Spray Theory and Applications
Prof. Mahesh V. Panchagnula
Department of Applied Mechanics
Indian Institute of Technology, Madras

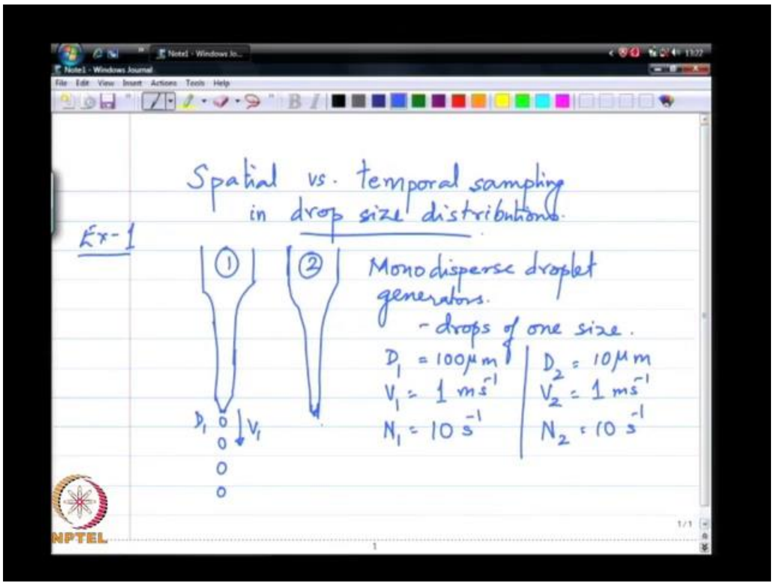
Lecture - 03
Spatial Vs Temporal Sampling example problem

We will continue our discussion of drops and drops size distributions in sprays, and we will begin with a few questions and try to answer them.

Student: Drop size distributions, we came across or a spatial drop size distribution under temporal drop size distribution. What is the primary difference between the two drop size distributions?

The question relates to different measurement techniques of measuring drop size distributions in sprays. We discussed spatial sampling and temporal sampling; we want to see if we can use an illustrative example to differentiate between the two in some. It will be a simplistic illustrative example, but sufficient to understand the complexity is involved.

(Refer Slide Time: 01:14)

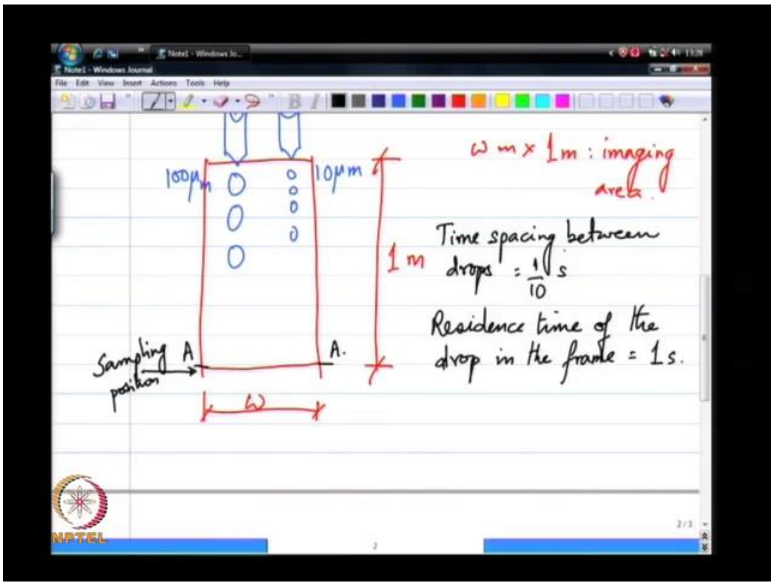


Let us take couple of mono disperse droplet generators. So, these are mono disperse droplet generators, I will number them 1 and 2. The whole idea of mono disperses means that they are producing drops of one size. Let us say these drop the generator 1 and let us will the call this example problem number 1 for now. Let say droplet from generator 1 is 100 micrometers.

