Fundamentals of Operations Research

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Lecture No. # 13

Transportation Problem, Methods for Initial Basic Feasible Solutions

In this session we continue our discussion on the transportation problem.

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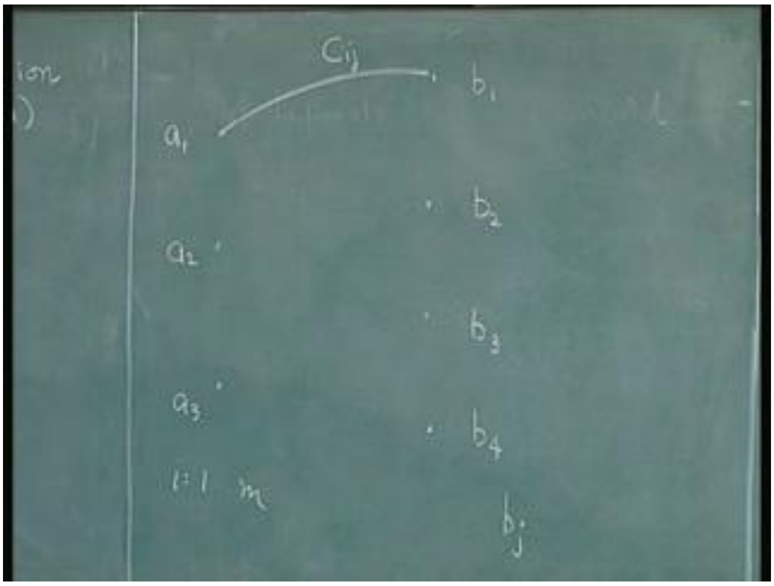

Transportation problem talks about transporting a single item from a give set of supply points to a given set of destination points. The supply in supply point i is a_i and the requirement in demand point or destination point j is b_j and the problem is one of finding a least cost transportation from the supply points to the demand points where C_{ij} is the unit cost of transportation.

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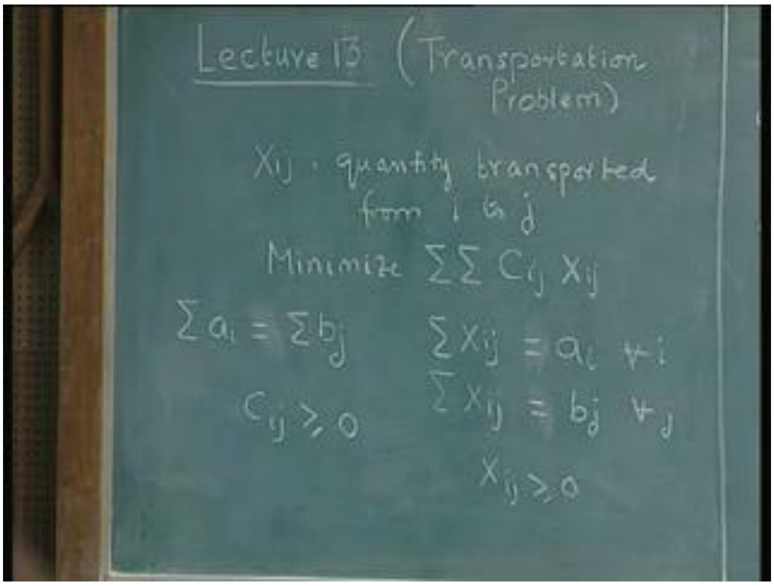

The problem is formulated as X_{ij} the quantity transported from supply point i to destination point j. The objective function is to minimize the total cost of the transportation C_{ij} X_{ij} subject to supply constraints. So for every supply point total quantity that leads the point should be less than or equal to what is available. So a_i is the quantity available in the supply point i and as far as every demand point or destination point is concerned, they are greater than or equal to b_j for every j, X_{ij} is greater than or equal to 0. This is the linear programming problem but we will not be solving this directly using the simplex algorithm.

We will see that this problem has some more structure which we will try to exploit and use an algorithm which is a slightly different version of this simplex algorithm. Now sigma a_i represents the total availability and sigma b_j represents the total requirement. If sigma a_i is greater than or equal to sigma b_j which means the total availability which is the sum of all these is more than what is required. It is then possible to transport the entire requirement such that the demand of every destination point is met. If sigma a_i is strictly less than sigma b_j which means that the total ava$_i$lability is less than the total requirement. Then clearly all the requirement cannot be met.

So we right now assume that sigma a_i is greater than or equal to sigma b_j so that the requirement of all these points are met. The next thing would be assuming the total will be more than the total demand. We will be sending quantities from each of the supply point to one or more of the destination points so that we meet the demand as well as minimize this transportation cost. It is very reasonable to assume that the transportation cost C_{ij} is greater than or equal to 0 for all ij transportation cost cannot be negative. When the transportation cost is non-negative then we will only end up sending exactly the amount that is required and none of these destination points will receive even 1 unit more than what it requires because that extra unit has to be transported from one of these supply points and that can only increase the cost of transportation. For example if this requires b_1 we will send exactly b_1.
We will not send b_1 + 1. The extra unit would increase the total cost of transportation if it is strictly greater than 0 or if it is 0 it would still retain.

The optimal solution will send exactly the quantity that is needed in each of these destination points. We would solve this problem where the greater than equal to inequality has now been converted into an equation.

If we consider a very special case where sigma a_i is equal to sigma b_j then what will happen is we will not only meet the exact requirement of every destination points but we would also be utilizing all that is available as for as each of the supply point of the system which means this inequality also becomes an equation. Now the transportation problem is under the assumption that sigma a_i is equal to sigma b_j which means total supply equals total requirement reduces to the same problem except that the inequalities are all now equations.

This transportation problem which has all equations is called a balanced transportation problem.

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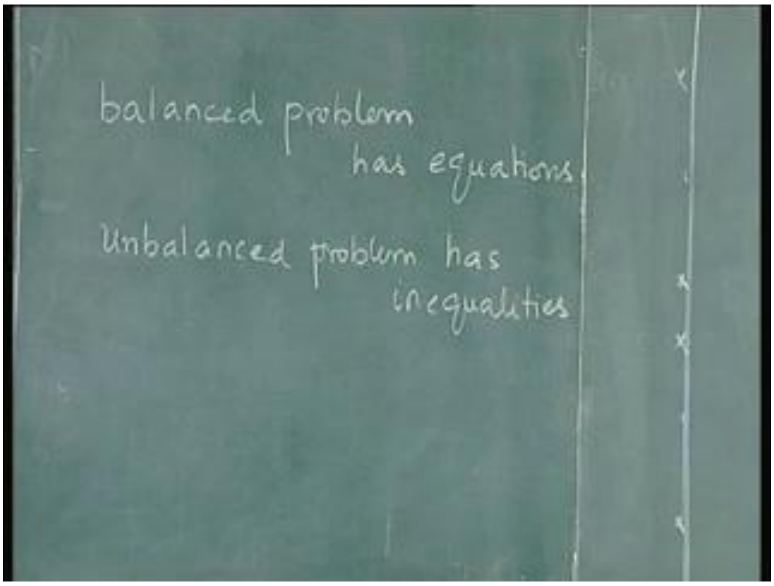

A balanced problem has equations and the original unbalanced problem has inequalities. Now we have defined two types of transportations. Problem one is called the unbalanced problem which has inequalities less than or equal to here and a greater than or equal to here and a balanced problem which has equation which will satisfy the condition sigma a_i equal to sigma b_j. Now as far as solving the transportation problem is concerned we will solve the balance transportation problem. We will show that it is easier and better to solve the balanced problem and then we will also show a way by which every unbalanced problem can be converted to a balanced problem and solve it. Let us look at solving a transportation problem.

