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Lecture 13

Numerical solutions to the Blasius equation and Similarity solutions to heat transfer

Good morning all of you. Yeah so, in the last class we had derived the Blasius similarity solution I hope all of you can recollect how we derived it based on the similarity variables we reduced the PDE to an OD and also we have seen the way blushes did the solution I think some of you are not there but to summarize you can I will just tell you we were blushes use the series expansion technique where we assume the series to be a power series and he substituted that into the Blasius equation applied the boundary conditions and this series expansion was valid for small values of η .

Okay so, if your η was very large the other boundary condition says that η going to ∞ your f prime should be 1 so we cannot apply that to the convergent series therefore he had to come out with another series which is actually convergent for large values of H okay so that is an asymptotic expansion he did and then he matched the coefficients of these two series subject to these boundary conditions three boundary conditions.

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Vunctical Solution to Blasius opn (3)

And therefore he finally calculated the slope at the wall that is f double Prime at 0. So once he got the slope you all the coefficients in that series were a function of that slope and he calculated the F basically. Okay so that that is how Flash is done it and we have discussed that in the last class so any questions on that. I hope you know it is the standard mathematics nothing more than that so today. What we will see is what is more practical nowadays it is to use numerical methods to solve any ordinary differential equation at the time and blushes did it he did not have any access to these computers or whatever even by hand he could not think of solving this so therefore he resorted to this no analytical expansion techniques.

So I am going to talk about a very important numerical method to solve the higher order ordinary differential equation and please take note of this method. Because, I will not be repeating this again and the rest of the similarity solutions. You will be solving this yourself you know you will be writing a program to solve the ODS by this method. Okay so therefore you play a piece please pay close attention to what I am going to describe. Here so the technique that we are going to solve used to solve the OD will be called the shooting technique.

Okay so I have any one of you had a numerical methods course where the shooting technique was explained so you know the shooting method okay so how anybody else. Okay so I think then I will pretty much go through the thing again if you find something different then you can also bring in your ideas okay so basically this is the OD which is a third order nonlinear OD so therefore you cannot find a closed form analytical solution subject to these boundary conditions so what the shooting method does it is very simple you have to first reduce nth order audio to n first order Rudy is okay so in this case you have a third order ode body so reduce it to three first order Rudy is so you can just say if one equation will be directly F prime should be = to

some variable G prime should be = to η and finally you can cause this equation in terms of η F and G okay so these are the three OD's.

Which satisfy this particular blushes equation and they are all three first-order bodies okay so once you reduce that to three first order for first order ode ease of this form now the next thing is how do we solve this first order Rudy. Okay numerically what you have to do you have to discretize the governing equations on the physical domain now this is one-dimensional therefore you have to discretize into points. Okay and you have to integrate these OD over these points you start from some location save it typically at $\eta = 0$. And from there you keep marching in space till whatever your extent of the domain is.

Okay so typically you want to have a domain large enough know the large domain here means the η should be not ∞ practically but something which is quite large which satisfies the other boundary condition where your F prime =1. So we will we have already seen from the blaster solution that your boundary layer thickness extends to some value of η which is close to five point five eight okay beyond that you find that your F prime is almost 1 it does not change therefore anything above five six should be good enough.

Okay so far safer said you can take value of H up to ten okay so now next what you have to do so you can use either you know if you want to go for higher order accurate you can use runge-kutta but just to make it simple you can use a very first order Euler method okay so you can just discretize the ordinary differential equation based on the Euler method okay so what it says is any derivative like DF by D η can be written as FI-1 byFI-1 D η . Okay so that is a simple first order upwind differencing okay so that you do that and you can now express your f of i okay now this is based on the current point so that depends on the value of the previous point okay plus of course all the things on the right hand side are based on previous points okay so therefore you start this loop from 2 to n okay where 1 I gives you the boundary condition okay you know the value at the boundary right that is the boundary condition.

