

**Indian institute of technology madras
NPTEL**

**National programme on technology enhanced learning
Video lectures on convective heat transfer**

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Lecture 11

Boundary layer approximation

Good morning again out of you so today we will continue on the derivation of the non-dimensional form of the navier-stokes equations particularly we are interested because as we said as I said before the course deals mostly with two dimensional incompressible flows so therefore if you I think in the last class I just gave you the non dimensional variables which we are putting into the dimensional navier-stokes equations.

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where

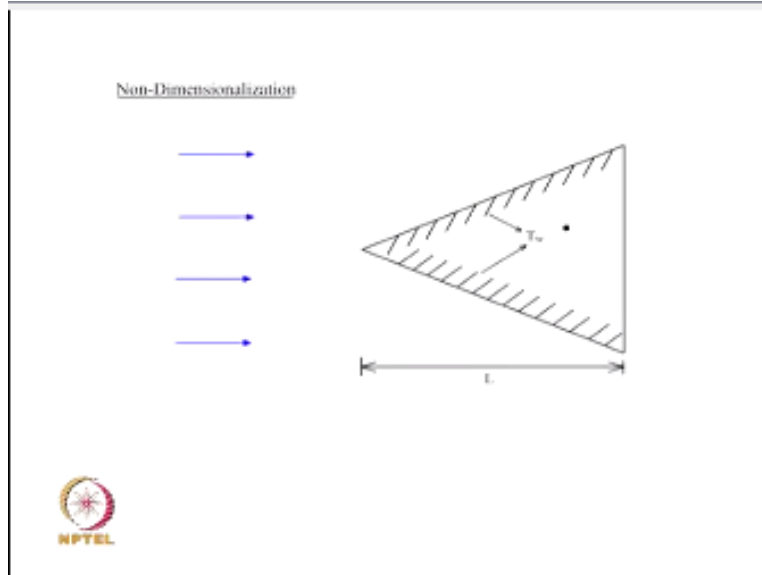
$$\Phi^* = \lambda \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right]$$
$$U = \frac{u}{U_\infty}, \quad V = \frac{v}{U_\infty}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{L}$$
$$P = \frac{P}{\rho_\infty U_\infty^2}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

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And if you plug in I leave that as a nice exercise you can plug it into the steady state two dimensional incompressible dimensional navier-stokes and finally you end up getting this set of non dimensional steady state incompressible navier stokes equation for two dimensions so now this is how your non dimensional numbers come out okay so all these are capital letters which means they are non dimensional okay UVXY capital P okay so and the way that we are non

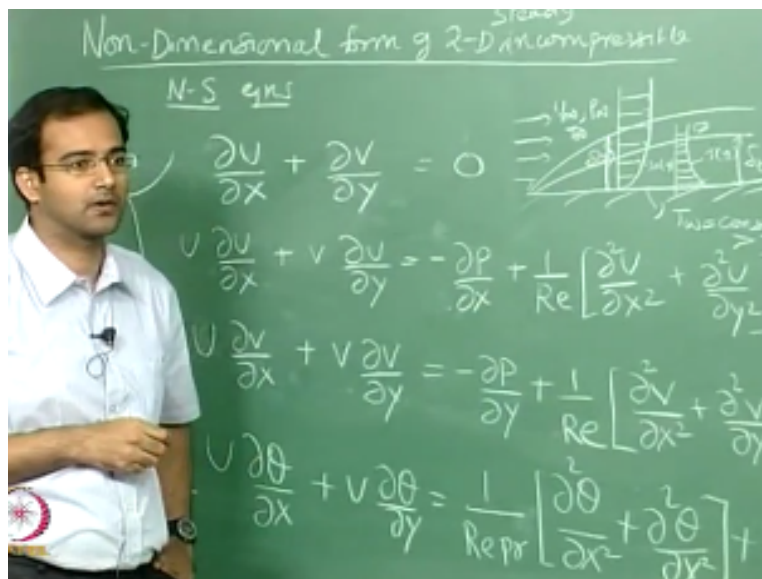
dimensionalizing is shown here this was discussed last class I think all of you can probably reach to this particular point.

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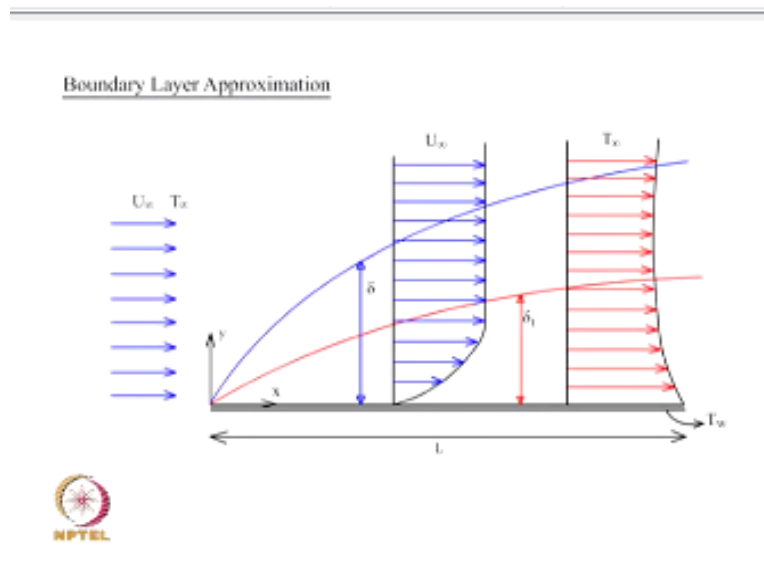
You now the following non dimensional numbers are propping out of this particular non-dimensional set of equations okay. So we will discuss the non dimensional numbers.