So, essentially it is producing a stream of droplets that are all diameter D 1, moving it on velocity V 1. And they are produced at let say I will give them some numbers let us say 1 meter per second. And the number rate of production is let us say 10 per second. So, we producing 10 drops of 100 micron diameter per second and each one is being ejected out from the needle at a speed of 1 meter per second in the axial direction. One should note that the number frequency is nothing to do with velocity; they are completely different measures. How pass the drop may be moving.

Likewise, we will say D 2 is much smaller let say it is only 10 micrometers. We will start by looking at the situation were V 2 and N 2 are the same. We will see what this generator would give us for a spray.

(Refer Slide Time: 04:09)



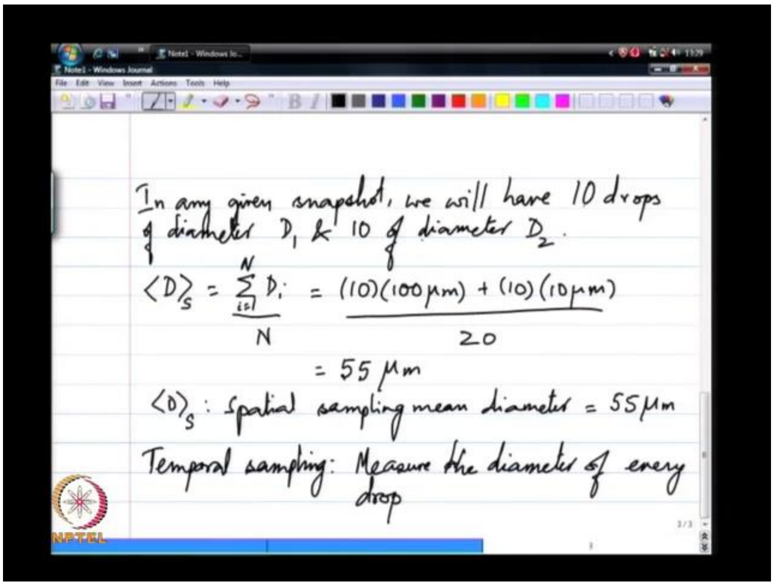
Essentially, my atomizer is composed of two little needles; one producing 100 micron drops and another producing 10 micron drops. If I take a region of interest that includes the tip of this nozzle, let us say something that is 1 meter in diameter and the width really does not matter; this is imaging interrogation area or imaging area. So, that is the area that my camera is captured.

So, as simple way could be that I have a back light and the black light is illuminating their drops and when I view from the front I see the shadow of the drops. So, the individual shadows have from circles in a snap shot, in a frozen picture and from those individual circles I can get the size of its droplet. Since I am producing; so let us see is each droplet is moving at 1 meter per second and the spacing between droplets is one tenth of a second. So, that production spacing between droplets is one tenth of a second. So, if I take one snap shot that is 1 meter in height, let us say the time spacing between drops is 1 by 10 seconds. This is coming from the fact that N 1, I will call this N 1 dot where is number production rate.

So, if I take N 1 and N 2 both are 10 per second, this means I am producing drops; one drop every one tenth of a second which is the same as saying the time spacing between the drops is one tenth of a second. Since the velocity of the drop is 1 meter per second, if the interrogation area has a height of 1 meter the drop is going to be resident in there for 1 second.

So, the second thing is to do with a residence time is 1 second.

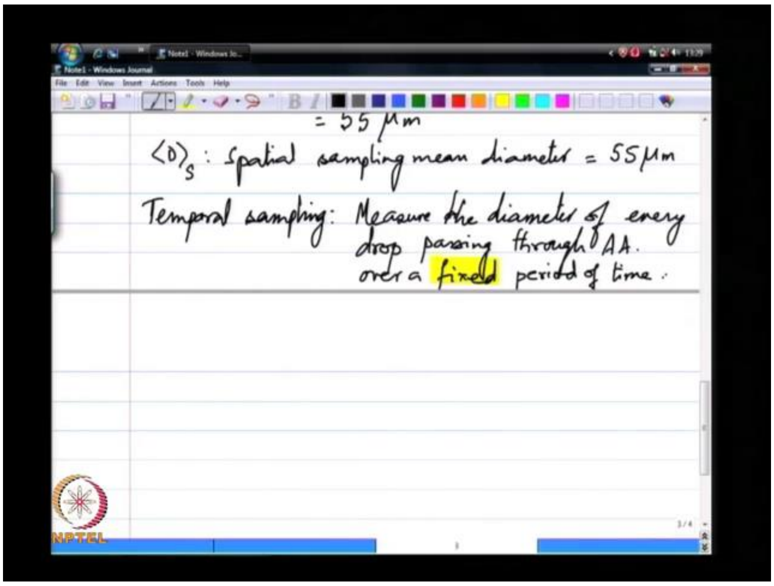
(Refer Slide Time: 08:24)



