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Video Lectures on
Convective Heat Transfer

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Lecture 15
Falkner skan solutions for heat transfer

Good morning all of you so we have completed nearly about 10 classes so far I hope most of you are following are what is happening in all these 10 classes and being the first tutorial is due on 14th of this month and I will also most likely early next week upload the second tutorial on similarity solutions ok so most likely will complete the similarity solutions in the next couple of classes and start the integral analysis or external flows okay, so we have quite a few to π cs to cover so I have to move a little bit fast and I hope your earlier knowledge of fluid mechanics will help you in covering all the things that we would not be discussing here ok.

So let us continue with the Falkner skan similarity solutions that we had derived yesterday for these are for flows with.

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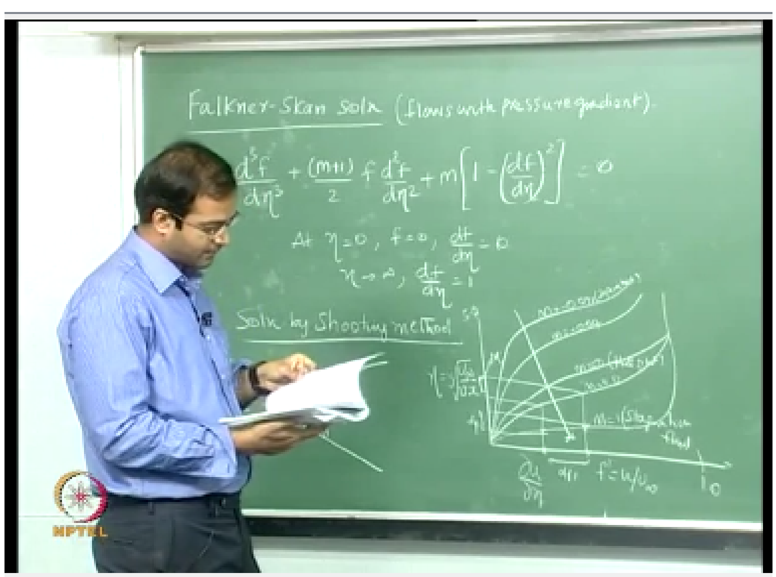

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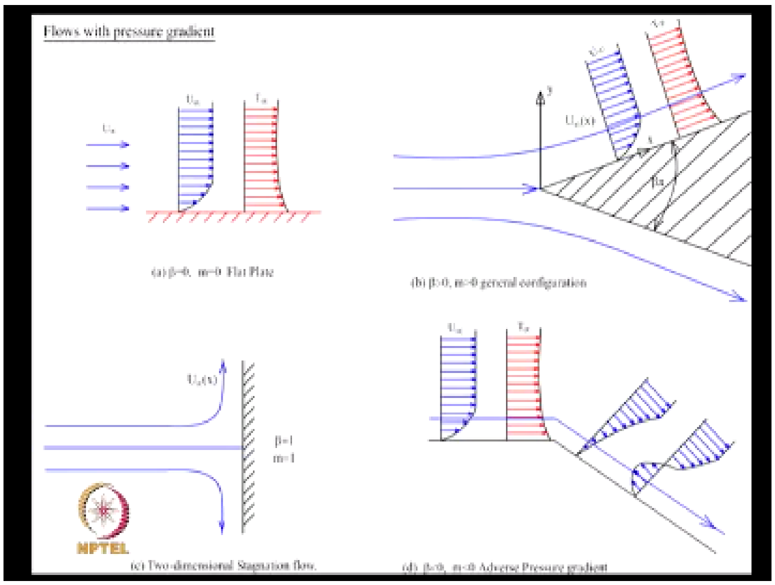

Pressure gradient you and the final similarity differential equation appears in this particular form and these are the boundary conditions and once again you have to use the shooting method to solve the OD in numerically I have also given you the three first-order Ode's to which it can be reduced and the same boundary conditions like that of the Blazes equation.

So once you solve this for okay so one more thing is that so these are some of the velocity profiles if you solve the OD by the shooting technique and this is how the velocity profiles look and these are the similarity profiles so I am plotting H on the y axis and F prime which is nothing but the non-dimensional velocity on the x axis and I can substitute different values of M into this and for each value of M I can solve the OD by shooting method.

And I can plot the profiles for example M = zero this is the flat plate case you get a profile something like this exactly similar to the Blazes profile and for positive values of M greater than zero some say like 0.33 something like this so one it is something like this so if you look at M = 1 this is the stagnation point flow rate and this is your flat plate alright okay.

