Engineering Fracture Mechanics

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Module No. # 01
Lecture No. # 05
Fracture Mechanics is Holistic

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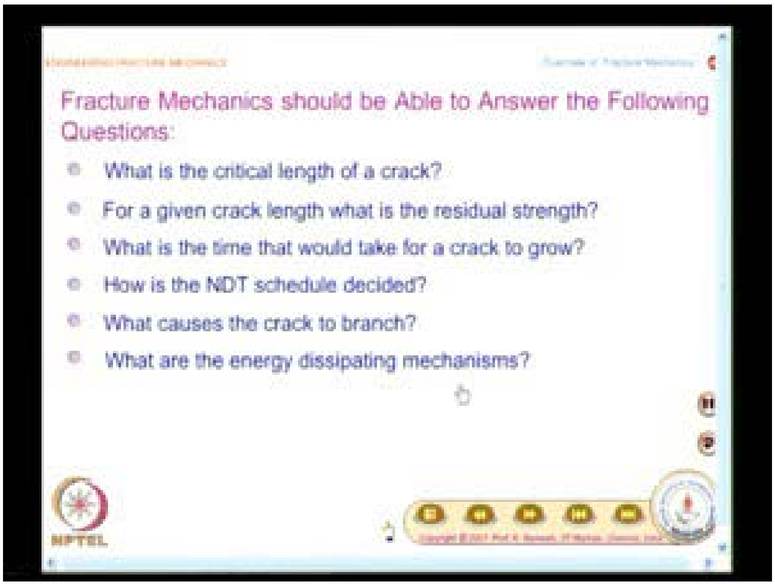

Fracture Mechanics should be Able to Answer the Following Questions:

- What is the critical length of a crack?
- For a given crack length what is the residual strength?
- What is the time that would take for a crack to grow?
- How is the NDT schedule decided?
- What causes the crack to branch?
- What are the energy dissipating mechanisms?

In the last class, we had raised a few questions that fracture mechanics should be able to answer; these were: what is a critical length of a crack? For a given crack length, what is the residual strength? What is the time that would take for a crack to grow? How is the NDT schedule decided? For all these four questions, you cannot find an answer by performing only a tension test or a fatigue test. You need to have an additional test to find answers for these questions.

The last two questions were: what causes the crack to branch? What are the energy dissipating mechanisms? These two are very fundamental in nature. When we develop the concepts related to energy release rate, we would be able to appreciate how these questions can be answered.

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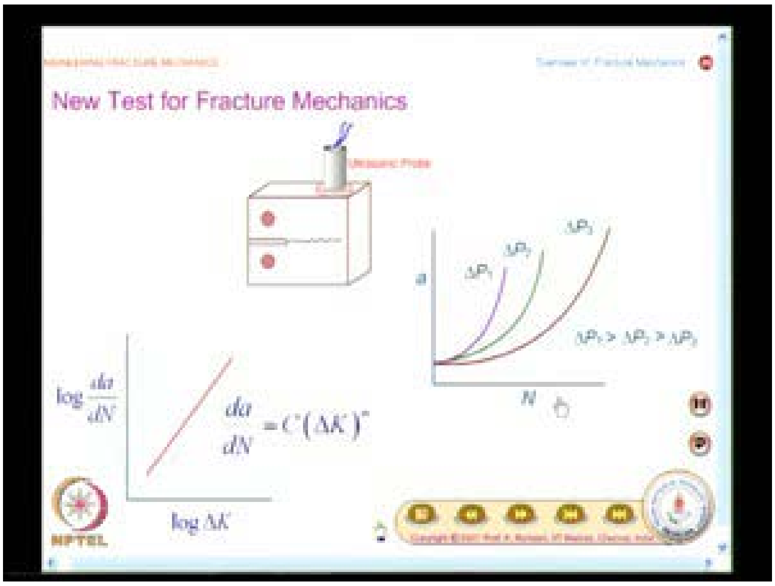

New Test for Fracture Mechanics

$$\frac{da}{dN} = C(\Delta K)^m$$

So, what we will now look at is, if I have to know, what is the critical crack length, how long the crack will take to grow, I need to have a new experiment for fracture mechanics. And this is what is schematically shown. Here you have a specimen, which is subjected to fatigue loading. Try to make a neat sketch of this. Because of the fatigue loading, you find the crack grows in service and you also monitor the growth of the crack by a suitable probe. This is only a schematic. There could be many methods by which they could monitor the growth of crack. And you essentially cull out the data of crack length versus number of cycles. And, you do this for various amplitudes of the load applied delta P 1, delta P 2 and delta P 3.

