Fundamentals of Operations Research

Prof. G. Srinivasan

Department of Management Studies

Indian Institute of Technology, Madras

Lecture No. # 21

Inventory Models- Deterministic Models

In this lecture we continue our discussion on deterministic inventory models. We have already defined inventory control.

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We have various cost associated with inventory systems these are cost of the product ordering cost carrying cost and shortage cost.

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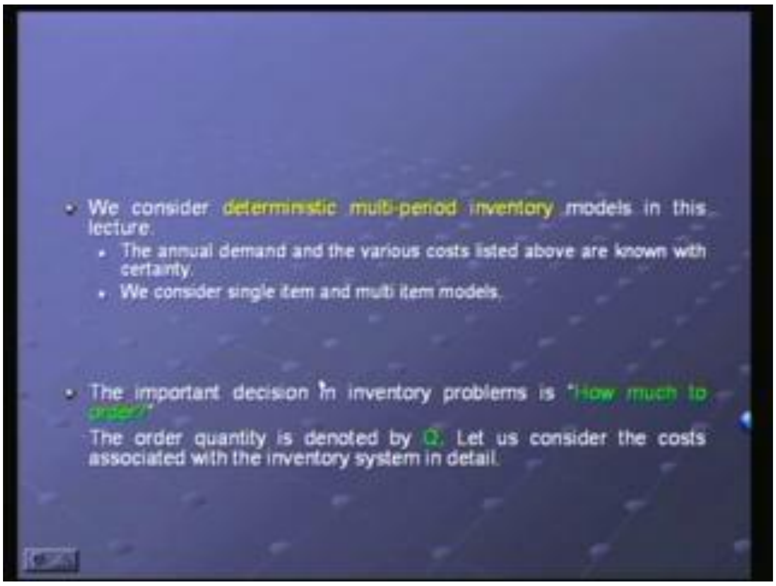

We have also seen that we will be addressing deterministic inventory models in this lecture and the most important decisions in inventory problems is how much to order. Now the solution to this is called the order quantity which is denoted by the capital letter Q.

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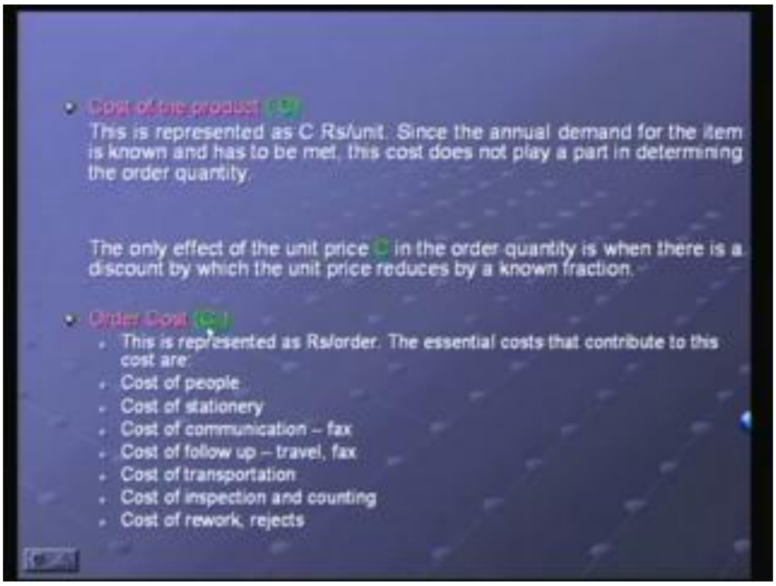

The costs have been discussed in more detail. We have seen some aspects of the cost of the product as well as the ordering costs C_0.

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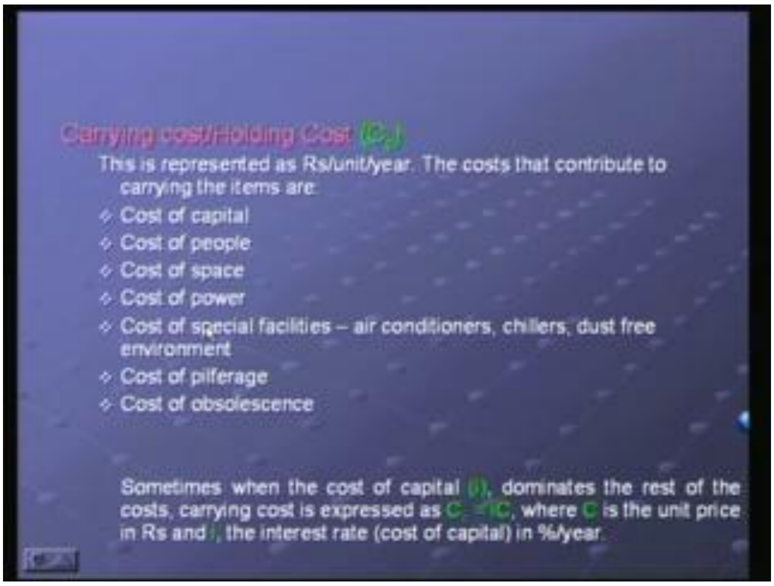

Now let us look at the various aspects of the carrying cost which is C_C. Now the carrying cost is represented as rupees per unit per year. The various costs that contribute to carrying the items are cost of capital. In order to buy these items we need to borrow money and there is a certain amount of money that is spent as interest on the money borrowed. So that is reflected in the cost of capital cost of people. The items that have to be carried are stored in warehouses and we require people to handle and to control these warehouses. We require space to carry these items and we pay rent and we incur other cost associated with space. There is also the cost of power which is needed to maintain the warehouse in addition, we might require some very special facilities such as air conditioning, chillers and dust free environment for certain items and therefore the cost of carrying such items increases. In addition to all these, there is the cost of pilferage where it a small portion though of the items may be stolen and there is also an associated cost of pilferage and there could be cost of obsolescence items that are bought in large volumes and stored.

There could be occasions where by the time the item is taken to use, the item would have become obsolete and therefore would become useless. So these are the various costs that contribute to the carrying cost or the holding cost represented in rupees per unit per year or money per unit per year. Sometimes when the cost of capital dominates, the rest of the cost, i.e., the first cost which is the interest paid on the money borrowed, dominates the rest of the cost. The carrying cost is represented as C_C = i into C where i is the interest rate and C is the unit price. C, the unit price is represented in rupees or money value and i the interest rate is represented as percentage per year. So the product will be rupees per unit per year.

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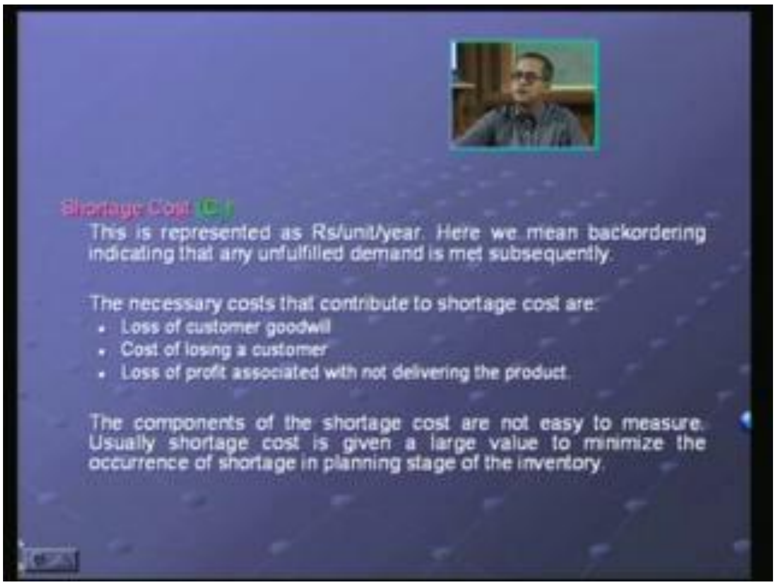

The last of the cost, we will be looking at is called the shortage cost C_S, C subscript S and is also represented in rupees per unit per year. In this lecture when we say shortage, we mean back ordering indicating that any unfulfilled demand is to be met subsequently. The necessary cost that contributes to shortage is loss of customer goodwill, Cost of losing a customer and loss of profit associated with not delivering the product. So when there is a shortage or when a particular product is not delivered in time, the first thing the organization looses is the loss of profit associated with not delivering the product. In addition there could be loss of goodwill, the customer may not like the organization, if the organization continues to deliver late and then over a period of time the organization may end up losing the customer. So these are the important components which contribute to C_S cost of shortage. The components of the shortage costs are very difficult to measure. They are not as easy to measure as the other costs. Usually shortage cost is given a very large value. We normally assume a large value for the shortage cost to minimize the occurrence of shortage in the planning stage of the inventory. Among these 3, loss of profit associated with not delivering the product is easily measurable while loss of customer goodwill or the cost of losing a customer cannot be measured that easily. So C_S is usually is not a measured quantity but it is a quantity assumed to be very large, so that shortages are minimized during the planning stage.