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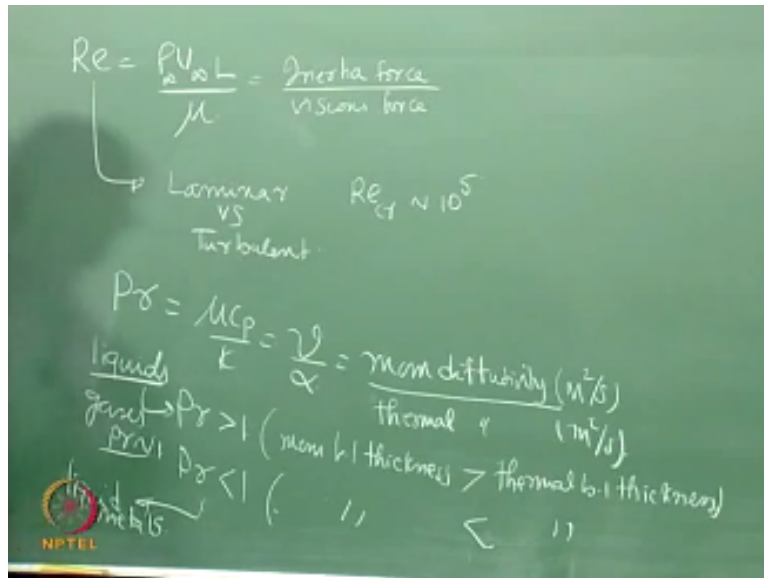
Which are coming out of this okay so for deriving the non-dimensional form you can consider a flat plate boundary layer equation such as what is shown here you are assuming a free stream velocity which is uniform which is not varying along the x axis X direction and you have also temperature boundary condition which is applied to the flat plate so this will induce no a thermal boundary layer developing along with the momentum boundary layer okay so the thermal boundary layer thickness is Δ D function of X your momentum boundary layer thickness is Δ which is also a function of X so this is how the velocity and temperature profiles at any location will appear okay.

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You so for this is a typical case where you want to finally reduce your navier-stokes equation to what are called as boundary layer equations okay so the boundary layer flow is an approximation to the complete full solution of the navier stokes equation okay before we do that approximation let us look at the non dimensional numbers in word okay so first is your Reynolds number which is your $P \propto u \propto L$ by μ where L is your characteristic length here we can use the characteristic length as the for example length of the plate okay so this denotes the ratio of your inertial force to viscous force so if you look at the non-dimensional form of the momentum equation so how can you use the Reynolds number.

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To classify the flows for example so if your Reynolds number very small. Then this term this predominant is dominant over the other term so you can neglect for example the inertia compared to the diffusion terms right here okay so under the limit of small Reynolds numbers you can have a viscous diffusion dominant flow okay at very high Reynolds numbers you can neglect the diffusion terms compared to the inertial terms okay now of course you know you want to operate in a regime where you are not too extreme you know you are not going towards very small Reynolds number or very high Reynolds numbers.

So in those practical situations so you have both these terms which are dominant however the Reynolds number will also be helpful in classifying the flow regime okay so if depending on the Reynolds number you can classify for external flows for example laminar versus turbulent okay if you take the example of a flat plate okay so as you start from the leading edge right here initially it could be a laminar boundary layer and after some certain extent okay your instability start appearing and you have a critical Reynolds number beyond which your instabilities will grow and the flow becomes turbulent and that is classified based on the local Reynolds number okay so typically your local Reynolds number that is the critical value based upon.

Which you classify whether the flow is laminar or turbulent in the case of flat flow is approximately 10^5 to the power 5 okay so this is a good example how you are using the Reynolds number as a means of classifying the flows of course you know you can also make approximations to these terms based on the Reynolds number very small Reynolds number you can neglect the inertial terms extremely high Reynolds numbers you can neglect the diffusion terms so that is one utility of Reynolds number now let us move to the other non-dimensional number okay so what is the other number that is appearing here Prandtl number okay so your Prandtl number here is basically your μC_p by K .

