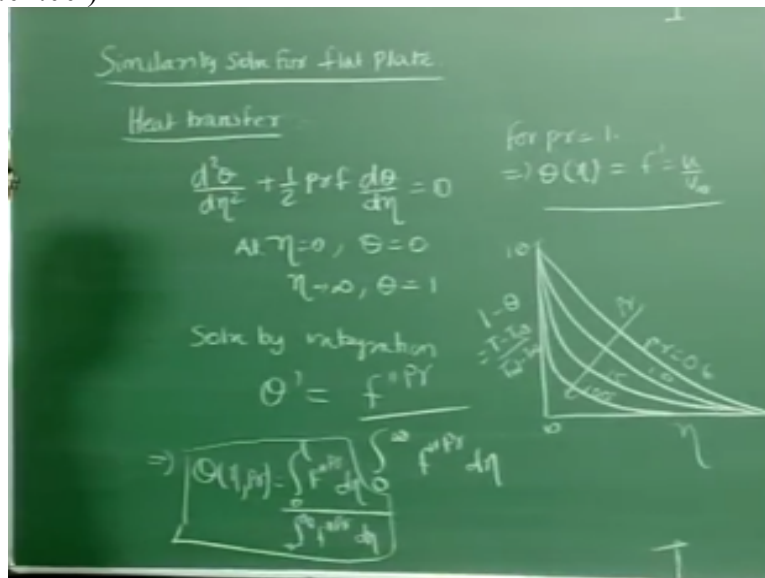


**Indian institute of technology madras**  
**NPTEL**  
**National Programme on technology enhanced learning**  
**Video lectures on**  
**Convective heat transfer**  
**Dr.Aravind Pattamatta**  
**Department of mechanical engineering**  
**Indian institute of technology madras**

**Lecture 14**  
**Pohlhausen similarity solution and**  
**Flows including pressure gradients**  
**(Falkner-Skan)**

(Refer Slide Time:01:00 )



Okay so where we stopped yesterday we were looking at the solution for heat transfer a similarity solution and this is also called the pohlhausen and similarity solution after named after pohlhausen and who continued the velocity boundary layer similarity solution of Belasis and we extended that to thermal boundary layers and we saw the derivation of this particular expression subject to these boundary conditions and of course you know from them from the

intuition that for Prandtl number equal to 1 if you replace the temperature variable non-dimensional temperature  $\eta$  with the non-dimensional velocity the momentum and the temperature equations are identical exactly so pohlhausen got the idea that if for Prandtl number = 1 they have the same solution so for other Prandtl numbers there should be some kind of a similarity solution possible so then he intuitively substituted the same similarity variable

which was used for velocity boundary layer and finally he finds that this falls into the category of similarity solutions ok and of course we have seen direct integration you can reach to this particular step where this gives you the solution of course you need the  $F$  from the solution to

the velocity similarity solution and from there you plug it in and you can numerically integrate it or we have also seen the standard way of doing all the solving all the ODEs will be

hereafter by using the shooting technique so you solve the ODE directly numerically so that also will give you the same solution for  $\eta$  but for all this the solution for  $F$  is required okay so you have to solve both of them simultaneously so all the momentum similarity solution once you get the value of  $x$  you substitute into the energy similarity solution find the solution for  $\theta$  so all this also depends upon the Prandtl number therefore your  $\theta$  is a function of your

location as well as your Prandtl number so for a given Prandtl number you substitute and you integrate it out and you find a solution so for different values of Prandtl number you will get different values of  $\theta$  as a function of  $\eta$  so therefore if you plot the similarity solution coming out of this if I plot  $1 - \theta$  that is I just convert it as  $(t - T_\infty) / (T_{\text{wall}} - T_\infty)$  as a function of  $\eta$  you see for each Prandtl number the slope of the curve changes higher the Prandtl number greater is

the slope so that which means now now you all know that the slope of a temperature profile at the wall directly governs your near wall heat transfer rate okay so we are now going to calculate the heat flux which is carried away from the wall and we all know that the heat flux is directly related to the slope of the temperature profile at the wall so greater the slope greater will be the heat flux carried away and therefore your heat transfer coefficient which is defined

from this also will become higher so higher the Prandtl number the higher the heat transfer coefficient okay so now you can probably once you know the solution you can convert this in terms of the  $\theta$  and derivative with respect to  $\eta$  and then plug in those values okay can you have you know we have defined  $\theta$  in this manner  $(t - T_w) / (T_\infty - T_w)$  so this can be written as  $-K$  so  $dt/dy$  will be  $d\theta/d\eta$  so  $(T_w - T_\infty)$  into  $d\theta/d\eta$  I am transforming the variables.

(Refer Slide Time: 04:05)

here into  $dh/dy$  at  $y$  equal to 0 or in fact I should apply this condition to  $d\theta/d\eta$  at  $\eta = 0$  okay now I also know my similarity variable  $\eta$  is  $y \sqrt{u_\infty / \mu X}$  okay so I can just substitute this should give  $-K (T_w - T_\infty)$  into what is  $dH/dy \sqrt{u_\infty / \mu X}$  into  $D\eta / DT$  at  $\theta = 0$  now  $d\eta/dh = 0$  so this is my  $D\theta / D\eta$  if you integrate this once you end up with this solution for the slope  $d\theta/d\eta$  okay so you want to evaluate this at  $\theta = 0$  so we will say that my  $d\theta/d\eta$