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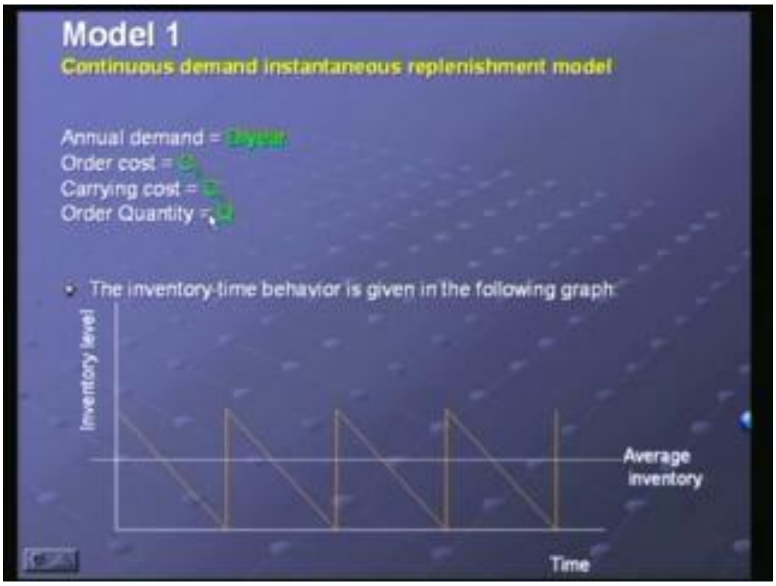

Now let us consider the first model that we are going to see in the inventory. In this model we assume that there is a continuous demand and there is an instantaneous replenishment. Now the annual demand is given by D per year. The ordering cost is rupees C_0 per order. Carrying cost or the holding cost is rupees C_C unit per year. The order quantity is given by capital Q. Now when we say that the demand is continuous we make an assumption that the annual demand is D per year. Then a monthly demand is D/12 and the daily demand assuming 365 would be D/365 and so on. The graph here shows the inventory versus time for the item that is chosen. This is actually the graph of the inventory versus time. Now we assume that an order quantity Q has an order for quantity Q, has been placed and because of instantaneous replenishment. We assume that the order has come instantly because of instantaneous replenishment we will assume that we will place the order when the order when the stock on hand becomes 0. So we assume that the order has just arrived and the stock position is capital Q which is this position and because of continuous demand is there at every moment of time the stock position or the inventory position reduces in a linear fashion and it reaches 0 here. This is a cycle.

The cycle is the time taken for the inventory to reach 0 from Q. This is called one cycle and at the end of that cycle an order is placed for the same quantity Q and it is instantaneously replenished, so the stock position goes up from here to this position and then the cycle repeats and so on which is shown in this. The average inventory is also shown here. The question is out of these 4 what this order quantity Q is the variable and the other 3 parameters namely annual demand, order cost and carrying cost are known, so what is the value of Q which minimizes the total cost?

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The 2 cost that we will consider in these are the ordering cost and the carrying total annual ordering cost and the annual carrying cost. Now if the annual demand is D, the ordering quantity is Q which means every time we place an order, we order for Q items. The total number of orders per year is D/Q. The annual order cost will be D/Q into C not or DC not by Q where there are D/Q orders and every time we place an order, we incur an order cost C_0. The average inventory in the system is Q/2. This is seen from this figure. We can compute the average inventory in more than 1 way. An easy way is to do an average inventory. The total inventory is the area under the curve divided by the period which would give the average inventory. The area under this triangle in a cycle is 1/2 into base into height, 1/2 into T which is the cycle into Q divided by T will give us Q/2. We can also look at it as if the cycle begins with the quantity Q ends with the quantity 0. So average inventory in a system like this would be beginning inventory + ending inventory/2 which is Q + 0/2 which is Q/2. So the average inventory in this system is Q/2. Average inventory carrying cost will be Q/2 into C_C. We have already seen C_C in rupees per unit per year as the cost of holding 1 item per year so; total inventory carrying cost is $QC_C/2$.

So the total cost, annual sum of the total ordering cost + carrying cost will be DC_0/Q + $QC_C/2$. Now we want to find out the value of Q. Q is the only variable in this total cost expression. We want to find out the value of Q that minimizes the total cost. Now that value of Q is obtained by setting the first derivative = 0 and solving, we would get DC_0/Q square. This 1/Q will give us – 1/Q square. So – DC_0/Q square, $QC_C/2$ will give us $C_C/2 = 0$ from which Q star, the best value of Q is root of 2 DC not/CC this is called the optimum order quantity or the economic order quantity Q star is given by root of 2 into D into C_0/C_C square root of 2 times demand into ordering cost by carrying cost and then when we substitute the value of Q star in the total cost expression. We get the minimum total cost as root of 2 into D into C not into C_C. So we have now solved the first model to find out the value of economic order quantity. Given the demand, the order cost and the carrying cost, we will be able to find out the economic order quantity and the minimum cost associated with the economic order quantity.

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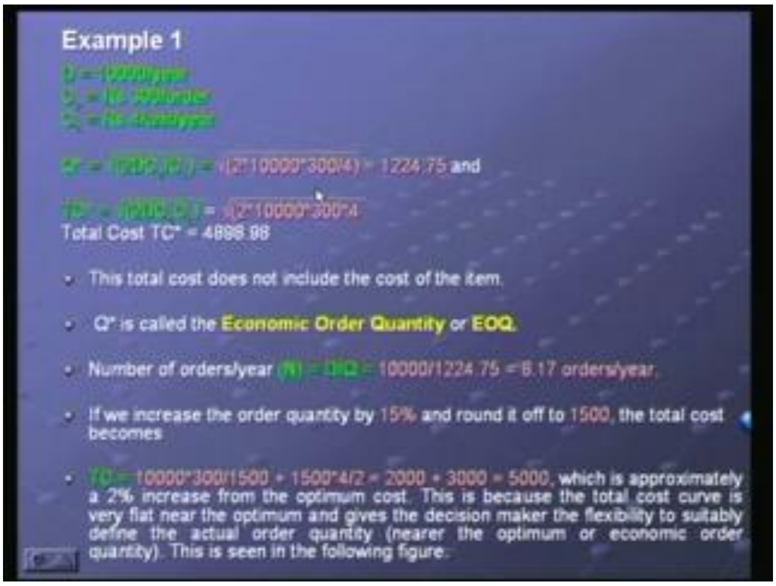

Now let us illustrate all these using an example. Now let us assume that D is 10,000 per year. The item has an annual demand of 10,000 per year. Order cost is a rupee 300 per order, carrying cost is rupees is 4 per unit per year. From the equation Q star is = root of 2 into D into C_0/C_C which we derived here which we derived here. We substitute to get 2 into 10,000 into 300/4 would give us 1224.75 as the economic order quantity. So for this example, every time we place an order, we would be ordering 1224.75 items but before we do this we need to go back here.

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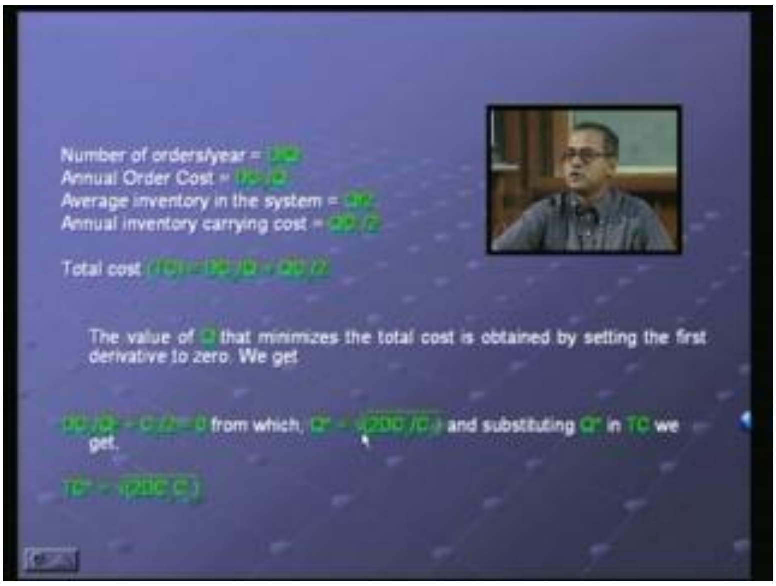

Look at Q star = root of 2 into D into C_0 by C_C which we derived by setting the first derivative of this expression to 0 and we also need to actually verify that this economic order quantity gives us a minimum total cost. Now that has to be done by setting the second derivative to 0 and evaluating the second derivative at the point Q star = root of QD. C_0 by C_c. We find out the second derivative. For this we realize that the second derivative is positive indicating that there is a minimum. So Q star = root of 2 DC_0/C_C gives us a minimum value of the total cost.