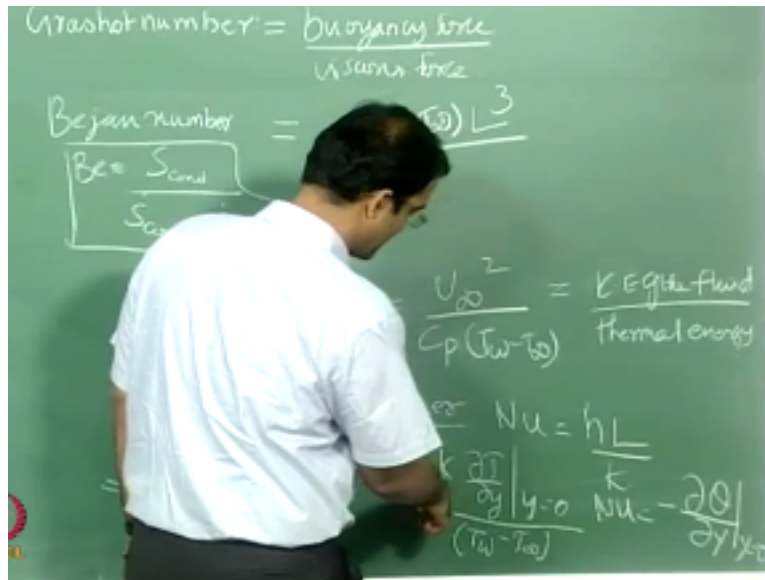
Which is nothing but if you write in terms of diffusivities momentum diffusivity by thermal diffusivity so you can divide and multiply by P so this μ by P is your new and K by P CP by K so will give you $1/\alpha$ basically so this is your momentum diffusivity so what are the units of diffusivity meter square per second okay so now the Prandtl number is a very useful parameter especially when you have flows with heat transfer okay so Prandtl number appears in the energy equation along with the Reynolds number so if you are looking at this kind of is this figure right here so Prandtl number decides how far what is the rate of growth of the velocity boundary layer with respect.

To the thermal boundary layer okay so in this case it shows that your velocity boundary layer is growing at much higher rate than the thermal boundary layer okay so your Prandtl number greater than one signifies that your momentum diffusivity or momentum boundary layer thickness which is related to the diffusivity has to be greater than your thermal boundary layer thickness okay and of course vice versa if you are looking at smaller Prandtl numbers your momentum boundary layer thickness should be less than the thermal boundary layer thickness okay so with that you can look at some approximations that you can make when you are solving the incompressible boundary layer equations.

Wherever you have a very small Prandtl number approximation for example when your momentum boundary layer thickness is much smaller okay so you have to make certain approximations which will simplify your flow so these are again some kind of classifications where you can identify how the boundary layer thickness velocity boundary layer thickness grows with respect to the thermal boundary layer thickness and typically for gases you will have about Prandtl number close to one okay typically for liquids you will have Prandtl numbers much greater than one okay so gases have a Prandtl number approximately one and if you are looking at liquid metals okay so they fall under the category where your Prandtl number is much lower than one okay.

So this is the other non-dimensional number appearing in the energy equation now when you go to Flo's suppose you have a body force here okay which we have not included now okay you can have basically buoyancy which is driving the inertia okay so that is the natural convection so even though you don't have a force convection you don't have a velocity which you supply by means of an external blower or pump you can have the natural convection happening due to the buoyancy forces and that if you include now instead of the Reynolds number defined for force convection you can define another non-dimensional number for the case of natural convection okay.

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So that is let us call the grashof number okay so strictly speaking I have not included here but if you include the external buoyancy and non dimensionalize it you get a number which is nothing but G/R^2 the ratio grashof number two Reynolds number square that is also called as Richardson number so your grashof number is now since you are defining your Reynolds number as inertia force to viscous force when you have buoyancy driven flows you don't have inertia dominance but buoyancy dominance therefore this should be buoyancy force buoyancy force by viscous force okay.

So your by Anzhi force the net force the net buoyancy force which is acting so it is basically your force acting downwards due to the weight of the fluid the gravity minus the force which is acting upwards to it to by on so that is the net force okay so that is $P - P \infty$ you can say into acceleration due to gravity so if you replace the busyness if you replace the density difference apply the boussinesq approximation and replace it with temperature difference so you can write it as $G \beta$ into t minus $to \infty$ so this is your force per unit volume so if you want to convert it as force so that will be in $x l^3$ divided by the viscous force which is nu square okay so this is the exact way that you calculate your rash of number once you know the temperature difference and your coefficient of so this is your 1 by $P D P$ by DT .

Okay so now we will move on to the other non dimensional numbers one which you are the last one which is appearing in that particular non-dimensional form is your Eckert number okay this is actually defined as the ratio of your kinetic energy of the fluid the exact expression that comes out will be $u \infty$ square by CP into $T - T \infty$ so this is nothing but you can say the numerator is the kinetic energy of the fluid and the denominator is your thermal energy so what it means is if you have a high Eckhart number okay so a portion of your kinetic energy actually gets converted into the thermal energy okay.

So this is especially true if you have high speed flows okay or if you also have very high viscosity dominated flows there you have a lot of viscous dissipation effects coming into picture and a portion of the kinetic energies converted into thermal energy so even though if you maintain this wall as adiabatic okay so due to this viscous dissipation effects you can have a temperature profile okay so the temperature profile can set in a sense because of the occurred number which is coming into the viscous dissipation term all right so this is these are some of the non dimensional numbers we will go ahead and define one or two more which are important for heat transfer although they are not appearing in the governing equations.

