Fundamentals of Operations Research
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Lecture No. # 2

Linear Programming Formulations

In this second lecture we will be looking at two more formulations in linear programming.

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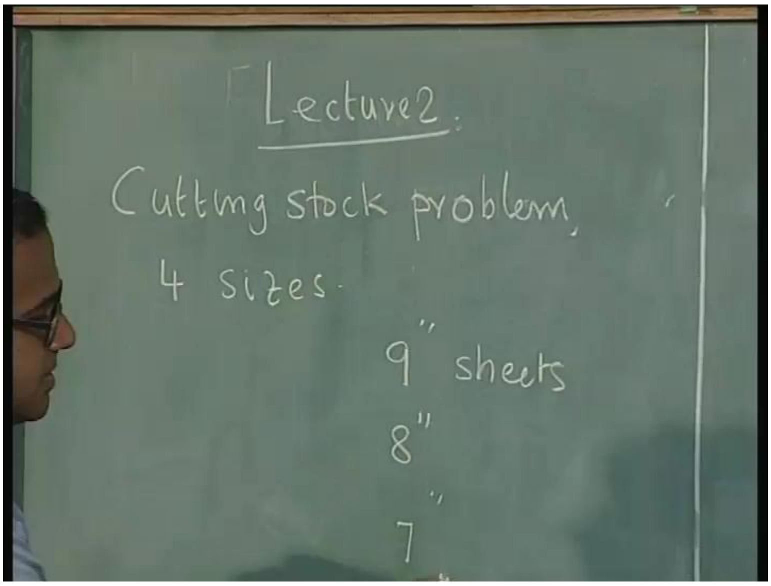

We look at a third formulation which is the cutting stock problem and we will be looking at another formulation where we formulate a problem from Game theory. So let us move to the cutting stock problem. Now in the cutting stock problem, we are talking about cutting stock of 4 sizes from a sheet. We want to cut 9 inch sheets, 8 inch sheets, 7 inch and 6 inch from a 20 inch sheet.

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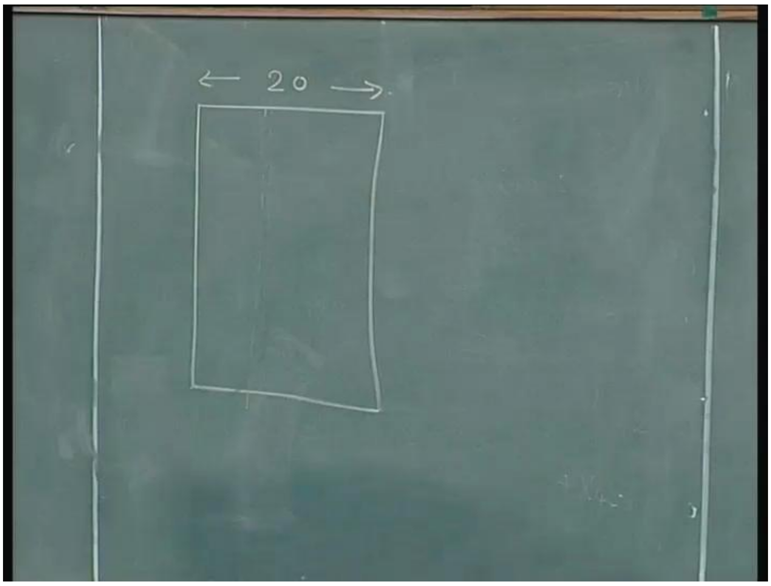

For example we assume that we have a 20 inch sheet like this (Refer Slide Time 2:32). Now from this 20 inch sheet, we want to cut sheets that have for example, 9 inches or it could have 8 inches or 7 or 6 inches. Now there is a requirement for each of these. So we need to go for this.

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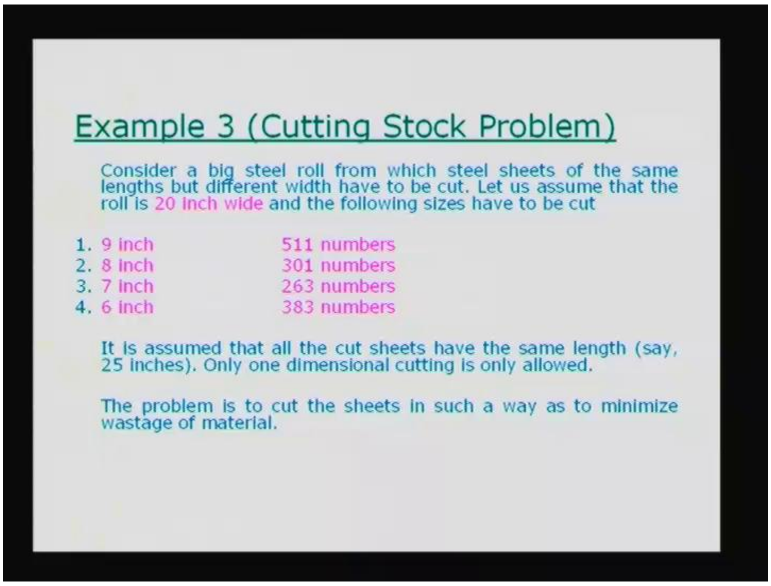

Now we need 511 sheets of 9 inch, 301 sheets of 8 inch, 263 sheets of 7 inch, and 383 numbers of 6 inch sheets. You can actually assume that either we have about 10,000 such 20 inch wide sheets with something like a 50 inch length and you can go back and say you want 50 by 9 (511), 50 by 8, 50 by 7 and 50 by 6. We could do that. Or we could think in terms of an infinitely long roll of 20 inch width from which we want a cut 50 by 9, 50 by 8, 50 by 7, 50 by 6.This length is not important to us. This width is important and we are looking at one dimensional cutting. For example we do not allow cutting this (Refer Slide Time 3:57) way. We allow the cut only along the width. Now the problem is to cut in such a way that we get

511, 9-inch white sheet 301, 8-inch white sheets 7 and 6 inch and minimize the waste. So the first thing we need to do is to define what this waste is. Now before we go into the definition of waste, let us see number of ways by which (Refer Slide Time: 04:12) 2 inch will go as a waste. So this will be a waste if we cut 2 into 9. If you cut 2 into 8 then you realize that you have a 4 inch that will go as a waste and so on. And so the first thing we need to do is to try and find out how many cutting patterns are possible. So typically the first cutting pattern would be like, you can define a pattern for example [2 0 0 0]. It means that from a 20 inch sheet, you are cutting 2 sheets of 9 inches.