In general, what we will have is, when you change the amplitude of the loading, these graphs also will differ. And you start with the initial crack. And these are called as crack growth curves. So, if I have to answer questions related to crack growth, I need to collect data; I need to do a series of experiments and experiments is costly. So, if I have to adopt a fracture mechanics methodology, the whole program becomes expensive. Unless you conduct additional tests, you will not be in a position to answer the questions that we have raised. If you recall, if you look at the fatigue test, the test was conducted to record after how many cycles the specimen has failed; it has not really monitored the crack growth.

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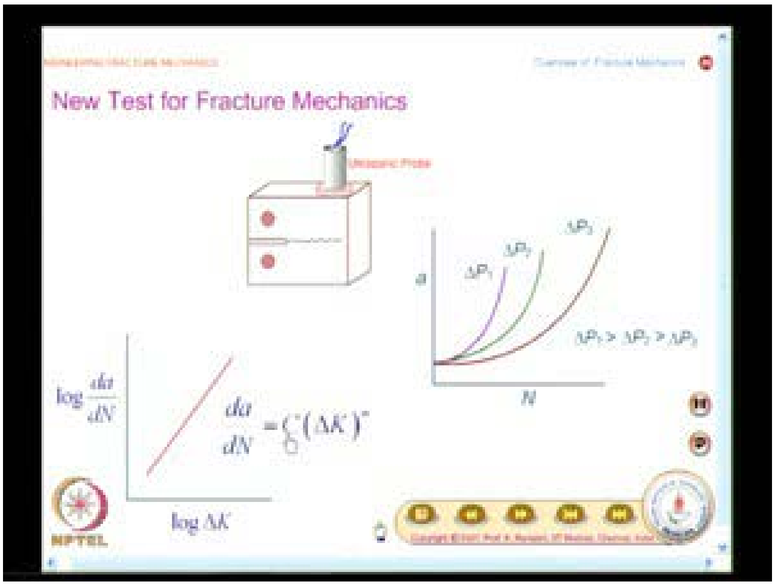

New Test for Fracture Mechanics

$$\frac{da}{dN} = C(\Delta K)^m$$

Another important aspect is, for one material, if I have number of such graphs, it would not be convenient from a design point of view. This data was analyzed and reduced very intelligently by Paris by re-plotting whatever the data collected, by a different graph with log delta K in the x axis and log of da by d N in the y axis. And you get a straight line, and this is known as same as Paris law. We would see in detail during the later part of the course. And what you have here as delta K, is the change in the stress intensity factor. And what you have as c and m are material constants.

So, you have to perform this kind of a test; on the basis of the test find out c and m. If you really have c and m, it is possible for you to find out what way the crack will grow. So, that is the success of fracture mechanics. You are able to predict how the crack will grow and when does it become critical. That is possible only when you do additional test.

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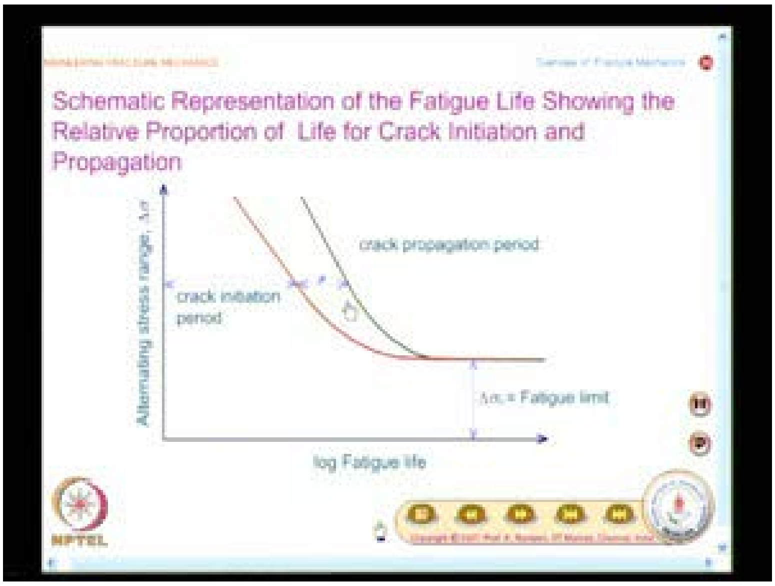