So therefore if you look at your values of M which are basically increasing okay so the profiles are in this particular fashion and for the values of β the red jangle I mean let me once again draw the representation so that you can understand okay so can you tell me how the M and β related or β π is your wedge angle β by 2 - β okay so for negative values of β that is I was giving an example when you roll this in the anti-clockwise direction so that they both coincide and that is the m = 0 case and if you roll it further roll this particular surface further down it becomes negative okay.

For the negative values typically if you make it more and more negative you will get adverse pressure gradient flows right now in the example that we have seen yesterday and for a particular value for M which is exactly - point 0 9 1 you will see the flow is separating okay so at this particular location the boundary layer theory will not be valid anymore so there is no point in going further less than - point 0 9 1 so this is the limit of M where you can up to which you can go negative and the value of M up to which you can also go on the other side for typically we are more interested up till the stagnation flow so we can go up to m = 1 okay.

So what is happening to the velocity gradients so typically if you want to calculate say D u by D θ as you keep increasing your M what is happening to the U by D θ huh smaller hmm so typically if you if you are saying okay so I say this is this is my θ range okay if you compare 0.32 so this is my change here okay this if I if I look at F for this value so in which case your gradient is higher whether is it for 0.32 or whether it is four so he in this case your θ value are this is your D θ here for this value of D F Prime and this is your D θ here okay in which case the D θ will be more hmm.

For this case so therefore in which case the gradient will be higher for this case okay so as you keep increasing your M okay the velocity gradients become more and more steeper okay so therefore so this is to do with the particular kind of flow that you are looking at so typically what you can do I will give you some problems where you can substitute the corresponding configuration according to the value of M and you can get these profiles and you can try to see how the variation in the slope appears to you.

Okay so this is all with respect to the solution we are more interested in the derived quantities like the skin friction coefficient for the flow as far as the flow is concerned.

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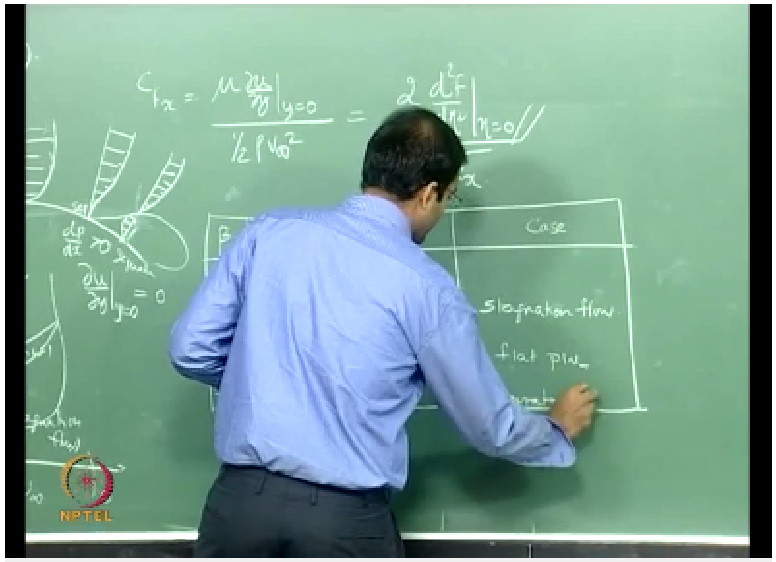

So we can calculate the skin friction coefficient locally so this is nothing but u D U by DY okay now if you substitute everything in terms of the similarity variables so you will be getting an expression which is like this in terms of F and θ okay so this depends on the curvature at the wall this I know you all know but the case of flat plate the value of this is 0.332 so this leads to the familiar expression CF is 0.66 word by ^{2} of last number okay .

So now all we need to know is for a given configuration what is this curvature at the wall once we know that we can calculate the skin friction coefficient locally for that particular configuration so once you solve the equations by the shooting method for different values of M we can have a nice tabulation where we can grab relate the curvature terms for different configurations so the value of β this is the value of M and this is D square F / D θ^{2} θ = 0 and this is the particular configuration or case that it corresponds to okay.

So β equal to 1.6 okay so 1.6 times π so you will find that there is something like this okay or maybe you can say it becomes like this you have a flow which is like this right it is much more than one so it is much more than your rectangle is more than π all right so in this particular case you can you can have flows like this and the corresponding value of M will be = 5 if you substitute into this expression okay if you compute the slope the curvature at the surface this will be 2.6344 exactly in fact you can do it in fact I will give you an exercise where you can do it and compare with these values okay should be getting the same values.

And now the case of β equal to 1 M equal to 1 and 1 point 2 3 to 6 what is this case stagnation flow all right so β is 0.5 M is 1 over 3 and the corresponding value is 0.75746 β = 0 m = 0 what will be the value 0.332 this is the flat plate and 0.14 the corresponding value of M blue 0.0654 and the corresponding value is 0.16372 ok and finally 0.1988 okay corresponding value of M can you guess what is the corresponding value of M -0.091 okay that is this particular case that we have so what do you think will be the curvature for this, this is a separated flow okay.