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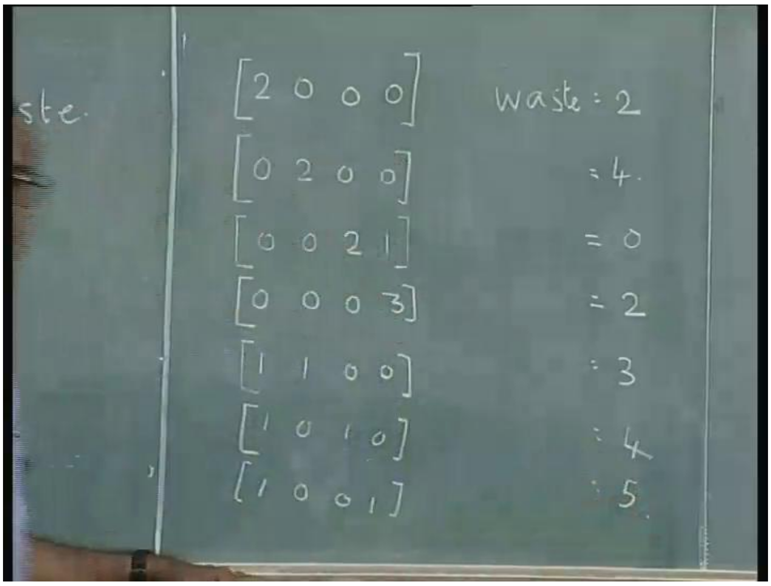

So this has a waste equal to 2. Now you could think of another pattern which could be [0 2 0 0] which means 2 into 8 = 16 which would have a waste of 4. Now you could think in terms of [0 0 2] which means we are trying to make 7 inch cuts, so maximum of 2 is possible with a waste of 6 and since 6 is a already a requirement here, we do not treat this 6 as a waste. You could think in terms of [0 0 2 1] with waste = 0. You could think in terms of a fourth pattern which could be [0 0 0 3] with waste equal to 2 because 6 into 3 = 18 and remaining 2 inch will go as a waste. There could be more patterns. For example we could think in terms of [1 10 0] which would give a wastage of 3, which means there is a 9 inch and an 8-inch cut and there is a wastage of 3. One could think in terms of [1 0 1 0] with wastage equal to 4. We could think in terms of [1 0 0 1] which could have wastage of 5. One 9 inch and one 6 inch would give us 15 which would give us wastage of 5.

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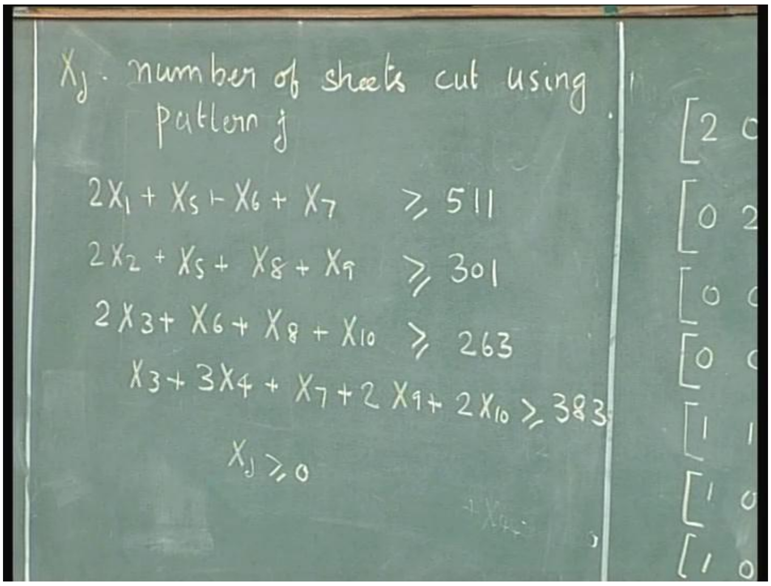

Now we could go back also think in terms of [0 1 1 0 9], 8 + 7 = 15. So wastage is equal to 5. One could think in terms of [0 1 0 2]. So this is 8, 6 into 2 = 12 and wastage equal to 0. Also, one could think in terms of [0 0 1 2]. This is 7 + 6 into 2 = 12 which is 19 with wastage equal to 1.So we have 3 + 3 equal to 6, 7 + 3 = 10 different patterns that we have. Now those 10 patterns are also shown on the other side on the screen. Now one important observation in these patterns is that we have made sure that the waste is less than the minimum thickness that is required. We will not consider, for example in this formulation [0 0 2 0] with the waste of 6. We would rather use that remaining 6 to meet this requirement here. So the important thing is that the wastage is less than the minimum quantity that is needed which is shown here as 6. Now what we want to do is if we have many sheets like this and 10 patterns are possible then we want to find out how many sheets are we going to cut using pattern 1 how many sheets with pattern 2 and so on.

So decision variable X_j is the number of sheets cut using pattern j so X_1 to $X_1$0 will now represent these 10 patterns and the number of sheets cut using these 10 patterns. So now if we look at the requirement for the 9 inch, we can get 9 inch sheets if we cut using pattern number 1. We call these patterns 1, 2.3,4 up to 10. So if we cut pattern 1 this pattern this would give us 9 inch. So, 9 inch sheets are cut by using patterns 1, 5, 6, and 7. Similarly 8 inch sheets are cut using patterns 2, 5, 8, and 9 and so on. So if we decide to cut X_1 sheets using pattern 1 and so on then as far as 9 inch sheets are concerned, we will have $2X_1 + X_5 + X_6 + X_7$. Now this many sheets we will get of 9 inch. Now this has to be greater than or equal to the requirement of 511.The question is whether this is an equation or an inequality. Now what can happen is sometimes, not necessarily in this case sometimes if these coefficients are not one it may be possible that we end up getting more than 511.So it makes sense to model this as an inequality of the greater than or equal to type rather than fore set as an equation. So we put an inequality here and say the number of sheets that we get through this cut is greater than or equal to 511. Now similarly for the 8 inch I get $2X_2 + X_5 + X_8 + X_9$ is greater than or equal to 301. For the 7 inch I get $2X_3 + X_6 + X_8 + X_1$0. 10 is greater than or equal to 263 and for the 6 inch, I will get $X_3 + 3X_4 + X_7 + 2X_9 + 2X_1$0 is greater than or equal to 383. The non-negativity restriction is X_j greater than or equal to 0. So we have written the constraints as well as the non-negativity restriction for this problem which you can see is shown on the top portion of this.

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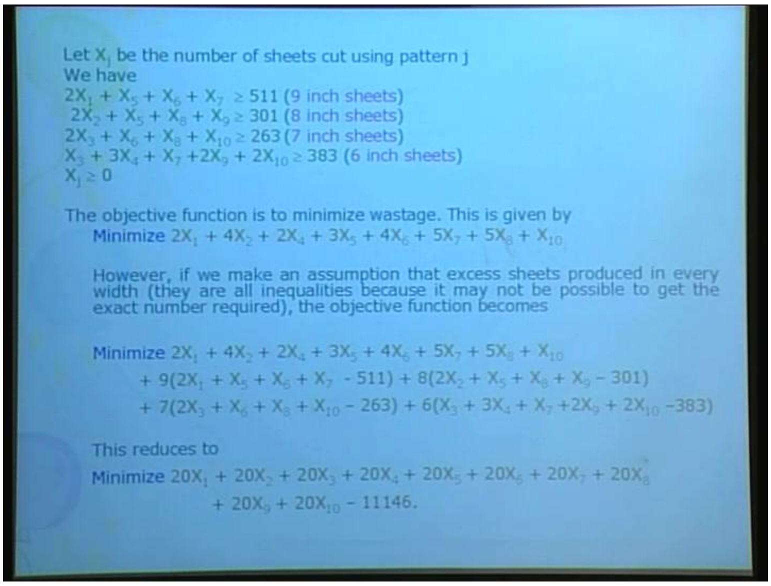

Now we need to write the objective function. The objective function is to minimize the waste. So let us write the objective function now. What is the waste? When I use pattern 1, my waste is 2 inch.