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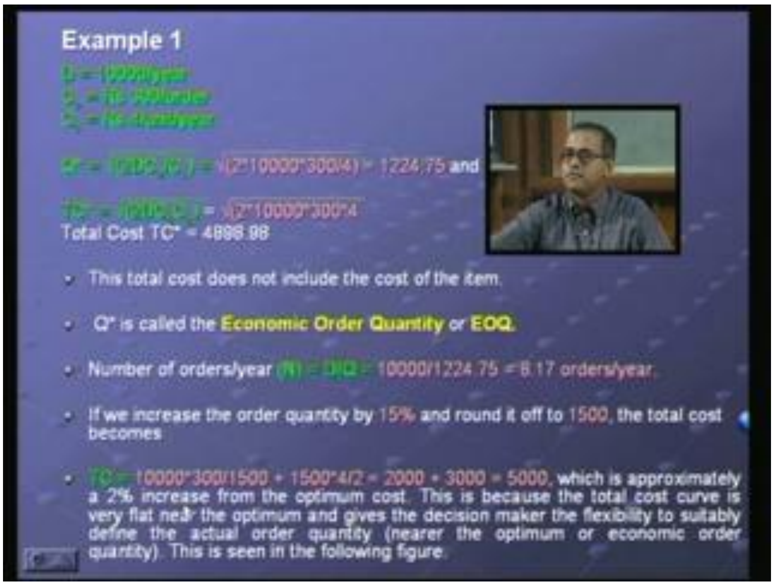

Now when we compute Q star = 1224.75. DC star the minimum value of the total cost is root of 2 DC_0/C_C which we have seen here. It gives us 2 into 10,000 into 300 into 4 square root of this expression gives us rupees 4898.98. Now what are the other things that we need to look at from this example? The total cost that we have computed here does not include the cost of the item. In fact in this example, we have not even defined C which is the unit price of the item. So the total cost, the economic order quantity and the total cost do not include or consider the cost of the item. This is because we are looking at an annual cost and the total cost of the items will be D into C which is a constant and does not depend on Q. So Q will not have C in any of the equations. Now Q star is called the economic order quantity called EOQ for short. It is a very commonly used term EOOQ represent the economic order quantity or Q star. The number of orders per year for this example would be N is = D/Q annual demand is 10,000 order quantity is 1224.75. So we will be having 8.17 orders per year. Now the number of orders per year need not be an integer.

It can be a fraction. This would simply mean that we would be placing an order every 365 divided by 8.17, roughly every 46 days, we will be placing an order if we assume that a year consists of 365working days. Now what happens how sensitive is the economic order quantity or how sensitive is the cost with respect to the economic order quantity. Suppose we want to round off the economic order quantity to a value 1500. Suppose we assume that the item is available only in say packets of 500 or multiples of 500 and we decide to round it off to 1500 which means we have roughly increased the order quantity by about 15%. If we do that and round it off to 1500 and go back and substitute, total cost is DC not/Q + $QC_C/2$ and we substitute 1500 here then our total cost becomes. When we substituted 1224.75, the total cost was 4898.98 but when we substitute 1500, the cost which is approximately less than a 2 %, increases from the 0.2 percent increase from the optimum cost. Now this is because the total cost curve is very flat near the optimum and therefore gives the decision maker the flexibility to suitably define the actual order quantity which could be near the optimum or the economic order quantity. Now this is shown in this figure.

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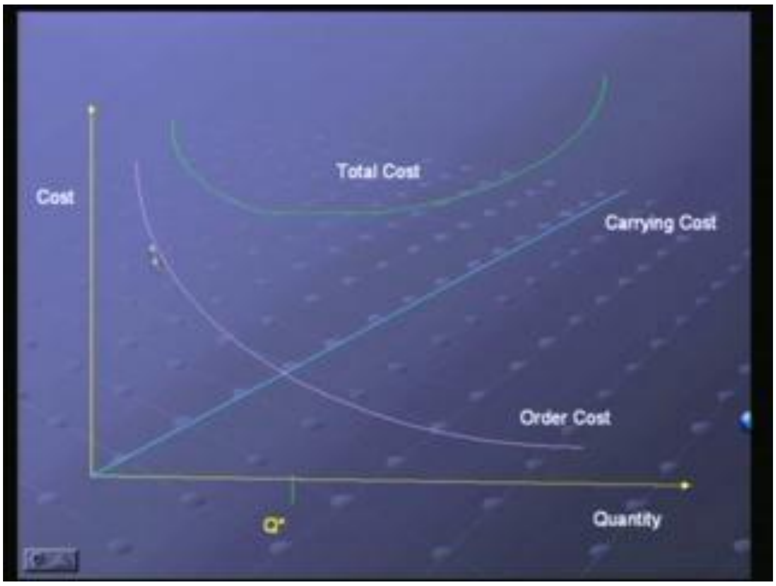

Now the order cost curve is a D/Q into C_0 curve. So it is a rectangular hyperbola which moves like this. The carrying cost curve is Q/2 into C_C which is a straight line. It is a linear curve and the total cost is the sum of this D/QC_0 + 2/2 C_C and we get a curve which is like this and it is very flat near the optimum. This is the optimum, Q star it is very flat. At the optimum, we will also find that the total order cost is = the total carrying cost and the total carrying cost is 2 times the order cost or the carrying cost. It is very flat near the optimum so even if we increase Q by a certain quantity or decrease Q by a certain quantity, we realize that the total cost actually does not increase very significantly.

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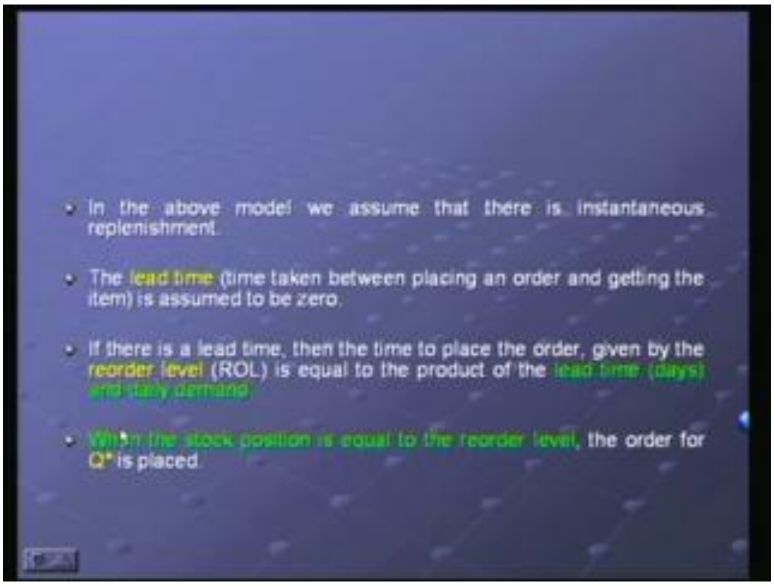

We also assumed in this model that there is an instantaneous replenishment.

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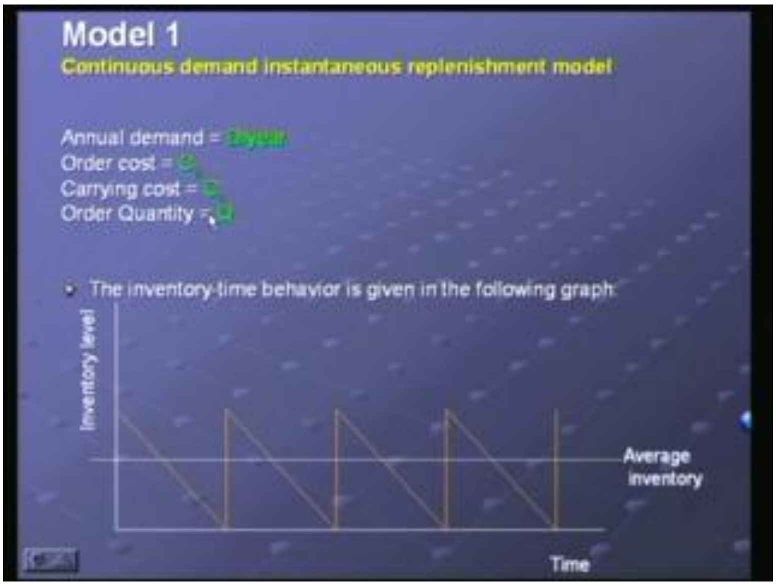

For example we place an order here, instantly the quantity becomes Q. There is an instantaneous replenishment therefore we wait for the inventory level to reach the 0 and then we place an order so that the inventory raises to Q. Now there is something called as the lead time which is the time taken between placing an order and getting the item. Because of instantaneous replenishment, the lead time is 0 in this example, but if there is a leaf time then the time to place an order given by reorder level or the stock position at which the order is placed is called reorder level will be equal to the product of the lead time and the daily demand. In this deterministic model the lead time into daily demand would give the reorder level or the stock position at which an order is to be placed so when the stock position reaches the reorder level the order for Q star is placed.