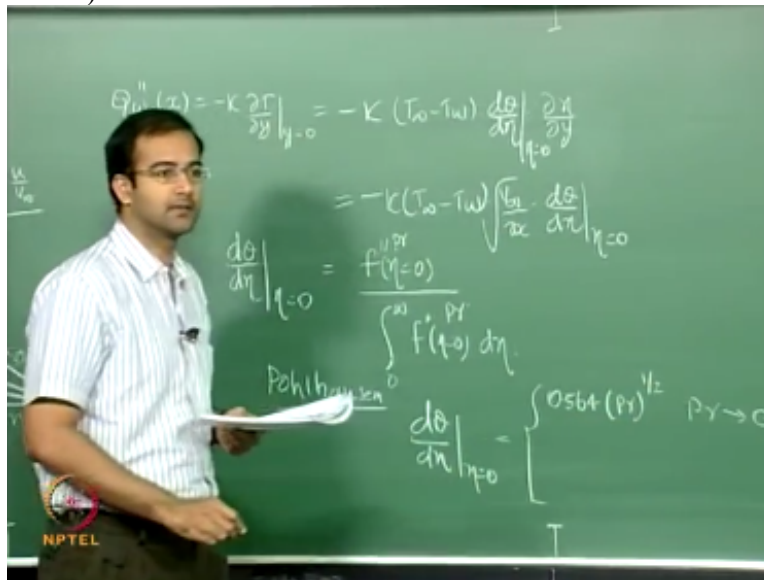
equal  $H = 0$  should be the value of this  $F''$  okay at  $\theta = 0$  okay so  $F''$  at  $\theta = 0$  to the power  $P$  at  $\theta = 0$  to the power  $P$  right since your  $F$  is a function of  $\eta$  so the value of slope from that equation will be evaluated exactly at  $F''$  at  $\eta = 0$  okay so I mean when you are doing this numerically so all you have to do is you have to substitute the value of  $F''$  from the Blasius solution that you got right

so you know the value of  $F''$  at  $\eta = 0$  so that is nothing but the curvature okay you substitute and then you integrate it okay so then that should give you the value of these temperature profile temperature slope a slope of the temperature at  $\eta = 0$  okay so when you do

this numerically once you have a complete Belasis solution it is just a matter of substituting the curvature for in terms of  $F$  and then calculating the slope in terms of  $\eta$  okay so once you do this

you will find out in fact this was done numerically Paul Hassan and he has actually fitted a nice curve so he gets different sets of values of  $d\theta / d\eta$  for different values of Prantle number so depending on the Prantle number you get different values of this okay so now he fitted a nice curve for different ranges of Prantle number for each range you know he found the curve fit which is sufficiently good enough to describe it and this is what Paul Hassan did it so

(Refer Slide Time: 07:44)



Paul Hassan's solution was expressed as  $d\theta / d\eta$  at  $\eta = 0$  for the case where the Prantle number was extremely small okay so I say Prantle number approaching 0 okay so for such a case he found the curve fit like 0.564 times Prantle number raised to the power half this was the best fit which can describe the approximation for low extremely low Prantle numbers okay for moderate Prantle numbers between 0.6 and  $1 < Pr < 15$  he found 0.33 to be  $Pr^{1/3}$  okay

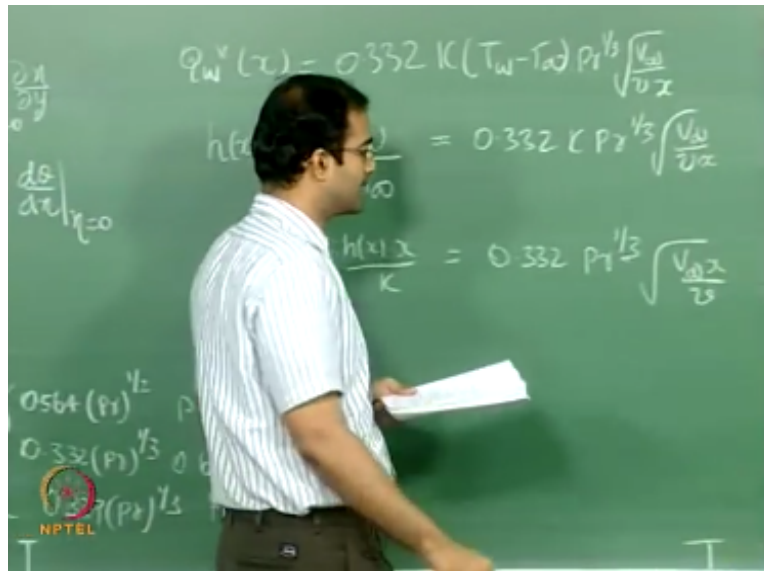
and for Prantle number which is very large  $0.339 Pr^{1/3}$  in fact it is very close you know even this could have approximated all the way from Prantle number 0.6 to  $\infty$  it is only the third decimal point which is just changing but for Prantle numbers which are very small the functional dependence on Prantle number itself is different okay so in fact this is a very useful information for us he has completely covered that entire Prantle number regimes okay so in fact

it is a good exercise you can do when you are solving this for example shooting method you can you will be directly solving for  $d\theta / d\eta = 0$  right we have seen this just like the case of Blasius solution where you do not know the curvature you guess it and try to match the other boundary condition ok  $f'(\infty)$  should be equal to 1 the same way here you guess  $d\theta / d\eta = 0$  you guess this and you finally match this solution ok so ultimately when

you converge finally you will directly converge to the correct value of the slope at  $\eta = 0$  okay and if you probably check with the Pohlhausen since correlation you can get a good idea how good this correlation works ok so you can repeat this four different Prantel numbers right ok for different Prantel numbers you solve this equation you get the slope and then you substitute and check whether it satisfies the Pohlhausen fit ok so that is a good way of for you to learn and

understand whether this fit is very accurate or not ok so this is what and got will let us accept it for the time being so most of the times we are interested in fluids in the kind of intermediate Prantel numbers ok we are not going into liquid metals or fluid Pohlhausen with the extremely high viscosity but with intermediate Prantel number between 0.6 and 15 so we will take that value and then substitute for  $D \theta / D \eta$  for calculating the heat flux okay so therefore

(Refer Slide Time:11:07 )