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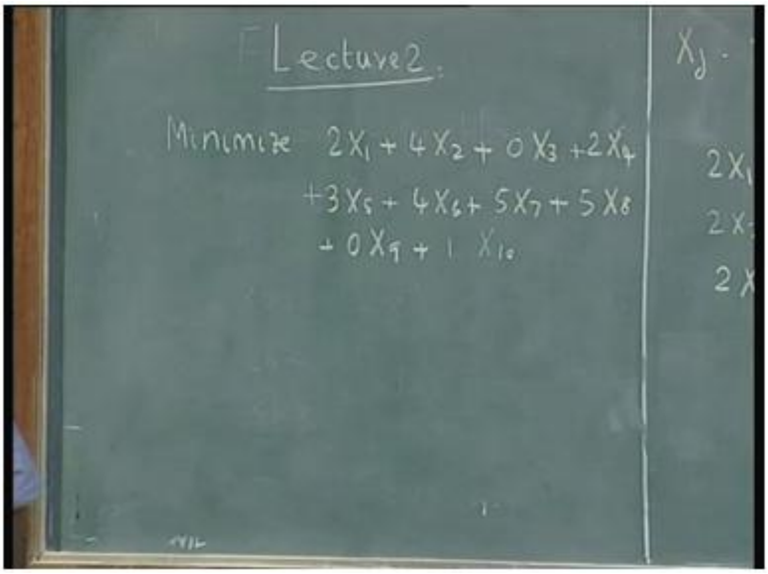

So I can write $2X_1$ waste or the objective function is to minimize $2X_1 + 4X_2 + 0X_3$ which can be left out of $+ 2X_4 + 3X_5 + 4X_6 + 5X_7 + 5X_8 + 0X_9 + 1X_10$. So we have completed this formulation. This formulation of cutting sheets is over. We require 4 different sizes of sheets. 6 inch, 7 inch, 8 and 9 and we have 4 constraints corresponding to this non-negativity restrictions and an objective function that minimizes the waste. The only difference between this formulation and the previous one is that the decision variables were not apparent as they were in the previous two formulations. 1. Now the decision variables depend on the patterns that we are able to generate. 2. The number of decision variables is not fixed in the sense that, for a different problem you could have different number of patterns possible unlike in the first two when we said that there are 4 month's demands. So you know that there are 4 production quantities here. It does not happen that way, so you have to do something first and then based on what you have worked out, you write down the decision variables. So the important learning in some sense here is that there could be problems where the decision variables are not very apparent. You will also realize by now that if you had addressed this problem in a different direction and if you had not thought of the possible patterns that are there, then the written the decision variable, formulation would become very difficult. The first step in any formulation is to identify the decision variable. In fact once the decision variable is identified almost half the formulation is over. Rest of the constraints and objective come along with the decision variable. Now what are the other things that we can learn from this formulation? Let us do something more. Let us assume for example, that if we end up making more than this 511, then we will assume that those excess sheets that are cut over and above the requirement are also a waste. Therefore we add those excess sheets into the objective function which we did not do earlier. Now let us see what happens. If we add the excess sheets into the objective function. To do that, let us go back and understand this. We are having a 20 inch sheet. Let us assume that this has a constant length of say something like 50 which is required.

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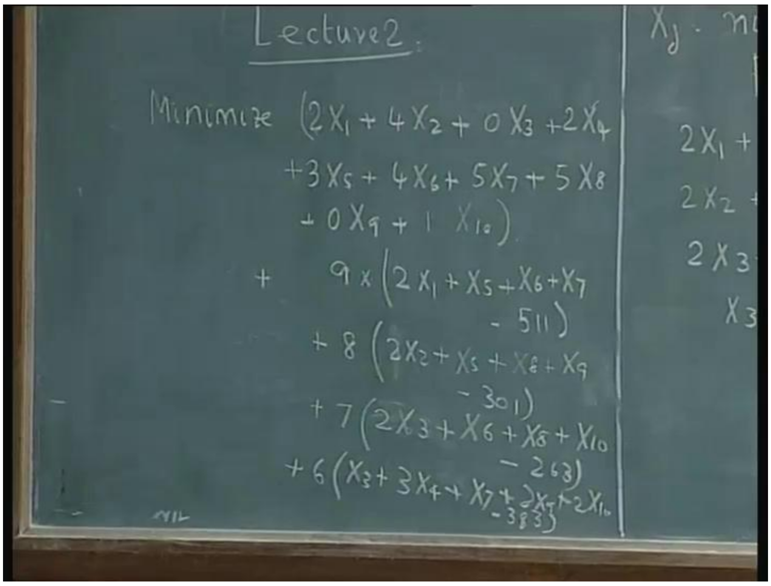

So actually speaking what is the waste? The waste is not 2 but the waste in terms of area, is actually 50 into this (Refer Slide Time 15:51), if we look at and represent the waste as an area and not as a length. Now to this we are going to add the excess as wastes. So let us see what happens if we add the excess as a waste. If we do that then the waste here would be 9 inch, i.e., 9 into $2X_1 + X_5 + X_6 + X_7 - 511$. This is the excess number of 9 inch sheets which is multiplied by 9 which is the length quantity. Here we have also multiplied by 50 to make it area. So to be consistent, we retain the length dimension we do not make it area and therefore so we retain it as length + 9 into $2X_1$. This is the excess number of 9 inch sheets that have been made + 8 into $2X_2 + X_5 + X_8 + X_9 - 301$ + 7 into $2X_3 + X_6 + X_8 + X_10 - 263$ + 6 into $X_3 + 3X_4 + X_7 + 2X_9 + 2X_10 - 383$. So this is the waste. Now you realize that something interesting will happen if we try to simplify this function. So let us simplify this function and see what happens. Right now we will leave this out and simplify. So when you simplify this function, you realize that you get to minimize $20X_1 + 20X_2 + 20X_10 – 1000$ or 11,146.