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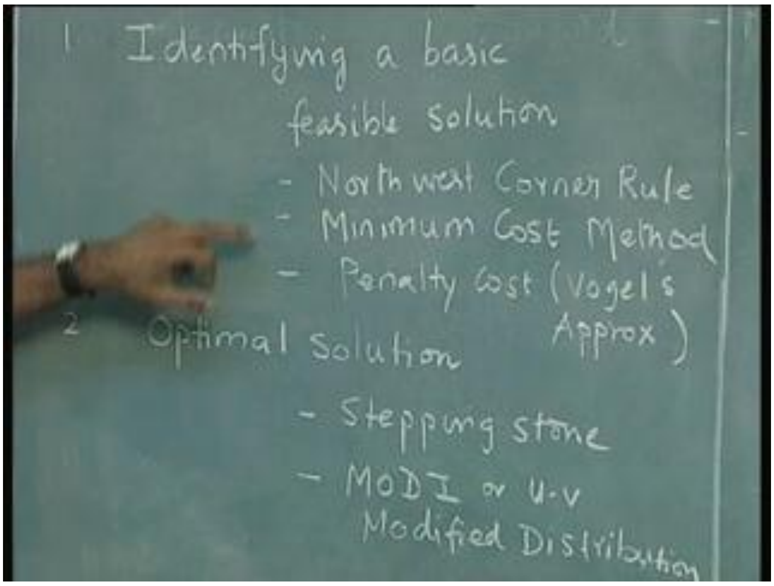

Now the solution to a transportation problem has 2 stages. One is called identifying a basic feasible solution and the second is to get the optimal solution. Now we look at 3 different ways of identifying a basic feasible solution. They are called the North West Corner rule, Minimum cost method, Penalty cost method also called Vogel's approximation method. We will be looking at 2 different methods to get to the optimal solutions. One is called stepping stone method and the other is called MODI (modified distribution) method or U V method. So we will be looking at all these 5.

We should remember that these 3 North West Corner rule Minimum Cost Method and Vogel's approximation method help us get a basic feasible solution while we can use any one of these to get to the optimal solution after we get an initial basic feasible solution. We should also remember that this is a linear programming problem but we do not use the simplex algorithm directly. We use an algorithm which has all the properties and characteristics of the simplex but it is a slightly different version of the simplex algorithm.

Normally if we take any linear programming problem, if we take the standard problem that we have looked at, a maximization problem is all less than equal to constraints. Then the first basic feasible solution will be very weak with respect to optimality. If we solved a maximization problem with all less than or equal to constraint and all greater than or equal to variable. The starting basic feasible solution with all slack variables in the basis gives us an objective function value of 0 for maximization. Even though it is a basic feasible solution, it is not a very good basic feasible solution with respect to the objective function. The very nature of the transportation problem as we will see will help us get a good basic feasible solution rather than arbitrarily or simply starting a solution with slack variables. It is easy and it is possible to get a good basic feasible solution so that once we concentrate on this and get a good basic feasible solution, the number of iterations to get to the optimal solution reduces. Therefore we follow a separate algorithm where we concentrate on getting a good basic feasible solution and then apply the optimal algorithm to the basic feasible solution to get to the optimal solution.

We will first look at each one of these through a numerical example and then we also explain these two with the numerical example. Before that we also represent the transportation problem in a very convenient form. We do not normally represent it this way. We represent a transportation problem in a very convenient form.

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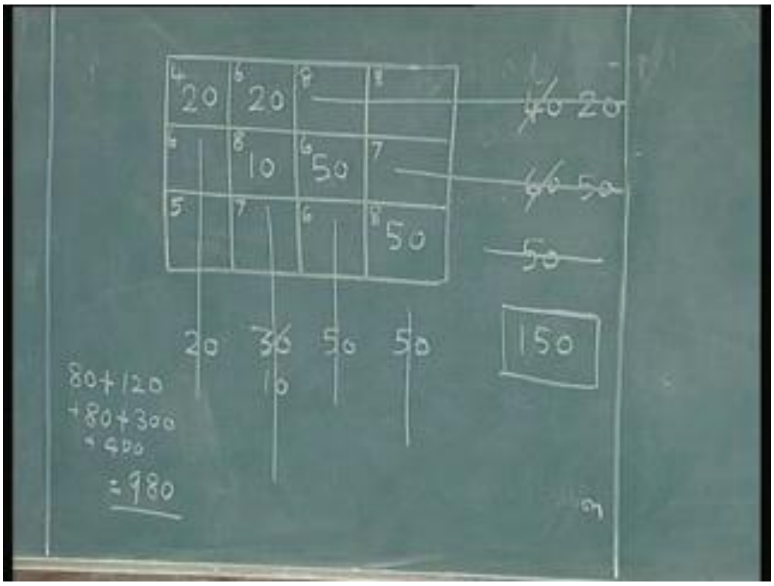

These three rows for column matrix represents a transportation problem with three supply points and 4 destination points. The suppliers in these three points are 40, 60 and 50. The requirements are demand in the four points is 20, 30, 50 and 50. So these are the a_i's a_1, a_2 and a_3. The supplies are the b_j's b_1, b_2, b_3 and b_4. The total supply is 150 and the total demand is also 150. Therefore this is a balanced transportation problem. The unit cost of transportation from i to j, C_{ij} is written as follows. You have 4 6 8 8 6 8 6 7 5 7 6 8. It is compulsory to write the unit transportation cost in the top left hand corner of the individual allocation in a smaller size so that it represents the transportation cost. This in total represents the transportation problem with three supply points with supply 40, 50, 60, four destination points or demand points with 20, 30 and 50 and 50 respectively. Balance indicated by the total and unit transportation cost are given here. Let us try to get a good feasible solution to this problem.

We are going to look at three methods North West Corner Method, Minimum cost method and the Penalty cost or Vogel's approximation.
First let us look at the North West Corner rule to solve this. Given the transportation matrix, we first look at the North West Corner Method or top left hand corner. As far as this allocation is considered there is a supply of 40 and there is a requirement of 20. Try to put as much as possible into this. So 40 can go in only 20 is required. So take the minimum and allocate 20 here so from the first supply point to the first requirement point we have allocated 20.

We have met the requirement of this 20. So just put a dash indicating that this has been met and because 20 has been allocated, out of this 40 becomes 20 and a balance of 20 is available. Now once again go to the top left hand side corner or the North West Corner. Now this is the top left hand or the North West Corner. The supply is 20 and requirement is 30. Only 20 can

be put here. Put the minimum now. This 20 is exhausted. Put a line indicating that this supply is exhausted and this 30 now requires a further demand of 10 because 20 has been supplied. Go back and identify the North West Corner or the top left hand corner which is this corner. In a supply of 60 there is a requirement of 10 so you can meet only 10. This 10 is exhausted so this is over. So this 60 becomes 50. Once again go to the top left hand corner.
There is a supply of 50 there is a requirement of 50, so you can meet a 50 here and both these are exhausted. Once again there is only one point available which is a North West Corner. Once again available is 50 and requirement is 50.