Which means, I take a snap shot I am going to have 10 of diameter D 1 and 10 of diameter D 2; so if I now look at in any given snap shot I want to calculate, let us say this simple arithmetic mean diameter which is sum over all the drops D i divided by N. If I do this I have 10 times 100 micrometers plus 10 drops of 10 micrometers divided by 20 which happen to give me a number like 55 micrometers. This is the diameter I get; I will put this D s for spatial sampling.

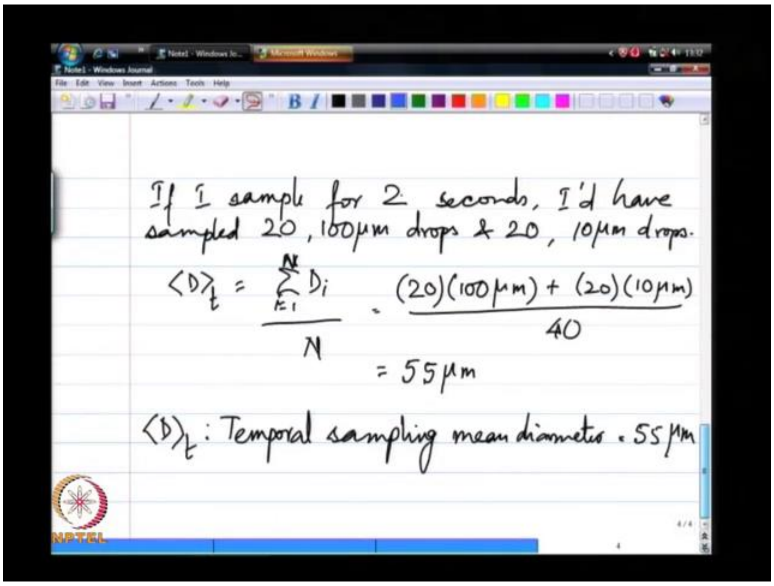
For the same situation if I do temporal sampling, so this is spatial sampling mean diameter equal to 55 micrometers. For the same spray if I do a temporal sampling what do I get. So, the whole idea of temporal sampling is that I am going to remain at some cross section here let us say, this is my sampling position and I am going to accrue statistics of drops passing through this point, I will call this cross section A A. I am going to measure.

(Refer Slide Time: 11:06)



So, the next thing will talk about is temporal sampling. The steps involved are; measure the diameter passing through A A over a fixed, this is very important; period of time. So, what happens if I do this?

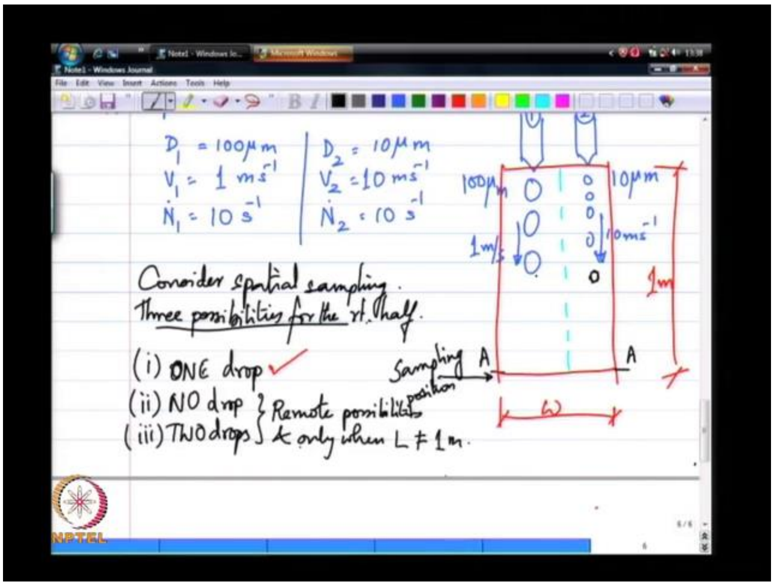
(Refer Slide Time: 11:39)