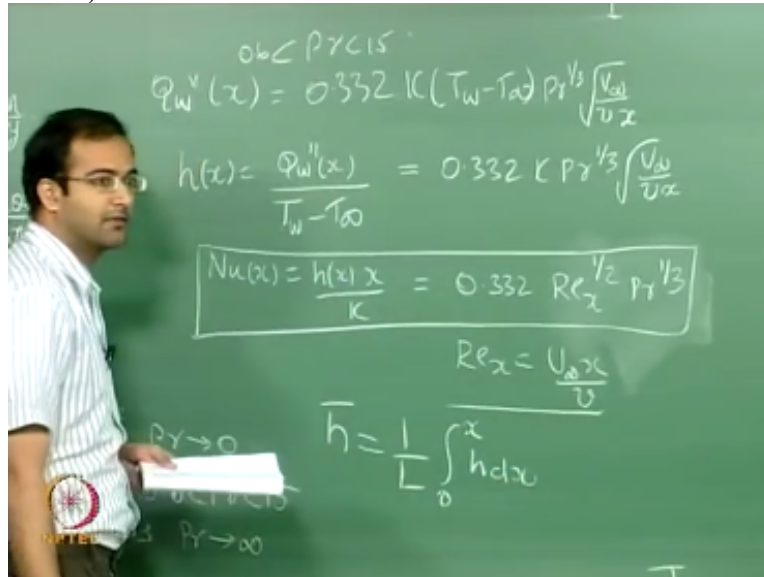


my heat flux will become now  $0.332 (k \text{ times } P_{\text{wall}} - T_{\infty})$  and just flipping the signs here into Prantel number power one / third square root of  $u_{\infty} / \mu X$  so this is the equation for the wall flux I want to now go one step forward and calculate what is the heat transfer coefficient okay so my heat transfer coefficient as a function of X will be the wall flux by how do i define it wall temperature - reference temperature is my free stream temperature okay so this will be

$0.332k$  here power  $1/2 \sqrt{u_{\infty}}$  now I will non dimensionalize the heat transfer coefficient in terms of nusselt number local nusselt number which is defined this way okay so this will be  $0.332 \text{ to } Pr \text{ power } 1/3$  so I multiplied / X here so I will be getting  $\sqrt{u_{\infty} X / \mu}$  right so which is nothing but the Reynolds number okay where your local Reynolds number is  $u_{\infty} X / \text{the kinematic viscosity}$  okay so this is the final expression that you probably find in the textbooks

if you are taking a basic heat transfer course you are all straightaway given the final expression for flat plate on a cell number okay so this is how it is coming out so this is a very straightforward correlation and probably most of you know by heart and this is valid for the intermediate prantle number range that we are talking about that is Prenatal number  $< 0.6$  and  $> 15$  okay so we can also do one more thing rather than into being

(Refer Slide Time:13:34 )

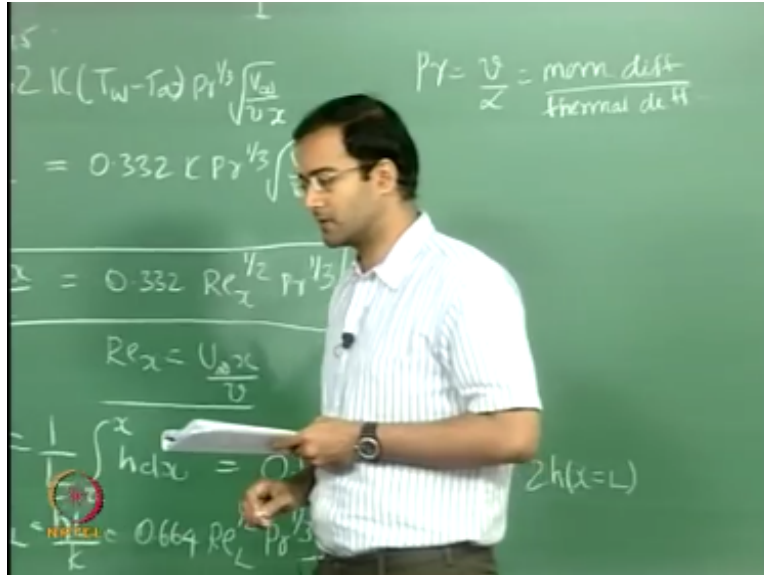


interested in the local variation of heat transfer coefficient our nusselt number we can kind of calculate an average value of H for the entire plate so therefore we can also define an average value  $\bar{H}$  over bar which is one over  $\int_0^L h dx$  okay so this is the way I define my average over the entire plate length okay so if I substitute you can probably do that as a nice exercise you can substitute and integrate it with respect to X you will find that the resulting expression will

be  $0.664 \sqrt{k Pr} / \sqrt{\nu L}$  okay which is exactly  $2 H(x=L)$  but here X will be  $= L$  okay so this is exactly twice of the heat transfer coefficient corresponding to  $X = L$  okay now same way if you define an average nusselt number for the entire plate okay depending on the average heat transfer coefficient and based on the length instead of the local coordinate so you will be getting  $0.664 \sqrt{Re} Pr^{1/3}$

okay so this is this is exactly twice that of a new at  $X = L$  all right okay so these are some I think correlations which are familiar to you so I am not going to spend too much of time and so in all these cases you know that