We will use them very routinely so one is your nusselt number yeah I think this should be so we characterize the heat transfer rate by means of a non-dimensional number okay so which is usually used in convection and that is called the nusselt number the nusselt number you can define based on the heat transfer coefficient which is dimensional multiplied by the length divided by the thermal conductivity and where your heat transfer coefficient is defined as $h = \frac{q}{T_w - T_\infty}$ right so this is your heat flux at the wall divided by in this case $T_w - T_\infty$ okay now if you use the non-dimensional temperature based on.

This definition right here can you tell me what will be the nestled number in terms of the non-dimensional temperature gradient you can plug in further non-dimensional form into the dimensional form and then you can finally calculate your nusselt number in terms of non-dimensional temperature gradient so nice exercise I think you can try out will give you a couple of minutes yes so what should be the expression okay into L by K .

Now I want to calculate the Nestle number can thermal conductivity cancels and you can also write in terms of non-dimensional why h cancels so that will be $h L / k$ at capital y equal to okay so this is the advantage of casting in non-dimensional form your expression gets much simpler alright so now you understand why we define a certain number in this particular way if you have a dimensional form of heat transfer coefficient which is related to the temperature gradient your nusselt number is exactly nothing but your non-dimensional temperature gradient okay and if you look at the physical significance of nusselt number you can also write your nusselt number you can multiply the numerator and denominator by $T_w - T_\infty$ and you can write this as $h L / k = \frac{q L}{k (T_w - T_\infty)}$ okay so this is nothing but what is the numerator here this is your convective heat flux by conduction heat flux.

So what does this represent the non-dimensional number tells you what is the actual contribution of advection okay over conduction okay conduction as I said you know convection has contribution from conduction as well as from advection okay so if this number is greater than 1 okay so if it is equal to 1 then there is no contribution of advection okay so the heat transported from the wall to some distance okay let us say some L okay in fact I should not maybe use L here let me use some H okay just to give you a non-dimensional physical representation of this non-dimensional number.

So it tells you if you have a fluid element vertical distance of H from the flat plate okay so you are calculating what is the contribution of both advection and conduction over conduction okay if that value is equal to one so it means that all of this is happening only by conduction okay and if it is greater than one you know that there is advection which is helping in the transport of heat okay so therefore you are looking at when you are looking at convection you are looking at nusselt numbers greater than one where you are where you know that higher the value of nusselt number better is the heat transfer rate augmentation due to convection alright.

So these are some of the non dimensional numbers of course we have also come across Bayesian number the last class when we were looking at the entropy generation okay so do you remember how we define the Bayesian number in fact I have also given one problem in your assignment to calculate the irreversibility is due to viscous dissipation and yeah ok so this is your you can say your entropy generation due to conduction heat transfer by entropy generation due to conduction + viscous dissipation ok so you can substitute the appropriate terms and you will you will arrive at what we have derived in the last class so essentially it tells you characterizes quantifies okay.

The entropy generation are contributed by conduction over the total entropy generated okay so this is a very useful number ok so far entropy generation people have been talking about qualitatively but now you can calculate the actual values from conduction from viscous dissipation that is the friction of the fluid which is generating entropy and you can actually calculate the contribution of each and then you can take the ratio and that gives you the Bayesian number so these are some of the non dimensional numbers I think more or less if you know these non dimensional numbers this will keep coming through the rest of the course okay.

So I think most of the non dimensional numbers are covered you know you know you won't need anything apart from these numbers okay can you think of any other non dimensional numbers in heat transfer convective heat related to convective heat transfer for example okay so the Stanton number again it is an extension of the way that you are defining nusselt number okay it's simply ratio of nusselt number to the peclet number okay sometimes the Reynolds number times Toronto number is also referred to as the peclet number alright so this is nothing but your $P u \propto P \propto u \propto L$ by what α okay.

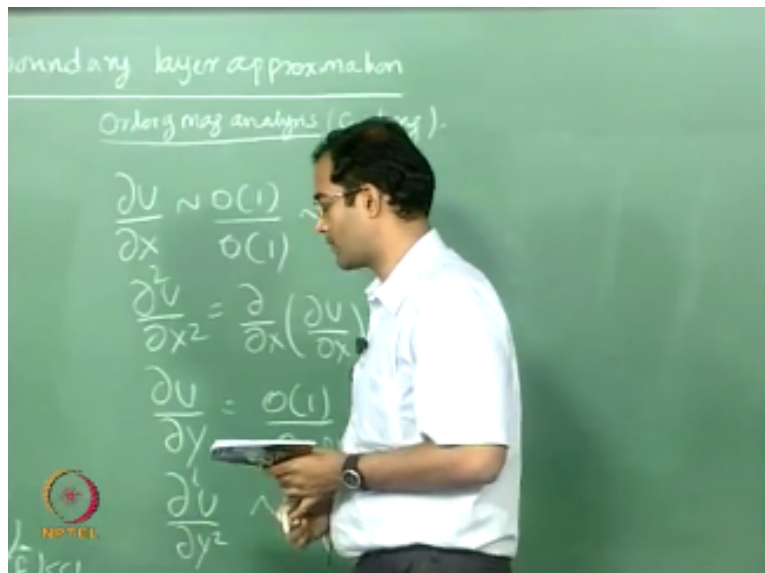
So this is also referred to as the peclet number alright so most of the times you have this pair coming in to many of your expressions so therefore they have combined that and it's denoted as a peclet number so any other non dimensional numbers relay numbers okay so the relay number again is basically nothing but your rash of number times your Prandtl number okay so these are the basic non dimensional numbers you can of course group them together and again give a give a different name to that okay so far as far a single phase convective heat transfer is concerned these are the ones you know if you go to multi phase heat transfer say boiling or so

on and then you will have other non dimensional numbers you know bond number your towards number of a capillary number okay and Weber number.