So 50 is allocated so all these supplies and demands are met and we have got the basic feasibility solution with this. In fact as far as the matrix is concerned even though we clearly said the supplies are 40, 50, 60 and demands are 50, 30, 50, and 50. Even interchanging the supply and demand will not change anything because it is a balanced solution. However as far as this problem is concerned they would represent supplies here on the right hand side and the demands here below the transportation matrix. This is a basic feasible solution.
We will later define what exactly the basic feasible solution to a transportation problem is. At least this is a feasible solution because this solution meets all the supply demand constraints and has all allocations greater than or equal to 0. This is feasible because it satisfies all the constraints. The cost associated with this allocation for example, this solution says 20 is allocated from the first supply to be the demand. As far as the second demand is concerned, allocate 20 from the first 10 from the second. As far as the third demand is concerned allocate 50 from the second and as far as the fourth demand is concerned allocate 50 from the third. So all the 40 is allocated, 60 is allocated, 50 is allocated and all demands are met. Cost of transportation would be (4 into 20 = 80) + (6 into 20 = 120) + (8 into 10 = 80) + (6 into 50 = 300) + (8 into 50 = 400), so this is 280 + 700 is equal to 980.

We have the starting solution the North West Corner rule which has an objective function value of 980. So we just note it down here. North West Corner rule as objective function value of 980 and that is shown in this example. The same allocation that we have here is shown here.

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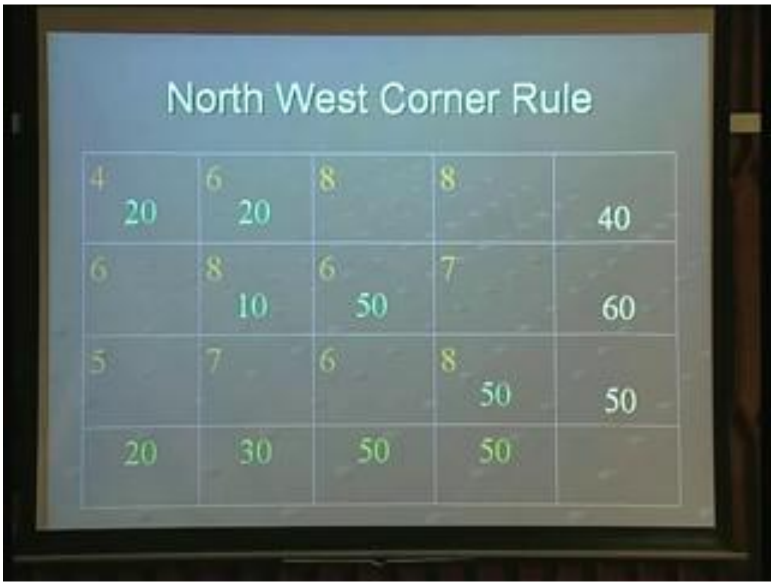

See 40, 60 and 50 are the supplies and 20, 30, and 50 are the requirements. 20 and 10, 50 and 50 are the allocations for a total of 980. This is the one of the ways of getting the basic feasible solution with the problem. Now look at the second method which is called the minimum cost method.

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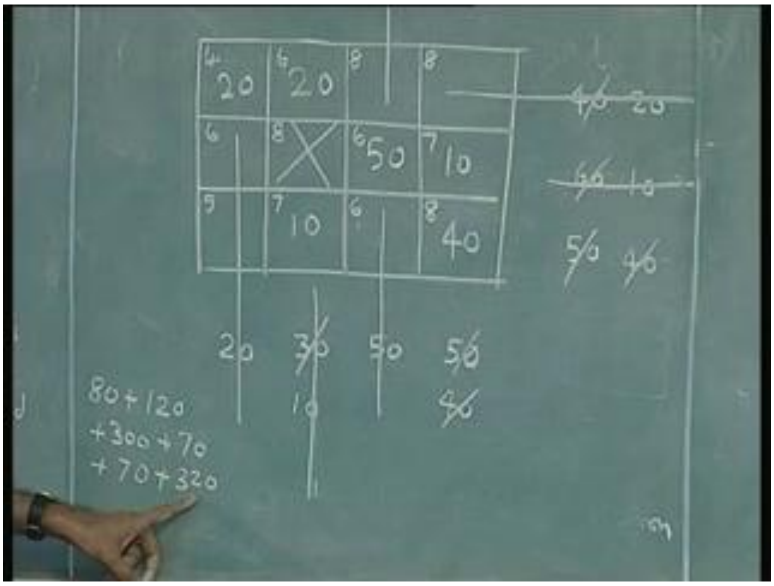

We represent the problem again and we start the allocation using the minimum cost method. What is the minimum cost sell? The minimum cost ij that you have is 4 and it happens to be for this particular position. Identify this position which has the minimum cost. We identify this. The supply is 40 the requirement is 20 so allocate 20, a minimum of possible supply and possible demand, so this is exhausted and this 40 becomes 20. Look at the remaining minimum cost. We have a minimum cost of 6 here. So there is a tie between this and this for the minimum cost position. So we can break the tie arbitrarily to choose between this and this. Let us assume that we pick up this 6 arbitrarily.

This would mean there is a supply of 60 and a requirement of 50. So allocate 50 here. This is over and this becomes 10. Once again look at the minimum cost, minimum cost is either here or here. Once again we can break the tie arbitrarily and we choose to allocate this position. As far as this position is concerned, there is a supplier of 50. There is requirement of 10 so allocate 10 here. This is over. This goes, this 50 becomes 40. Once again the minimum cost position is here, supply 10, requirement 50, and allocate 10. This gets exhausted this becomes 40. This is the only one that is available. Supply 40, requirement 40, allocate 40. This goes. This is another basic feasible solution. Once again we haven't yet defined what the basic feasible solution is but it is a feasible solution where all the supply demand constraints are satisfied. All allocations X_{ij} are greater than or equal to 0. Let us find out the cost associated with this solution. This is (4 into 20 = 80) + (6 into 20 = 120) + (6 into 50 = 300) + (7 into 10 = 70) + (7 into 10 = 70) + (8 into 40 = 320), 200, 500, 570, 640, 960.
The minimum cost method gives us solution with 960 for us.

Between the Vogel's methods and between the North West Corner rule and the minimum cost method, we realize for this example minimum cost is able to give us a slightly lesser objective function value compared to the North West Corner rule. The problem being a minimization problem, we would we interested in the solution when compare to this solution. There is also one slight difference between this and the previous one. If you could go back to the North West Corner rule, the only difference that we have is we have 6 allocations in this in the minimal cost allocation whereas we have 5 allocations in the North West Corner solution. For example there is 20 here, there is another 20, 10, 50 and 50. There are 5 allocations here while there are 6 allocations here but then when compared to the cost; this has a lesser cost than the North West Corner rule.

This understandable because the method is based on progressively identifying least cost positions whereas in the North West Corner, we did not consider the cost while making the allocation. Any method which considers cost while making the allocation is expected to perform better than the one which does not consider cost. It is only obvious that in this case minimum cost has given a less cost than the North West Corner solution.

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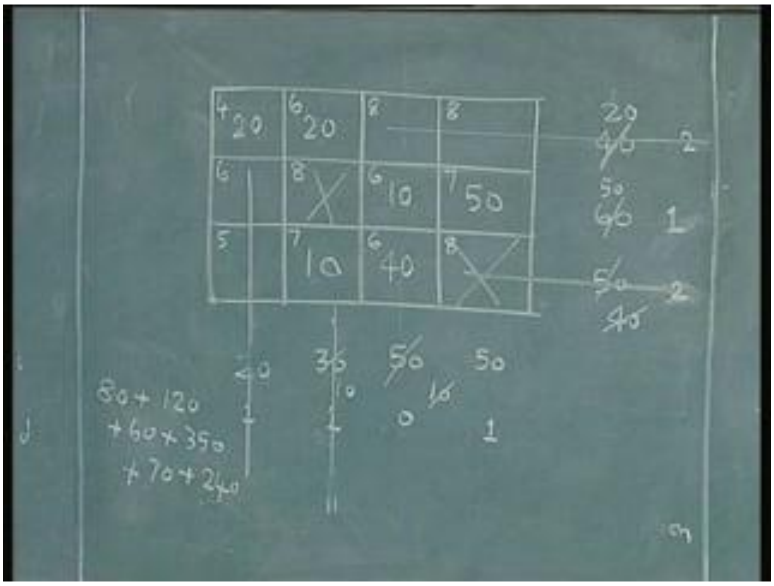

Now let us look at the first row. This has 40 because we want to minimize the objective function. We would ideally like to put as much as 40 as possible to this because it has the least cost. If we are not able to allocate some or all of this 40 to the least cost position, then there will be an additional cost or penalty associated with that. There will be an increase in the objective function because we are not able to allocate to the least cost. The penalty we will incur per unit for not being able to put in the least cost position is the difference between the least and the next least. There will be a penalty of two associated with this row.
If I am not able to allocate here then definitely my cost will increase by two for every unit that is allocated because I may do it here.