Schematic Representation of the Fatigue Life Showing the Relative Proportion of Life for Crack Initiation and Propagation

You may think, now I have fracture mechanics. So, I know how to monitor the crack growth; you should not be relaxed on that; you should find out, during the service life, what is the component of life before crack initiation and what is the component of life when the crack grows. If you really look at the subtle aspect like this, you will find the crack initiation period is generally much longer than the crack propagation period. This is again your SN diagram re-plotted with different axis. I have log of fatigue life here; here I directly put the alternating stress. That is why this is a non-linear curve rather than a straight line. And what does this give? For a given alternating value of stress, the crack initiation period is much longer than the crack propagation period.

So, you really take an advantage, if you delay the crack initiation phase. And this is what metallurgists focus upon. They find out methodologies, how the crack initiation can be delayed. In fact, if you have blunting of the crack-tip, it would delay the crack growth. In general, you may say plasticity is bad. From fracture point of view, the local plastic deformation is beneficial. The local plastic deformation will make the crack blunt. All these issues will take it up later, but you will have to have a mental picture, though you have a model to predict crack growth, it is only secondary.

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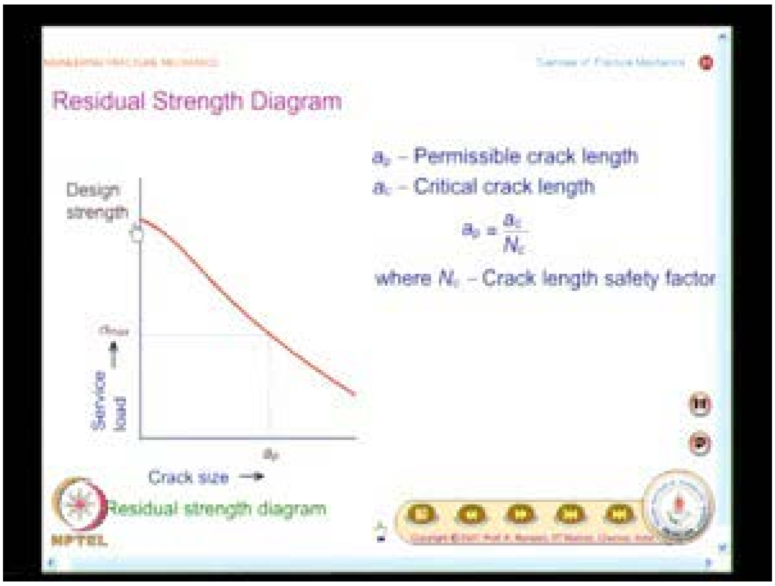

Residual Strength Diagram

a_p – Permissible crack length
a_c – Critical crack length

$$a_p = \frac{a_c}{N_c}$$

where N_c – Crack length safety factor

Residual strength diagram

If you are able to address the issue of crack initiation, then you are much better off. And if you look at this is also a function of the operating alternating stress. So, there could be a point where crack initiation period and crack propagation period may be 55. So, it depends on the given application. So, you have to work upon crack initiation as well as crack propagation. Working only on crack propagation will not help you for your design scenario. And we have already said, that we need to find out, for a given crack length, what is the residual strength. And this is again shown schematically. You have on the x axis crack size is plotted.

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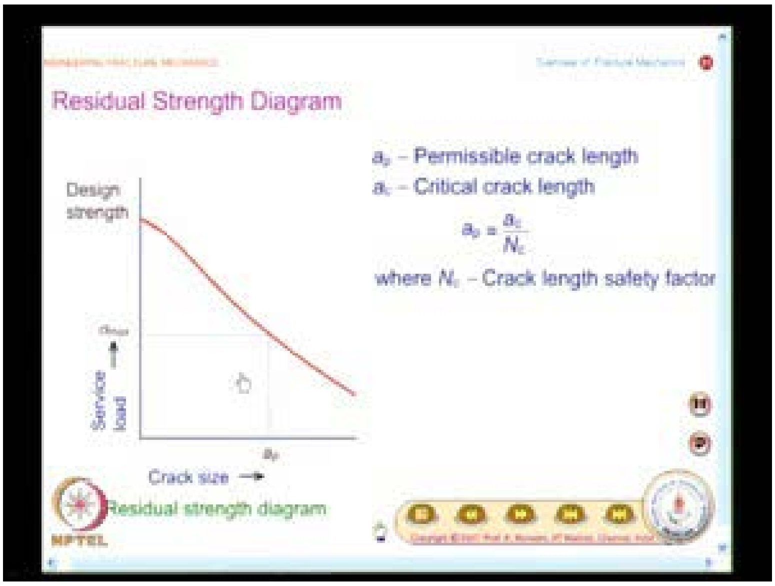