So on and so for depending on the forces that you encounter okay so any questions on this Richardson number is again as I said it you feel group your grashof number by re square and you group them together as another non-dimensional number that is called as Richardson number okay Chael disappeared this is oh so this is HL by K so KK should cancel them I think it's straightforward right okay KK cancels if you already hear from H if you divide it by KKK directly cancel okay so now what we will do next is to we will use the non-dimensional form from here we will go one step forward as I said we will approximate the navier-stokes equations to much simpler set of equations.

Which we can for which we can find analytical solution okay so that we will do for this particular boundary layer problem okay so we will look at what we call as boundary layer approximation to the navier-stokes equations okay.

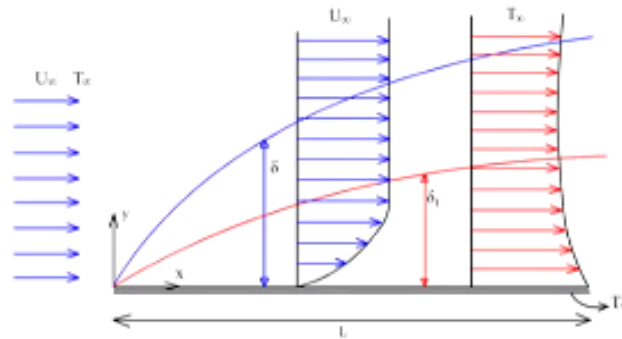
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So the reference figure which we will be using is still the same whatever. I have drawn here.

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Boundary Layer Approximation



You we will be taking any boundary layer flow for simplistic reason you can assume that the free stream velocity does not vary with the axial location but it could also vary still the boundary layer approximations are valid okay so you have a momentum boundary layer you have a thermal boundary layer both simultaneously occurring due to a heat transfer.

Which is taking place we will start with the non-dimensional form of 2d incompressible steady state navier-stokes equation and from here we will apply the scaling analysis okay so when you want to reduce this full equation you want to neglect some terms so for that we have to understand the order of magnitude of each of these terms you know so there has to be a particular way that you are doing it so this is done by the order of magnitude analysis so the order of magnitude can be decided only if you scale the equations okay, so you should know to what extent the dimensions can go for example okay and what extends the velocity can go so once you know that once you know the scale so that gives you the order of magnitude of each term from which you can neglect some terms for this particular problem.

Okay so the order of magnitude is basically decided through scaling so let us introduce some order of magnitude of certain quantities here so before that we will introduce what is called as a non-dimensional boundary layer thickness for velocity let us call this Δ over bar which is nothing but your Δ by L and obviously for the boundary layer flows this quantity should be what very small okay so this is where the scaling starts okay so very first approximation that you introduced in the scaling is when you scale your boundary layer to the length of the plate obviously this is very small quantity okay.

So we will use this as the starting point and we will find the order of magnitude of other terms also when you look at the new velocity that is the non-dimensional U velocity what should be the approximate order of the magnitude that should be approximately but very small so what is the maximum value till it which it can go one rate so the order of magnitude should be

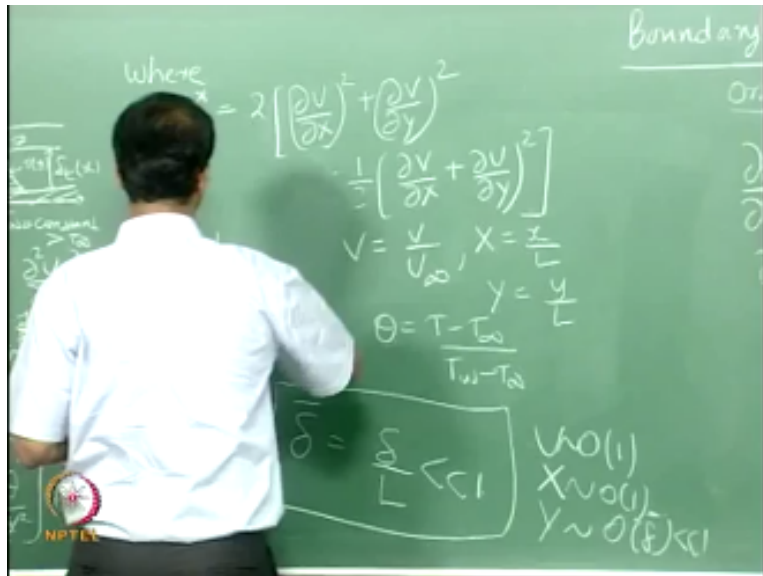
approximately that okay you look at the maximum value and you know what is the order of magnitude of that particular quantity okay now we do not know whether we could be neglected whether we is of the order of one or whether we could be neglected so that we will have to decide okay so we will first write down the okay.