Now I know that I am producing 10 drop per second which means if I sample for 1 second, I will do just to glue away from 1. If I sample for 2 seconds, I would have sampled 20, 100 micron drops and 20, 10 micron drops. So, again I can calculate the temporal mean diameter.

So, as it turns out the temporal mean diameter is also 55 microns. Now we have two parameters characterizing; two separate measures of a time associated with a drop. One is the velocity another is the frequency of production. So, we will keep the same frequency of production and see what happens if we changes the velocity. That is our first test.

(Refer Slide Time: 14:34)



So, I will now take the exact same situation. Except I am going to make this V 2, so I call this my example 2. Example 1; first fairly straight forward we got the same mean diameters, whether you sampling the time, in time domain or whether you sampling the spatial domain. So now, if the drop is moving fast, and I will go back to the same exact spatial. Now the drops are moving fast, these drops are moving at 10 meter per second. In relation to these the drops that only moving at 1 meter per second, but that time spacing in the production is exactly the same.

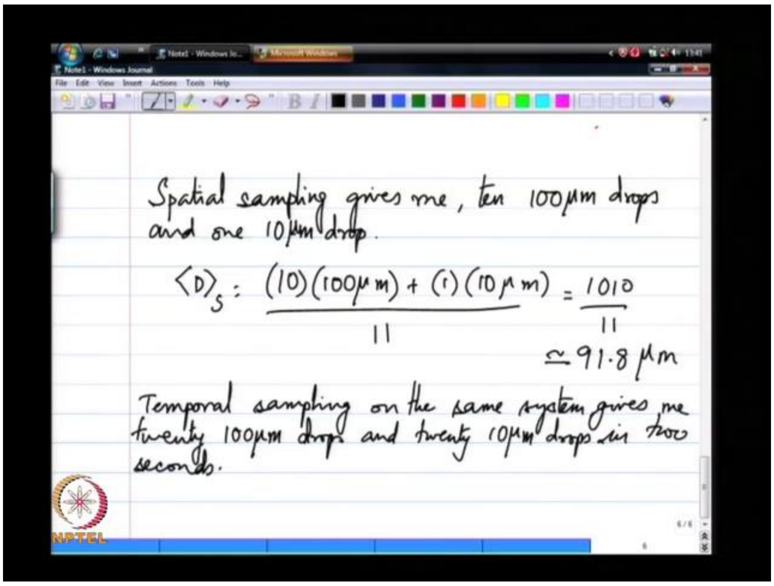
So if I did spatial sampling, just like I did before consider spatial sampling first. If the drop is moving at 10 meters per second and the length of this whole thing is 1 meter, it takes one tenth of a second to travels the whole meter. Whereas, I am producing one drop of 10 microns every one tenth of a second. So, what are all the possibilities for how my right side of the image will look like? Hence, I want to distinguish between the right and left side, because it is like a very simple spray, like a two mono disperse sprays super posed. And if we understand how this works we can get a feel for how the real spray works.

So, the drops that are 10 microns in diameter are moving at 10 meter per second; that is a given drop is in this frame for only one tenth of a second. So, I have 3 possibilities for the right half depending on how my; one depending on whether I am when I take this snap shot, I could have one drop somewhere in the middle of the frame like that, and the time before the next drop is produce this like further in the future compare to my frame, and the previous drop is already exited the domain. So, I could have one drop in the frame or there is also a very remote intense, say for example if this was not 1 meter but 0.9 meters, or slightly less than 1 meter, I could also have the instance where the drop has just exited and I have not get produce the next drop on the right half. So, I will have no 10 micron drop.