At the point of separation what is the slope what will be DU by DY for separated flow so if you have a if you have a flow suppose you have a gradient like this and here you plot they have profile like these okay now at the point of separation the profile becomes like this and after that in fact so this is DP by DX greater than 0 okay this is separation point this is separated flow so you can visualize the flow coming like this and at this point dηching and then you have a nice big separation bubble here okay.

So what, what how do you check that flow has separated or it is about two separate huh what is that not curvature first you go to the first derivative before you go to the second derivative now DU by DY for the separate four condition what is the condition for separated flow DU by DY at Y = 0 , this is the case okay once it is separated what is the condition there it should be negative okay therefore what will happen to curvature when it is at the separation state 0 right so this has to be 0 and this is a separation case separated flow all right so I think this is giving you final summary.

And special case where we can apply this stagnation flow if you look at flow past a circular cylinder.

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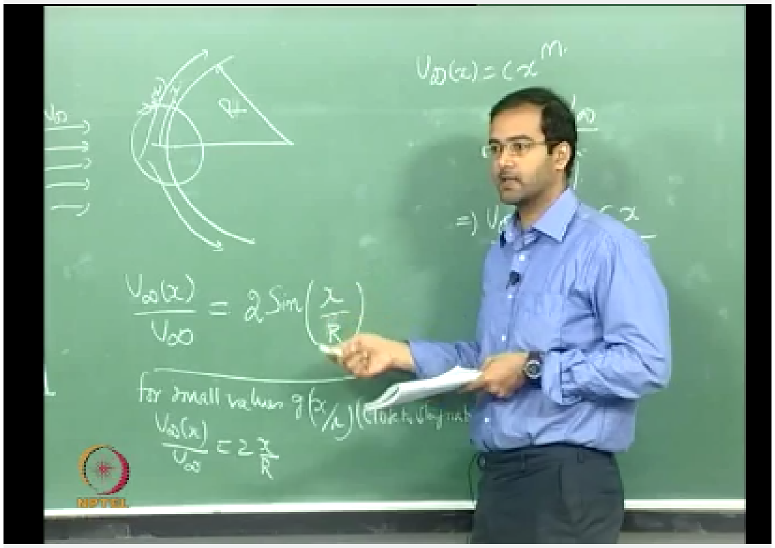

Okay so this is your radii R and this is you are approaching free stream velocity which is constant and then the free stream now when it travels over the surface it becomes a function of your local coordinate X and where X is defined in this particular fashion okay that is the sector location sector distance starting from this point where it is 0 okay.

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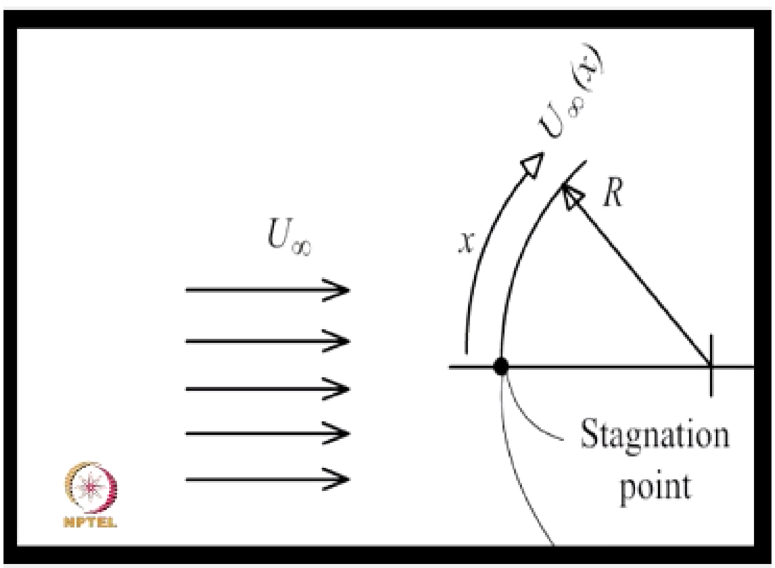

So if you look at the case of flow past a circular cylinder from the potential flow theory you can actually calculate how the U ∞ is very locally okay so the profile is given I think you must have studied this in your incompressible flow course so u ∞ by you any guess how it varies if you go along the periphery or the circumference of the particular cylinder how does the local free stream vary what , what is the value of velocity here it has to be 0 and where does it reach maximum where your $\theta = \pi /2$ okay and X is = R θ basically so X / R should be equal to $\pi /2$ at that location it becomes the maximum okay so it should be a sinusoidal variation okay only you have sine 0 is 0 sine π / 2 is 1.