Residual Strength Diagram

a_p – Permissible crack length

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where N_c – Crack length safety factor

So, when you have no crack or crack length is 0, you have the conventional design strength. So, obviously, when you have a crack you cannot operate with that strength. In fact, if you look at some of the old bridges, they would have discarded it; they would say that its permissible to allow only two wheelers; they will not allow heavy truck to go on that bridge.

So, that means, you have reduced the load that would be taken by the structure. I need to have a scientific methodology. And this is what, one of the important results that we want to get from fracture mechanics. So, you need to get what are known as residual strength diagrams. So, this shows, for different crack lengths, how the service load should be limited. And you also have a concept, which is again an extension of what you have been doing it in strength of materials. Though I have what is known as critical crack length, we do not want to operate or go up to a critical crack length. We want to define what is known as a permissible crack length. And this is dictated by a factor that is similar to our safety factor. You call this as N c and you name it as crack length safety factor. So, using this, what you want to do is, you want to find out what is the maximum stress that you can go, and what is the corresponding permissible crack length.

So, you borrow the ideas from your conventional design methodology. You have a factor of safety; you do not operate at the calculated the failure stress; you operate much below that by bringing in a factor of safety. Similarly, by a fracture mechanics calculation you find out what is a critical crack length, and in order to take care of uncertainties, do not operate that crack length, but operate at a crack length below that. How below, depends on what kind of a confidence level you have in your analysis. And the other question that we would like to know is, what is the time available for inspecting the crack? We will have a look at it.

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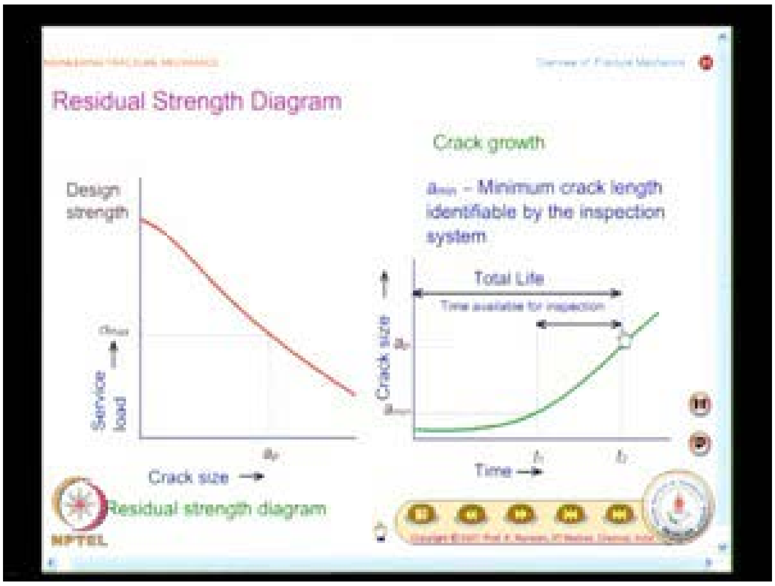

Residual Strength Diagram

Crack growth

a_{min} – Minimum crack length identifiable by the inspection system

Residual strength diagram

So, we want to look at the crack growth. So, you have a crack size versus time. And we start with a premise that, you will have a minimum crack that would be detectable by your nondestructive testing methodology. And we have already noted that, there is something called permissible crack length. And you have the crack growth curve. On the crack growth curve you can find out, when does the crack grows to the length a p.

So, that would give you, what is the time available for inspection. Only on this basis, you would be in a position to decide on the NDT schedule. In fact, if you look at, in conventional design also, when you look at railway stations, when the train stops, there would be someone sitting below and then hitting the bottom of the bogies at appropriate places. He is actually doing a vibration test to find out whether there are any structural damage. And in fact, for aircrafts, when they land, every time they land, they do certain minimal inspection. And aircrafts go through exhaustive structural monitoring during its service life.

