ∞ okay.

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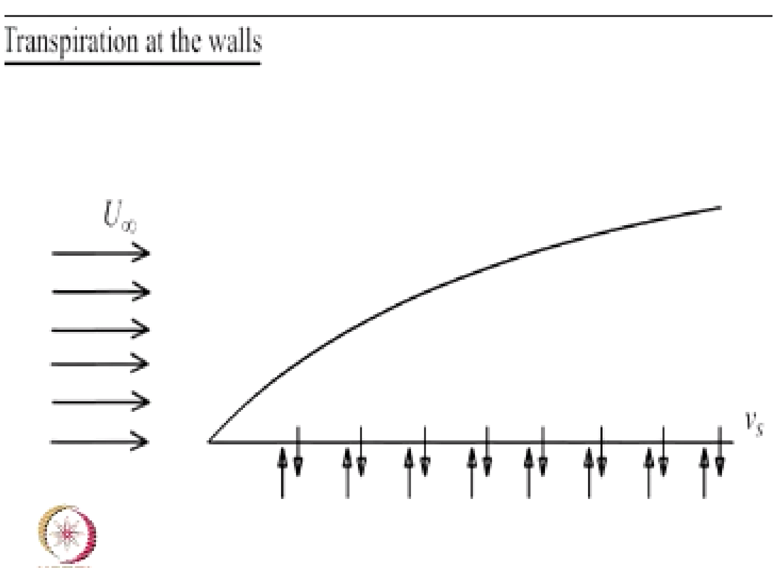

You so whatever equation that we derived last time for Falkner Scan the same thing remains and the boundary condition also the same okay so the only thing this solution changes because the value of f changes now okay and that is used to calculate the non-dimensional temperature okay asses the equations and boundary conditions are the same but the solution will be a different solution.

So therefore you are more interested in the Nusselt number if you are looking at Nusselt number this is d the η by D H at H = 0 into re X power half this is what we derived finally so this is how the Nusselt number is related to the non-dimensional temperature gradient okay so all we are interested is the η ′ at 0.

So let us tabulate okay for some more I think this is coming from you from one of your textbooks. So prantle number = 0.7 and also M = 0 so for this case you're blowing ratio versus your D then η by D H th η = 0 so 0 this is only for blowing not for section 0.375 0.5 so once again just like your velocity gradient keeps reducing your temperature gradient also has to keep reducing okay so 0.292 0.166 0.107 0.0517 okay so this with this information you can plug in for the say Prantle number of 0.7 for a blowing ratio of this you can plug.

In this value and that is the relationship of Nusselt number with respect to Reynolds number all right so the same thing applies the only thing you have to use the value of blowing ratio calculate different values of F and therefore solve the equation again by shooting method and from that you get your non-dimensional temperature gradient at the wall okay the same procedure whatever you did earlier same shoot shooting technique applies okay so any questions so far okay so one more last topic we will we will see for similarity solution before we move on to integral methods.

So this is for so far in the energy equation we have not accounted the viscous dissipation effects we have neglected them safely because we said we are looking for low-speed flows what if the viscous dissipation effects are also considered do we find a similarity solution for that case yes okay even for this Falkner Scan solution if you include the viscous dissipation effects into the energy equation we do indeed find a similarity solution okay so let us not do it for the general case let us do it for a flat plate case and I just want to tell you that that the same thing can be done for the case of Falkner Scan solutions also.

Because it is only the energy equation which is affected okay the flow remains the same alright so let us move on to that topic where we consider viscous dissipation effects any questions on this so far because we are not considering variable properties okay we are assuming that constant property assumption is still valid if your property is a function of temperature then yes then you do not have similarity solution in that case okay in all these cases we are assuming that the temperature the flow field they are decoupled therefore you first solve for the flow and that is used into the temperature equation alright.

So it is a constant property assumption okay so of course you know most of the flow flows where you have strong heat transfer at high temperatures and strong temperature gradients you will have considerable variation okay so that that is the assumption that we are using in all of this .So let us look at some of the assumptions before.