(Refer Slide Time:15:17 )



the Prandtl number is a very important parameter which is the ratio of  $\mu / \alpha$  so which gives the ratio of the momentum diffusivity / thermal diffusivity okay and you also know from the scaling analysis which we did when we derived the boundary layer equations the relationship between Prandtl number and  $\delta$  if we showed that my  $\delta T / \delta x$  is approximately Prandtl number to the power  $(Pr)^{-1/2}$  for one case for higher Prandtl number this relationship becomes

number to the power  $(Pr)^{-1/3}$  but nevertheless you can see that the thermal boundary layer thickness is inversely proportional to the Prandtl number okay so therefore if you are saying that my Prandtl number is much  $< 1$  so your thermal boundary layer thickness is much greater than your momentum boundary layer thickness and for Prandtl number  $= 1$  both are equal and vice versa ok so so this is the kind of observation for higher Prandtl numbers for Prandtl

numbers greater than 1 of the order of 1 and greater than 1 you can clearly show that  $\delta T / \delta x$  scales as Prandtl number to the power  $(Pr)^{-1/3}$  okay so this something which was concluded once you calculate your thermal boundary layer thickness from here and the momentum boundary layer thickness from the Blasius solution you can simply take the ratio and you find exactly scales / this Prandtl number power  $- 1 / 3$  this is something that which you can observe

yourself ok for higher Prandtl number for the low Prandtl number regime it scales as Prandtl number power  $(Pr)^{-1/2}$  so in fact you can see here itself ok the slope for the low Prandtl number regime is of the order of PR power half here for higher Prandtl number it is PR power  $1 / 3$  ok so these are some some of the observation that you can make and most of the times if you are looking at oils which are very viscous you are looking at high Prandtl number or if you

are looking at liquid metals you are looking at very very small Prandtl numbers most of the practical fluids air water and so on they fall in this intermediate Prandtl number regimes so that is why we have specially derived the expression for Nusselt number for those majority of the

fluids which fall within that Prandtl number range okay so in fact this will be also very nice exercise which I can give you probably in your homework you can calculate the thermal

boundary layer thickness once you get your velocity profile right so that is that is the value of  $y$  where your  $\eta$  corresponding to the  $\eta$  where your  $u / u_{\infty}$  goes to one or .99 or  $F'$  that you calculate should be approaching .99 name the corresponding value of  $y$  will be  $\delta$  the same way thermal boundary layer thickness is defined as the point where you are  $\eta$  goes to 0.99 okay the corresponding value of  $y$  so you can calculate both you can take the ratio of that and you

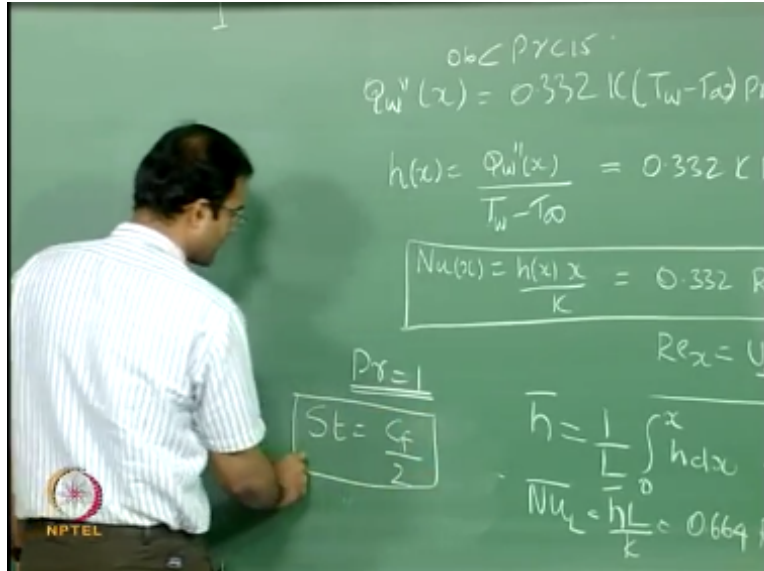
can see how it scales you check you check the scaling with respect to Prandtl number okay so you can keep doing this for different Prandtl numbers and you will find that it exactly scales in that manner alright so so with this we will move on to the next topic but but actually what could have been done I could have actually started off from the other the next topic that I am going to talk about now rather than doing the flat plate solution because the flat plate solution is

a special case of that particular type of flows okay so this this type of flows are called as Falkner Skan flows okay so let me just give you an introduction to the Falkner Skan type of flows before we go into the similarity solution so any questions so far on the flat plate flat plate is a very basic case that is why I had to start so that you can understand how the similarity solution is obtained how we can solve the ordinary differential equations and so on the other

cases are little bit more not that difficult but it is just one order of approximation higher and the flat plate case is going to be a special case of those kind of flows I did not want to directly start off from there and show you the flat plate I could have done that to save the time but nevertheless it would have not been a very good learning experience for you okay okay one

more thing which I probably will just mention casually before signing off from this I think all of you have heard about Reynolds analogy okay I think in also knowledge is something which is taught in any heat transfer course so you can also show I mean for the case of Prandtl number = 1 okay there is a very good relationship between the Nusselt number and the skin friction

(Refer Slide Time:20:53 )



coefficient okay so it is expressed in terms of standard number is equal to exactly  $St = C_f / 2$  okay so this was called the Reynolds analogy because it was discovered by Osborne Reynolds and this relationship is very useful because once you know the skin friction coefficient you can directly calculate the corresponding nusselt number also okay your Stanton number is another non-dimensional number which which is a non dimensional group which defined based on the

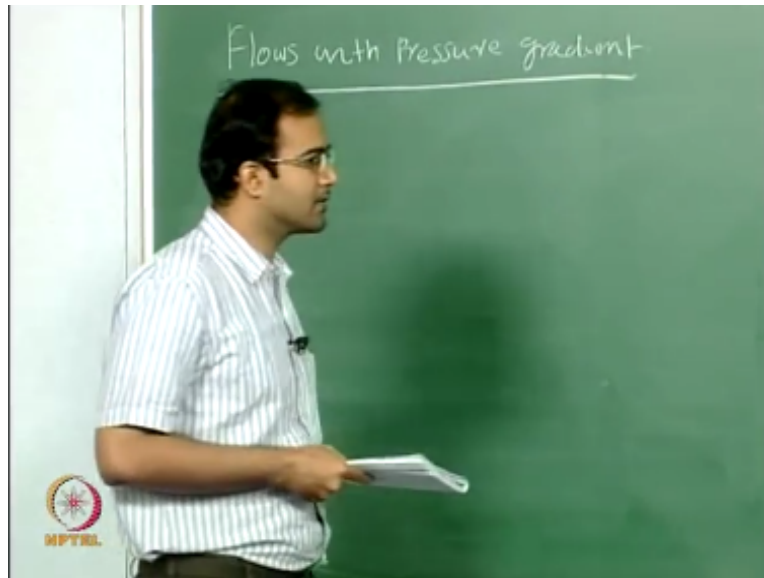
nusselt number Reynolds number and prantel number so all these are grouped and just another name is given to that now for prantle number not equal to one people have found that still you can use an analogy now that that is called as the Reynolds called burn analogy because it is an extension of the Reynolds analogy for Prantel number one according to that your Stanton number into  $St Pr^{2/3} = PL / 2$  so wherever your Pantone umber is not equal to 1 you can still