So if you say u ∞ x by u ∞ it has to be a sine variation and what should be the variable X /R all right maybe I can use capital R because this is your radii of the cylinder and what should what factor should come here at X by R = π / 2 this will become 1 does u big u ∞ become u ∞ there it should be 2 because it becomes exactly twice the because it has to accelerate again from here once it accelerate it has to go more than the free stream velocity there ok.

So this is your in viscid velocity profile for a circular cylinder now if you are looking at region values of X which are which are very small that means close to the stagnation region okay so then you can approximate this sign X / R simple X/R for small values of X okay so for small values of X / R that is close to the stagnation you can write your u ∞ by constant free stream velocity is 2 X /R okay.

So therefore if you look compare this with your Falkner skan form of velocity profile which is like something like u ∞ of X is C X power M so what can you deduce what should be value of M what should be the value of C if you compare these two C is = 2 U ∞ by R okay and what is the value of M 1 okay so therefore this profile will be something like okay so what kind of flow does it mean stagnation point flow okay so when you are looking at region close to the stagnation even for a curved surface like cylinder okay you can approximate the flow pattern to be similar to the stagnation point flow for which we have already calculated the profiles and the curvature at the wall okay.

So this is a very important thing so it does not limit the Falkner Skan solution does not mean it is only applied to a wedge configuration like this it can even apply to any stagnation flow even for a bluff body like this not it does not need to have a sharp corner okay provided you are looking at only region close to the stagnation region so if you simply use C equal to 2 ∞ by R so this is nothing but the stagnation flow okay the same solution will hold for the cylinder also as well you okay.

So this is a very important useful correlation because to calculate for example the heat transfer in the stagnation region of a cylinder we can solve the energy equation from the Falkner Skan solution and you can apply that to get the local nusselt number profiles for the cylinder okay so for small values of X your sine θ can be replaced as θ right okay so therefore now we will move on to the heat transfer problem so the boundary layer energy equation when you write it down for the flows with pressure gradient or without pressure gradient they are both the same okay.

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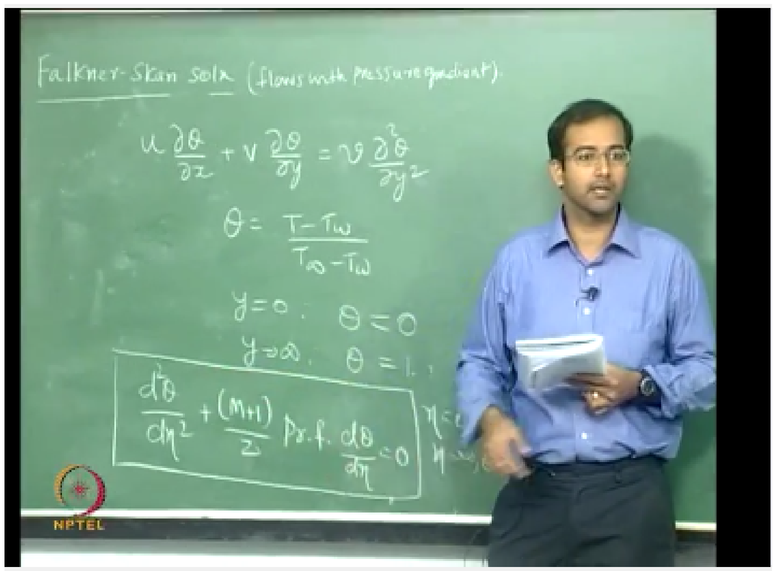

So without the viscous dissipation term this is your energy equation where your θ is defined how T - TW by T ∞ - TW so I want my temperature profile to look identical to the velocity profile alright and the boundary conditions at y equal to 0 your θ should be it should be zero okay if I if I had defined my θ this way okay it has to be similar to the velocity profile right that Y going to ∞ θ should be one okay.

Now what I can do from the definition of the similarity variable that I use also from the definition of stream function which is a function of the similarity variable I can plug in for u we convert all x and y in terms of H okay the same way that we did for the blush is energy you can you can apply that here because it is no different and except that when you write the similarity variable H here this is a function of U ∞ of x by nu X rather than simply u ∞ in brushes so when you differentiate this with respect to X you have to be careful now you have to account for the variation of this so you can substitute CX power M okay and then you can differentiate it okay.

so for example I will if you say that this is Y √ of CX power M by nu X so this can be written as Y √ of C by mu into X power M - 1 by 2 okay therefore if you say you are D H by DX so this will be Y √ of C by nu into M - 1 by 2 into X power what for differentiate what should I get exponent M - 3 by 2 so I can write that as M - 1 by 2 - 1 okay so once again why C by nu X power so this entire thing is what H so this will be M - 1 by 2 into H by X ok so you should take care when you are differentiating and transforming the variables now that your free stream velocity is a function of X ok so appropriately you do all the substitution and transform this in terms of the similarity variable and everything in terms of F and you will get the similarity equation for energy the ^{2} θ by H ^{2} okay.