The minimum penalty we will incur by not allocating in the least cost position is the difference between the minimum and the next minimum. The minimum is 2 (6 – 4 = 2). As far as this is concerned, the penalty will be 0 because there are two positions with the same minimum cost. If I am not able to allocate here then there is a chance that I might still do it here so my penalty is 0.As far as this is concerned the penalty is 1, minimum is 5, and next minimum is 6 so penalty is 1. As far as this column is concerned I would ideally like to get everything from this because of this 4, but if I do not then I will incur a minimal penalty of 5 – 4 = 1, at least for every unit.
Similarly here the penalty is 7 – 6 which is = 1, 6 – 6 = 0 in this case and 7 – 8 any of these 8 – 7 is = 1.

Mathematically we can define as the penalty as the difference between the second smallest number and the smallest number. Physically the penalty this index represents is the minimum additional cost per unit that we will incur by not being able to allocate to the least cost position. If it is a supply and we are not getting it from the corresponding least cost, if it is a demand, we compute this entire penalty. We look at these penalties and find out the maximum penalty. The reason is now, this is more important compare to the rest of them because here by not allocating into the least cost I am incurring maximum penalty as far as all the penalties are concerned. So I will look at the row or column that has the maximum penalty which is this and what does this mean? Considering that, this is the most important row because the penalty is very high. When considering all the rows and all the columns the first decision will be made on this row because this row has the highest penalty and now in this row I will try to put as much as possible in the least cost position because by not putting in the least cost position i will incur this much penalty. The algorithm is compute the penalties and then find out that row or column that has maximum penalty and having identified that row or column try to put as much as you can in the least cost position. Therefore, I look at this. This is the list cost position. This supply of 40 and demand of 20, I put 20, when I do that I have exhausted this 20 this 40 becomes 20. I have to compute the penalties once again.

As far as this row is concerned, minimum now is = 6. Next minimum is either of this, 8 so the penalty is 2. As far as this row is concerned, minimum is 6 next minimum is 7 so penalty becomes 1. The minimum is 6 next, 7, penalty is 1. Column penalties will not change and they remain as they are. Now once again this has the maximum penalty therefore go to this row and try to put as much as you can in the minimum position. The minimum position is here. Now supply is 20, requirement is 30, so put 20. This gets exhausted, this becomes 10. Compute the penalties again for this row. The minimum is 6, next minimum is 7. This is 1, minimum is 6, and next minimum is 7. This is 1. In fact the row penalties will not change and now go back and look at the column penalties. This is minimum 7, next minimum is 8. This penalty is 1, 6 and 6 will give a penalty of 0, 8 and 7 will give a penalty of 1. The maximum penalty is 1 1 1 and 1 here. There is the tie for the maximum penalty and the tie can be broken arbitrarily.

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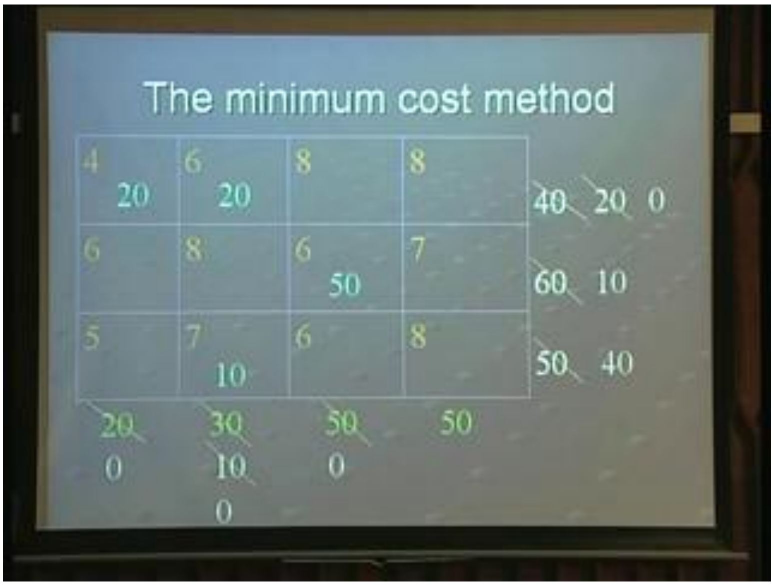

We try to look at this one and break the tie. So when we consider this column first for breaking the tie, the minimum position is this. We try to put the maximum so you get 10 here. This is exhausted. This goes and this becomes 40. Now once again, compute the penalties 7 – 6 is = 1, 8 – 6 = 2, 6 – 6 = 0, 8 – 7 = 1. Maximum penalty is here so, this is minimum position. We try to put the maximum here. This is 40. This goes. This is exhausted and this becomes 10. There is only one so 6 – 7, 1, so look at the minimum position that is the 10 and a 60 here. Put 10. There is remaining 50 and put this 50 here.

So you get this solution from the penalty cost method or the Vogel's approximation method. This also has 6 allocations like that of the minimum cost and let us finds out the cost associated with this. So (4 into 20 = 80) + (6 into 20 = 120) + (6 into 10 = 60) + (7 into 50 is = 350) + (7 into 10 is = 70) + (6 into 40 is = 240) so 200, 260, 310, 610 + 70 = 680 + 40 = 720 + 200 = 920. This penalty is cost also called Vogel's approximation method, VAM for short. This now gives us a basic feasible solution again with objective function value 920.

Now there are two aspects to it. Number 1 is this Vogel's approximation method is expected to perform slightly better than the minimum cost because not only does it do a minimum cost allocation after identifying the row or column but it also uses more information considering the penalties. It goes into one more level of detail in carefully choosing the rows or columns to be considered. Therefore it is expected to perform slightly better than the minimum cost method and it happened in this case where it gave a solution that is slightly better than that using the minimum cost method. Now there are two other dimensions. One is we have at one point of time, we had tie in 4 positions 1, 2, 3 and 4 and then we broke the tie arbitrarily by considering this. If for example we had considered some other position to break the tie then we may end up getting a slightly different solution for the Vogel's approximation method or for the minimum cost method as well even here we could get a tie when we look for the least cost entry we could get a tie.

Similarly with the Vogel's, the possibility of the tie is more because we are computing the penalties. Whenever we have a tie and we break the tie arbitrarily depending on the choice which is an arbitrary choice the solution will change the allocations will change as well as the cost would change. If we are lucky we could get something which is much lower so you are not really lucky then you would get something with the slightly higher value compared to this. For example we can go back and show one more solution as to what will happen when the tie is broken arbitrarily in the penalty cost method which we will do now. 4 6 8 8 6 8 6 7 and 5 7 6 8 Let us quickly compute the penalties. Penalty is 6 – 4 = 2; 6 – 6 = 0; 5 – 6 = 1; 5 – 4= 1; 7 – 6 = 1; 6 – 6 = 0; 7 – 8 =1. This has maximum penalty. Choose this row. This is the minimum cost position. 40 available, 20 required. Put 20. This goes, this becomes 20, penalty is 6 – 8 = 2; 7 – 6 = 1; 7 – 6= 1; 8 – 7 = 1; 6 – 6= 0 and 1.