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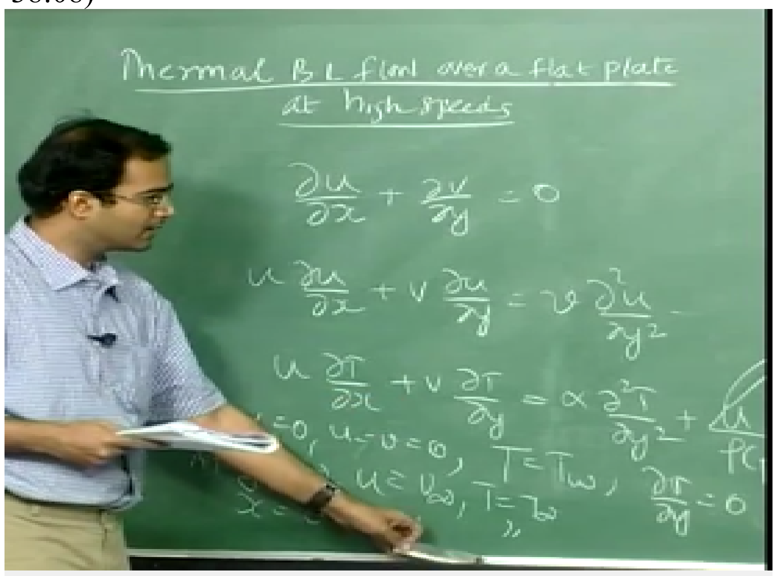

We the thermal boundary layer flow over a flat plate and although I am doing this for flat plate the same thing is applied to even wedge flows okay so when I say high-speed here. I am just indicating that the ratio of Eckert number by the Reynolds number is very important and therefore the viscous dissipation has to be considered so to write down the boundary layer equations in the case of flat plate anyway I am not going to consider the pressure gradient okay so these are your complete equations for flat plate or you do not consider pressure gradient term now we have also included the viscous dissipation if you non dimensionalized this you will get this as the ratio of Eckert number by Reynolds number.

Where how is your record number defined do you remember u ∞ square by CP say T wall - T ∞ okay so now this is subject to the boundary conditions that at y equal to 0 your U and V are 0 and for temperature you can say you can either have a constant wall temperature or you can also have an adiabatic temperature adiabatic wall I mean you are not allowing any heat transfer to take place at the wall but internally the temperature will vary because of the viscous dissipation you are understanding the fact.

So your risk is dissipation is an internal generation of heat okay you do not need to transfer heat externally with an internal gender because your kinetic energy is converted into this which is dissipated into heat at the viscous level at the molecular level so that is changing the temperature locally okay because this gradient of course varies inside so therefore there is a local variation of temperature so even if you maintain an adiabatic wall you can get a temperature profile if you have is this dissipation.

So therefore for high speed flows and considerable effects can be observed where your temperature will vary from the free stream temperature because of viscous dissipation itself okay so you can also have heat transfer case even if you do not have any heat transfer through the wall okay so therefore you can also maintain a boundary condition that you are DT by dy 0.

In either of these cases you can solve for heat transfer okay so similarly at why going to ∞ you are u = u ∞ and your t becomes to ∞ and same thing at X is = zero okay so these are your governing equations now you can use the same boundary layer similarity expressions and substitute for UV all the gradients I am not going to do that okay so if you substitute that you will get a final similarity expression which is something like .

So if you if you substitute and you clean up all the unnecessary terms this is the final similarity expression solution equation.

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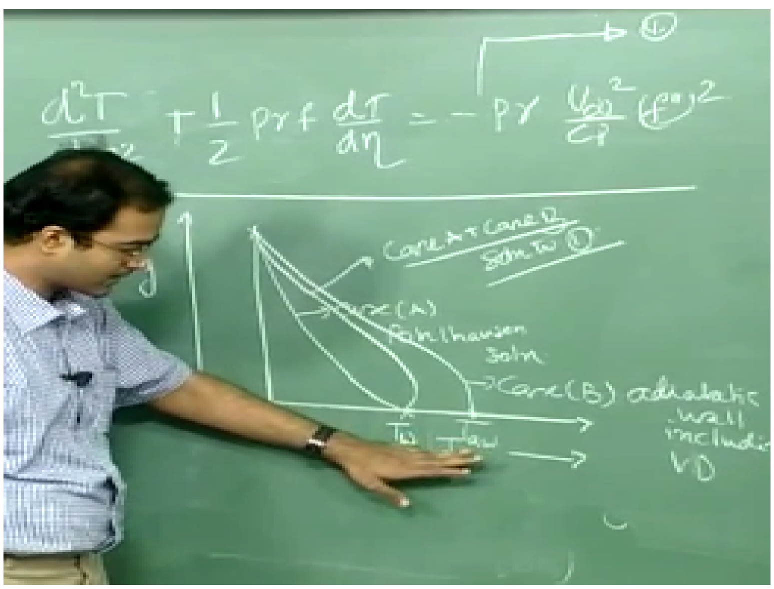