Okay and you start this marching from I = 2 that is from the second point till the last point okay now for this you need boundary conditions for F G and η all at I = 1 okay so you are I = 1corresponds to $\eta = to 0$ okay so you need all the three values our data =0to start this marching process and you have to solve this together you know first for the second point you solve all the three move to the third point to use that values so on and so forth but the problem is if you look at the boundary conditions we have these two boundary conditions F of I = to 0 G of I = to 0but we do not have the boundary condition for η of 1 right so rather we have a boundary condition for G but for large values of G we know that at large values of η G goes to 1 so what essentially you have to do here this is why this called a shooting technique.

So you shoot a guess basically okay so you guess the value of η at 1 and you just substitute and then you do the marching process go all the way till $\eta = to 10$ and then you find out whether G

at n so where n is corresponding to each I pull to 10 that is the last value of the point ok so you have to choose n number of points such that the end point corresponds to $\eta = to 10$ or so at the end point so the value of G should be = to 1 so if this satisfies that means your guess for η of 1 is correct right so this is why it is called a shooting method so you just do it by guess guesswork so you keep shooting values of η 1 and make sure that as and when you get the solution finally at the end point the other boundary condition.

Which is the third boundary condition is automatically satisfied if your guess for η is correct then this should give this satisfy this particular boundary condition if not you have to keep on doing this again and again till you land up the correct value of H which satisfies that boundary condition ok so G of n = to 1 nothing means F prime of n _ 1 should be = to 0 or so or the other better smarter way of doing it rather than just simply throwing wild guesses okay which is not probably going to lead to a converged value anywhere in the next 100 or 200 guesses the more better way of approaching the correct value will be using Newtons method.

Okay so Newton rap son technique is a very powerful technique for solving any non-linear algebraic equation okay that is also numerically so what it says if you want to solve this particular algebraic equation okay this is the condition that you have to satisfy and you have to find the roots of η h1 okay such that this particular equation is satisfied okay so what does Newton rap son method say so you can choose a guess value such that you know the guess value is coming out of this particular equation which is h1 of K - this is the f of X by F prime of X okay so if you choose your guesses based on this rather than wild guess it is more likely to converge to the correct solution which satisfies this equation okay so it is based on the slope method you know it is a modified secant method.

I think I am not going to into η but you can always quickly derive that we are assuming a linear line and you are getting the slope of it and therefore you are getting a better guess of the route okay so now if you rewrite this now the thing is how do we know the slope of f Prime with respect to η okay so f prime this is at F Prime at N and the point four different guesses of η at the first point so how do you know the slope so initially you do not know the slope okay so what we have to do is you apply simple finite differencing scheme to calculate the derivative. okay so to do that what we will do is we will apply finite differencing scheme between two are two kind of two guesses so guess K _ 1 and K so between these two guesses.

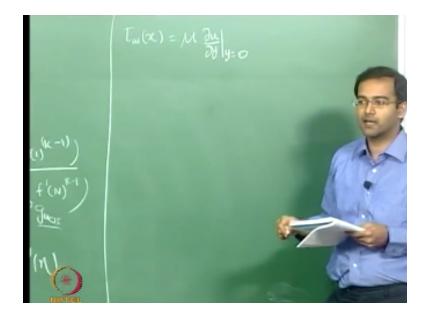
We will use the finite difference again and so we actually expand this numerically the same way we expanded this term right here no simple appoint differencing okay so here k-means guess okay so the first time you just give a wild guess okay the second time also you give a wild guess so now you have two wild guesses and two solutions from the third time you can calculate the right value based on the second wild guess and the first wild guess okay so you need two wild guesses because you need this particular.

DF by D η therefore you are first giving two wild guesses and taking the difference in the values of F of n for the two wild guesses corresponding to those wild guesses okay and from the third wild guess you do not it is not wild anymore so you use this and then you calculate the more appropriate guess value so like that you keep using the previous guess values and until you are satisfying the condition that F prime of n _ one = to zero okay or you would not achieve exactly zero numerically it should be a very small number something like 10 power _ 5 so once this condition is satisfied that then your guess is good enough you can stop there and you can say that you are iterations have converged.