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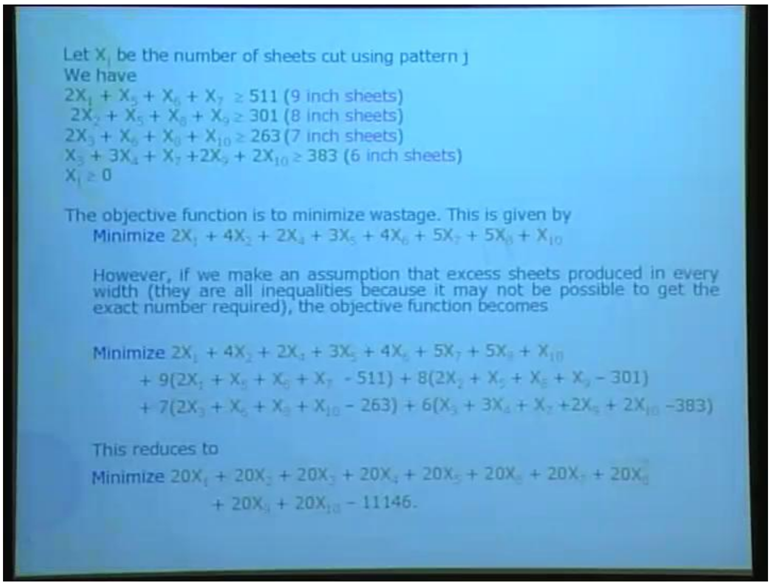

Suppose what happens when you simplify this. The objective function reduces to something else when you add the excess sheets into the objective function as waste and you simplify. The 20 comes in because 9 into 2 = 18 + 2 = 20 for X_1 and 8 into 2 = 16 + 4 =20. If you typically look at an X_9 or X_10, if you look at X_10 then you have, 1 + (there is an X_10 coming here), 7 + 2 into 6, 7 + 12 = 19. So it simply becomes 20 into this. Now what else can happen? Now this constant can be taken out of the formulation and this does not depend on the variables. So this can be removed similarly this 20 is a common factor to all the terms, a common multiplier with all the terms. So the 20 can also be taken out.

The objective function now minimizes sigma X_j. So the problem of minimizing the waste actually reduces to the problem of minimizing the total number of cuts. If we assume that the excess material cut is also treated as a waste then you can show that the cutting stock problem to minimize waste now becomes one of minimizing the total number of cuts. Simply because the way the patterns are written. For example if you take this (Refer Slide Time 19:49) pattern, this pattern has a waste of 5. This pattern has one 8 inch, one 7 inch and a waste of 5, so 8 + 7 + 5 = 20. That is how this pattern was created. Now when you write the excess there, it is only a constant and you get the same thing for every excess pattern. You get an 8 + 7 = 15. Plus, the waste file would make it 20.

So the important learning is the cutting stock problem reduces to one of actually minimizing the number of cuts and not minimizing the waste, provided you make an assumption that the excess is also treated as a waste. It is also interesting to note that excess need not be a waste physically only for the purpose of modeling you may assume that the excess is a waste. On the other hand, if you end up making more than 511, nothing prevents the person from using it again assuming that there is going to be a demand for the same 9 inch or 8 inch or 7 or 6 in subsequent days. We assume that this problem is some kind of a recurring problem that happens in sheet metal cutting or wood cutting or cutting a rectangular sheet of wood of various sizes in a typical manufacturing kind of an application. So the problem is expected to repeat. There is going to be daily demand for various sizes of sheets. So if we make the assumption that this is treated as a waste then the problem becomes minimize sigma X_j. This can be generalized as minimize sigma X_j subject to aij X_j greater than or equal to b_i. If I need sheet of type i, aij is what you get from the pattern so sigma aij xj greater than or equal to bi, X_j greater than or equal to 0. So it takes a very generic form where this co-efficient aij can be seen from the various patterns that you have here. There is one important thing which we need to clarify at this point. We have seen 3 examples. In the first 2 examples we learnt the various terminologies, different types of objective functions, constraints etc. Here we learnt that the decision variables may not be apparent and certain things have to be done before the decision variables are identified. Now we have to look at one more thing. Now we have said that this X_j is explicitly non negative. For example I cannot have – 5 cuts of pattern. It has to be a number greater than or equal to 0. Should this also be an integer?

For this problem the answer is yes. It has to be greater than 0 and for integer the same is true for the earlier formulations. You cannot make 2.5 tables nor can you produce 107.5 items to be regular time and so on. But in most of these problems we do not explicitly state the integer, for a different reason. If the problem has a linear objective function, linear constraints, non-negativity, it is a linear programming problem. All this plus the integer restriction would make it what is called an Integer Programming problem. So most of the times we leave it out because Integer programming problems are solved differently compared to linear programming problems. So we leave out the integer and we try to formulate it as a linear problem and then we solve it as a linear programming problem and if we still get integer solutions, then an integer programming problem is being solved. So even though most of these variables have to be explicit integers we do not state them as integers. When we formulate them as linear programming problems we them leave out.

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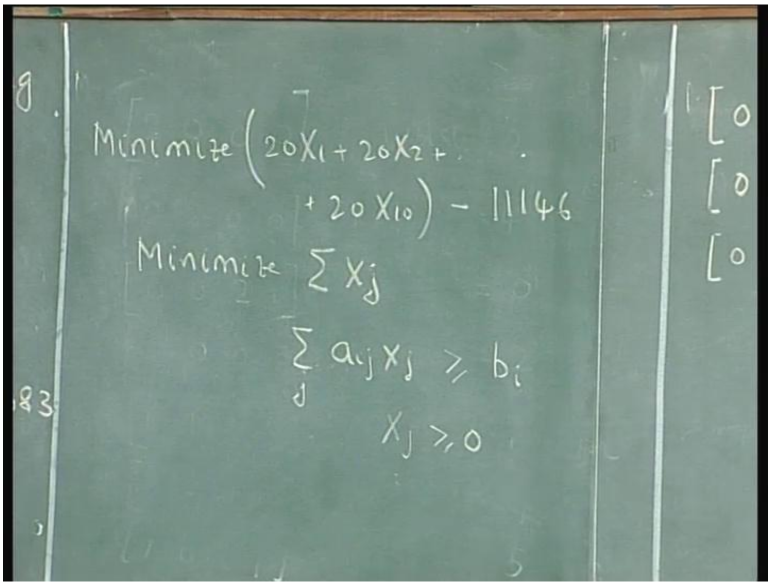

There is one more thing we need to look at in this formulation. Can I formulate this problem in such a way that I have equations? When we started doing this we learnt that the cutting patterns may be such that I may not be able to satisfy this as an equation. I would satisfy this more as an inequality and therefore we wrote a greater than or equal to in this case. The first pattern if you remember was [2 0 0 0] with waste equal to 2. Now can I consider a pattern which is [1 0 0 0] with waste equal to 11? So far we did not consider such patterns. We considered the patterns where the waste was less than the smallest thickness that was needed. Now can we consider a pattern like this? If we consider a pattern like this then the first thing that will happen is the number of feasible patterns or number of possible patterns will be definitely more than 10 and it will be a very long number. When we consider an exhaustive set of pattern, we can always go back and say that it will be more than 10. You may get some large finite number of possible patterns of 30 or 40. But you will still be able to write it as an equation. I will be able to cut in such a way that I exactly meet the demand.