use the Reynolds Colburn analogy to calculate the nusselt number from the skin friction coefficient so in any way I mean already when Pohaussen did it he understood the relationship between the nusselt number and skin friction directly for prantle number one okay for the other planter numbers it was extended based on the Reynolds Colburn analogy and therefore these are very useful expressions for flat plate is concerned you do not have to really

be bothered about the heat transfer solution once you get the fluid flow solution you can apply the analogy and calculate the heat transfer solution especially if we are interested in the heat transfer rate at the wall okay so therefore we are more interested in the nusselt number and things like that okay so with that I think you can show and derive this yourself it is not that difficult I think you can do it I am not going to spend time and therefore with this we will move on to the Falkner Falkner Skan type of flows ok so

(Refer Slide Time:23: 11 )



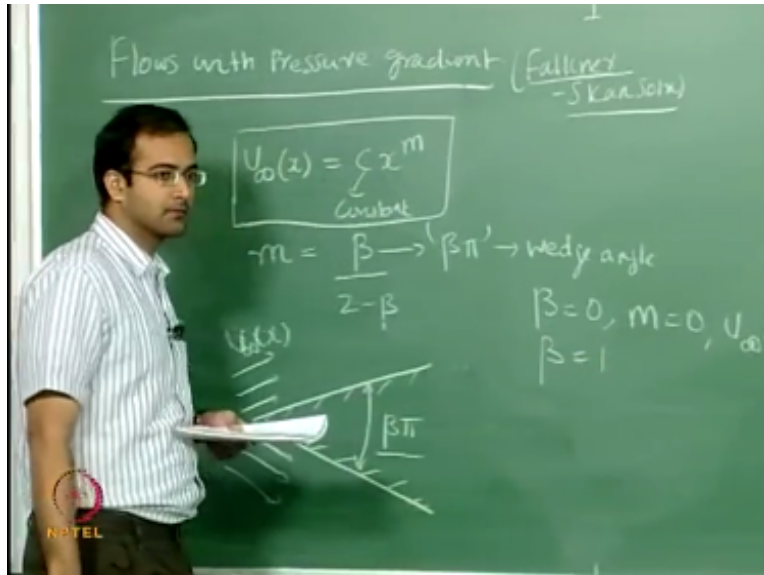


so these are flows with pressure gradient okay so so far as you all know in the flat plate case we have seen that the pressure gradient is neglected and therefore it becomes very simple to solve now what happens when you have flows with pressure gradient okay can we derive a closed-form analytical solution or maybe can we find at least a similarity solution which we can solve numerically okay so this was not addressed till the 1930s so when Falkner and Skan the extended they studied the similarity solutions derived for a flat plate and they try to extend that for flows with pressure gradient term also okay so in that that is why they have been called as Falkner Skan solutions at least the flow part okay the heat transfer part was added to the flow part because the heat transfer equation has nothing to do with the pressure gradient the heat transfer already has been solved okay so once you solve the flow that  $F$  that comes out of the flow goes into the similarity solution for temperature okay so that is going to be much simpler so the flow is the major complication here in fact Falkner and Skan they discovered that for flows with adverse pressure gradient they have in fact you can do this from the potential flow theory that is from the inviscid solution that you can derive the velocity profile the free stream velocity which is a function of  $x$  now it is not it is not a constant anymore so can be shown to be a relationship with respect to  $x$   $v(x) = cx^m$  okay where  $c$  is a constant and  $m$  is also a constant but it is related to what is called the wedge angle okay so  $m = \beta/2 - \beta$  and  $\beta \pi$  is your wedge angle okay now what does it mean by the wedge angle is if you draw very a general kind of a problem with describing this kind of a free stream velocity profile okay it will be something like this okay so you have a wedge okay this is a wedge profile okay so you stick this wedge into the free stream and so initially here your free stream which is approaching will be constant and now once it encounters this wedge so there is of course the pressure gradient okay so the flow will try to for for example here accelerate okay so therefore here the local free stream

velocity will be a function of  $x$  okay correct okay and this profile is described by this relation  $cx^m$  for this kind of a case what is the wedge angle so this is your wedge angle  $\beta \pi$  okay so this

is the so this is a general figure configuration for which Falkner and scan has shown the similarity solution and you can derive special cases of this particular solution okay now you can see if my  $\beta = 0$  what happens what happens to the wedge it becomes a flat plate okay so these two collapse and this is just simply flow past a flat plate

(Refer Slide Time: 27:38)



and if my  $\beta = 0$  what what is the value of  $m$   $m = 0$  right so therefore my  $u_{\infty} = c$  will be constant so which becomes basically the flat plate solution so flat plate solution is a special case of the wedge solutions are the falkon scan solutions okay now if my  $\beta = 1$  for example how does this flow look okay so let me draw the different cases  $\beta = 0$  as you can see that this will be a flat plate okay now if my  $\beta = 1$  what will happen to em 1 okay so therefore my  $u_{\infty}$  of  $x$  will be  $cx$  u

$u_{\infty}(x) = cx$  okay now how does how does the configuration look if my if my  $\beta = 1$  the wedge angle will be what  $\pi$  so it will be a vertical plate instead of a horizontal plate but the flow will be still coming along the horizontal direction so this will come and impinge the flow comes like this it impinges and it goes this way okay so this is called a stagnation point solution because you can see that the flow comes hits the velocity has to be 0 so this is a stagnation point okay so this is