So this is your energy equation okay for the case of M equal to zero this reduces to the flat plate energy equation similarity solution so on additionally here you have M plus 1 by 2 because of the factor of M which comes in the free stream velocity okay we substitute all of that you will definitely be able to reduce this and the boundary conditions as θ going to 0 θ equal to 0 it R going to ∞ θ equal to 1 alright okay so once you know the flow you know the value of F you plug it in for a corresponding value of M you can solve this once again by shooting method okay the same way that we have been doing and you can get the profiles for θ as a function of H alright.

So the same procedure repeats here now what I am going to give is just the way we tabulated the curvature at the wall for the velocity profiles I am going to give you once you substitute and get the values you can get the slope at slope of the temperature at the wall for different values of M okay for different configurations how does it look because this is required to calculate the nusselt number nusselt number depends off upon this temperature slope at the wall okay so if you do the tabulation.

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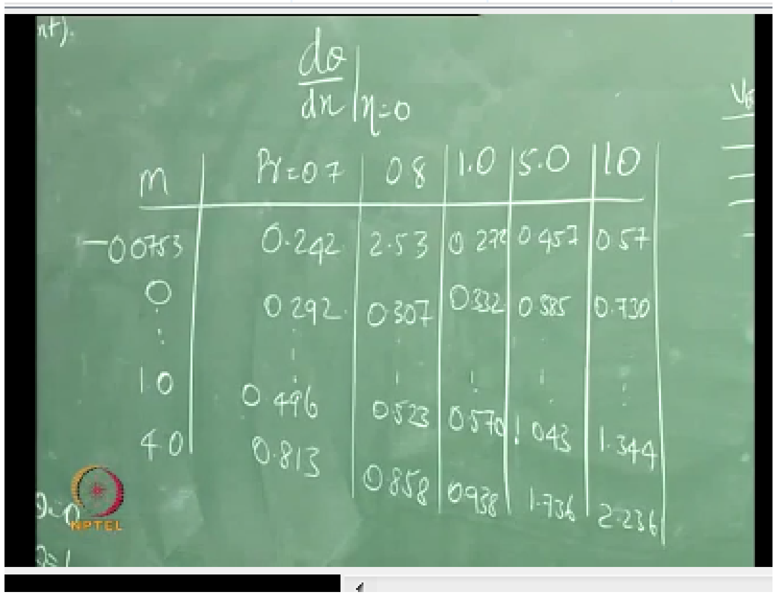

So D θ by D H at H equal to zero okay so M Prandtl number now you should realize the temperature profiles are now function of your velocity profile your prandtl number and your M okay so far a given value of M for a given value of M you know the velocity profile put that function the value of M and also the Prandtl number which you want to calculate so both of all the three how to be simultaneously fixed of course if you fix your M you fix your F also okay and also you have to decide which plan tail number you are calculating.

So you can tabulate this for different values of mantle number so for M is – 0.0753 okay I am just giving you some value of M here for which if you have Prandtl number of 0.7 this value becomes 0.242 okay I am just tabulating all the values here okay so anybody remember now value of M = 0 , flat plate what should be the value at prandtl number 0.7 probably a must-have yeah I think you can calculate and tell me.

What should be the value of D θ by D H for prandtl number 0.7 okay so for Prandtl number of 1 what should be the value 0.332 why yeah because for the case where your parental number = 1 I mean the velocity and the temperature profile should be identical so the curvature D cube d square F by D H square should be exactly equal to D θ by D so this should be 0.33 2 so this value should be 0.332 into Prandtl number power 1 / 3 okay so what should be the value.

So it will be something like 0.292 will be reduced so then this is 0.307 and 0.585 this is 0.730 so on so if you go to m equal to one the stagnation flow 0.496, 0.523 okay so this has some kind of values I am just giving you this because tomorrow when I ask you to compute using the shooting method you should be able to match with these tabulated values all right so why we are calculating the slope okay so because we finally are interested in the heat transfer coefficient and nusselt number.