For temperature because the flow still is the same as the flat plate blushes solution okay so only the energy equation will be changing a little bit so on that change will be a term which comes on the right-hand side because of the viscous dissipation you have now this additional term which comes into the similarity equation right now why do I write this in terms of dimensional temperatures why am I not directly non dimension ling here.

This is because now you have in fact two temperatures with which you can non dimensionalized okay so you can have your free stream temperature T ∞ and you have what is called as the adiabatic wall temperature okay so this is the wall temperature if you maintain an adiabatic boundary condition at the wall as I said you can have a heat transfer due to internal generation due to internal viscous dissipation and that will result in a wall temperature which corresponds to the case of no external heat transfer okay.

And that is defined as the adiabatic wall temperature okay so if you don't have any heat transfer instead of using a you know instead of using something like a free stream temperature you can also use the adiabatic wall temperature so therefore you have too many temperatures here and we don't know which one we have to use to scale or non-dimensionalized the temperatures therefore we will split this problem into two sub problems one in which we have T = T wall for which we have already derived .

The similarity equation the pole Hassan's equation okay the other problem where you have only adiabatic boundary condition okay so finally if you look at the energy equation since this is quasi linear and also the boundary conditions are linear we can say safely say that the solution can be also split into two solutions and we can combine them in some linear fashion okay so one solution will be for Paulo ocean solution the other where you have adiabatic temperature including the viscous dissipation adiabatic condition with the viscous so these two solutions can be then linearly combined and we can construct the final solution to this particular problem.

Okay so just to give you an idea how the temperature profile looks before we wrap up.If you plot your temperature in the x-axis and so somewhere so at Y going to ∞ your temperature will be your free stream temperature okay so that somewhere at Y going to ∞ that will be the free stream value and from there if you maintain as I said the problem number one is self solving this without the viscous dissipation with constant wall temperature that is the pole house and solution.

How will you get your temperature profile it will be like this right this is the variation for case one or problem one this is nothing but your polymers in solution okay now if you do case B that is you apply a adiabatic condition at the wall but you include the viscous dissipation term how does your temperature profile will look is something like this okay and the temperature value that you see here is the highest that is nothing but your adiabatic wall temperature and the condition that the flux has to be zero make sure that this becomes normal right at the boundary okay.

Now if you have a combined case the case that we are looking here has both the solutions which you can linearly combine okay you have a constant wall temperature but you also have a viscous dissipation so then it has to force if you forcibly returned to this wall temperature which is a boundary condition but it will go through a process something like this like this okay so this is your case B okay this is your adiabatic wall including stress dissipation now what I have drawn here okay this line this profile is for case a + case B which is actually the solution let us say to equation number one with the boundary condition that you can have either a constant wall temperature or the idea batting temperature so finally because of the viscous dissipation you force the temperature at the wall to T wall.

But the viscous dissipation effect will deviate the profile from the polar sand profile okay so as I said that is the clue to solving this problem which we will do tomorrow we will split the solution into two parts case a will be the polymers in which we already did case P will be the adiabatic case with the viscous dissipation which we will do and for each of this there is a way of to non-dimensionalized the η okay so in this case we non dimensionalized with T ∞ t - T ∞ by T wall - T ∞ in the other case we cannot use T wall.

Because there is no wall temperature boundary condition there so we have to use a different temperature scale which we will redefine okay and finally we will express the final solution in terms of the linear combination of those two solutions okay so we will stop here today.

Similarity solution for flow and heat transfer with transpiration at walls

End of Lecture 16
Next: Thermal boundary layer in high speed flows

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