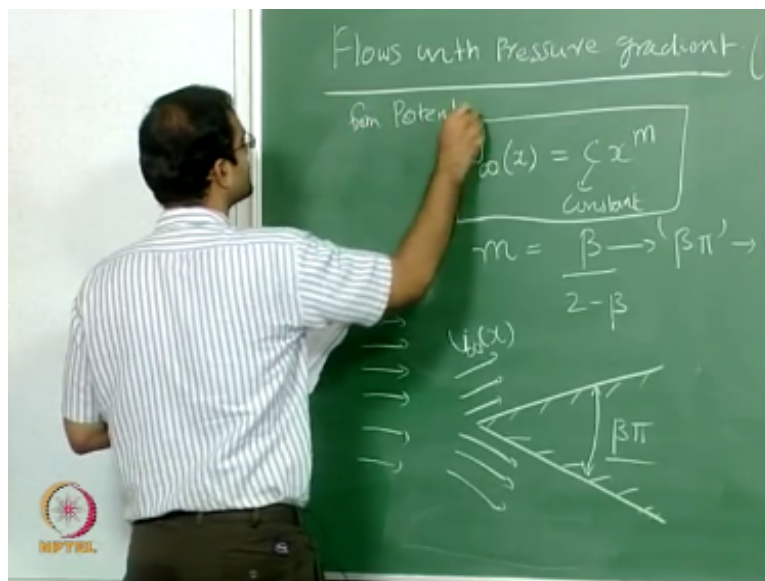
a stagnation solution stagnation point okay so this is I mean since it is 2d this is 2d stagnation flow or stagnation point flow okay so these are some special cases of the Falkner Skan now what happens if my  $\beta$  is negative so so far I have been discussing the case where is  $\beta = 0$  and  $\beta$  is greater than 0 so  $\beta$  going up to 1 now what what can have can can also you can also visualize another configuration where  $\beta$  is negative okay so how could how could  $\beta$  be negative so you

can for example take this you can wrap it around inside it becomes 0 when it touches here and again you turn it inside so that becomes negative and when you do that if you do completely wrap around this way you will find this flow cannot flow past the surface okay but instead this flow will have to come and then deviate this way okay so therefore I am just tilde it upside down if you if you tilt it upside down you can say for  $\beta < 0$  for example where your M also will be negative correct if my  $\beta > 0$  M also will be negative so I can have a flow which is something like this okay this is my wedge this is how my wedge becomes okay so I take this this will be 0 and again I put it this way and make this way horizontal okay so this will be horizontal and like this the flow will pass like this okay the same thing I am drawing upside down okay so the flow will be like this so what kind of a pressure gradient here it is it

favorable pressure gradient or adverse pressure gradient this is an adverse pressure gradient okay because you are because why your  $M > 0$  so your velocity has to decelerate correct so this is an adverse pressure gradient flow all right so where as defer the case where your  $\beta$  is  $< 0$  what should happen the flow should accelerate your M is positive okay so wherever your wet jungles are positive like this it is an accelerating flow where your wedge angle is negative like

this it is a decelerating flow R it is an adverse pressure gradient flow in this case it is a favorable pressure gradient flow okay so therefore you can see there is a family of solutions that can be derived once you have the basic solution for the free stream this is

(Refer Slide Time:32:22 )



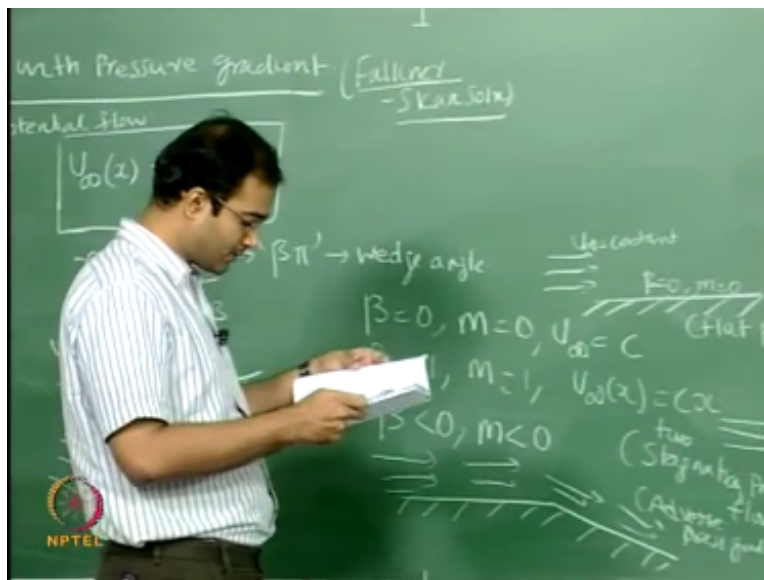
coming from the in viscid this is coming from the potential flow or the in viscid solution okay so for this kind of class of problems there are different configurations that are possible depending on the values of M which depends on the wedge angle okay  $\beta \pi$  and the limiting

cases some of the limiting cases that we can have a look at is the case of  $\beta = 0$  which gives you the flat plate solution  $\beta = 1$  which gives the stagnation point solution and negative values of  $\beta$

which is an adverse pressure gradient solution now if I reduce the value of  $\beta$  so small in fact if I make it to negative what will happen is beyond a certain point the flow will separate the adverse pressure gradient will be so strong that flow separation will take place and once the flow separation takes place the boundary layer theory is not valid anymore okay the boundary layer theory is valid only within the boundary layer once the flow separates there is no

boundary layer there okay therefore you have to be cautious there is the lowest low there there is a lower limit for  $\beta$  till which you can find solutions okay below that flow separation takes place and you cannot find a similarity solution to those problems okay you