Once again this is maximum penalty. Identify the minimum cost position. 20 supply, 30 requirements. 20 goes this becomes 10. Compute the penalties again 7 – 6 = 1; 7 – 6 = 1; 8 – 7 = 1; 6 – 6 = 0; 8 – 7 = 1. Now there is a tie between these four positions. Last time we took this one, this time; let us take this one so we break this tie arbitrarily. This is the least cost position 60 and 50. So I put 50 here so this goes this becomes 10. Once again you find out the penalties 8 – 7 = 1; 8 – 7 = 1; 8 – 7 = 1; 8 – 7 = 1. We could choose any one of these, for example if we choose this I get 50. If I choose this one, then this becomes the minimum cost position. So I get a supply of 50 requirement of 50 10, I put a 10 here. This goes and 50 becomes 40. We have only two positions. Our 10 and 40 so this 10 will go here + 40 will go here. Cost associated will be 20 into 4 = 80 + 120 + 300 + 70 + 70 + 320

So 200, 500, 570, 640 + 320 is 960. So we could have got the solution with 960. Different type of breaking rule can give a different solution. For example if we had got this solution then we realize, for this example it is given in the same solution as that of the minimum cost. There is absolutely no guarantee that all the time this will be lesser and so on. After all three are basically non-optimal they do not guarantee the best solution at all. They are approximate. Even though particularly the minimum cost and the penalty use cost information in getting the solution they are not optimal. They only provide us with a basic feasible solution. The reason is towards the end of the allocation.

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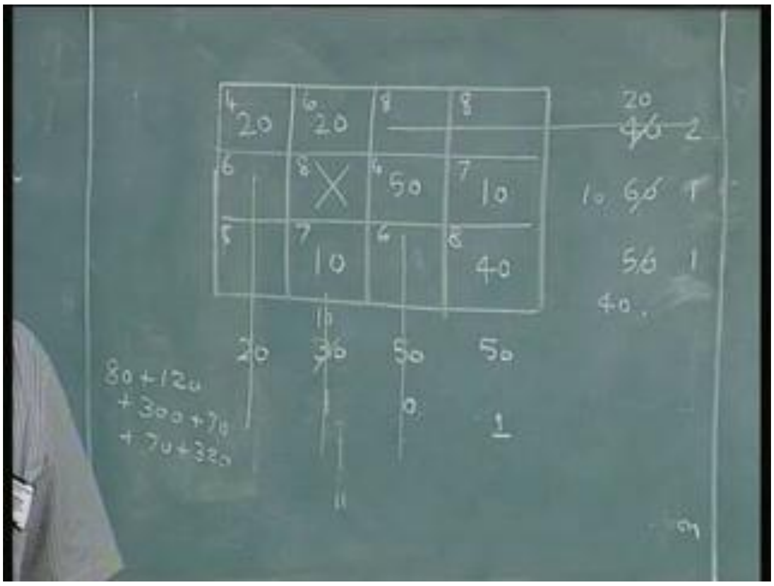

You would realize that you normally do not have a choice. For example when we went through the same calculation somewhere, For example when we had a supplier 60 and a requirement of 50 we put a 50 here and then we close this and we brought it here. Now the last two allocations, we do not have any control automatically a 10 comes here and 40 would come here. So towards the end of all the methods, we actually do not make a decision.
We identify the place to allocate at the very early stage of the allocation and towards the end we realize we actually do not have a choice and we are forced to put it in certain position and simply because of this they do not guarantee any optimality. Sometimes they can be very high. Sometimes, for example, if this had been 20 instead of 8 then this value would be very high.

We do not really have a choice with respect to the last allocations and therefore all the three methods are heuristic and non optimal in nature. They only provide a starting basic feasible solution to the transportation problem. We have to keep it in mind that all the three do not provide the optimal solutions. All these are not optimal solutions to the transportation problem. They only provide us with a starting basic feasible solution which is a good solution. For example we would be happy to start with the solution with the cost 920 and optimize the cost further rather than starting with the solution which has a much higher cost than 920.

We have seen three methods to get the starting basic feasible solution to the transportation problem and we saw we had 5 allocations here. We had 6 allocations here. In this solution we had 6 allocations but once again in here we do have 6 allocations in this. Let us try to understand the basic feasible solution to the transportation problem. All these are feasible solutions. But let us try to understand what the basic feasible solution to the transportation problem is. Next and then we will try to solve the problem optimally using any one of these three as the optimal solution and then also see how to solve the problem for any given starting solutions.

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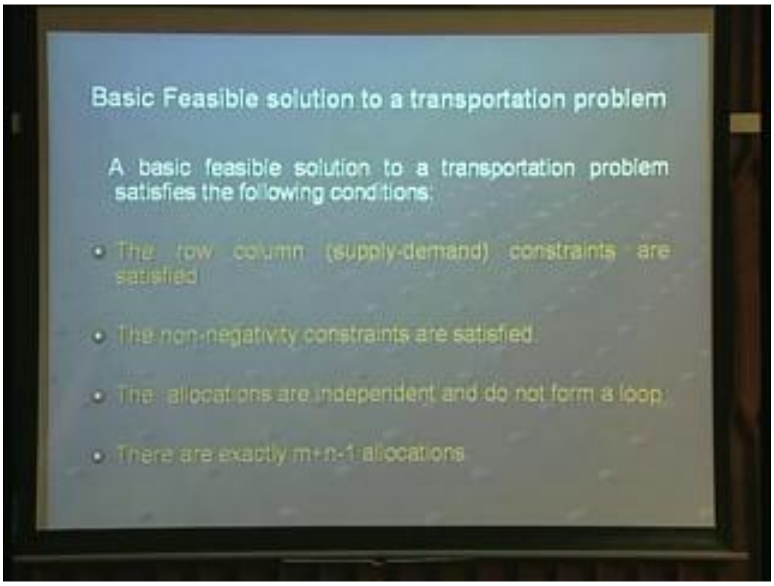

Now what is the basic feasible solution to the transportation problem? A basic feasible solution satisfies the following conditions:

1. The row and column constraints are satisfied.
2. The non negativity is satisfied.
3. The allocations are independent and do not form a loop.

We have to see what this is and there are exactly m + n – 1 allocations. All the 3, if we go back and observe all the 3 satisfied the first 2 out of the 4 conditions. We have to see whether all of them have a loop or do not have a loop. They have certainly different allocations. For example this had 5; the rest of them had 6. What is m + n – 1, there are m supply points there are n demand points. There are 3 supply points and 4 demand points m + n – 1 is 6. All these had 6 allocations this had less than that 5. In some, since all of them are satisfied m + n – 1, this did not but these two. None of them exceeded m – 1. They had m + n – 1 or less. Let us look at the third condition and first understand what this loop is and then go back and see whether this satisfies the loop condition or does not satisfy the loop condition.
For example if this satisfies the loop condition then this is basic feasible. Right now this is feasible because it satisfies all the supply - demand conditions. It has satisfied a non negativity condition. It has exactly m + n – 1 allocations.

The only thing we have to check is whether it satisfies the third condition. So let us first understand what this loop is. To explain the loop, let us consider the problem that is shown here.