Okay so now you have an iteration here this is an iterative loop where you keep changing the guess okay each time you change the guess you have to march the solution in space correct so for each guess you have to do this so if you are doing hundred guesses for each guess each of the hundred cases you have to march in space and check if your condition is satisfied for convergence okay once it is converged then you can now you have the solutions completely right numerically you have f so this is for F this is for F Prime this is for f double-prime so you have all the solutions at all the points which you can plot and you will be surprised to find you will be exactly matching with the blushes solution okay whatever he did it analytically so this is a very good numerical technique and we will have a few more bodies in this course which you are looking at you have to reduce that to n first order roadies and you will be using the same technique so I suggest strongly to understand this and you will be coding it yourself okay so you can probably use any program programming language of your choice.

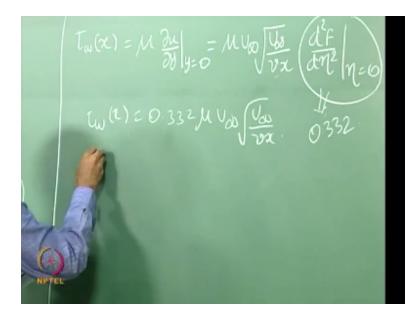
Fortran or C^{++} or mat lab B and you can implement this once the algorithm is done you can plug in your new ODS and then you can get the solutions okay so any questions I hope I try to make it as clear as possible so if you have some doubts you please ask me right now because I am not going to talk about the shooting method again is it clear okay so assuming that now you have the solutions to F as a function of H so now we will go ahead and we will now calculate the other derived quantities okay the first quantity that we are interested so this since this we are doing this for a flat plate we are interested in calculating the local shear stress okay that is tau wall as a function of X so the shear stress at the wall varying local.

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Okay so how do we get it you do you yeah with respect to what Y okay very good so if you go back to the similarity solution what is the expression for D u by DY that we derived okay in terms of stream function stream function was a function of H so we had derived an expression for D u by DY what? 0.332 that is that is not that is not the slope that is the curvature that is the value of F double Prime yeah you are right but that was what we derived with respect to H okay I am asking you to convert this in terms of no why so you have to do the transformation okay from H to y again okay so you can say that this is new can you tell D U by D Y in terms of F and H ok so I think we have already done this derivation I do not want to go through the steps please look into your earlier class notes and tell me the final expression ok you ∞ I think it was $\sqrt{}$ of by nu X okay and of course you know D u by D H we can write this as DS D square F by BH square okay and this is at H =0. Okay I hope all of you have this we have derived it okay so now as he said we have got this value either numerically or from the Blazes solution that a DT =0the curvature is 0.33 - okay so therefore your tau wall is from this you can also non dimensional the shear stress through what is called.

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As a skin friction coefficient and which can also vary locally so this is written as tau wall by half Rho u ∞ square okay if you plug in all the values you will be ending up with a nice expression like this as a function of Reynolds number okay and you can also now go ahead and calculate the drag force once you know the local shear stress you can calculate the drag force which is the total force acting on the plate so how do we calculate the drag force okay so you are right you have to integrate it locally over the surface area okay here of course we are assuming it's unit width in the third direction so this is tau DX 0 to L so this will give me the total drag force on the plate so if I substitute this will be 0.33 to which gives me the value point six four if I assume width of B this is the within the third direction perpendicular to the board okay so this gives me the expression for a drag force.

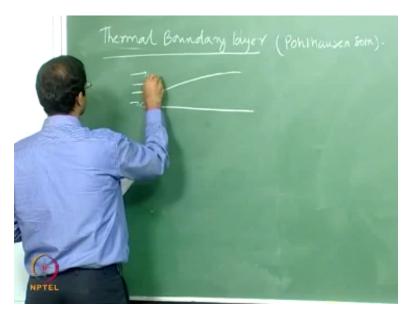
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Okay I can also rather than looking at the local variation of the skin friction coefficient I can calculate kind of an average over the entire plate okay sometimes people report in terms of the average skin friction coefficient so I would like to average it over the entire plate I will call this as C F based on X is = to L okay and over bar indicates average here so how do I have say one by L integral 0 to L C of X DX. Okay so this is how I average the skin friction coefficient over the entire plate if you do the averaging you will find out that you will get this nice expression okay right now your Reynolds number is defined based on X is = to L so Arielle here will be u ∞ L by u okay so I am sure that most of you have done this in the fluid mechanics but probably you did not know how exactly.