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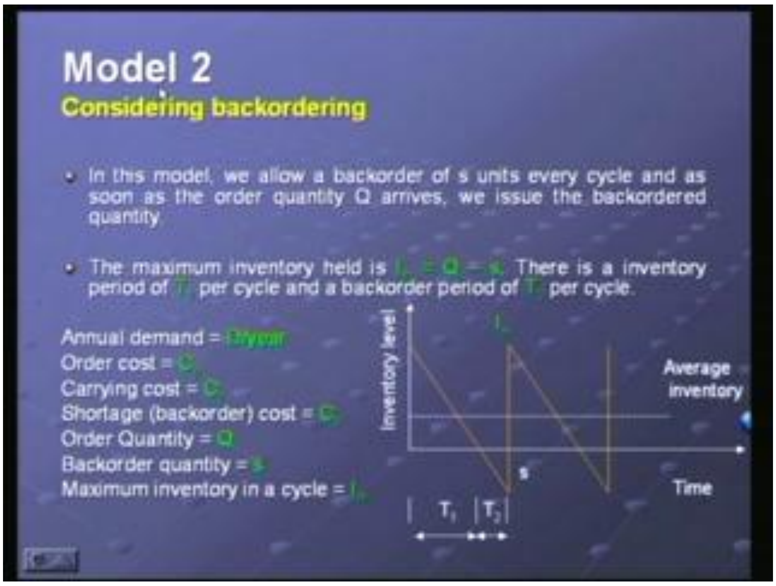

Now let us look at the second model which is an extension of the first model considering back ordering. In this model we allow a backorder of S units. Every cycle and as soon as the order quantity Q arrives, we issue the backorder quantity. You can see this in this figure. Now we start with a certain quantity which is IM. We will see that later and we start consuming it. Now here the start position is 0 but here we do not place an order. Here also we assume instantaneous replenishment. We do not place an order. We allow a backorder to build up to a quantity S, then we place an order for Q items, so instantly we get a Q but the moment we get this Q, the backorder is fulfilled. So S out of the Q is fulfilled and Q – S is the inventory position here which is given by IM or maximum inventory. The subscript M indicates maximum inventory so IM represents maximum inventory which is Q – S and then we consume from this IM till we reach 0 again.

We allow backorder up to a quantity S and then once again place an order for Q star and so on. Now in this case there are 2 cycles. The cycle is divided into 2 parts. One is the T_1 part and the other is the T_2 part. The T_1 part is the portion of the cycle where we hold inventory and T_2 is the portion of the cycle where we have backorder. So we use the same parameters but we need an additional cost which is called C_S which is the shortage cost but it represents the cost of backorder. We have already defined C_S in rupees per unit per year. Now we have 2 variables in this Q, the order quantity is a variable and S is the quantity that is backordered, S is also another variable. The maximum inventory held in this for example is IM which is Q – S. Now this becomes a dependent variable. Q and S are the independent variables and the 2 cycles T_1 and T_2 also depend on Q and S. So annual demand is D, order cost is C not carrying cost is C_C, shortage cost or backorder cost is C_S, order quantity Q and quantity that is backordered is S are the variables and the maximum inventory IM depends on Q – S. These are the various parameters that we will be considering in this model.

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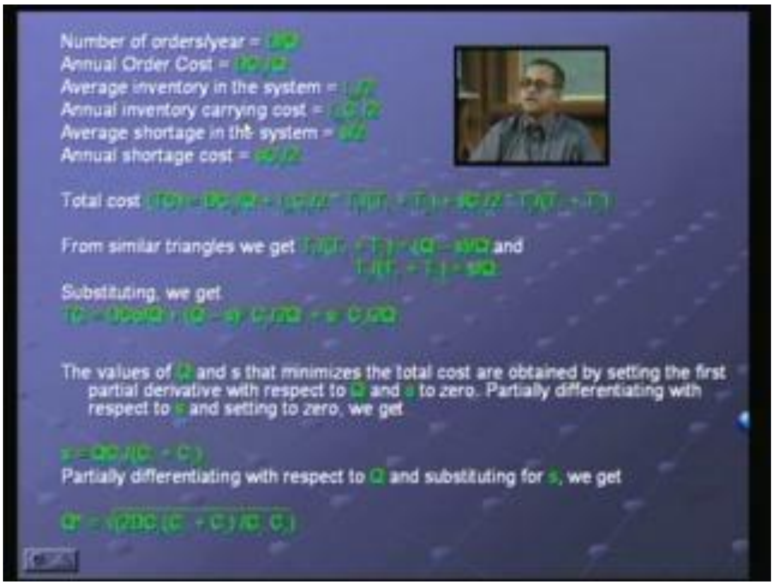

Now there are 3 costs that we have. There is an ordering cost, there is a carrying cost, and there is a shortage cost. So we compute all the 3 costs. Number of orders per year is D/Q as we have seen before. Annual order cost will be D/Q into C_0 or DC_0/Q. Now average inventory in the system, in order to calculate this, we need to look at this. Now if we look at the cycle, the total inventory that we have in a cycle is the area of the triangle, 1/2 into base into height which is 1/2 into T_1 into IM. So there is an average inventory. Now there is a total inventory of 1/2 into IM into T_1 which is held for a time period T_1 and there is a time period

T_2 where there is a backorder which means there is a 0 inventory. So we carry a total inventory of 1/2 into IM into T_1 but we carry it for a period $T_1 + T_2$. So the average inventory will be IM/2 into $T_1/T_1 + T_2$.

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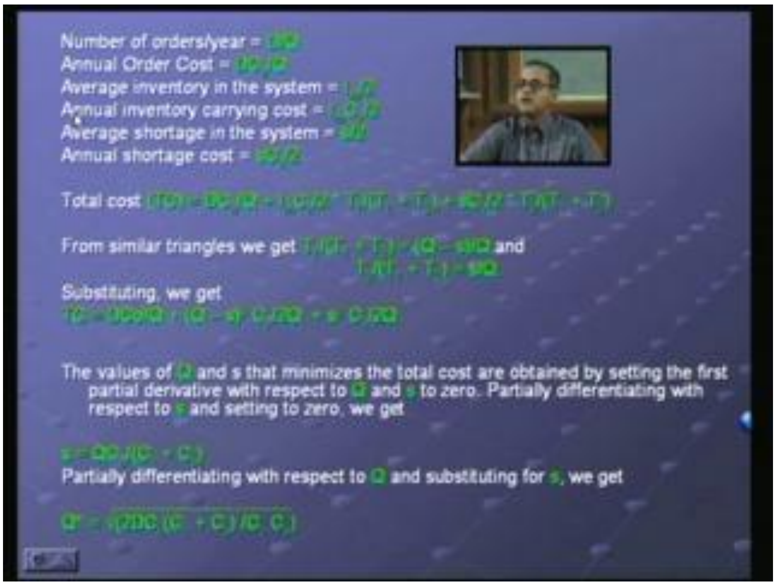

Now we look at this. Average inventory is IM/2. So we will actually have IM/2 into $T_1/T_1 + T_2$. That is the average inventory and C_C is the cost of carrying this inventory. So annual inventory cost will be IM/2 into C_C into $T_1/T_1 + T_2$. Similarly if we look at the second part of the cycle that is the backorder part, total quantity shortage or backordered is area of this triangle which is 1/2 into T_2 into S. So this quantity is for the entire period $T_1 + T_2$. So average quantity short will be 1/2 into S into T_2 divided by $T_1 + T_2$ which is shown here (Refer Slide Time: 24:38). S/2 into T_2 into T_1/T_2 multiplied by the shortage cost C_S would give us a total annual shortage or backorder cost of S into $C_S/2$ into $T_2/T_1 + T_2$. Now we have to optimize this function but then we have only 2 variables that is Q and S. So we have to write an M as well as all these T_1, T_2 quantities in terms of Q and S. So we go back to this figure and then we observe by similar triangles. Now this is T_1. This is T_2. Now this portion is IM. This portion is S. So from similar triangles and this total portion is Q, this is also Q. So $T_1/T_1 + T_2$ will be IM divided by Q and $T_2/T_1 + T_2$ is S divided by Q and IM is written as Q – S. So $T_1/T_1 + T_2$ will become Q – S/Q and $T_2/T_1 + T_2$ will become S/Q. So we go back and substitute here $T_1/T_1 + T_2$ is Q – S/Q and $T_2/T_1 + T_2$ is S/Q. So we go back and substitute to get total annual cost TC. DC/QC_0 remains as it is.

Now this becomes Q – S the whole square into $C_C/2$ Q because this will become IM/Q which is again Q – S/Q and since there is already a Q – S, therefore this becomes Q – S square into $C_C/2Q$ and this will become again an S/Q. There is already an S. So we will have S square $C_S/2Q$. So now we have written the total cost expression in terms of the problem parameters as well as the 2 variables Q and S that we have. Now we have to find out the values, the optimum values of Q and S. Now the values of Q and S that minimize the total cost are obtained by setting the first partial derivative of this function with respect to Q and S to 0. Now partially differentiating with respect to S and settling it to 0, we get S is = $QC_C/C_C + C_S$. We have not shown the intermediate steps in the derivation. We have just shown the final result here so you can derive this and verify this result. So S is = $QC_C/C_C + C_S$. This is obtained by partially differentiating this with respect to S and setting it to 0. Now we have to partially differentiate this with respect to Q and all the 3 terms involved in Q. So it is a slightly involved partial differentiation and we have to do that and then in the first derivative, with respect to Q, we have to go back and substitute for S = $QC_C/C_C + C_S$ and then when we simplify this. We get this expression Q star is = root of 2 DC_0 into $C_C + C_S/C_C C_S$ or 2 root of 2 DC_0/C_C into $C_C + C_S/C_S$. So this is the optimum or economic order quantity in the case that includes backordering. Once again we go back and we can show that the second derivative is positive at the optimum values normally that is not done because even in the earlier model we know that the second derivative is positive indicating minimum. The same thing continues in all inventory problems. So it is not customary every time to show that the second derivative is positive. So we can leave it at that and can assume that the first derivative indicates the minimum. Now we illustrate this through an example.