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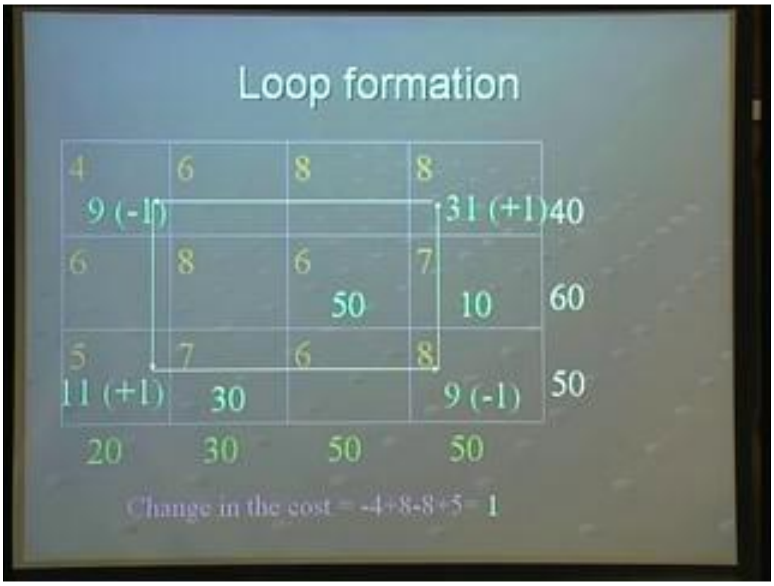

We show we have the same problem with 40, 60 and 50 supply points that we have here, same requirements 20, 30, 50 and 50 that are shown here; same cost structure but we show different allocations. So we now have a problem with 7 allocations. 10 here 30, 10, 50, 10, 30 and 10

Now let us look at this 7. Now these also satisfy the supply demand requirement. They also satisfy the non negativity requirement but then we observe something additional in this. If we start with the top corner 10 and then we observe something like this. Starting from this 10 top corner we can draw this line which that goes to this 10 and then we move horizontally to get into this 10. We move vertically to go to this 30 and then we move horizontally to come back to this step. This is called a loop. So given a solution starting from one of the allocations if we are able to move horizontally and vertically across allocations and we are able to come back to the starting point then this represents a loop and loop is some kind of dependence in the solution which shows that these 7 are not independent allocations. There is a dependency here and that dependency is given by this loop.

So the solution has a loop then we should break the loop and make it independent. What we do here is this, we try from this 10, we try to put a, – 1 here and see, we try to bring down the 10 to 9 by looking at a by putting a – 1. Now this minus would mean that we have to satisfy this 20 so this 10 becomes 11 and because we have added 1 here this 10 would become 9. We end up subtracting and because we have subtracted here, this 30 would become 30 –1 and we added it and so the net is 0. In a loop we should get net equal to 0. What is the effect of this change? Reducing one of the elements of the loop by 1 would now reduce the cost by 4 here which is shown as a, – 4 here. Cost increases by 8 here which is shown as a + 8. Cost reduces, – 1 will reduce the cost here which is – 8 and cost increases here, + 1 increase the cost by 5.

The net change in cost is equal to + 1 so the effect of reducing this 10 to 9 would actually end up increasing the cost by 1.So now what we need to do is if we had increased this 10 that was originally here to 11 and while the same loop, this 11 would have brought to 29. This to 11 and this to 9 then we would have a net change of – 1.

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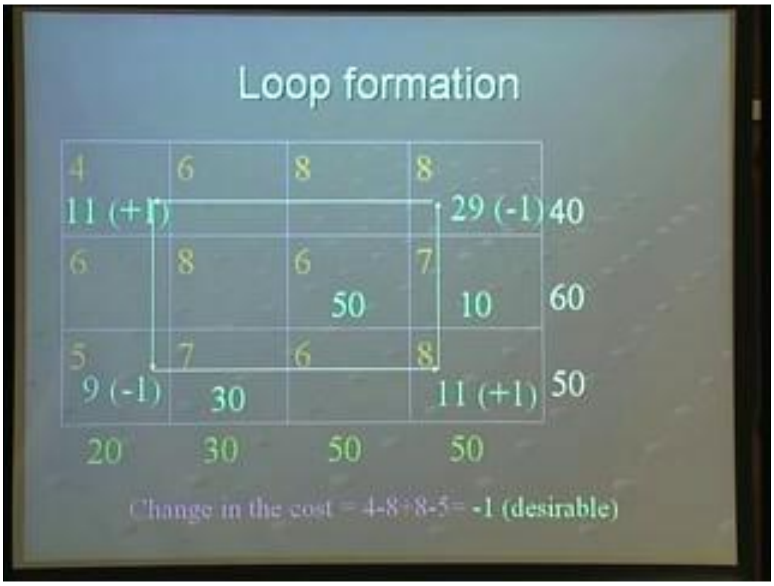

For example if we have done this then we would have a net change – 1. Now this is desirable because reduction in the cost associated with this. Being and being a minimization problem we would consider this to be the most desirable.

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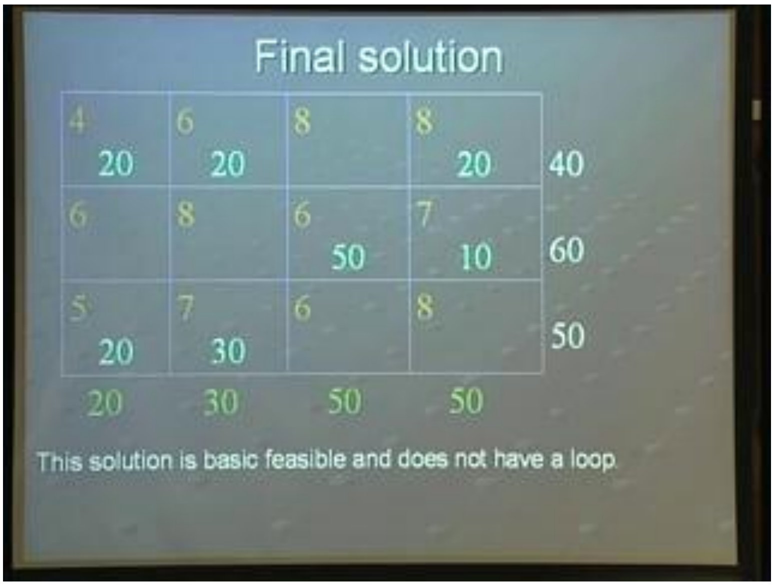

Every loop can be broken in two ways. One of which would give us a positive net increase and other would give us a negative net increase. This is desirable so we would consider this. It is now desirable to break the loop by adding to this position and not by subtracting to this position. Now we realize that every addition here would reduce the value by 1 so we want to add as much as we can here and that is given by this.

For example the maximum we keep on adding here, we make this as twelve and this will become 8, 13 would make this 7 and so on till this becomes 20 and the other one becomes 0 . So till this would be made 20 and this would be made 0. If we make this 20 then this would change to 20. This would change and this would become 0 and that is shown in the next table. A 20, 20, 20, 10, 50, 30, this 20 will become 0 and this is not 20. This would become 0 because of that change. We have 1, 2, 3, 4, 5 and 6 positions excluding this. We have 6 positions and we have exactly m + n – 1 allocations.