Those numbers were coming now you are already given an expression for shear stress and from there you were doing it I think now you would be better understand should be μ ch better okay so that is exactly the curvature term which blushes has determined you know so in both the way in both the ways in the way that blush has tried to do it by power series expansion and the way that we are doing it by the shooting technique in both the ways the struggle is to find the curvature okay so here also we do not know the value of η at 1 so η is nothing but the curvature basically right and blushes.

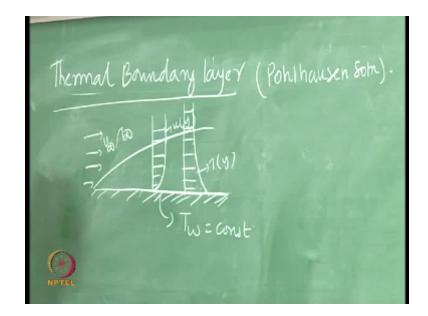
Also did not know that okay so we had to invent an asymptotic expansion and then match the solutions and finally find this curvature and we are also doing by means of guesswork we are calculating the curvature so the curvature is the key once you get the curvature everything just simply is solved ok so now the next what we will move on to the thermal boundary layer I think we have spent enough time on the fluid mechanics part so we will look at the solution to the heat transfer problem.

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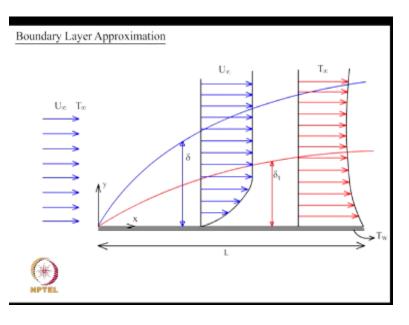
Okay so blushes did the flat plate solution for only the fluid hydrodynamics and stop there okay it was pole hasten who continued they extended the solution for similarity solution towards heat transfer so this is this is also called as poles and solutions okay so in the case of heat transfer you well very well know that earlier we had considered only the fluid flow and this was the velocity profile so now you are maintaining this surface at a constant temperature okay so you can also do a case with the constant heat flux I am not going to do it now but there is also similarity solution possible for that case I will leave that as an exercise to you so what Pole Hassan did was he took the wall temperature to be uniform okay and then he was interested in calculating the solution for the temperature profiles.

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You okay so now if you apply the boundary layer equations.

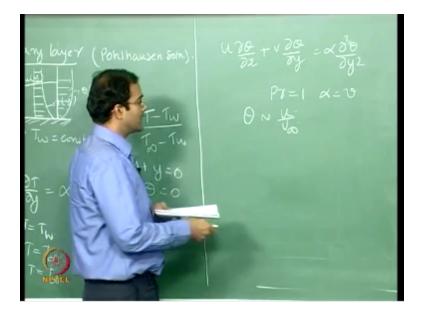
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That we derived okay so can you tell me the dumps in the boundary layer equation for temperature I am asking the governing equation for the energy yeah so u DT by DX okay is = to + UY μ by Rho CP into D u by DY the whole square okay this is your boundary layer approximation for energy equation correct so if you make an approximation that we are looking at only low speed flows you know and moderate Reynolds numbers okay so you can say that

safely the viscous dissipation term can be neglected in comparison it will not be exactly zero but can be neglected with respect to the order of magnitude of the other terms okay so therefore it gets μ ch more simplified okay now what we are going to do is so what are the boundary conditions for this at y = 0 T = T wall right and y = Y going to ∞ and at X is = to zero so now I am going to define a non-dimensional temperature Θ okay I want to define in such a way that a non-dimensional temperature is exactly identical to the velocity profile so how do i how do i define so I want to convert this profile into something like a velocity profile how do i define my Θ t _ T ∞ if I do t _ T ∞ at y =0 is it going to be 0 t _ T wall by what okay t ∞ _ T one so that at y = to zero my T = to T wall or Θ = to 0 at Y going to ∞ my T goes to T ∞ my Θ goes to 1 okay so if you plot the Θ you will you will get exactly profile like this right ok why do I want to do this way if you happen to compare now you write your equation in terms of Θ this is d Θ by DX ok suppose I compare this with the momentum equation that I had written and I assume that Pr= 1.