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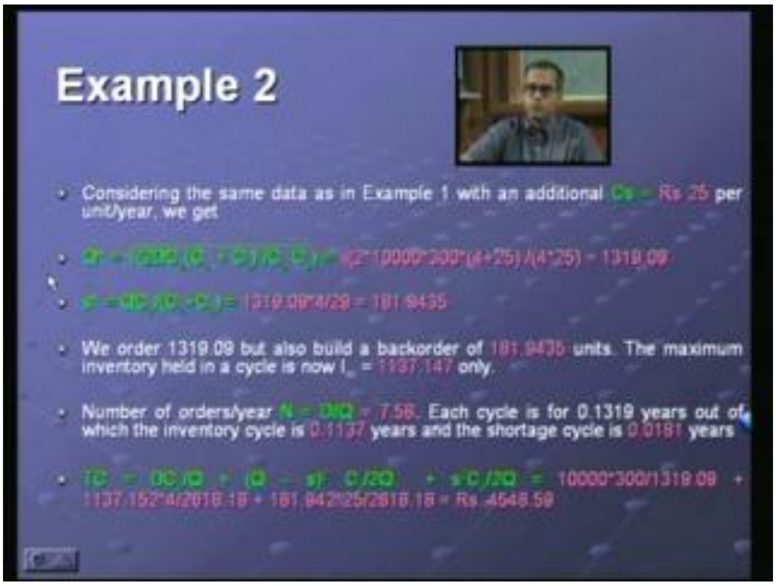

We use the same data as in example 1. The same data which is 10,300 and rupees 4 and we include an additional C_S shortage cost as rupees 25 per unit per year. Now when we substitute Q star is = root of 2 DC_0 /C_C into $C_C + C_S/C_S$, we substitute here to get 1300 and 19.09. S star which is the best value of the quantity backorder is given by $QC_c/C_c + C_S$ on substitution gives us 181.1395. Now let us go back and compare the results that we had in example 1. Example 1 is similar to example 2 in the sense that the parameters DC_0 and C_C are the same. The additional consideration in example 2 is the build up backorder, so if we go back and compare in the first model, the order quantity is 1224.75.

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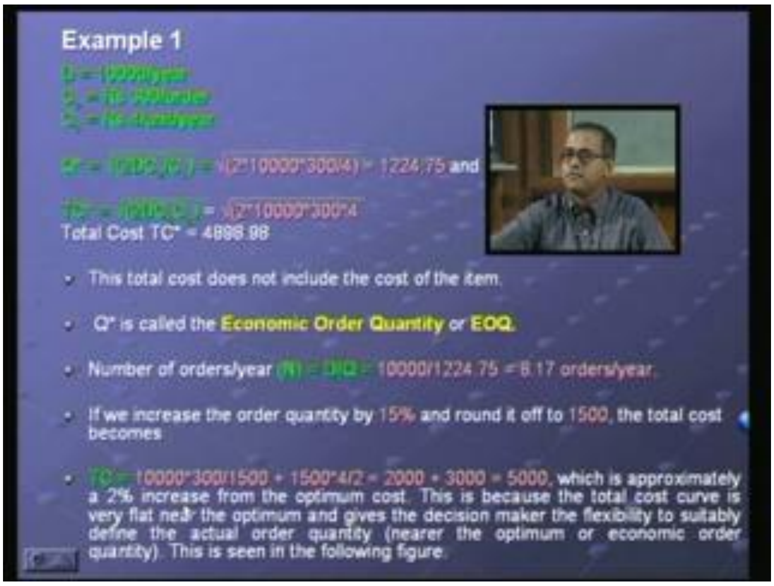

The minimum cost is 4898.98. Total order quantity is 1224.75 which was the maximum inventory that was held in the system. Now in this example, the maximum inventory that we have is Q – S which is 1319.09 – 181.94 which is 1137.147. So when we build a backorder into the system, our order quantity increases, but the maximum inventory reduces because of the backorder quantity which is 181.94. Number of orders per year will come down. It was 8.17 in the previous case because the order quantity has increased. Each cycle is for .1391 years out of which the inventory cycle is 1137 and shortage portion is 0.181. Now we substitute to get the total cost. Total cost is 4548.9 when we substitute for DC not $Q_S C_C C_S$ from all these we get 4548.59. Now when we compare this with the total cost in the first example, the total cost in the first example was 4898.98. The economic order quantity increases, the maximum inventory held in this system, reduce and the total cost reduces if we build backorder into the same system.

When compared to the previous model, we observe that the order quantity has increased but the maximum inventory has reduced. Maximum inventory decreases in this. There is a gain in the order cost. That is because the order quantity has increased. Number of order per year reduces. Therefore total annual order cost reduces. There is gain in the carrying cost because the maximum inventory in the system reduces therefore the average inventory in the system also reduces. The period for which we hold the inventory also reduces in this because there is a backorder cycle or a backorder portion so there is a gain in the total annual carrying cost but there is an additional backorder cost which was not there in the first example. Now there is a gain in the order cost, there is a gain in the carrying cost but there is an additional expenditure in the backorder cost. So the net will be a decrease in the total cost. There is a net decrease in the total cost that we have seen in the first model. It was 4898.98. Now it has come down to 4548.59. This also does not include the cost of the item.

So there is a net decrease in the total cost and this model would suggest that backordering is always advantageous because the total cost comes down. Now we go back to this. Q star is = root of 2 DC_0/C_C into C_S C_C + C_S/C_S. Now when shortage cost is set to infinity the order quantities are the same. We can go back and substitute. This can be written as 1 + C_C/C_S. So when C_S is infinity C_C/C_S becomes 0. So Q star becomes the same Q star of the first example.

So only when the shortage cost becomes the infinity both the models become equal. Now we also observe that when C_S is infinity, the order quantities are the same. For all other values of C_S however larger value we may give, the second model would yield a less total cost than the first and would always suggest that backordering is advantageous. Now what is the learning from this model? The learning from this model is that we should keep C_S to infinity. If we do not want backordering and eliminate backordering, the idea should be eliminate backordering by fixing C_S to infinity rather than encourage it by a wrong value of C_S. One should not exploit the fact that cost is minimum in this example and therefore one should not be carried away by the fact that backordering is advantageous. Backordering has other aspects which may not have been adequately reflected in the C_S that has been chosen here. For example it is very difficult to incorporate the loss of goodwill and the loss of customer in this value. So even though backorder by plan reduces the total cost.

One has to be very cautious about the extent of backorder that we allow when we accept backorder as part of plan. So we need to keep the backorder quantity minimum and in order to do that we need to set the C_S as large as we can. The issue is not evaluating the quantity S by defining C_S, but the issue is keeping the C_S large enough, so that the quantity backorder small s becomes small and manageable. So it is necessary that shortages in practice should happen only due to uncertainties in demand and if however if we want to plan backorder at the planning stage of the inventory then we need to keep a very large value of C_S. The value of C_S should be large enough to have a very small quantity backorder. So these are the things we learnt when we compare the first and the second models in inventory.

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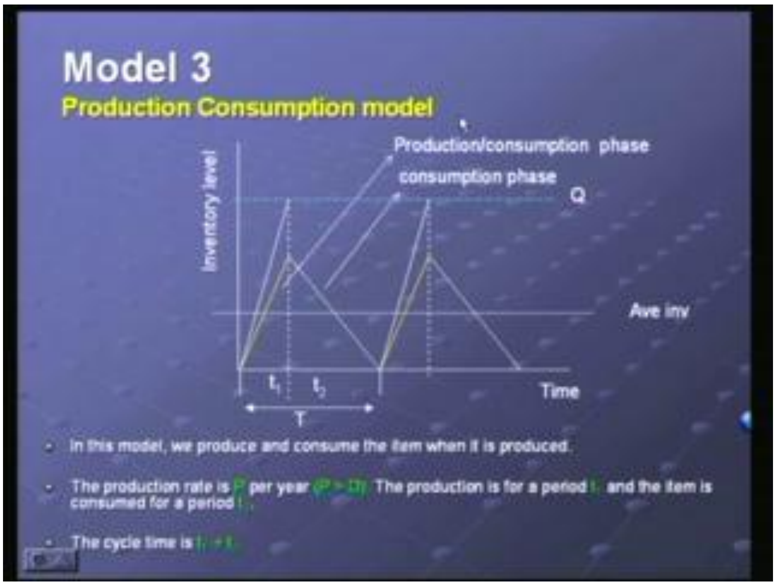

Now let us look at the third model, where we look at something called the production consumption model. Now in this model we assume an item that is produced in the factory and it is consumed either by assembly or by directly giving it to the customer. In both of the previous models we were looking at items that were bought out. We were looking at the items that were supplied by the vendors. An order was placed for these items. Now we are looking at what happens in the production systems. So we are assuming that these items are being produced. Now what do we do here? Now we assume that the system is capable of producing at the rate of P per year capital P per year. Production rate is P per year which is greater than the demand which is higher than, not equal to, but higher than the annual demand. Now because P is greater than D we do not produce all the time. We produce only for a certain period. So this is the period we produce. So when we produce, we also consume. So for this cycle T_1, the items is produced as well as consumed. It is produced at the rate of P consumed, at the rate of D. So inventory is built up at the rate of P – D. After a period T_1 production stops and the item is only consumed from the inventory that we have built up. So from this maximum inventory for the period T_2, the inventory is consumed and the inventory becomes 0. We may assume that the production facility is producing some other item during this period T_2. Once the stock position becomes 0, then we start producing the first item or the item under consideration and then we both produce as well as consume. So the inventory is rising at the rate of P – D here, once again it is consumed and so on.