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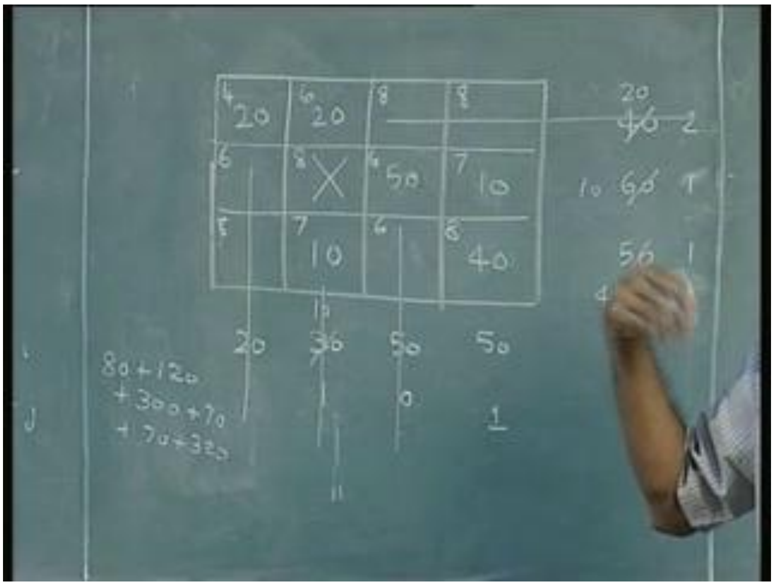

So breaking the loop should give us exactly m + n – 1 allocation. Let us move to another example.

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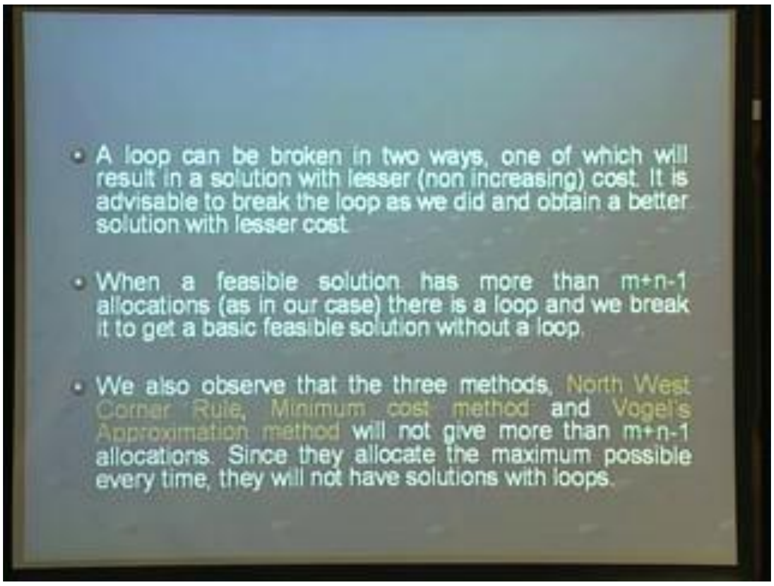

A loop can be broken in two ways. One of which will result in a solution with less cost. It is advisable to break the loop as we did and obtain a better solution with a lesser cost. When a feasible solution has more than m + n – 1 allocation as we had here, it is 7, and then it automatically tells us that that there is a loop somewhere and that loop has to be broken. When we have more than m + n – 1 allocation we can be confident that there exists a loop.

The solution is not basically feasible they have to break the loop and then make it basic feasible. We also observed that the three methods North West Corner rule, Minimum cost and Vogel's will not give more than m + n – 1.

The reason being whenever we made an allocation we allocated the maximum possible which is the minimum of the supply, the net supply and the net requirement by allocating the maximum possible which is the minimum of the 2. We make sure that we will not have a loop. Therefore these 3 methods are acceptable. These 3 methods are desirable simply because they don't give us solution with the loop. They give basic feasible solutions. They do not give more than m + n – 1. Since they allocate maximum possible every time, they will not have the solutions with loops.

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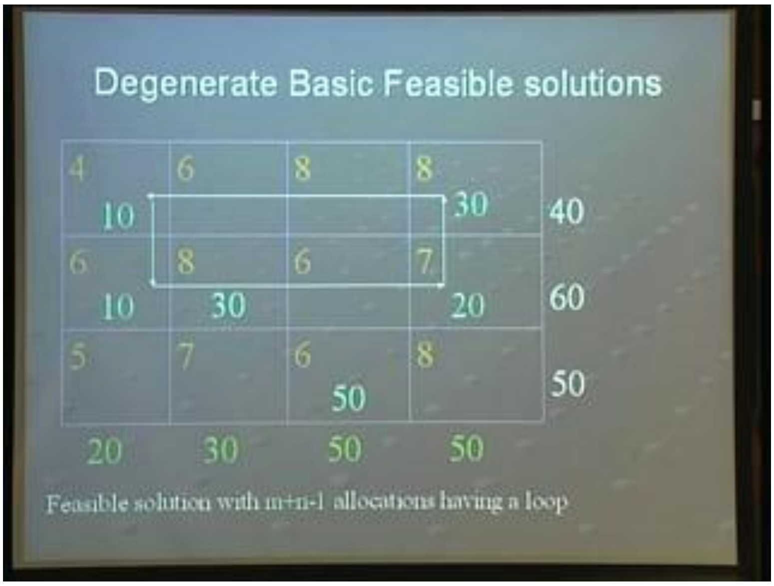

Now we need to look at one more aspect which is given by this example, once again for the same supply and requirement of 40, 60 and 50, 20, 30, 50 and 50 and the same cost structure that we have. We provide another solution here which is exactly 6 allocations m + n – 1 is 6. We provide a solution with exactly 6 allocations. The row and column constraints are satisfied 10 + 30 is 40. 10 + 30 + 20 is 60. 50 comes here, column additions are also satisfied. There are exactly 6 allocations but if we look at these allocations very carefully we realize that there is a loop. So on one hand when we know that if we have more than m + n – 1, there is definitely a loop. When we have exactly m + n – 1, it does not guarantee that we do not have a loop. We could have a hidden loop as we see in this. So when we have a solution that is obtained by any method other than this 3 or any method that does not guarantee a loop then we do not have to look at loops. For example if we had used any of these 3 or any other method which would guarantee a non loop solution then if we have a solution with m + n – 1 we can be confident that it does not have a loop.

We do not know how we got this solution but this solution is feasible. It satisfies the row column and it has exactly m + n – 1 but we need to check whether this has a loop because to know whether this was obtained using any of these 3 or a method which does not guarantee a loop. We verify and we realize that there is loop here. This 10, 30, 20, 10 is another loop so we follow the same procedure as we did previously. We had a, + 1 and 1 position.

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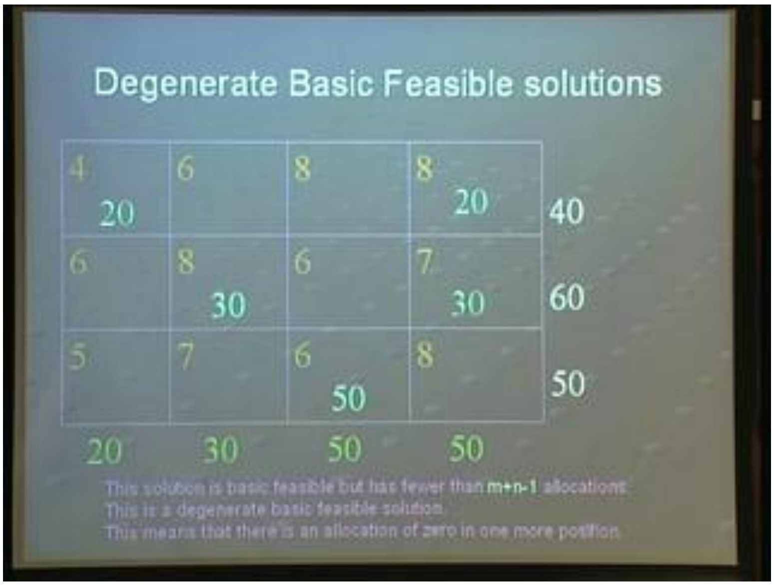

We break the loop finally after we put the maximum, after we identify the desirable, we allocate the maximum we break the loop and we get now 5 allocations here. Now these 5 allocations are independent. They do not form a loop but this is basic feasible because this does not have a loop. But it has fewer than m + n – 1 allocations. m + n – 1 is 6 but it has 5. This is very similar to the North West Corner solution which also did not have a loop but had 5 allocations as against the m + n – 1 = 6. When we have a basic feasible solution with no loop and with less than m + n – 1 then it is a degenerate basic feasible solution. This basic feasible solution is degenerate because there is a hidden 6 allocation which is not shown here which has a 0. This is the case where allocation has a 0 and allocation corresponds to a basic variable and it has 0 so this is a degenerate basic feasible solution.