I could also have again if it is slightly greater than 1 meter I could have the instance where I have one drop just about to exit and another drop just about to be produced. So, really speaking I have just one drop in the frame, somewhere in the frame or I could have no drop or two drops. These are all remote possibilities and only when L is not equal to 1 meter exactly. If L equal to 1 meter then if this is exactly mathematically correct then it has to I will only see one drop the drop has to exit before I see the other drop.

So, really speaking the only possibility we want to look at seriously is this one drop of 10 microns that is the only possibility. On the left hand side nothing has changed, I will still see 10 drops, because the time production of each drop is the time separation between each pair of drop is one tenth of a second. And there all cued of one behind the other. So, in 1 meter physical space I have 10 drops one behind the other if there all moving at 1 meter per second.

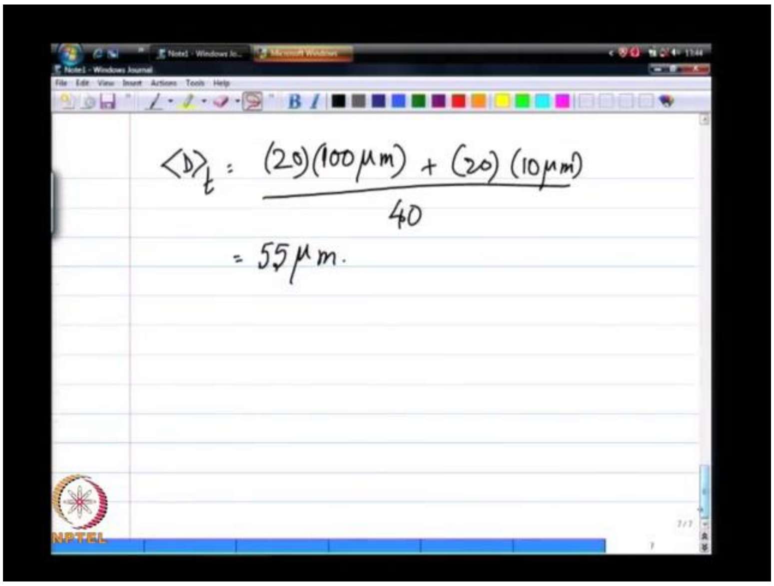
(Refer Slide Time: 20:32)



So, essentially if I do this spatial sampling correctly 10, 100 micron drops and one 10 micron drop. So, if I do my spatial mean average approximately 92 microns.

If I do the time sampling let us see what we find. So, temporal sampling on the same system is based on the fact that I have one drop coming through here every one tenth of a second, and so if I again sample of 2 seconds I will get 10 or 10 times to twenty 100 micron drops and twenty 10 micron drops, so sample for 2 seconds.

(Refer Slide Time: 23:42)

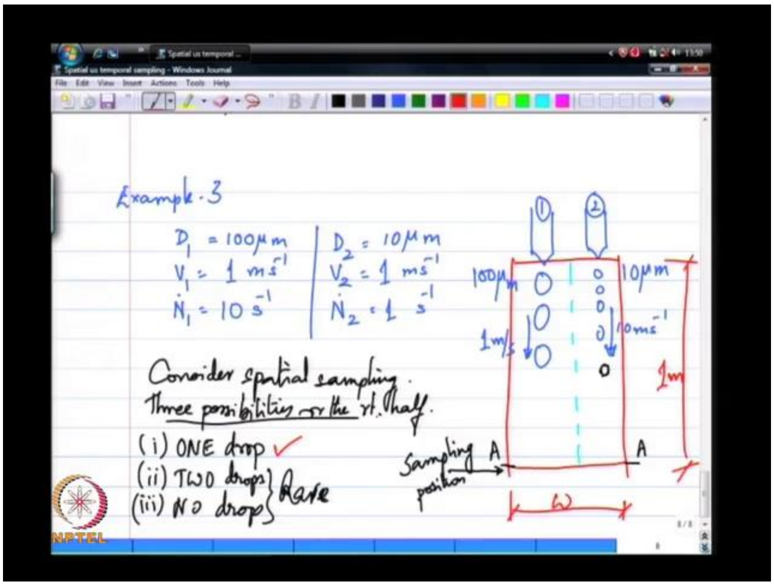


Which means, again if I look at what happens, what is it that I am measuring when I do spatial sampling, what is it that I am measuring when I do time sampling. If I time sample at for 2 seconds; that means, I am getting 20 drops that are at 100 microns. Now you have the sampling and it is like a sampling cross sectional area in which I am allowing these drops to go through.