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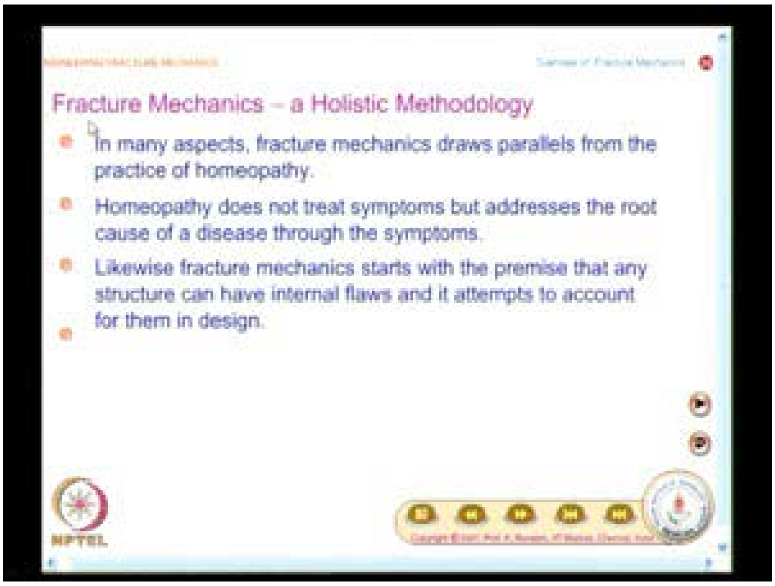

Fracture Mechanics – a Holistic Methodology

- In many aspects, fracture mechanics draws parallels from the practice of homeopathy.
- Homeopathy does not treat symptoms but addresses the root cause of a disease through the symptoms.
- Likewise fracture mechanics starts with the premise that any structure can have internal flaws and it attempts to account for them in design.

So, you need to know, how often you should see and what you should see, in each of those inspections. You may have a preliminary inspection; you may have an exhausting inspection in between. How do you go about collecting the information to decide on these issues? That you get from a graph like this. And you know, in a lighter vein, I would like to introduce the important aspect that fracture mechanics is holistic. You know, now holistic health is a buzzword in your fitness consciousness circles. You know, people want to be healthy. At the drop of a hat, you go and take a medicine. People want to remain healthy. There are a lot of alternative medicines have been focused upon. And some kind of an analogy exists, between fracture mechanics and homeopathy. If you look at, homeopathy does not treat symptoms, but addresses the root cause of a disease through the symptoms. See, if you go to allopathy, if you are vomiting, they would give you Domstal and arrest your sensation for vomiting. That is the way they look at, look at it.

They do not look at what causes vomiting, because the body wants to throw certain stuff from your system. You have to assess whether it is healthy or not. In fact, if somebody gulps a poison, one of the first aid is, you have to make him vomit, so that, whatever that the poison that has gone into the system, that should be expelled as quickly as possible. In some cases, there are medicines which would stimulate vomiting. In order to take that medicine, they suppress your sensation for vomiting. So, this is how they handle it in allopathy. Not only that, when you have a symptom, whether it is for individual a, individual b, or individual c, they may change the strength of the medicine, but the medicine will be more or less the same; that means, they are operating on symptoms. It is morely symptomatic rather than finding out the root cause. Liberty ships were made with the kind of design practice they had at that time. It was designed, tested, then only put into service, but it broke like glass. What was ignored?

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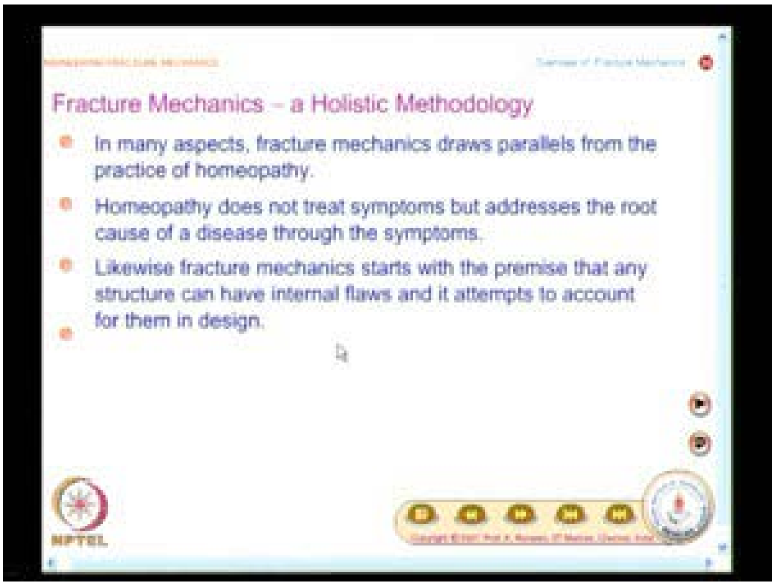