Now the rate at which it is produced is shown here. This is the production rate P at which it is produced. So at every cycle of production we actually produce for a period T_1 and only consume for a period T_2 it is called consumption phase. The period T_1 is a production and consumption phase. The total that we have produced is P into T_1 because we produce at the rate of P for a period T_1. So total production Q is P into T_1 and we consume the maximum inventory that we build is P – D into T_1. So in this model we have production rate is P per year P greater than D. Production is for a period T_1. Item is consumed for a period T_2 and the cycle time is $T_1 + T_2$.

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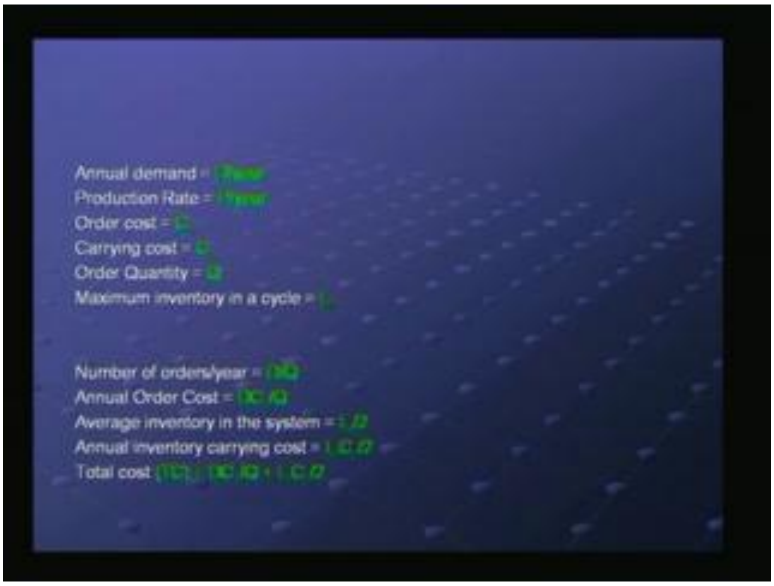

Now in this model we need to look at couple of things. Now this Q is called the economic batch quantity. It is not an ordering quantity it is a batch. It is a production batch quantity because we are looking at the items that are produced in house but we use the same notation Q for the economic batch quantity for production. We also realize now in this case, there is not going to be an ordering cost because we are not ordering any item here from the vendors. Instead of the ordering cost we have another cost which is set up cost, so every time the facility produces this item. There is a cost incurred just as every time we placed an order we had an order cost. Every time we set up this facility for producing we have the set up cost. Now set up cost, we use the same notation C_0 to represent setup cost in this model. So annual demand is D production rate is P per year. Setup cost is C_0 rupees per set up which is similar to the order cost but it is a different notation C_0 represents set up. Cost carrying cost is C_C order. There is not ordering quantity here the batch production quantity is Q and maximum inventory in a cycle is impedance. So number of batches produced per year is D/Q which is similar to the number of orders but it is a number of batches produced per year which is D/Q. The annual setup cost is D/Q DC_0/Q because we have D/Q setups in a year. Each setup is going to take C_0. So it is DC_0/Q. Average inventory in the system is IM/2 now that comes from this. Now the total inventory is the area of the triangle 1/2 into base into height 1/2 into IM into $T_1 + T_2$. Average inventory is total inventory by time 1/2 into IM into $T_1 + T_2$ divided by $T_1 + T_2$ which would give us IM/2. So average inventory will be IM/2 and the inventory carrying cost will be IM/2 into C_C. So total cost which is the sum of the setup costs and inventory carrying cost will be D/Q into C_0 + $IM/C_C/2$.

So in this model we are optimizing the total set up cost + carrying cost. There is also going to be a carrying cost of the inventory because inventory is built up from 0 to IM and then subsequently consumed. Then again, these inventories that is built up and consumed result in items that are going to be there in the shop that has to be carried. So there is an inventory carrying or holding cost in this model. Now there is no ordering cost but there is a set up cost because every time we set up or want to produce this item, we incur a set up cost so here. We optimize the total cost which is given by the sum of the set up costs and the inventory holding costs. So the total cost is D/Q into C_0 + IM into $C_C/2$. Now the variable is Q and not IM. So we have to write this IM in terms of Q.

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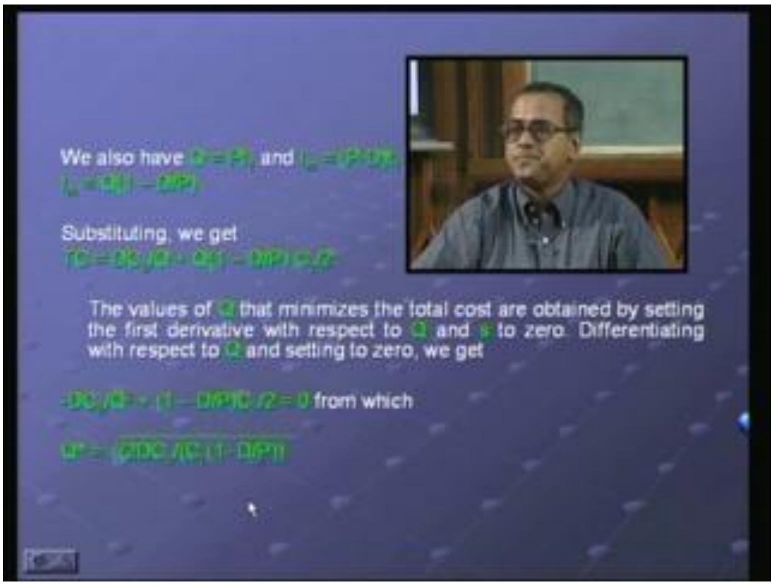

So do that we have Q = P into $T_{1.}$ IM is = P – D into T_1. This comes from this figure. We realize that for a period T_1, we produce every time; we produce of a period T_1 and at a rate of P. So P into T_1 is the quantity that is produced which is the production quantity Q. So Q is P into T_1 that is shown here. Q is P into T_1. Now what is the inventory build up? As we produce during this period T_1, we produce at the rate of P, we consume at the rate of D. So inventory is build up at the rate of T – D for a period $T_{1.}$ So total inventory is build up is P – D into T_1 which is IM. So IM is P – D into T_1. Now dividing one by the other, we get IM = Q into 1 – D/P or Q into P – D/P which is Q into 1 – D/P. So substituting we would have total cost TC. We go back and substitute for IM is = Q into 1 – D/P. So we substitute. We get total cost is =

D/Q into C_0 + Q into 1 – D/P – into C_C/4. Now we want to find out the best value of Q that this TC. The value of Q that minimizes the total cost is obtained by setting first derivative with respect to equals Q to 0 differentiating with respect to Q and setting it to 0, we get – D/Q square C_0 + C_C/2 into 1 – D/2 = 0 from which Q star is = root of 2 DC_0/C_C into 1 – D/P. Now we look at this equation carefully. It is very similar to the first model except that we have an additional term 1 – D/P. We also realize something common. We have seen 3 models and there seems to be some relationship. The first model also had root of 2 DC_0/C_C. The second model had root of 2 DC_0/C_C into C_C + C_S/C_S. The third model has root of 2 DC_0/C_C into 1 – D/P. So the term root of 2 DC_0/C_C keeps coming again and again. So for the third model we have QC star = root of 2 DC_0/C_C into 1 – D/P.