You can go back and say that if for example this is 512 against this 511 and 1 sheet is carried. That one sheet becomes a waste in this formulation. If I write this as an equation and have an exhaustive set of formulations then it means for that one sheet I am using this(Refer Slide Time: 25:32)pattern instead of this pattern. So there is nothing wrong, provided we can formulate it that way.

So if we exhaustively enumerate all the patterns, and there are many more, for example you could have [0 1 0 0] with 12 and so on and in fact you can even think of [0 0 0 0] with waste equal to 20 as a pattern and we can still do it. And if you do that then you end up getting equations here in all these four and you will have ax = b. Now this apparently is an inferior formulation because it has fewer variables compared to the earlier one. But you will realize much later in advanced course in Operations Research that the cutting stock problem is actually solved using this formulation and not the formulation with inequalities.

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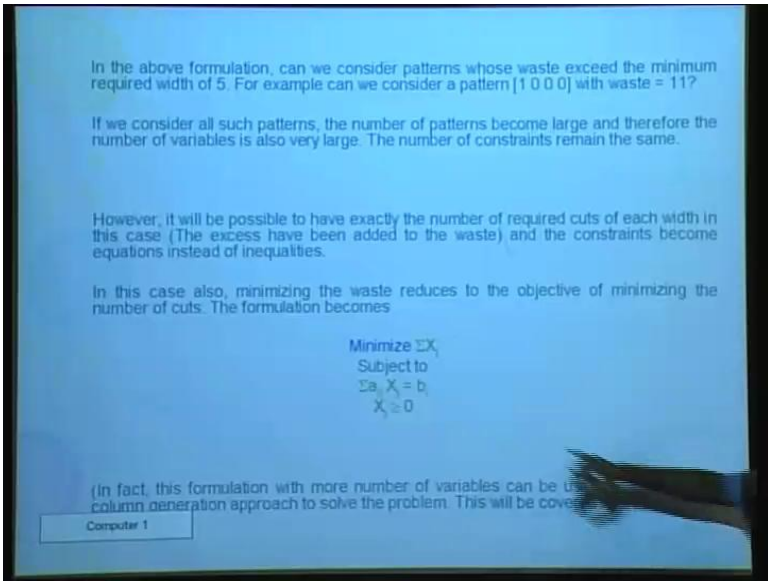

If you go back and see, this is what we have tried to show. You get subject to a_x = b and if you use exhaustive set of patterns and X_j greater than or equal to 0.

In fact formulations with more number of variables can be used if we develop a column generation. It is too early to look at column generation now but much later in advanced course we will see column generation method. And the one dimensional cutting stock problem that we have just now formulated is actually solved using a column generation and more importantly considering this equation and not inequalities. But you get the equation if you look at more variables and more exhaustive formulation such as this. So this brings us to the end of the third formulation in this course. We now look at the fourth formulation before we complete the formulation topic.

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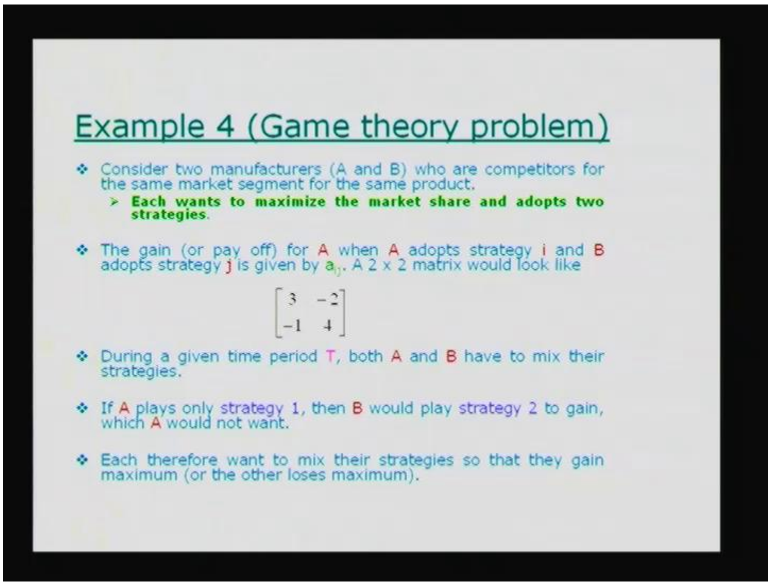

So we look at a fourth formulation here and that is also an interesting formulation. This is a problem from game theory and this formulation goes like this.
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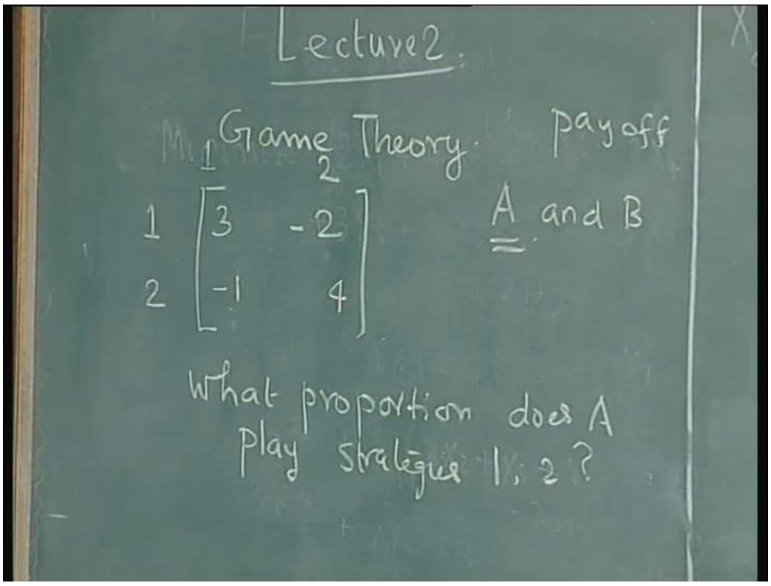

Now let us look at a problem from game theory which is also formulated as a linear programming problem. Let us assume that there are two competitors. We call them A and B - competing for market for the same product. We can assume any two from any industry that you know. Now both these people A and B want to have a higher marker share and both of them have some strategies with respect to promoting their product. For example typical strategies would look like a discount you could go back and say strategy 1 would mean I give 1 Re discount on the product and a strategy 2 could be that you buy 2 and get the third free. A third strategy could be that you get 10% more for the same price. A fourth strategy could be that you buy this and you get something else free. So people have different strategies that they use over a period of time to promote their product. So we see that that A and B have now sought 2 strategies for each. We do not know what these 2 strategies are. The same two are not handled by A and B. For example A could handle a different one and B could handle another. The information that we have here is called a Payoff matrix for A.