(Refer Slide Time: 33:39)



so now what we are going to do is for this kind of class of problems we will write down the try to reduce the partial differential equation to a similarity equation so can you all try to write down the governing equations for this kind of problems okay so for flows with pressure gradient so how does the boundary layer equations look okay the

(Refer Slide Time: 34:13)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$$

continuity is still the same how about the momentum equation everything is the same except only one term which is the pressure gradient term and we are now neglecting the okay now when we are not still written the energy equation first we will take the flow solution and then we will apply that to the energy okay so this is your governing equation now we all know how do we calculate the pressure pressure gradient okay I probably mentioned this in the very third

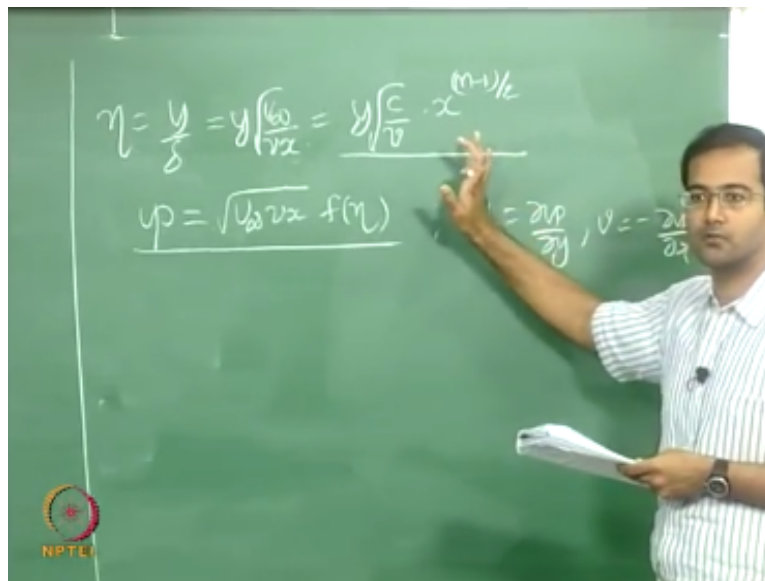
or fourth class okay when we derived the governing equations for boundary layer so how do we evaluate the pressure gradient inside the boundary layer upstream so since we have shown that the pressure is invariant of Y okay so we can evaluate this outside the boundary layer and the same value should be valid inside also okay so therefore if you apply the governing equations outside the boundary layer so you will get  $u_{\infty} \frac{D u_{\infty}}{D X}$  okay only your free stream

velocity is there your V velocity is 0 so that should be equal to  $-1 / \rho \frac{D p}{D X}$  and there are no viscous effects also there ok so you can simply replace your pressure gradient with your equivalent velocity gradient therefore this term will become  $u_{\infty} \frac{D u_{\infty}}{D X}$  all right so now for the class of problems we are looking at the wedge wedge flow problems okay this is the form of the velocity profile see  $X^M$  okay so therefore we know the functional dependence of  $u_{\infty}$  on X we can just simply substitute it can you tell me how does it look since my  $u_{\infty}$  is equal to  $C X^M$  my  $\frac{D u_{\infty}}{D X}$  will be  $C M X^{M-1}$  okay so therefore this becomes  $u \frac{D u}{D X} + V \frac{D u}{D y}$  is = I can write again  $u_{\infty}$  okay I can say that this  $C X^M$  is again nothing but  $u_{\infty}$  okay so how one  $u_{\infty}$  here  $u_{\infty}$  here which I clubbed together as  $u_{\infty}^2$  and apart from that I have M and I have X power - 1 okay so I can write this as  $M / X$  plus  $\mu$  ok so this is the form of the

momentum equation that I will be working with for the Falkner Skan flows okay now what are the boundary conditions at  $y = 0$   $u = v = 0$  no slip boundary condition and  $y$  going to  $\mu_{\infty}$   $u$  approaches my free stream velocity now be careful your free stream velocity is function of X it

is not a constant anymore ok so which is actually of the form  $CX^M$  ok so now how do we reduce this to an ODE the same approach that we did with the Blasius flow you assume a similarity variable let us assume the same similarity variable that we got for the flat plate flows okay because functional dependence of boundary layer thickness on  $X$  is going to be the same whether you have a pressure gradient term or not therefore your

(Refer Slide Time: 38:29)



$\eta$  which is a function of  $Y/\delta$  the functional dependence of  $\delta$  is still the same therefore you can assume the same similarity variable holds good okay only thing here  $u_\infty$  is a function of  $X$  okay so therefore you can write this as  $Y \sqrt{C/\mu}$  into this is  $X^M$  this is  $-1 X^{M-1/2}$  okay  $M - 1/2$  and also we can define the stream function and show that stream function is related this is a function of  $\eta$  the same way that it was there for the flat plate also so those

things remain unchanged where your stream function is defined such that  $U = \partial\psi/\partial y$  is equal to  $u_\infty$  ok so till now this is nothing new ok this is the same thing we did for the Blasius solution so all you need to do is substitute for the derivatives now ok in terms of the stream function now stream function is a function of  $F$  so therefore you can calculate  $UV$  and you can substitute all of them here now only thing is you have this  $m$  and  $u_\infty$  is a function of  $X$  ok so

once you substitute into this you will find that that I will leave as a nice exercise to you it will reduce to a ODE in terms of  $f$  and  $H$  without any terms from  $Y$  and  $X$  appearing ok so therefore this confirms that the similarity solution to this class of problems is possible okay it is not too difficult to show once you plug in all the velocity gradients and your velocities into this expression let me call this as number two your stream function already satisfies the continuity

okay so you have to just plug it in here and substituting into two you get your final ODE which is free of  $x$  and  $y$  terms it is only function of  $F$  and  $\eta$  okay so therefore this shows that for this kind of flows similarity solution is perfectly possible okay and this is the Falkner Skan similarity solution so now you can see now here it is also a function of  $M$  okay now if you put  $m = 0$  okay what happens this term completely vanishes you have  $1/2 F d^2 F / D \eta^2 = 0$  so this is nothing