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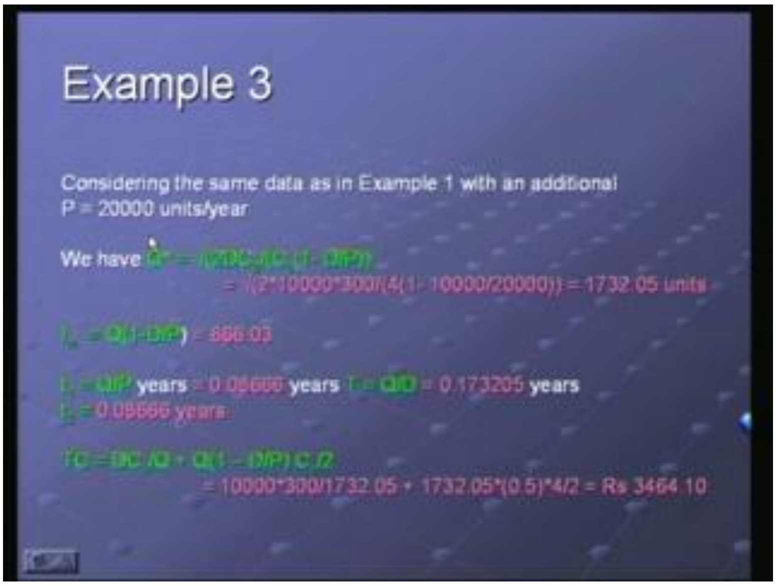

Now we consider an example to illustrate this. Let us look at the same data as in example 1. Demand 10,000 per year instead of order cost C_0 is the set up cost, is considered to be 300 per set up. The inventory carrying cost is rupees 4 per unit, per year and the production rate P is 20,000 units per year. P is greater than D. D is 10,000 per year. P and D are expressed in the same unit which is units per year. So Q star is root of 2 DC_0/C_C into 1 – D/P, 2 into 10,000 into 300. C_0 is 300. It is assumed to be the set up cost. Set up cost per set up is very similar to the ordering cost rupees per set up. 300 C_C is the same 4 into 1 – D/P 4 into 1 – 1/2 which is ½. This gives us 1732.05 units. The maximum inventory is Q into 1 – D/P which on substitution gives us 8 66.03. T_1 production cycle is .08666 years and consumption cycle is also .08666 years. Total cost is D/QC_0 + Q into 1 – D/P C_C/2 which is 34664.10.

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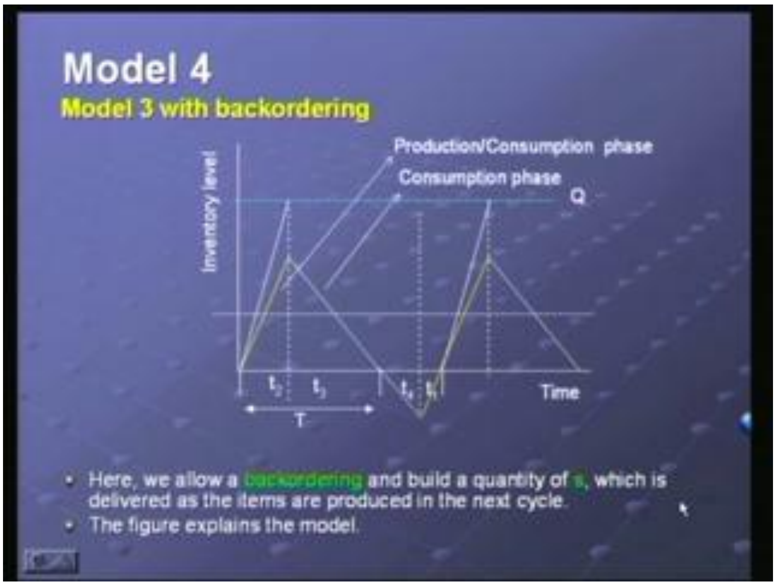

Now let us extend this model to backordering. Just as model 2 was an extension of model 1 considering backordering, model 4 is an extension of model 3, considering backordering so they are very similar except that we introduced backordering. So we introduced a C_S which is the shortage cost or the backorder cost and the inventory figure looks like this (Refer Slide Time: 46:42). Now we start with inventory = 0, produce at the rate of P and consumed at the rate of D. So inventory is built up at the rate of P – D. We build a maximum of IM which is = P – D into T_1. Now once that inventory IM is built up, we stop the production. We assume that the facility is producing some other item during this period and then we start consuming at the rate of D till the inventory reaches 0.

We do not start the production immediately as we did in the previous case. We allow for some backordering for a period T_4 and we build a backorder = S and once the backorder is built. We start producing again and then we produce at the rate of P. Inventory increases at the rate of P – D because we consume and now it comes to 0. The backorder is all met and once again the cycle completes. So now the cycle has 4 components T_1, T_2, T_3 and T_4. T_1 and T_2 are the production components and T_3 and T_4 are the entire consumption components but we have a T_1 and T_2 because T_1 production is all consumed and backorders are met. Now T_2 is consumed and inventory is built up. So we have a lot of parameters in this model. We allow a backordering and build a backordering quantity of S which is delivered or given as items are produced and inventory is built up and then inventory becomes 0 after the backorders are given and then the cycle begun and then the cycle begins. So this figure adequately represents this model. So let us derive.

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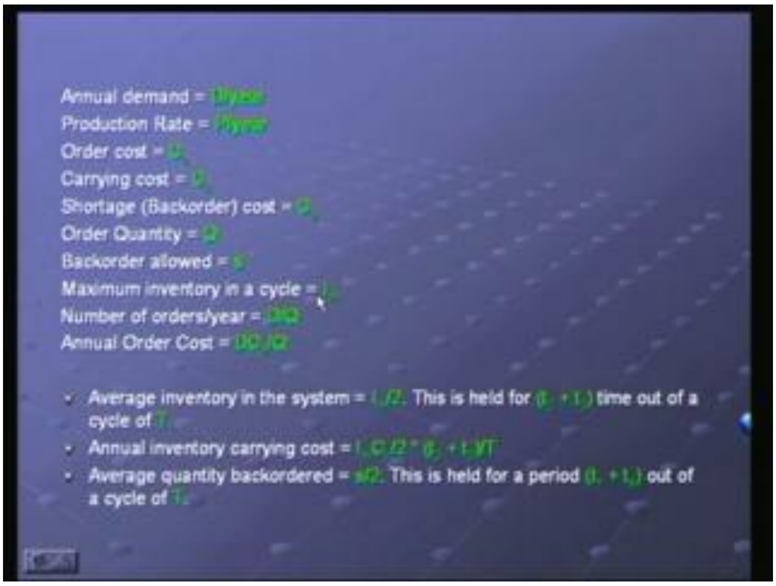

Now we have 2 variables Q and S in this model. So annual demand is D, production rate is P set up cost is C_0; ordering cost set up cost is C_0, very similar to the ordering cost. Carrying cost is C_C shortage cost or backorder cost is C_S. Production batch quantity is very similar to the order quantity is Q. Backorders allowed is S. Maximum inventory in the cycle is IM. Number of set ups in a year is D/Q. Annual set up cost is D/Q into C_0 when every time we produce Q. So number of set ups in a year will be D/Q. Annual set up cost will be D/Q into C_0. Now average inventory in the system is IM/2 but held for a period of $T_2 + T_3$ out of the cycle T. Now we go back here. Now there are cycle time T but the total inventory of this quantity that is 1/2 into IM into $T_2 + T_3$ is held for a quantity, Big T or capital T. So average inventory is IM/2 into $T_2 + T_3$/T. So IM into C_C/2 into $T_2 + T_3$/T. Average backorder quantity is S/2 but held for $T_1 + T_4$/T. Once again it can be seen from this example. Now this is the total quantity backorder for a $T_1 + T_4$ but it is held for an entire period capital T.

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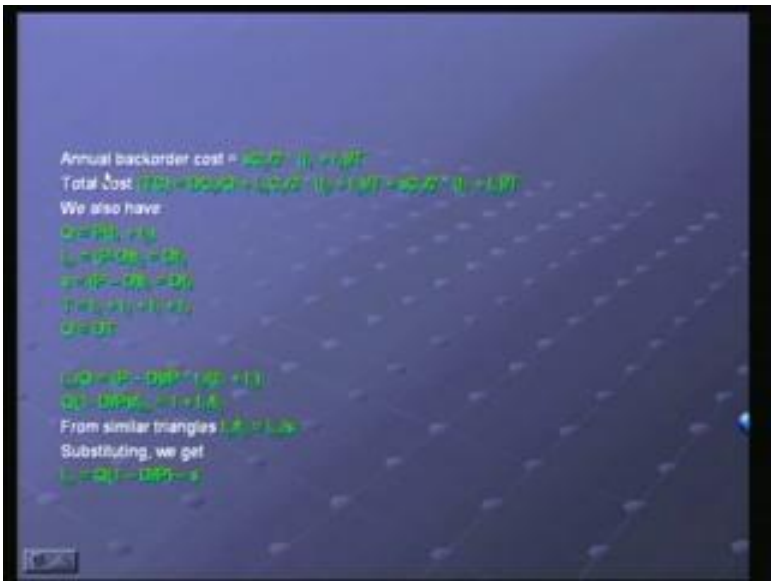

So we go back and we substitute here to get this expression. So total cost is D/Q into C_0 + IM/2 into C_C T_2 + T_3 by capital T + S/2 into C_S into T_1 + T_4/T. But we have only 2 variables that we are going to have which are Q and S. Therefore the rest of the things have to be written in terms of Q and S. IM T_2, T_3/T, T_1, T_4/T all have to be written in terms of Q and S. Now let us do that. Q is P into T_1 + T_2. Now Q is the quantity that we have produced so whenever we produce, we produce for a period of T_1 + T_2, and so Q is the total quantity produced or the batch quantity which is P into T_1 + T_2. IM is P – D into T_2 that is the inventory buildup, built up for a period T_2 at the rate of P – D which is = P – D into T_2. It is also = D into 3. Similarly S is the quantity built for backorder which is P – D into T_1 which is = d into T_4. If we go back to this figure we can understand this. Now S is = D into T_4 which is P – D into T_1. Now capital T is the cycle T_1 + T_2 + T_3 + T_4 and Q is D into T because T is a cycle. So we produce enough to meet the demand during the cycle, so Q is D into T so substituting and simplifying we get an expression like this IM/Q is P – D/P into T_2 T_1 + T_2. So from what we have, we use from similar triangles T_1/T_2 is = IM/S which we can see here. We have this quantity which is IM. This quantity is S, so we go back this is T_1. This is T_2 so from similar triangles IM/S is T_2/T_1. So we can go back and substitute to first, we get IM/Q dividing one by the other here and then substituting from similar triangles, we get IM is = Q into 1 – D/P – S. This is an important equation which we need to substitute here for IM and then we go back and we also get this T_2 + T_3/T is P into IM/Q – D or IM/Q into 1 – D/P and T_1 + T_4/T is S/Q into 1 – D/P.