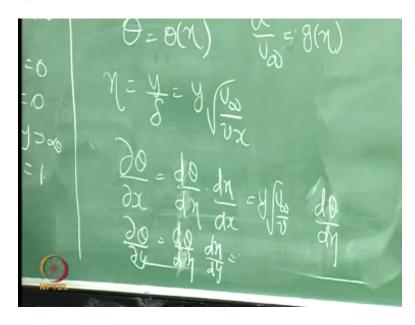
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Exactly that is my alpha = to M okay and I just replaced my Θ with u by u ∞ ok so what do you find this is exactly identical to the momentum equation ok so except for u by u ∞ you have in terms of Θ okay so it is exactly identical to the momentum equation therefore for this case where a number = to 1 so the solution for Θ as a function of say x and y should be exactly identical to u by u ∞ rate as a function of okay so this is a very important conclusion you know for a flat plate case in the momentum equation you do not have any pressure gradient and if you non dimensional the energy so the non-dimensional forms of the momentum and the energy equation are exactly identical when you Pr= to 1 ok now when Paul Hassan looked at it he knew directly he had the solution for Pr= to 1 ok the profile that you have already derived for velocity is the same internal number = to 1 however he also guessed that for Pr number not = to 1 we can still find some kind of a similarity solution ok so what he did as usual so now he has guessed that Θ is a function of H ok similar to the way that my u by u ∞ is a function of some G of b η ok the same analogy applied and the same similarity variable also he used ok where my η is y by Δ which is y $\sqrt{}$ of u ∞ by nu X which we have derived in the last class ok you use the same similarity variable and you assume.

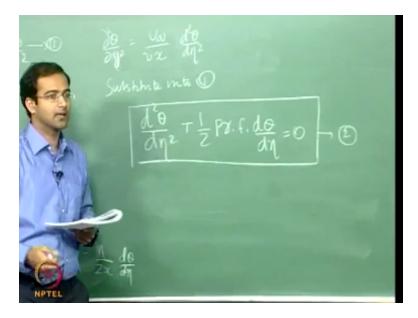
That if there could be a similarity solution for Θ which is a function of H so the proof of this is that if you substitute for Θ as a function of b η into this it should reduce to an OD perfectly ok if it does then there is definitely a similarity solution okay so now he calculated all the other terms which are required you already know U and V okay so D Θ by DX will be $\tilde{\eta}$ by D η and similarly so this will be what can you work out and tell me your eat as a function of X here so you have D Θ by Θ as a function of η okay so I am replacing everything with because Θ is a function of η only okay so this will be D η D Θ by D η and D η by D X so this is y $\sqrt{}$ of into X power differential of x power _.

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Half okay that is _ half so you have X power _ 3 by 2 which I can again write it like this like this okay and this already is my η okay this can be written as _ η by 2 X into D Θ by D η so D Θ this one is straight forward D η by D Y is directly this so this is nothing but u ∞ by nu X into DT Θ by okay so now it you can also find the second derivative d square Θ by D η square or DY square here so that will be u ∞ by μ X into d square Θ by D okay so all this can be substituted let us call this as equation number one so you already know expressions for U and V okay similarity expressions which we had derived before if you substitute all of this into one so I am not going to once again do the substitution and making a nice form of the this thing but finally you get a neat expression which is actually an OD perfectly all the other terms get cancelled off so you get only a function of H everywhere so there are no terms in terms of x and y coming out so therefore pole Hassan's guess that $\eta \Theta$ should be a function of H was absolutely correct okay so this is the similarity solution.

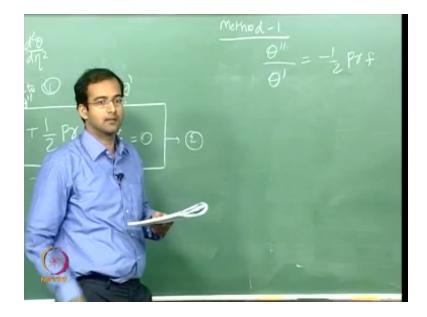
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That you can get for temperature now the boundary conditions for this once again at H = 0your Θ should be 0 right and H going to $\infty \Theta$ should be = to 1 so H going to ∞ means two things whether your y going to ∞ or X going to 0 so in both cases your Θ has to become 1 okay so therefore so now we know everything is getting familiar ok you have a nice OD which is second-order is it linear or non-linear why linear why your f is a function of η .