So your u goes to the order of one what about your X order of one okay now I am going to scale my Y non-dimensional e so what I am going to do in the boundary layer I am looking at X which is varying from zero to all the way till the length L and as far as the y coordinate is concerned I am mostly concerned within the confines of the boundary layer okay the assumption here is that the boundary layer is the region of interest because the viscous effects are very important here outside the boundary layer you can treat this as an inviscid potential flow all right so you do not have to solve the full Navier-Stokes equation so therefore I will confine my domain to the extent of the boundary layer so my in that case my Y extent should be of the order of Δs this should be of the order of Δ bar which is of course much lesser than one all right.

Okay so I will start with the terms one by one now you can tell me what will be the order of magnitude of $D u / DX$ so U is of the order of magnitude of 1 X is of the order of magnitude of 1 okay so this will be from the order of magnitude 1 now how about $D^2 u / DX^2$ that is nothing but d / DX of $d u / DX$ so already this is of order of magnitude 1 divided by 1 so this should also be order of magnitude 1 okay so similarly we will do that in all the direction so you have $d u / dy$ so use of the order of Y is of the order of magnitude of Δ okay therefore this will be order of magnitude of 1 over Δ okay.

Similarly your $d^2 u / dy^2$ will be order of magnitude of 1 by Δ^2 .so now we will have to decide what is the order of magnitude of the V velocity okay so that we cannot directly hypothesis so we have to use the continuity equation.

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Okay so we can say my DV by dy okay is of the order of magnitude of D u by DX correct so this should be exactly minus of this right so I know my DV by DX is of the order of magnitude of one and my V is of the order of magnitude Δ that therefore yeah this is you right so therefore we should be of the order of magnitude okay so when I use Δ here it is the non-dimensional Δ so you can put an over bar because all these are non-dimensional variables $\partial u / \partial x$ ∂u by ∂y yes we will come into that okay we will come so first we just write down and then we will later say which order of magnitude is higher which of the terms are probably more important than the other terms so therefore we know so you are DV by DX now should be what should be the order of magnitude and your d square V by DX square again Δ so d square V by dy square Δ by Δ Square 1 by okay so you have all the derivatives okay so we will just substitute into the equations one by one and you know the order of magnitude okay.

So let us start with the continuity here so this will be of the order of magnitude one by one right this is of the order of magnitude what is the order of magnitude of V Δ by Δ so both are of heart of 9q1 okay so of course because we have only two terms and the two terms should balance each other right so they are both of the same order of magnitude so coming to this term use u into D u by DX so use of the order of one by one now here this is of the order of Δ by Δ we do not know the order of magnitude of this okay but you can quickly see that the order of magnitude of these two are same which is of the order of magnitude one therefore in order to balance this should be of the order of magnitude.

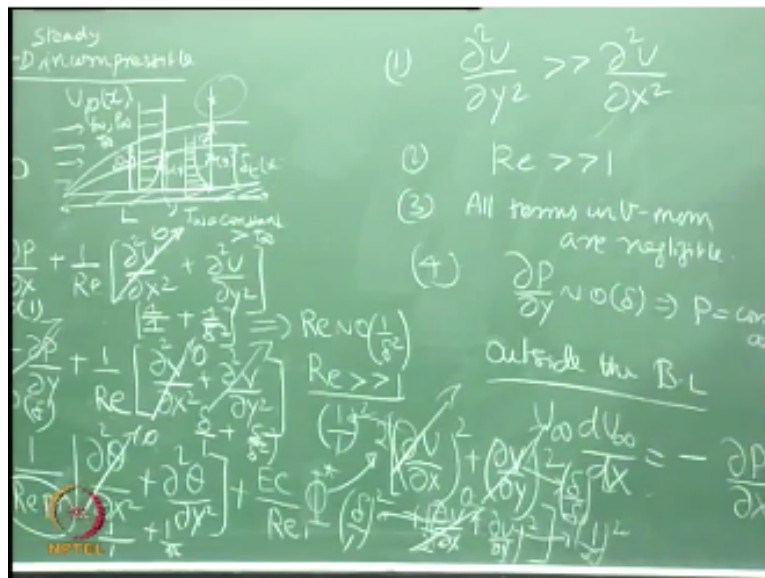
Okay now coming to this d square u by DX square this is of the order of magnitude 1 upon 1 + this is 1 upon Δ square ok now therefore if you compare these two terms since your Δ is very small naturally this term has to be much more significant than this term right so you can say that you can neglect this term directly okay so therefore if you neglect that term so what should be the order of magnitude of Reynolds number such that all the terms have the same order of

magnitude Δ^2 one over Δ^2 okay so therefore from here your Reynolds number should be of the order of magnitude one over Δ^2 what square or Δ so what does it mean so now this puts a constraint on the Reynolds number.

So my Δ is much lesser than one so therefore my Reynolds number should be so only for this particular case this approximation is valid so whenever I look at boundary layer flows I am looking at very high Reynolds numbers okay so now coming to this second the Y momentum equation so what is the order of this Δ by one and this Δ^2 by Δ okay so now so this you can see how the order Δ and Δ^2 therefore this should be of the order Δ coming to these two terms you have this is Δ by one + you have Δ by Δ^2 so this is one over Δ this is Δ okay so which is a more dominant term one over Δ .