but the Belasis solution ok so in fact without even touching the Belasis solution we can straight away started you could have started from Falkner Skan and showed Belasis solution as a limiting case ok so all kinds of problems can be approached in this manner you can put any value of  $M$  that you are interested in for that particular configuration and get the similarity profiles for that particular configuration so this is a function of  $M$  right here so what are the

boundary conditions the same boundary conditions that that that we have used for Belasis solution apply here  $\eta = 0$  you are  $F = 0$   $Y$  you are  $F = 0$  which boundary condition does it satisfy no sleep correct but exactly which weather weather does it correspond to  $u = 0$  or  $V = 0$   $u = 0$  what is  $U$  in terms of  $F$  what is the relation between  $U$  and  $F$   $u / u_\infty$  therefore  $F = 0$  does it correspond to this so then you equal to 0 should be what  $F$  prime should be 0 so then what what

does it correspond to  $v = 0$  so go back to your the way that we derived the Belasis solution so cut the expressions for  $u$  and  $V$  ok so if you put  $F = 0$  then you then only we become 0 because already from the condition that  $u = 0$  we know that  $F$  prime = 0 so far we have  $V$  to be 0 then this has to be 0 okay and what is the remaining boundary condition heat are going to  $\infty$  the same same thing that we have done for the flat plate okay so what is the condition of  $\theta$  going to

$\infty$   $u / u_\infty$  should be 1 or  $DF / D \eta$  our  $F$  prime should be = 1 okay so so this this is this is called the Falkner Skan similarity solution so now how do we solve it shooting method okay exactly the same same technique that you read for Belasis solution okay you have a boundary condition for  $F$  you have boundary condition for  $F$  Prime but you do not have a boundary condition for  $F$  double Prime at  $\eta = 0$  so you have to guess something and finally match this

satisfy this condition okay you don't worry about because even if you have this equation I will just write down if you apply the shooting method it will simply reduce to 3 ODE is which I will just write and stop there and maybe I can write it here

(Refer Slide Time:45:39)

$$\eta = \frac{y}{\delta} = y \sqrt{\frac{U_{\infty}}{\nu x}} = y \sqrt{\frac{c}{\nu}} \cdot x^{(m-1)/2}$$

$$\psi = \sqrt{U_{\infty} \nu x} f(\eta) \quad ; \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

Substituting into (2)

$$\frac{d^3 f}{d\eta^3} + \frac{(m+1)}{2} f \frac{d^2 f}{d\eta^2} + m \left[ 1 - \left( \frac{df}{d\eta} \right)^2 \right] = 0$$

$$\eta = 0, \quad f = 0, \quad f' = 0$$

so the first whoa d will be what  $f' = G$  right second M odi will be  $G$  prime equal to  $H$  and what is the third OD you substitute in terms of  $F$   $G$  and  $H$  here okay so that will be  $H H'$  will be equal to - you have  $M + 1 / 2$  into  $F$  into  $G G'$   $H d^2 F / D H^2 H$  okay this has to be  $f$  into  $h$  plus  $m$  into  $1 - DF / D H$  the holes this is yeah so this has to be  $f'$  square so this has to be  $G$  square okay so this is my third ODI so these are the three first-order ode is ok all i have to do is

match from  $\eta = 0$  till  $H$  some value of 8 or 10 okay and the same ball set of boundary conditions you know you guess by Newton's method successively you reach to the better guess and then check the condition that at  $\eta$  equal to large values of  $\eta = 8$  or 10 your  $f'$  prime which is nothing but your  $g$  should should be equal to 1 okay so that equation has to start so till then you keep on hydrating by Newton's method you keep guessing better values and you have to solve the set of Odie is for all the points in the domain okay the same exactly the same technique all you need to know is the value of  $M$  so which value of which configuration that you are looking okay so once you know the configuration the solution procedure is identical for different for each configuration you get a class of solutions so we will stop here so tomorrow we will complete the heat transfer part of the Falkner Skan solution you.

**Pohlhausen similarity solution and**

**Flows including pressure gradient**

**(Falkner-skan)**

**End of lecture 14**

**Next: Falkner skan solutions for heat transfer**

**Online Video Editing / Post Production**



M. Karthikeyan  
M.V. Ramachandran

P.Baskar

Camera  
G.Ramesh  
K. Athaullah

K.R. Mahendrababu  
K. Vidhya  
S. Pradeepa  
Soju Francis  
S.Subash  
Selvam  
Sridharan

Studio Assistants  
Linuselvan  
Krishnakumar  
A.Saravanan

**Additional Post –Production**

Kannan Krishnamurty & Team

Animations  
Dvijavanthi

**NPTEL Web & Faculty Assistance Team**

Allen Jacob Dinesh  
Ashok Kumar  
Banu. P  
Deepa Venkatraman  
Dinesh Babu. K .M  
Karthikeyan .A

Lavanya . K  
Manikandan. A  
Manikandasivam. G  
Nandakumar. L  
Prasanna Kumar.G  
Pradeep Valan. G  
Rekha. C  
Salomi. J

Santosh Kumar Singh.P  
Saravanakumar .P  
Saravanakumar. R  
Satishkumar.S  
Senthilmurugan. K  
Shobana. S  
Sivakumar. S  
Soundhar Raja Pandain.R  
Suman Dominic.J  
Udayakumar. C  
Vijaya. K.R  
Vijayalakshmi  
Vinolin Antony Joans  
Adiministrative Assistant  
K.S Janakiraman  
Principial Project Officer  
Usha Nagarajan  
**Video Producers**  
K.R.Ravindranath  
Kannan Krishnamurty

**IIT MADRAS PRODUCTION**

Funded by  
Department of Higher Education  
Ministry of Human Resource Development  
Government of India

Www. Nptel,iitm.ac.in  
Copyrights Reserved