Payoff matrix for A, for example if A plays strategy one and B also plays strategy one, then A gains Rs 3 or you could keep it as 3 lacs or any amount. If A gains 3 and A plays strategy1, B plays strategy 2. A gains – 2 which means A loses 2. Similarly A plays strategy 2 and B plays strategy 1 then A loses 1 and for 2 and 2 A gains 4. Now the question is this. Let us assume a certain period of time, say 1 month. We will also assume that the person can instantly switch from one strategy to another. B can also instantly switch from one strategy to another. If you look at a situation where A is playing strategy 1 for some time, what will happen is B is smart enough to understand that A is playing strategy 1, so B will start playing strategy 2, such that A loses. A loses 2 and B gains 2. So if A continues to play strategy 1 all the time, then B will only play strategy 2 so that B gains. Once A knows that B is playing strategy 2 with A, game A is also smart enough to switch to strategy 2 so that A gains 4 and once B knows that A is switching to 2 and B will switch to 1 and so this keeps going on. The question is, given a certain amount of time, what is the proportion of times A will play this and this? What is the proportion of times B will play this (Refer Slide Time: 30:46) and this B will play this and this such that there is a net situation that happens. So this is the problem we are trying to formulate in this. Now let us look at A. Now this is called a payoff matrix for A. Exactly opposite is the payoff matrix for B. If A gains 3, B loses 3. If A loses 2, B gains 1.This are matrix for A. You can write an equivalent payoff matrix with the signs reversed as payoff for B .Now look at A's problem alone. A's problem will be, what is the proportion does A play strategies (or options) for 1 and 2? So the decision variables for A are, let p_1 and p_2 be the proportion of times so A plays 1 and 2. So A is going to play this B one time. A is going to play this B two times.

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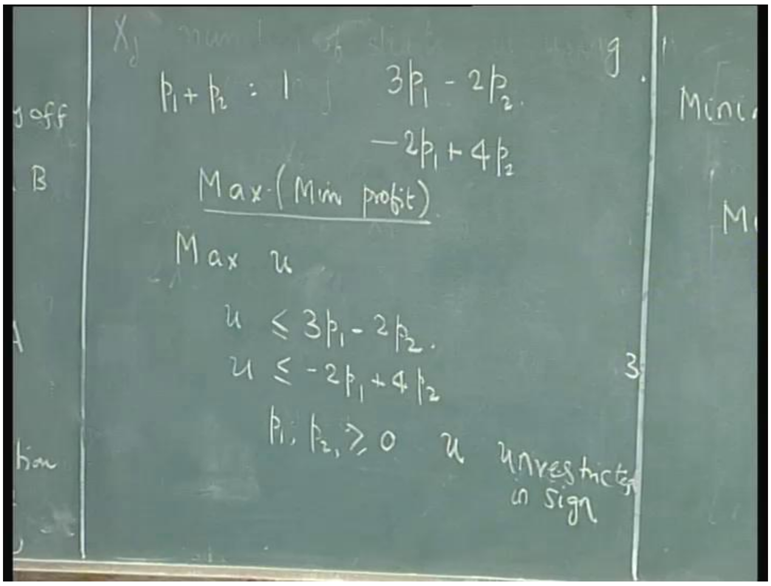

First and the simple thing is $p_1 + p_2 = 1$. p_1 and p_2 are defined as Proportions. So $p_1 + p_2 = 1$ now. A has to decide on p1 and p2. We know that A will play this but let us assume that if B plays this first strategy all the time and A continues to play these 2 strategies with p_1 and p_2 respectively, then A's gain will be $3p_1–p_2$. If B plays this all the time and A continues to play this with P1and P2, then A's gain will be $– 2p_1 + 4p_2$. We have made an assumption that A and B are equally smart. Therefore what A will try to do is A will try to play these in proportions p_1 and p_2 such that A would like to maximize the profit. Now B is intelligent enough and B will not consistently play this but switch the strategies in such a way that B is going to allow A to get minimum profit because B is an equally smart fellow. So what A would do is A would rather not try and maximize the profit but A would try to play p_1 and p_2 in such a way that A maximizes the minimum profit that B is going to allow A to get. So A would like to maximize the minimum profit that B would allow A to get. So what is A's problem now? A will allow you to maximize sum u and B will allow you to be minimum of these two. So B has to be minimum or lesser between these two. So u less than or equal to $3p_1 – p_2$, u less than or equal to $–2p_1 + 4p_2$ and $p_{1,}$ p_2 greater than or equal to 0 and u we will come to that.

Let us go back and define this again. Now A is supposed to be the following something called a Maximin strategy. A would ideally let to maximize his complete profit but B will not allow A to maximize it endlessly. B will play his cards in such a way that A gets minimum profit and therefore A will come to realistic terms and say that "I would now like to maximize the minimum profit that the B will allow me to get". So A's strategy is called a Maximin strategy and we formulate the problem for that strategy. So A will try to maximize a u, where u is a minimum profit that B would allow A to get. So u has to be the minimum of these two. These are the extremes though. u can be somewhere in between. u will have to be less than or equal to the minimum of these two and then p_1 and p_2 have to be determined in such a way that u is maximized. So this is the formulation for A’s problem. Now let us go back now. In this formulation we have defined p_1 and p_2 as proportion of times A plays these two strategies. p_1 and p_2 will have to be greater than or equal to 0. Now what about this u? Now u is the minimum profit that B would allow A to get which A tries to maximize. So u is some kind of a profit term. Can the problem be in such a way that A ends up making a loss possible? Hence we are not sure if u should be greater than or equal to 0 while you are sure that p_1 and p_2 are greater than or equal to 0 you are not sure whether this u is greater than or equal to 0. u could be greater than 0 if A ends up making a profit at all. u could be exactly 0 if A gets into a 0 situation and u could be negative if the maximum profit that A could get, (based on the minimum that B would allow A to get), turns out to be negative. So this u can be either greater than 0 or equal to 0 or less than 0. This u is called ‘Unrestricted in sign’. So this is A's problem and A's problem is called a Maximin strategy which you will see there (Refer Slide Time: 37:31). So A tries to maximize u.

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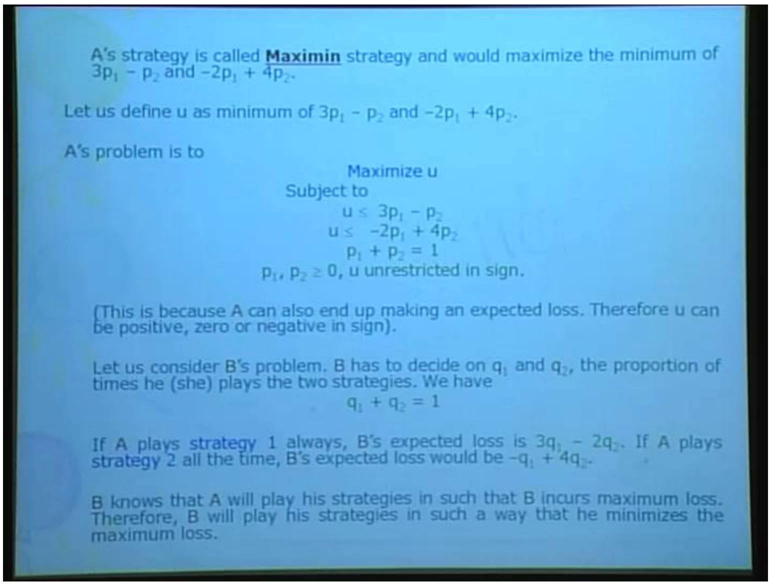

You can go back and look at this A's strategy is called Maximin and would maximize the minimum of $3p_1 - p_2$ and $-2p_1 + 4p_2$ and therefore we define u as a minimum of these two and we try to maximize u for A and you get the same problem. This u is unrestricted in sign. You can see that this is something that we are introducing for the first time.