So, every time a drop goes through I have a way of getting its diameter, measuring its diameter. So, the rate at which drops can go through here in a steady spray has to exactly equal the production frequencies, whatever be the production frequencies that is what I end up measuring here. So, what you get in the time sampling in the time domain is basically related to your N 1 dot and N 2 dot.

We can look at an example 3 where we go back to V 2 being 1 meter per second and N 1 dot or N 2 dot being let us say 100 per second or 1 per second. You will see that. So, let us actually complete that example, just so we gain or complete understanding of the whole problem.

(Refer Slide Time: 26:00)

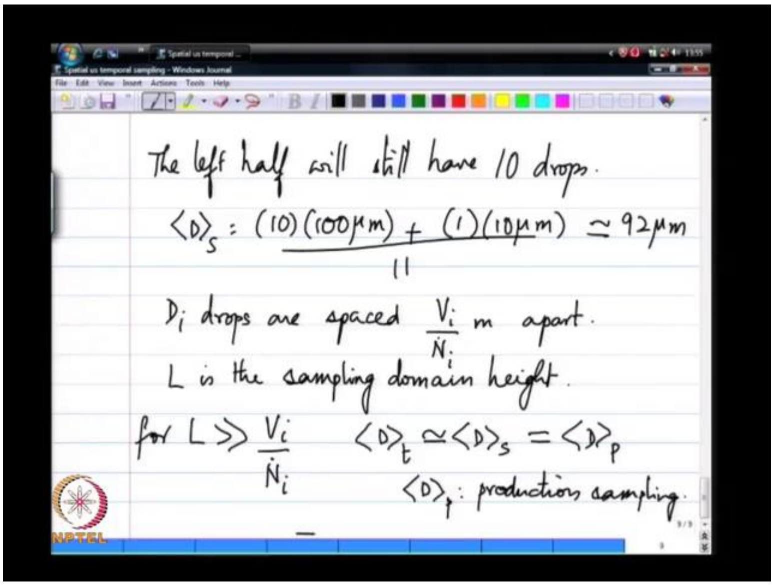


I want to go back to example 3 where V 2 is back to being 1 meter per second, but N 2 is now down to 1 per second. When you do that you end of seeing; now I am going to erase this part here because it corresponds to the previous example. But what could I possibly look for when I do spatial sampling in this. The fact that I am only producing whether all the drops are moving at 1 meter per second.

So, in any the drop is resident in this frame for 1 second by an end I am producing only one drop per second. So, if I look at what I will end up observing (Refer Time: 27:58). So, if we look at the rate of production of the drop are sized to in this example 3, it is 10 times slower than the rate of production of drops are size 1. If I do spatial sampling I am producing one drop per second and that one drop is going to move at 1 meter per second. That is, the drop that is produced will only have one possibility that there only have that there will be one drop in the frame. The other three where it is just about to exit or just about to be produced are sort of rare events. So, two drops and no drops are both rare events.

So, in the 1 meter frame that I have the only possibility is that I will have one drop resident in there, it is moving at 1 meter per second and it going to take 1 second to traverse through the frame and it is going to be 1 second after this drop is produced that the next drop will be produced, whereas that is the story for the right half.

(Refer Slide Time: 29:50)



For the left half the story is not changed. So, if I do this spatial sampling what I have is 10 drops. Now, does the size of the domain make a difference? So, if I have instead of 1 meter if I sampled over 2 meters, will that make a difference? Chances are again the drops are spaced so for any given drop D i.