So it is not completely non linear it is not linear also it is a quasi linear okay your F is still a function of H right it is not of course you do not have a Θ term here but it is still a function of H so this is a quasi-linear OD. okay so therefore you cannot still find an analytical solution to this so you have to once again go for a numerical approach okay of course you know this equation is easier to solve even numerically by integrating it rather than the shooting method that we had used okay so I will I will give you that method first before we find the solution by shooting method so I can cast right this is the solution method one.

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Okay although it looks like an analytical method finally we end up with a expression which has to be done numerically okay so I can write this in my convention which I am more familiar with I am writing as Θ double Prime and this is Θ prime just like I used F double Prime and F Prime so this is Θ double prime by Θ prime should be = to _ half PR into okay I am just rewriting the similarity equation from the Blazes equation I also know that if I rewrite the Blazes equation I can say half of F is = to _ F triple prime by F double Prime.

Correct so if I rewrite the Blazes solution I can write like this now you can see I can substitute for _ half into F in terms of F triple prime so I can link I can link these two equations I can write this as Foretimes F triple prime by okay so correct so if I can now integrate it twice I directed the solution for Θ ok if I if I now integrate once with respect to η so integrating once so this will be what Ln of Θ Prime okay there should be = to ln of F prime to the power P R so PR into this I can put it as f prime F double prime to the power P R plus some constant which I will use Ln of c1 therefore my Θ prime should be C 1 F double prime power P R okay this I will call that number 3.

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Okay so if I integrate once I get this solution where this constant is not known if I will integrate it again okay in fact I can do that here if I integrate it once more I will get Θ is a function of b η and PR what should be on the right hand side so I integrate it from 0 to some value of $\eta C \ 1 F$ double prime power PR ok D η + some C2 okay so this is my final solution for Θ okay so I integrate once I integrate it twice alright so this is f double prime here you know you do not see so now I have to find the two constants C 1 and C 2 how do I determine the two constants boundary conditions okay so $\Theta = \text{to } 0 = \text{to } 0$ okay so if I apply this here to equation number 4 so what will I get edit a = 0 if I integrate from 0 to 0 this is what 0 okay so therefore my C C2 has to be and the other boundary condition ηR going to $\infty \Theta$ should be = 1.

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Q(N=0)=0,=) (z=0 6(n-10)=1

Okay so my η going to ∞ so this is 1 = to 0 to ∞ okay so I take c1 which is a constant outside this is your f double-prime PR okay into D η therefore your C 1 is nothing but 1 by 0 to ∞ F double prime to the power.

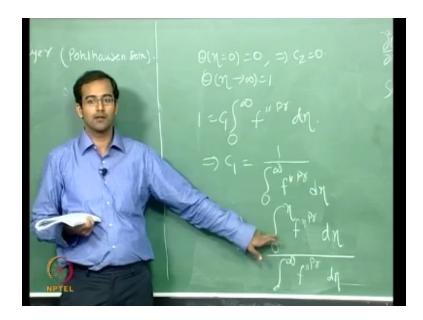
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Q(n=0)=0,=) (2=0 6(n-)~)=1

Okay so this gives my final expression for Θ as a function of H and PR so that is c1 which I have calculated now so in the numerator I have 0.2η F double prime to the power PR D η divided by 0 to ∞ F double prime to the power okay everybody is convinced okay if you have

problems in integrating you have to go back and revise how do you do the integration I cannot now spend half a class teaching that okay so finally you get the solution now although this looks like an analytical solution finally if what you are getting?