So far we have had variables that were always greater than or equal to now we look at this ‘unrestricted in sign’ that can happen. So this is because A can also end up making a loss so u can be positive 0 or negative depending on the situation. Now let us go back and do something for B and see whether these two problems are actually one and the same or whether they are different so let us now look at the problem purely from B's point of view. Now let us erase this (Refer Slide Time: 38:37). Now what is B's decision now? B's decisions are here. B would like to find out with what proportion B has to play these two strategies.

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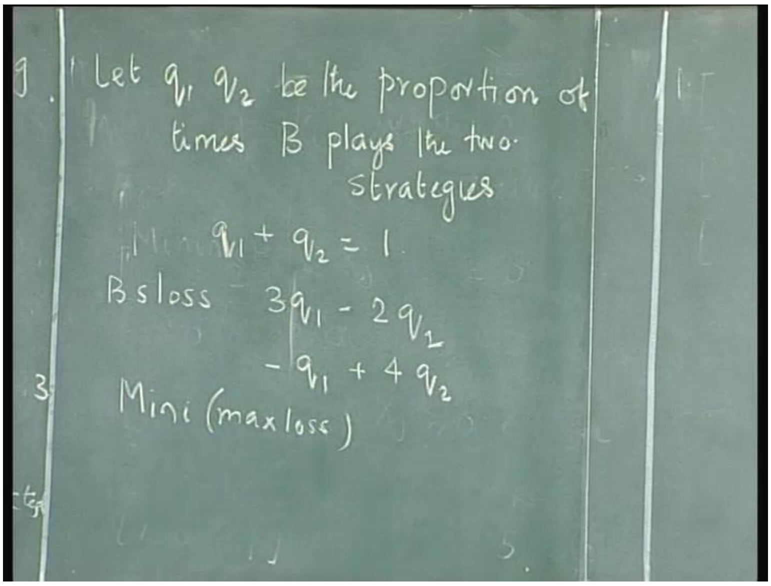

B's decision variable would be let q_1 and q_2 be the proportions of times B plays the 2 strategies. Now naturally, $q_1 + q_2$ will be = 1. Now this is where you define your q_1 and q_2. Now what do we do? This should be read as – p_2.

So if B plays these 2 strategies with proportions q_1 and q_2 respectively if A consistently plays this strategy then B's loss will be $3p_1$. B's loss will be $3q_1 – 2q_2$. Let us go back. B plays these 2 strategies with proportions q_1 and q_2. So if A plays this then B's loss will be $3q_1 – 2q_2$. If A plays this strategy consistently then B's loss will be $– q_1 + 4q_2$. Remember both are loss to B and they are gains to A. Now what would A and B do? Now A is not going to play one strategy consistently. A will switch the strategies in such a way that the gain to A is maximized or the loss to B is maximized. Now B will have to play the strategies in such a way that that loss which A wants to inflict on B is minimized so B will play strategy called Minimax or minimizing the maximum loss. A is going to allow B or would want B to have maximum loss and B would now play the strategies in such a way that this loss (maximum loss) is minimized. If A plays a 'Maximin' strategy, B would play a 'Minimax' strategy to minimize the maximum loss that A would like to inflict on B.

Both can what we are trying to do is we are trying to follow a very conservative strategy for both because we want to look at this problem more as a linear programming formulation we are not looking at it to try and solve a game theory problem. We could have situations where both play different strategies. This linear programming formulation is made under the assumption that A would play Maximin and B would play a Minimax. If they play different strategies, B would get different formulations. That is it. So now what we want to do is we want to minimize a v. v is the maximum loss that A would inflict on B. Now these are the two extreme scenarios for the losses. So v should be greater than or equal to $3q_1 – 2q_2$ and v should be greater than or equal to $– q_1 + 4q_2$ (q_1 and q_2 greater than or equal to 0) and v again can be unrestricted because we do not know (if for example A makes net profit then B would make net loss and if A makes net loss B would make net profit) therefore v is also unrestricted in sign. Now this (Refer Slide Time: 43:42) is B's problem and this is A's problem. We will have to quickly come back to A's problem to make the small correction which we did. A's problem would mean that A plays this with p_1 and p_2. So when B plays this strategy consistently A's gain will be $3p_1 – p_2$. If B plays this it will be $– 2p_1 + 4p_2$. Therefore this correction comes into play. Now we look at these two problems. Now immediately we get a feeling that these two problems are related because for the same data we have looked at A's problem and B's problem. Somewhere we also know that when A solves the problem and gets this u which is the profit that A makes. It is very likely that it will be the same v that B loses. So both these problems are actually related. And in fact it is enough to formulate only one. If we formulate A's problem and solve it and we get this u (Refer Slide Time: 44:39). We know this will be = v which is the B's problem. And then from the $q_{1,}$ the q_2 can also be determined. So what have we learnt in this formulation? We have actually learnt two things. One is the first simple learning that you could have variables that are unrestricted in sign so this brings us to the summary. The objective function can be of two types, maximization and minimization. In the first 3 examples we saw 1 maximization and 2 minimization problems. In this example we saw both maximization and minimization. Here constraints can be of three types:

(i) Less than or equal to
(ii) A greater than or equal to and
(iii) An equation

We have seen all three types of constraints in all the problems that we have seen. Until now the decision variables were of the greater than or equal to type in all the 3 formulations. In this formulation we have introduced something called an unrestricted variable. We could also have variables that are less than or equal to. So you could have 3 types of variables, 3 types of constraints, 2 types of objective functions and we have seen all these. What we have also seen in this formulation are 2 things. If we look at this very carefully, we wrote down two expressions: $3p_1 – p_2$ and $– 2p_1 + 4p_2$ and understood that the objective is actually a Maximin strategy which means we want to maximize the minimum of certain functions. We then wrote as we introduced another variable u. We said maximize u subject to minimum of something. In this we said that the objective is to Minimax the loss. So we defined another variable v which was not originally in the problem and then we represented the objective of Minimax by minimizing v and v greater than or equal to this. We then learnt how to formulate situations wherein we want to maximize the minimum value of certain expressions or to minimize the maximum value taken by certain expressions. So that is another thing that we have learnt in this formulation. The last thing that we learnt (which is the most important thing, very peculiar to this formulation) is the formulation is actually of 2 formulations, one for A and one for B which we independently did. We had a formulation for A which is this .We also had a formulation for B which is this (Refer Slide Time: 47:28) that we independently did. We then we realized that for this problem, the 2 players A and B are explicit and apparent so we looked at 2 problems.