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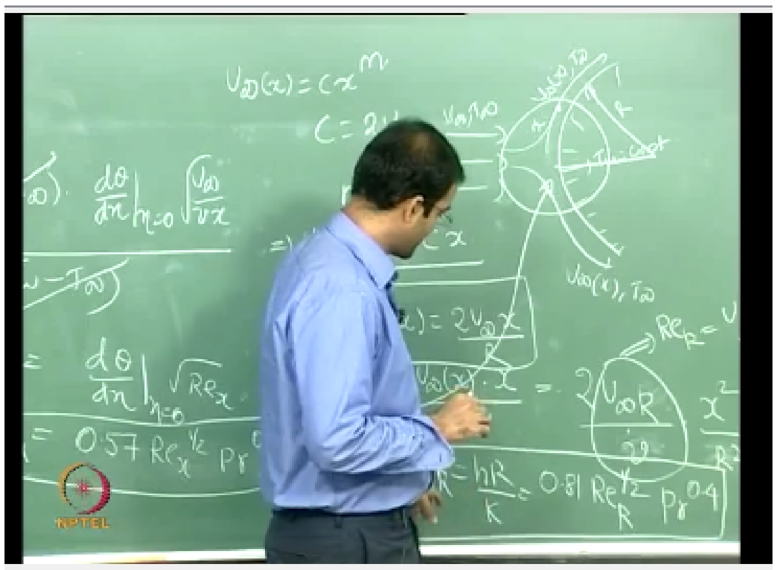

So therefore you can define your local heat transfer coefficient the same way the wall heat flux divided by T wall - T ∞ okay if you substitute for - K DT / DY at Y = 0 and write in terms of D θ / D H you should get a t1 - T ∞ into so you can write your DT / DY as D θ / D H at H = 0 into D H / DY okay which is nothing but $\sqrt{}$ of U ∞ by mu X divided by T wall - T ∞ okay so finally if you define your nusselt number local nusselt number as HX by K so that will result in D θ by DT at H equal to 0 into so you have X here so $\sqrt{}$ of u ∞ X by nu which is nothing but your if you if you divide it if you divide multiply H into X by K so what happens to this particular term Reynolds number ^{2} of Reynolds number okay.

This is the same as what we did for the flat plate okay there is nothing new here only thing you should now know for which configuration the value of D θ by D H you have to pick put it there and then you will get the nusselt number profile for that particular value of M okay and also now it is a function of Prandtl number so you may have to fit a curve as a function of Prandtl number and you should bring the prandtl number dependence okay now for the stagnation point flow if you are interested in values of Prandtl number about 1 okay.

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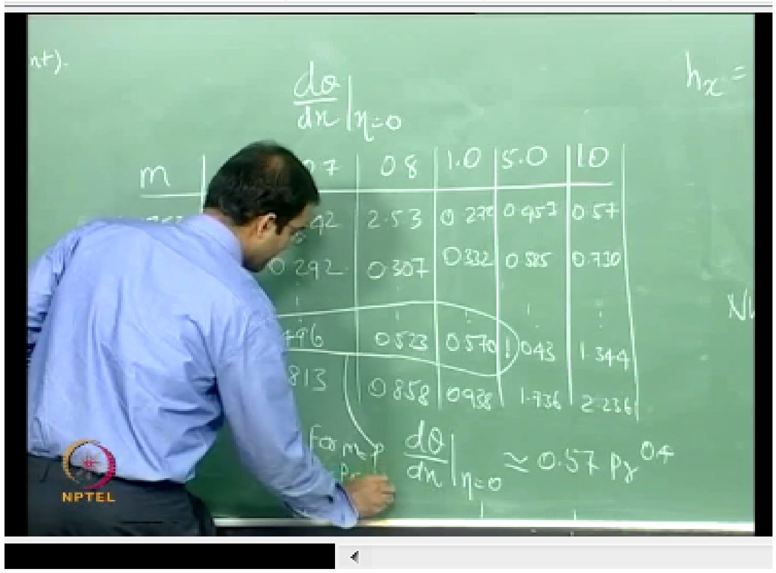

That is something in this particular region right here okay you can in fact fit a curve to the values of D θ by D H in this prandtl number regime close to one and you will get a nice curve fit of this particular form which is 0.57 times Prandtl number to the power point 4 and you can you just check it can you substitute prandtl number as 0.8 and check whether you are getting something close to 0.523 okay so , so this is this is the kind of fit that you can do for m = 1 around prandtl number close to 1 okay .

So you are not saying here parent a number is exactly 1 it has some prandtl number dependence but I mean the dependence is relatively kind of similar you know you have about 0.5 to 0.57 variation here okay so therefore if you substitute for that so your D θ by D η is a function of prandtl number now this becomes 0.57 into REX power half prandtl number to the power 0.4 so this is the case for m = 1 prandtl number close to one okay so this is the local variation that you find for the stagnation point flow case of course you are you know the variation with respect to Crandall number for m equal to zero.