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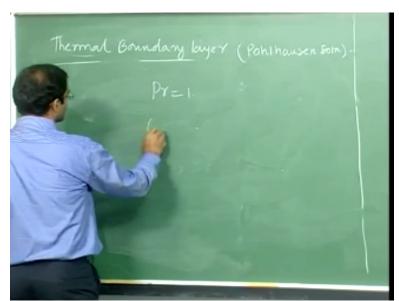
Is not a continuous function right now what you got from the Blazes solution okay numerically was four discrete points therefore now F is available only for discrete points so what you can do you can either do a curve fitting kind of a thing make a continuous function out of it and integrate it here okay so even the integration has to be done numerically okay you can use Mathematical or whatever Mathematical is a very good software where you can do symbolic manipulations now you can directly say that give this equation and f as a function of H and it will integrate and give you the value of Θ as a function.

Of okay or you can also write another program where you can evaluate this integrals numerically you can either use a simple rectangular rule or trapezoidal rule okay so these are very basic numerical integration procedures which you can apply at those discrete points and you can determine these integrals and therefore for any value of prantle number you substitute that you do this integration numerically for the numerator and denominator and you get the solution for Θ . okay so that is that is I mean that is up to you left up to you or the other way what is the other way of finding the solution okay so this is the equation that you know so now I have given you one method where you it looks like an analytical solution but finally you end up doing a numerical integration here okay so what could be the possible method two shooting method okay so which now I hope you have already become so familiar that you would like to use only this method so now you reduce the body into two first order ODS so for example you can say your Θ prime = to X this is one body and based on that you can say X prime = to _ PR by 2 F into X so these are the two first-order ODS okay you have reduced this in two so this let me call this as equation number 5 okay so and you have the boundary conditions for Θ at $\eta = 0$ right so Θ at zero =0.

But you do not have the boundary condition for X at zero right but you know that Θ going to ∞ should be =1 okay now this is the same kind of problem that you did for the Blazes case you did not know the boundary condition for η at 0 the same way here you know you do not know for X okay so you do the same shooting technique you guess the value of x at 0 you discretize it and March you check whether your Θ at large value of η = to 1 okay so you keep doing this hydrate of 1 Y till it finally matches that boundary condition okay so the same shooting technique can be applied for this okay the only thing is for this solution you need F at that point so first you have to solve the Blazes equation.

And use that value of f into this because before solving the energy equation you need to know the flow field correct so you have to integrate the code where you are solving for f with this code you get the value of F and put it into this and then you again apply the same Newtons technique with the shooting method and get the solution for Θ okay so I will just one more thing with which I will stop today if you look at this particular solution which we have derived by method one okay for a particular case where.

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Prantle number is =1 okay what happens so this Θ so I can directly integrate it out now I can do analytical integration if rental number =1 okay so 0 to η F double prime will be F prime correct divided by this is again F prime between the limits 0.1 ∞ so I can say F Prime at ∞ F Prime

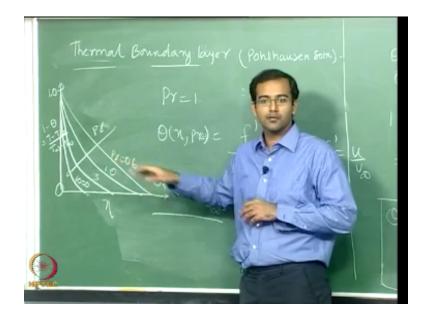
at 0 so f Prime at ∞ what is the value 1 and this 0 so this is essentially F Prime now what is F Prime. Okay so therefore for the particular case for Pr=1.

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Thermal Boundary

Your Θ is exactly = to in fact if you go ahead and complete the solution for different values of prantle number you can do this numerically as a nice exercise and plot 1 _ Θ which is t _ T ∞ by T wall _ T ∞ as a function of η okay you will find curves like this so this is all for increasing values of this could be prantle number 0.6 then this could be 1 3 something like thousand and so on okay so it will start from so at η going to 0 so I have plotted 1 _ Θ so T should be = to T wall therefore it goes to 1 and η for large values of η T should approach to ∞ and 1 _ Θ goes to 0 okay so this is how qualitatively you can sketch the similarity profiles for Θ ok and you can find that for the exact value of PR 1 both the velocity profile.

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And temperature profiles are identical alright so we will stop here today and tomorrow we will continue on calculating the heat transfer coefficient from the temperature profiles.

Numerical solution to the Blasius equation and Similarity solution to heat transfer

End of lecture 13

Next: Pohlhausen similarity solution and Flows including pressure gradient (Falkner-Skan)

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