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They ignored the presence of inherent flaws. So, that has to be looked at. So, in fracture mechanics, we do that. It starts with the premise, that any structure can have internal flaws and it attempts to account for them in design. If you do not account for this at the design phase, whatever the cosmetic steps that you take, will not yield a permanent solution. You will have surprises. You may have solved one problem, but that may lead to some other problem.

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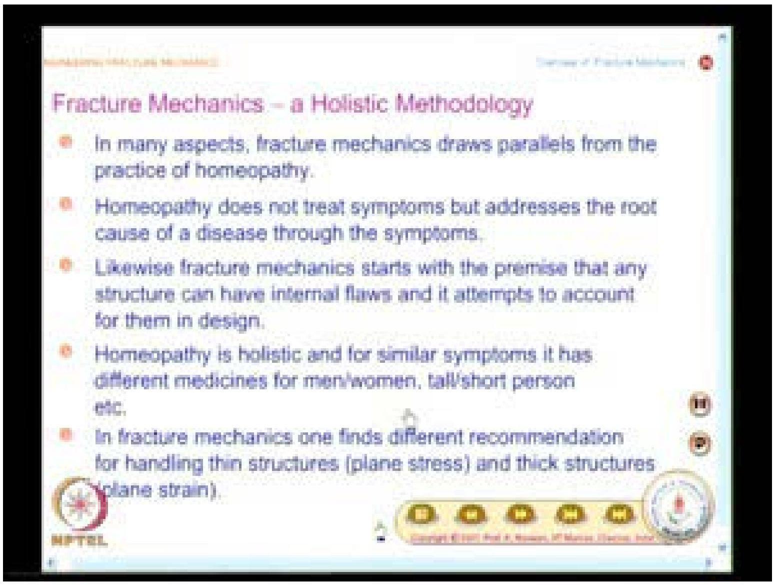

Fracture Mechanics – a Holistic Methodology

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- Homeopathy does not treat symptoms but addresses the root cause of a disease through the symptoms.
- Likewise fracture mechanics starts with the premise that any structure can have internal flaws and it attempts to account for them in design.
- Homeopathy is holistic and for similar symptoms it has different medicines for men/women, tall/short person etc.
- In fracture mechanics one finds different recommendation for handling thin structures (plane stress) and thick structures (plane strain).

So, you have to look at what is the root cause and try to address the root cause. And what is the other important aspect? See, if you look at a homeopathic doctor, for the same or similar symptoms, they will give different medicines for men, women, tall person, short person. There are so many subtle classifications; the medicine is individualistic. Something very similar to that happens in fracture mechanics also. You have recommendation separately for thick specimens and for thin specimens. So, you have thin structures which are under plane stress; you have a different recommendation. And for thick structures which are in plane strain, you have a different recommendation.

Say, suppose I want to have a underwater pipeline below the sea. You have not done a course in fracture mechanics; you are trained in conventional design. And you have a discussion in your design team and the design team member suggests, we should go for as high a wall thickness of the pipe should be possible, we should go for such a design. And the design team would also accept, yes, because it is a very critical location and any damage would be difficult for you to even repair.

So, it should be sturdy and heavy. If you have a very thick one, it will behave like a plane strain situation. And if you have done a course in fracture mechanics, which is what you are going to learn now, a fracture mechanics person will come and say, do not go for a very thick wall; go for a reasonable thickness, so that, you stimulate a plane stress condition rather than a plane strain condition. And in plane stress condition, you would see later, a fracture toughness is much higher than a plane strain. See, whenever the so called commonsense fails, science has to come to your rescue.