Pole houses already did that you did the curve fit for different values of Crandall number small planted numbers intermediate and large and you can use those values okay does it does it make sense okay so that this is a reasonably good fit okay for prandtl number close to one okay now so we this is one example to show you for the stagnation for example stagnation flow how we can define the local variation in nusselt number okay so all this can be also equally verified by you can calculate the values of the slope compare that for different values of M and Prandtl number with the tabulated values and you can yourself correlate with these values right here okay so now one more thing as we said if you look at the flow past a cylinder okay.

The flow that we are looking at right here so apart from u ∞ suppose you heat this particular surface so you are maintaining this t wall equal to constant okay so this is a flow apart from the flow you also have a temperature profile okay now this u ∞ is a function of X here are still it is having some temperature T ∞ we have a velocity boundary layer you have a thermal boundary layer which is simultaneously growing so if you are interested in the stagnation region for the cylinder what is the variation in the nusselt number okay so now as we has we already shown you can describe that by the stagnation flow m equal to 1 the same correlation will apply for this region as well okay now X is defined in this particular manner okay where this is your R this is your origin okay.

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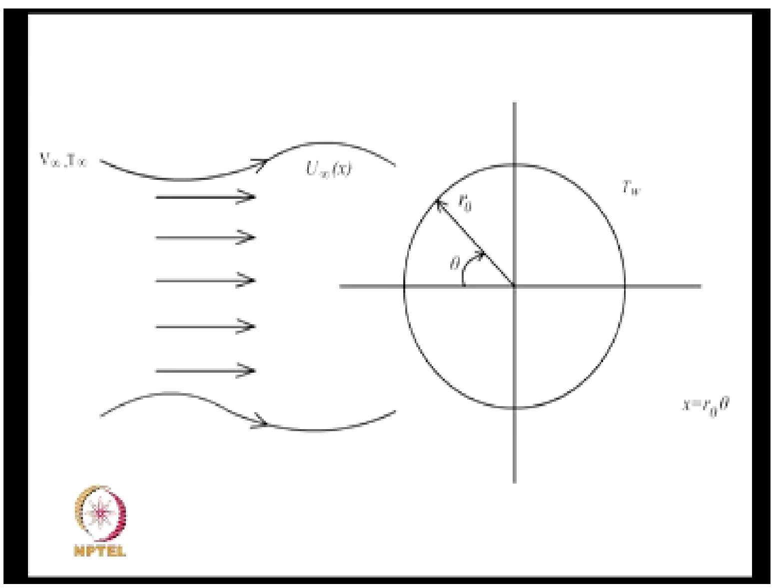

You so now it is it is not so convenient to operate in terms of local X for a cylinder and sphere what is the more convenient characteristic length instead of X diameter okay so usually when you talk about cylinder flow past circular cylinder the characteristic length that is chosen is the radius or diameter of the cylinder okay so therefore we can transform your local X into terms of a R similarly in the reynolds number also and we can define a reynolds number based on the characteristic length.

Which is the radius of the cylinder okay so if you do the transformation so you already know so how do you do this transformation you already know that your re X is defined based on U ∞ X into X by nu so this is the definition of your local Reynolds number right now so where you are defining based on your local velocity yeah.

That so that is how we have to transform we will see how we will transform it okay so now you can replace this as you can write this as 2 U ∞ you can you can you can write your U ∞ in terms of u ∞ of x in terms of 2 U ∞ X by R so this is your local velocity profile related to the free stream velocity profile for the stagnation region okay so you can substitute for u ∞ in terms of the constant free stream velocity X and R okay if you do that this will be X / R into X / mu okay so now what i can do is i can multiply and divide by r so i can say that this is R and there will be an X^2 by R^2okay so therefore this is how my local Reynolds number is related to now I can define I can define this as my Reynolds number based on the radius of the cylinder okay which is nothing but u ∞ R by mu.

This is now this is the constant free stream velocity okay and the characteristic dimension is the radius so now I have transformed from the local velocity and the local coordinate I have transformed that to constant velocity and a fixed coordinate so the fixed dimension here is our okay and you have a factor X^2 / R^2 therefore if you substitute you can you can find that your current nu X is also HX by K right if you substitute for re X from there you can finally write nu in terms of the radius which is nothing but HR by K so you have a 2^2 which comes out and multiplies with 0.57 that becomes 0.8 1 and this will be RE R to the power half and your prandtl number to the power point 4 okay.

So this is what you finally get in terms of fixed dimensions are and so this is the expression for the cylinder when you look at the stagnation flow okay so in terms of the cylindrical dimensions okay so you can simply transform from your local coordinate to the cylinder is fixed dimensions which is basically the radius okay the same way you can also do that for sphere okay for the case of three dimensional wedge flows okay now whatever we were discussing so far are our two two-dimensional which flows the same two-dimensional which flows can be transformed using what is called as a manger less transformation okay thank you if we happen to go through the boundary layer theory by listing okay.