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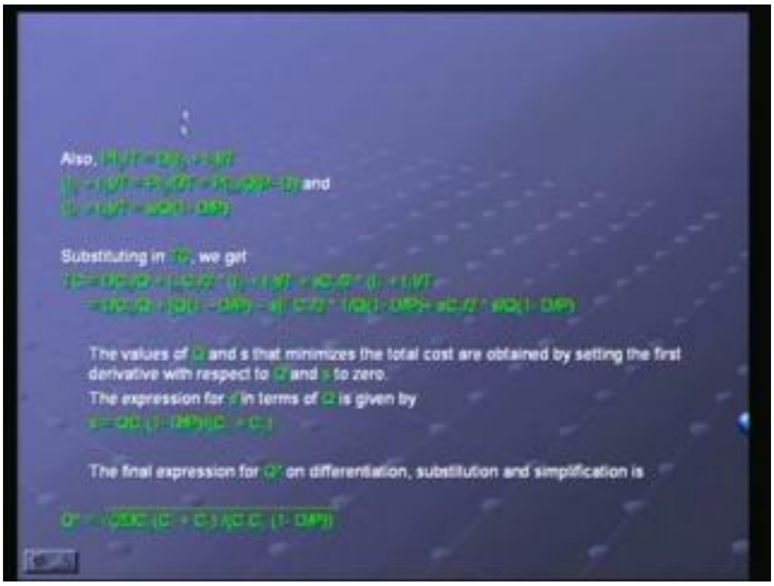

We need these 2 terms T_2 + T_3/T here and T_1 + T_4/T here so both are now written as, IM/Q into 1 – D/P and S – Q into 1 – D/T, so we have written everything in terms of the variables Q and S and the relevant parameters which is here and we have to substitute for IM square. We will get an IM square because there is an IM term here. T_2 + T_3/T also have an IM term.
So we get IM square, Q into 1 – D/P – S the whole square and we also have an S square term which is here. Now we need to differentiate partially with respect to Q and S and set derivative = 0, substitute and get and then we do that we get S. The first equation we get on partially differencing with respect to S would give us S. S is = QC_C into 1 – D/P/C_C + C_S. Then we differentiate this expression with respect to Q. Partially set it = 0 and substitute for S here. It is a very long lengthy involved derivative and substitution which we have not shown here. We have only shown the final result and the final result will be Q star is = root of 2 DC_0/C_0 into CC + C_S/C_S into 1 – D/P. Now this is a very interesting term because this Q star for the forth model actually has all the terms that have appeared in models 1, 2 and 3. So the first one was root of 2 DC_0/C_C, root of 2 DC_0 /C_C. Second term was root of 2 DC_0/C_C multiplied by C_c + C_S/C_S. We have the term C_C + C_S/C_S also coming here. The third term did not have C_C + C_S/C_S but had 1 – D/P we also have that 1 – D/P and the fourth model has all the terms root of 2 DC_0/C_C into C_C + C_S/C_S into 1 – D/P.

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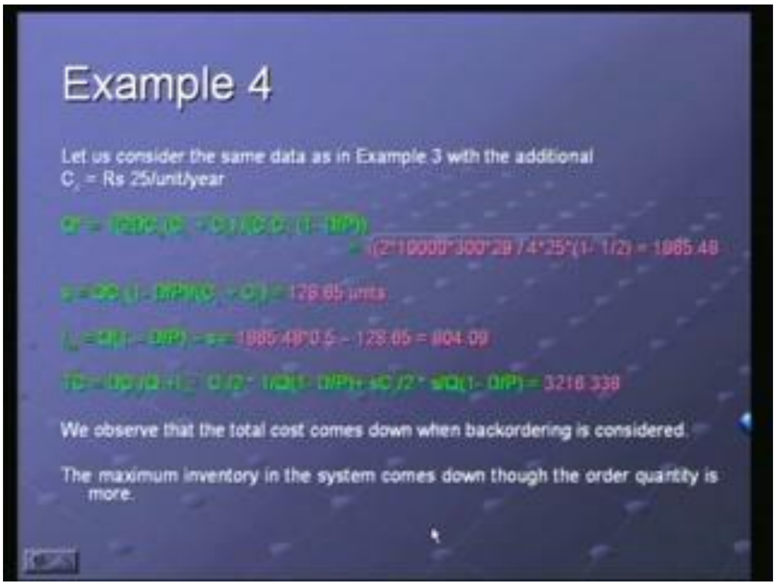

Now let us consider an example to see or compare between models 3 and 4 because they are very similar. So we consider the same example with an additional data that C_S is rupees 25 per unit, per year in the same C_S that we used in an example 2. Now going back and substituting Q star, the same economic batch quantity is now found to be 1865.48. The quantity backordered is 128.65 units going back to the same equations. Now IM the maximum inventory will be Q into 1 – D/P – S which is 804.09 and the total cost on substitution is 3216.338 or 334. Let us compare the corresponding values in the third example that we have seen. Now with the backorder the economic order quantity is 1865.48. In the previous example it turned out to be 1032.0 5. So we produce more per batch in the fourth one which was expected. Maximum inventory is 866.03 which is actually lesser than what we have in the in the third model. The maximum inventory is 866.03. Now in forth model the maximum inventory is 804.09 which is lesser, which is understandable. We produce more but we hold fewer inventories because of the backorders that keep coming in. So maximum inventory held is less total cost is 3216. In previous model the total cost is expected to be higher. So in the previous model total cost was 3464.

In the model with back order it is 3216. So very similar to the behavior that we saw between models 1 and 2, we see the same behavior between 3 and 4. So total production quantity increases, maximum inventory reduces and the total cost comes down. There are 3 costs because the total production quantity is more and number of setups reduces. So total setup cost reduces because the average inventory is less. The inventory holding cost reduces but there is an additional backorder which increases the cost of backordering but the net result is a decreasing total cost. So the total cost comes down. So again we get an impression that backorder provides us with a mechanism by which we can reduce the cost. We observe that the total cost comes down when backordering is considered. Maximum inventory in the system comes down though the order quantity is more. That is because of the backordering that takes place.

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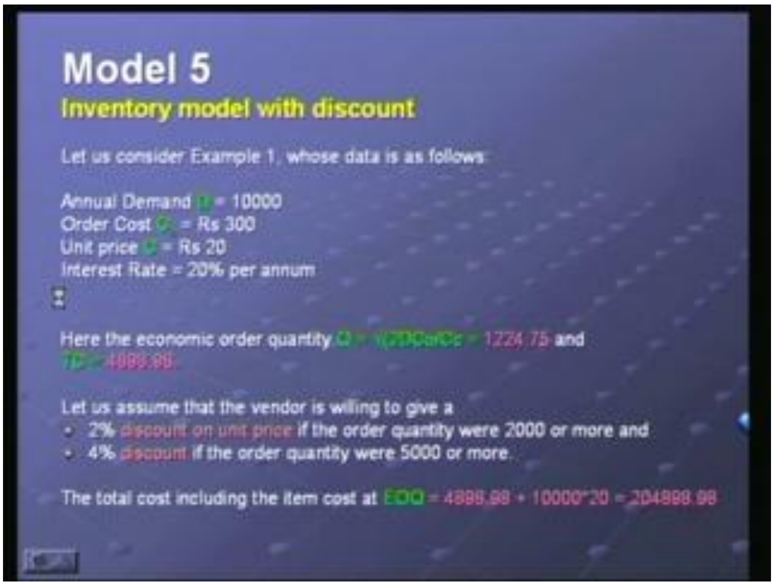

Now we would be tempted in this model to once again consider backorder favorable because the total cost comes down. When we look at backorder, we can again go back and show that only when C_S is = infinity this model will become model 3 which is the third model and there are 2 issues once again. For a given value of C_S however large is the value being this will have a backorder quantity S and we will have a system wherein the total cost will come down. So the real issue is not to find out the S, for a value of C_S but to define the C_S large enough so that the quantity S backordered is small and it is manageable. It is preferred that backorder is not assumed in the planning stage because if we assume backorder in the planning stage we will get into a situation where it is favorable. So if we are however going to still assume backorder, if we want backorder by plan, the issue is to define the C_S suitably high so that the backorder quantity S comes down and the system can be managed in the practice. So we have seen these 4 inventory models now and we will look at inventory models with discount as well as multiple item inventory models in the next lecture.