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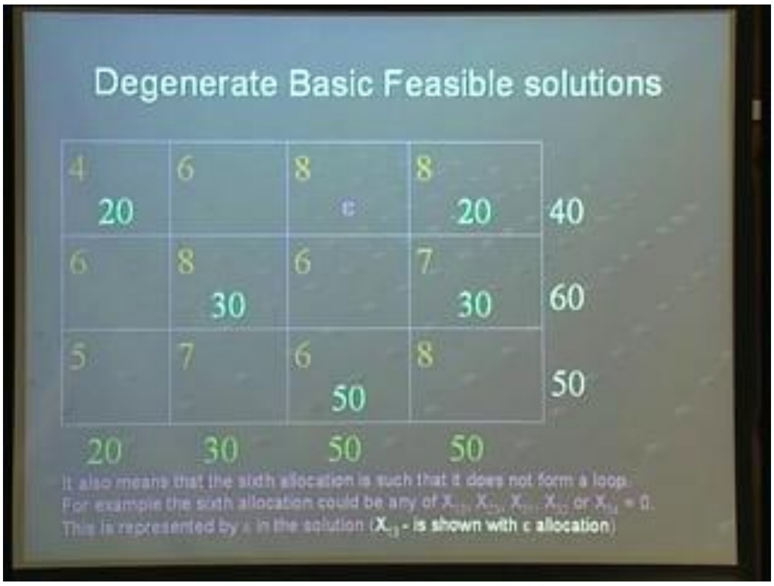

It also means that 6th allocation should be such that it should not form a loop.

It is very important that we have to identify the 6th position and we have to make sure that the 6th position does not create a loop because we do not want a loop.
For example we cannot identify this position as the 6th because this would form a loop. We wouldn't want to do that. We can identify any position. We could identify $X_{1\,3}$ or $X_{2\,3}$ or $X_{3\,1}$ or $X_{3\,2}$ or $X_{3\,4} = 0$. We should not look at 2 1. 2 1 would give a loop so we could put an epsilon. A small epsilon indicates that it degenerates a basic feasible solution and this epsilon can be put in any position such that it does not form a loop. Every degenerate basic feasible solution will have an epsilon and that epsilon is this 6th or the m + n – 1 allocation in this case.

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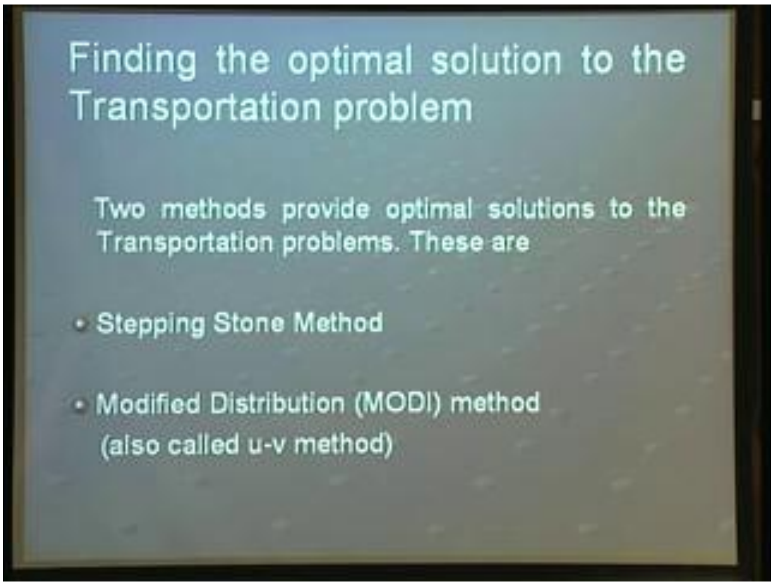

Now this kind brings us to a discussion on getting a basic feasible solution to the transportation problem. What have we seen in the initial? In a nutshell, we have seen 3 methods to get the starting solution. All these 3 starting solutions are basic feasible they are basic feasible because they satisfy the supply demand requirements. They have all X_{ij} as greater than or equal to 0. They don't have a loop. These 3 will not have a loop because of the way the allocations are made. Whenever we allocate it we try to put the maximum which is the minimum of the supply demand and because we put the maximum in that position there will not be any loop with solutions of these 3.

In order to get basic feasible solution we have to guarantee that the rule or the method that we use to get our basic feasible solution should not give us a loop as it happened in these 3.
A basic feasible solution is also characterized by m + n – 1 independent allocations.
Sometimes we could get a basic feasible solution out of these three with less than m + n – 1. Now that means we have a degenerate solution. A degenerate basic feasible solution has fewer than m + n – 1. It satisfies all the supply demand conditions. It does not have a loop.
Whenever we have a degenerate basic feasible solution it is necessary for us to introduce an epsilon in such a way that it does not form a loop. These 3 are commonly used methods to get the starting solutions, commonly used methods to get the basic feasible solutions and they do not have solutions with loop and they are desirable. It is not always necessary to use only these 3.

We could for example use arbitrary starting solutions. Arbitrary solutions created using different algorithms and different methods and we could use them also as starting solutions but the only thing we have to ensure is that any solution that we use as a starting solution to the transportation problem is basic feasible. If the starting solution is obtained using any of these 3 and it is very customary to use the Vogel's approximation method to begin or the minimum cost method. It is very customary to use one of these then we are sure that we do not have a loop so we will have basic feasible solutions which will have exactly m + n – 1 or may be degenerative.

If the solution is obtained by any other method other than these 3 or by any arbitrary method then the first thing we need to do is to check the number of allocations.
If the number of allocation exceeds m + n – 1 then there is definitely a loop and that loop has to be broken and even if the total allocation is exactly m + n – 1, we still need to check whether there is a hidden loop and by breaking that loop we will end up with lesser than m + n – 1 and a degenerate.

So the important catch is to be able to get a basic feasible solution to the transportation problem which is preferably a non degenerate. These methods are commonly used to get those basic feasible solutions. Sometimes we may also get the degenerate solutions out of this. If we get a non degenerate basic feasible solution with exactly m + n – 1 allocation, it is extremely desirable. If we get a degenerate a basic feasible solution with less than m + n – 1, then we need to add an epsilon suitably to make it basic feasible and start. The first part of solving the transportation problem is to try and get a starting basic feasible solution preferably using the minimum cost or the Vogel's approximation.
The advantage of using these methods are that, they provide basic feasible solutions. They do not give loop. They may give a degenerate basic feasible solution but if we get one we have accept it.

If we are using any methods other than these, any method on which we are not confident of its ability to give a non loop solution, then we need to verify whether the given solution has a loop or not. If it has a loop then that loop has to be broken to get a basic feasible solution.
A basic feasible solution to the transportation problem is characterized by satisfying the supply demand restrictions, satisfying the non negativity restrictions, having m + n – 1 independent allocations and not having a loop. So we know that these 2 methods would give us a solution which satisfies all these basic feasible.
Next thing we need to do is to try and get to the optimal solution from these either using the stepping stone method or the modified distribution method and we will see that in the next lecture.