So which we can this neglect and we already know the Reynolds number should be of the order of 1 over Δ^2 if you put that you will find that now all the terms are of the order of Δ okay so all the terms in the Y momentum equation are very small already your Δ is very small right so therefore you can safely say that you can neglect all the terms without making much difference all right if you are operating at high Reynolds number all the Y momentum terms are extremely small of small order of magnitude therefore what comes out of this but color analysis the order of magnitude analysis is that two number one.

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You are $\frac{d^2 u}{dy^2}$ is much greater than what right this is the first coming from the X momentum equation that is the first term that you neglected second what it also means your Reynolds number should be quite high okay greater than one and number three okay all terms in V momentum are negligible so what is the particular implication of this particular statement is that you can safely say that my $\frac{DP}{dy}$ is of the order Δ so therefore the gradient is very

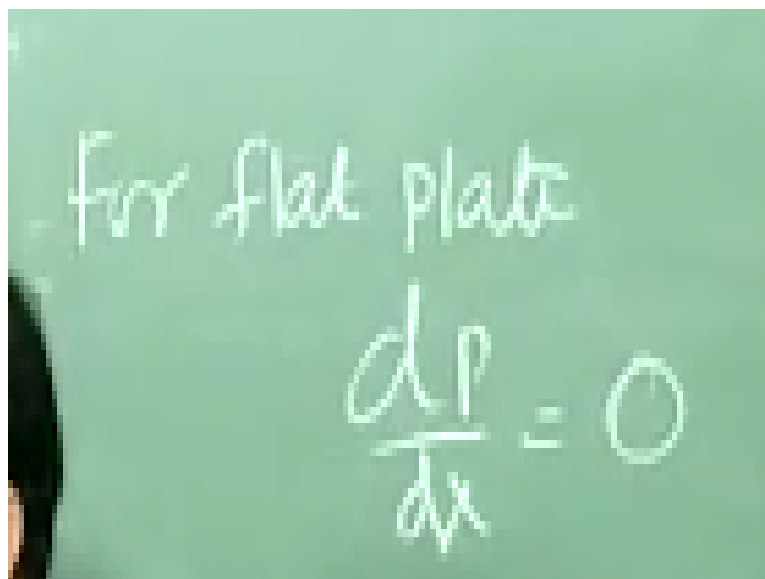
small along the Y direction okay or in other words my pressure is approximately constant along Y okay so this is a outcome of this particular this as $2 \frac{d^2 u}{dx^2}$ the whole square + $\frac{DV}{dy} + \mu \frac{d^2 Y}{dx^2} + \frac{DV}{dx} + \frac{d^2 u}{dy^2}$ I can divide it by half I can multiply by half the statement number three so this has actually saved a lot of my computational effort okay so what it is going to do is that I can safely say if I take any X location for example.

So this states that the pressure here has to be the same pressure here same here same here okay so what this means I can calculate the pressure at this point based on in viscous flow analysis and I can use the same pressure even inside the boundary layer okay so if you apply the navier-stokes equation to outside the boundary layer what will be the what will be the navier-stokes equation when you apply to the bar so if you look at the momentum equation do you have any V momentum equation outside no so we are assuming the flow is in the X direction so now the X momentum equation you have only the u velocity V velocity is not there.

So this term is zero okay and how about your discuss effects okay so it is negligible and therefore you have only two terms one is your inertia term $\rho u \frac{\partial u}{\partial x}$ of course your U is nothing but u_{∞} outside okay, so that should be equal to minus $\frac{DP}{dx}$ Oh square see all of this are nothing but your viscous effects okay so if your viscous effects are negligible outside in the free stream so therefore you can safely neglect all of this all right so now based on this we can calculate do P_{∞} yeah because in this case it does not matter P_{∞} and your P here they are going to be the same so you do not want to differentiate your free stream pressure from the pressure inside of the boundary layer okay.

So if you say that you do not have any pressure gradient or in other words you do not have any velocity variation that is for flat plate flows that we are looking.

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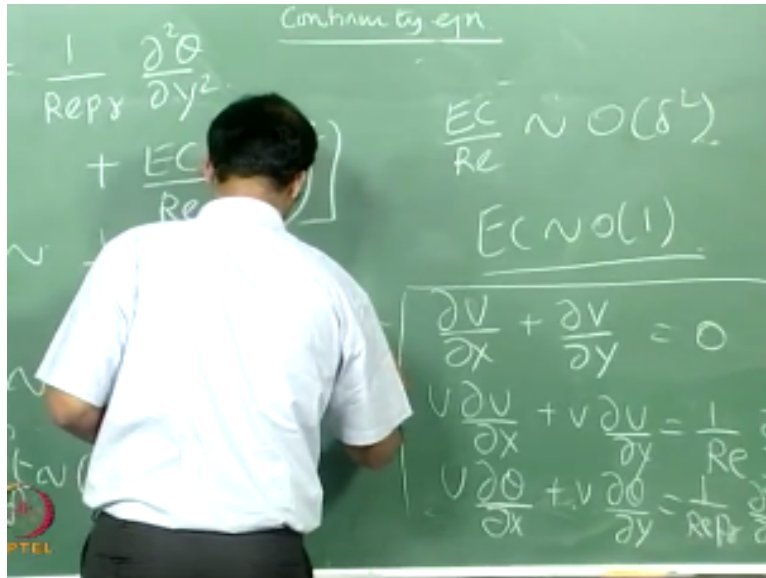
Now you can safely say $\frac{DP}{DX}$ this is what zero because my velocity free stream velocity is not going to vary okay so for cases where your free stream velocity varies so if your free stream velocity is a function of X you can have two kinds of flows one if you have adverse pressure gradient so in this case your $\frac{DP}{DX}$ should be the pressure should keep increasing okay so this should be greater than zero therefore your velocity will keep decreasing okay.