And we said somewhere that these two problems look similar and it is actually enough to formulate one and we can end up formulating the other. In reality what happens is we will show later that every linear programming problem also has an associated linear programming problem which was very evident in this example not so in the earlier 3 and then we will go back and say that every linear programming problem has an associated problem and if you solve one you could go back and solve the other indirectly. So that is something which we will see later in this course. So let us see the summary.

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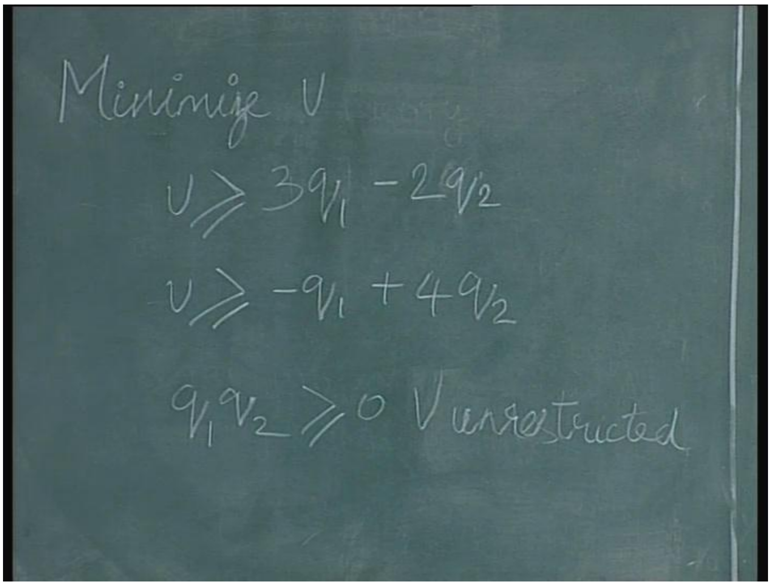

Now we go back to this problem which is minimize v. v is greater than or equal to $3q_1 – q_2$ and so on. Since A's gain is B's loss and vice versa, we know that the optimum A's problem and B's problem has the same value as the objective function that we saw.

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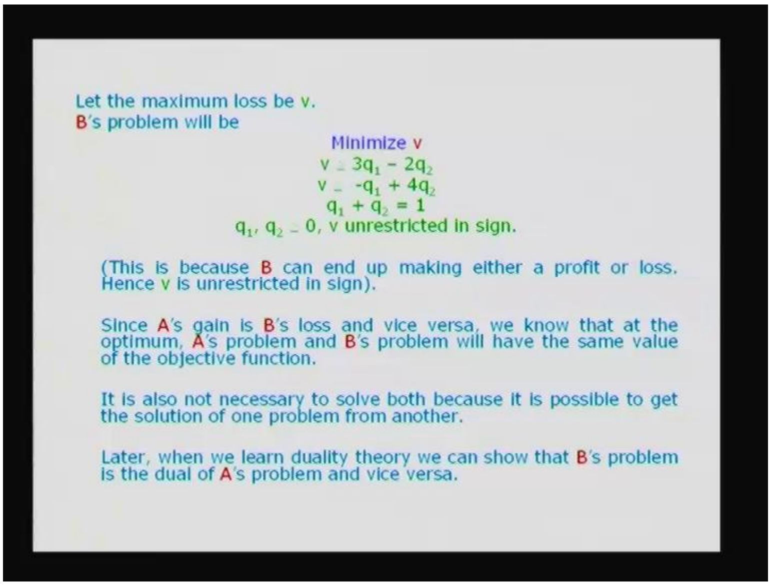

It is not necessary to solve both. It is enough to solve only one. We could get the other from it. Later when we do something called duality theory, we will notice that B's problem is actually the dual of A's problem and vice versa. Also we will show that every problem has an associated problem and by solving one we can actually solve the other.

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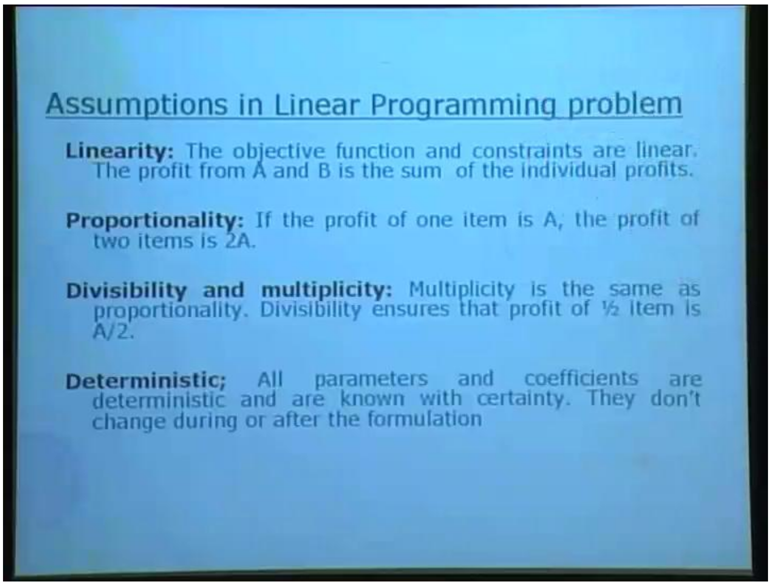

Now we have seen four linear programming formulations. We have seen different types of objective functions, constraints and learnt few things. What are the assumptions? There are some assumptions that we have made while we have formulated all this. The first one is called linearity.

1. Linearity: The Objective function and the constraints are linear only then it is a linear programming problem. The profit from A and B is a sum of the individual profits and we use a $6X + 5X_2$. We analyzed that the total profit is actually the sum of the individual profits.

2. Proportionality: Profit of one item is A and two items is 2A. We said if with each item we can make 5 and with two items you would make 10 and so on.

3. Simple Divisibility and Multiplicity (same as proportionality): Divisibility also ensures that it is proportionally divided. Most important, we have deterministic assumptions.

All parameters and coefficients are deterministic. They are known with certainty right at the beginning and they do not change during or after the formulation. We are not looking at any probabilistic situation where we define a profit function which is a distribution. Here we assume that all coefficients and parameters are deterministic. So at the end of the formulation in these two lecture sessions that we have had, we have seen four examples. One can go on and on and create different situations for formulations (endless in fact). With every formulation you can actually learn something new. What we have tried to do is, using four examples, we have tried to show you the various aspects of problem formulation i.e., the terminologies, and the definition in terms of objective function, constraints, and variables, different types of objectives, constraints and variables, different situations, where in some situations, the formulation will be explicit, the variables will be obvious and in some other where something more have to be done to get to the variables.

Lastly a problem wherein we defined both the problems and we say that solving one is enough and we can get the solution to the other. So with this we end the linear programming formulation of this course. We will do the linear programming solution in the next class.