So let us say in this case I take a situation where V i divided by N i dot is; and let us say L is the sampling domain height. I will take a situation where I have L is much, much greater than any of V i divided by N i dot that. So, let us say I will I will recreate the situation where V i is not 1 meter per second, but let us say 10 centimeter per second or 1 centimeter per second and N i dot is like 1 per second or 10 per second and we end up packing the drop much closure now. If L is very large in comparison to the max of V i divided by N i dot. What you end up seeing is that you will have these kinds of possibilities of whether I have 99 drops or 100 drops on the right hand sides has supposed to 1 or 2.

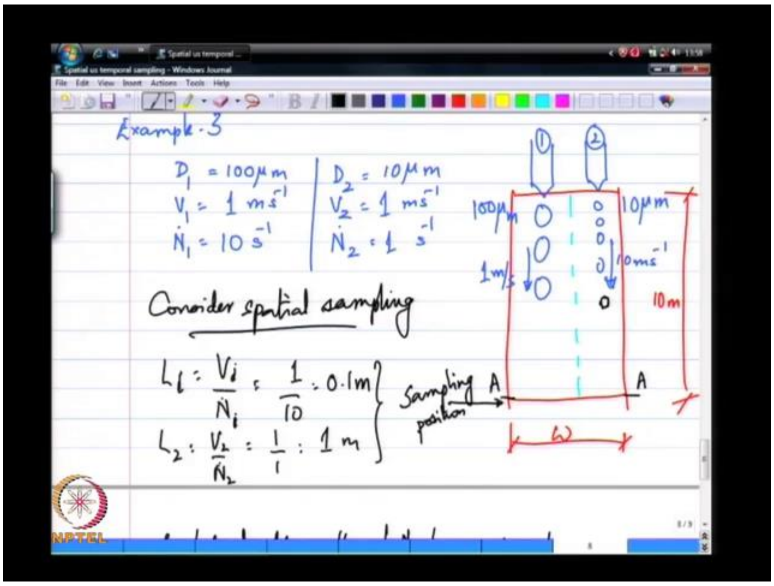
So, the percentage error associated with number of drops being slightly different on one side versus the other keeps going down. So, the spatial sampling gives you a result that is closer to the production rate sampling to the time rate sampling. Think of an extreme situation where I could sample an infinitely long domain, in which case every drop produced is at the moment present in the domain. So, if I took a snap shot of this domain it cannot be different from having sampled every drop produced in time.

So, essentially when we say spatial sampling and temporal sampling they are different as far as finite time of sampling are concerned. It is only when I say I am sampling over 1 meter, sampling over 2 seconds that you will get two separate answers. If you go to the extreme case of sampling over an infinitely long domain or and an infinitely long time those two answers are exactly the same. Those are true representation of the production rate; that is the SMD or that is a mean diameter based on the production rate is what I can completely try to the atomizer performance, everything else could be an artifact of the way I am sampling in the spray.

Essentially, if I take these two sprays and if I have L be much greater than V i divided by N i dot then the spatial sampling and the temporal sampling become very close to each other. And I will say that both of these will end up becoming equal to the production sampling. Actually, production sampling is something that is already exactly equal to the temporal sampling, but just too sort of has a physical feel for sampling in time being different from sampling at the nozzle exit.

If I now look at, I can now come back and quantify this effect of the size of the domain.

(Refer Slide Time: 35:41)



So, in go and let us say, instead of 1 meter if I went up with 10 meters. Same exact system what are the possibilities that I have here, instead of three possibilities I actually have many more but we will rewrite this. If I am sampling over a 10 meter long domain same rest of the performance characteristics are the same and producing one drop of every second. And I have 10 drops of N 1 dot being produced.

So, essentially I have 10 drops that are 10 microns in size and 10 drops that are 100 microns; sorry one ten, so I will have 100 drops that are 100 microns in size. The spacing between a pair of drops being the same for L 1 is 0.1 meters I call this L 1. So, if the size was 10 meters I will still end up getting the proportion of the drops D 1 to D 2 would still remain the same, because the physical spacing between them is still the same.

So, the mean drop size tends towards the temporal sampling mean drop size. The spatial mean drop size tends towards the temporal drop size remains distinct as long as the sampling domain is of a finite size. As you go to a larger and larger domain size cut they the two values converge to a particular unit.