So that is the outcome you can have favorable pressure gradient where your $\frac{DP}{DX}$ should be less than zero so that means your velocity keeps increasing okay so you can have either of this depending on the kind of flow externally that you are looking at if you ∞ is equal to a constant okay then this is the best approximation it is going to knock off the pressure term completely otherwise you have to consider either of these cases so therefore so that is the approximation to the momentum equations coming to the energy equations okay so what will be the order of magnitude of θ what do you think is the order of magnitude of θ one right you are scaling it between your wall temperature and the free stream temperature so it has to be of the order of 901 so therefore they should be of the order of magnitude one by one and how about your V here this should be $\Delta \theta$ is already of the order of magnitude one so V is of the order of Δ by now when we are considering the temperature gradient okay.

Which boundary layer we have to look at thermal boundary layer so for temperature gradients in the thermal boundary layer our Y scales with ΔT that so therefore we have to use this as the scaling parameter okay so we will say that this goes as ΔT bar all right so coming to this term right here so this is of the order of 1 by this is of the order of $1 + 1$ by ΔT^2 okay I do not have space I am just squeezing in and coming to this term right here the order of magnitude of V^* this is nothing but you are I can write whole square so this is my viscous dissipation terms and viscous dissipation terms now if you look at it this is of the order of 1 divided by 1 this is of the order of Δ divided by Δ this is of the order Δ by 1 and this is $1/1$ by Δ .

So all this the whole square basic right they are all inside so now which are the terms that we can neglect okay so first coming to these two terms between these two terms which is the smaller term this is of the order of $1/\Delta^2$ okay so this is going to be larger than okay so this can be neglected coming to the viscous dissipation term here so this is of the order of 1 this is of the order of 1 okay now this is of the order of Δ^2 this is of the order of $1/\Delta^2$ so only so this is the most significant term so all of the terms can be safely neglected so finally if you write for temperature you end up with the following equation.

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Which is $UB \theta$ by $DX + V D \theta$ by $D Y$ will be equal to 1 by re PR into square + you are required by Reynolds number times your 2 so 2 by 2 cancels there you have what is the only term that is important $d u$ by dy (2) okay so now what should be the order of re PR okay re PR so this is of the order of magnitude of 1 over ΔT Square right so for all the terms to balance re pier should be of the order of magnitude 1 over $d \nabla \cdot T^2$ correct understood so already you know our ease of the order of 1 over ∇^2 therefore PR should be the order of ∇ . By Δ -t the whole square or in other words your ΔT by Δ should be on the order of Prandtl number power minus half okay.

So this is a very important conclusion so this is the relationship between the ratio of your thermal boundary layer thickness to momentum boundary layer thickness with the Prandtl number okay and what should be the order of magnitude of Eckert number by Reynolds number so if you look at this term this is again one over ΔT^2 right this should be what just one small thing when you are looking at velocity gradient you look at the momentum boundary layer thickness right when you look at the temperature gradients you look at the thermal boundary so this should be of the order of Δ^2 knot ΔT^2 okay.

So now Eckert number by Reynolds number what should be the order for all the terms to be equal is it $\Delta^2 \Delta^2$ everyone agree okay so we already know Reynolds number is one over Δ^2 therefore the order of Eckert number should be one okay so this finally will give you the boundary layer equations which I am going to summarize and stop here so after all these approximations this is the final terms set of terms that you are going to retain so your X momentum of course for a flat plate you can also have you won't have this term but for favorable and adverse pressure gradient so you can have $d p/d x$ and your energy equation okay so this are called the boundary layer equations okay.

So any questions I think I had been a little fast in the end but I think you can fill up the I think we did almost you have to just go and revise okay how we have introduced the order of magnitude and we have skipped all the terms the important conclusion now you have a direct relationship between the ratio of your thermal and the momentum boundary layer thickness to the Prandtl number and of course you have the order of magnitude of Reynolds number and the occurred numbers all right this point okay so this is coming directly from the Y momentum equation right.

So this is this is correct so what directly tells you is that your P should be invariant along Y okay so therefore if you calculate your pressure somewhere outside the boundary layer using the Euler equations okay the same pressure will be valid even inside the boundary layer okay so we will stop here.

**Boundary layer approximation
End of Lecture 11**

Next: Laminar External flow past flat plate

(Blasius Similarity Solution)

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