So he talks about three-dimensional boundary layer so there one way of deriving the similarity solution for three-dimensional which flows is to take the two-dimensional which flow solution apply what a particular kind of transformation called mangle less transformation and you will get the similarity solution for 3-dimensional which flows and for that particular case where m equal to 1 for 3-dimensional which flows that is the stagnation region stagnation flow for 3-dimensional which flows and that will be similar to the stagnation region flow for sphere in 3d okay like we have equivalent to a 2d which flow stagnation region we can correlate that to stagnation region of a cylinder.

Same way the three dimensional ax symmetric which flow stagnation flow can be correlated to the stagnation region of a sphere okay so you can also have a similar relationship for nusselt number for a sphere from the applying the magnetization anyway manglers transformation is beyond the scope of this class so I am just giving you an idea that you can also do that for three dimensional which flows okay so I think with that we have more or less covered the flows with the pressure gradient terms any question so far whatever we have done yes.

So this just only says the nusselt number in the stagnation region that is that is only for that okay so it is not really you are not really going varying the X because you are a variation in the X is actually confined to a small region near the stagnation point so you just say that what is the stagnation point nusselt number for example okay so based on the free stream reynolds number and for a particular value of prandtl number you directly get the nusselt number in the stagnation region of a cylinder okay you do not once again look at the local variation of the nusselt number and things like that because the stagnation region is a very small region correct so the next class on Tuesday what we will do for the same which problem okay.

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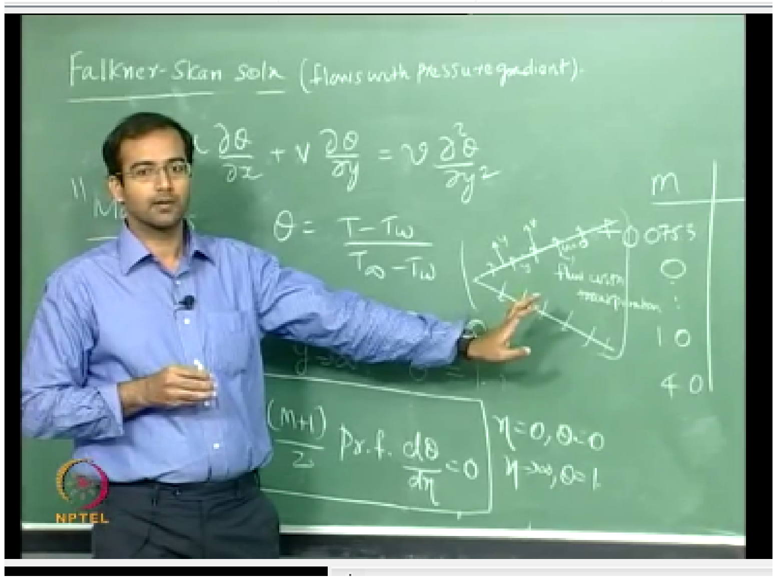

Now the which problem that we have taken we can add one slightly one complicated boundary condition which is so far we have assumed that your no slip exists in the wall which is correct but it is quite possible you can have some small vertical velocities at the wall correct so this is your local coordinate X and Y so therefore you can have your V velocity and you velocity at the wall anyway your u velocity is equal to 0 because that it cannot slip tangentially.

But it is quite possible that you are blowing or you can suck you know this is called flows with transpiration okay so this is a typical flow with transpiration so in that case you can have a small value of vertical velocity at the wall it will not be that much but it will be small enough so typical applications are if you are looking at boundary layer separation control okay .

So typically you can blow some small velocity okay in order to control the separation point or if you have a massive separation you can suck the separation bubble by means of flow suction and therefore you can avoid separation so typically in like air foils now you can do this kind of flows to control the drag and no stalling of the airfoil and so on okay so these are these are extension of the same Falkner Skan solution the same solution or same equation the similarity equation what we derived will exist and you hold true for this case also only the boundary condition will now change okay.

So far we have said at y equal to 0 V = 0 but now we will be V has a small component so that has to be included and we can use that as a more generalized solution okay so if you do this solution this will be the most general solution for whatever we have seen till now okay that includes all kinds of configurations also different kinds of boundary conditions for limiting case where V equal to 0 it becomes the solution that we had derived till now so okay we look at this particular case in the next class and we that will complete the similarity solutions okay.

Falkner skan solutions for heat transfer
End of Lecture 15

Next: Similarity solution for flow and
Heat transfer with transpiration at walls

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