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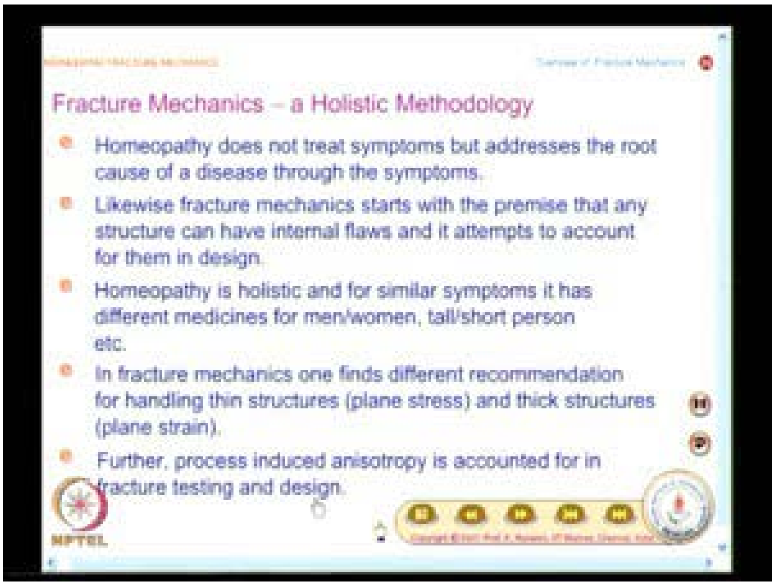

Fracture Mechanics – a Holistic Methodology

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- Homeopathy is holistic and for similar symptoms it has different medicines for men/women, tall/short person etc.
- In fracture mechanics one finds different recommendation for handling thin structures (plane stress) and thick structures (plane strain).
- Further, process induced anisotropy is accounted for in fracture testing and design.

So, the recommendations are different for thick specimens and thin specimens in fracture mechanics. Not only this, you have several manufacturing processes. Suppose you do an extrusion, it induces an anisotropy, because of the process. And this is accounted for, in fracture testing and design. See, this is not commonly seen in you, your conventional design. To some extent, they understand if you go from a bulk material to wires. The strength is very high, because that is how you do the spring design. If you really look at the design codes, the failure strength will be much higher when you use the same material as a wire; that is accounted for in spring design.

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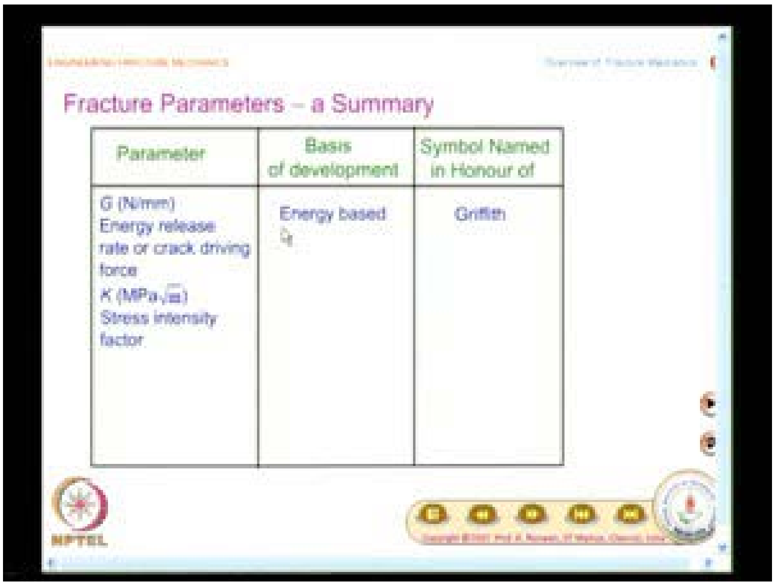

Fracture Parameters – a Summary

Parameter	Basis of development	Symbol Named in Honour of
G (N/mm) Energy release rate or crack driving force	Energy based	Griffith
K (MPa$\sqrt{m}$) Stress intensity factor		

But general conventional approaches, they do not bring in process induced to anisotropy into the testing itself. Whereas, in the case of fracture mechanics, when we take up a chapter on fracture toughness testing, we would take out the specimen appropriately to account for process induced anisotropy in our design calculations. So, this is another specialty when you are looking at fracture mechanics. You have several parameters in fracture mechanics and one of the earliest parameters that was used was energy release rate. You should also look at the units attached to this. This is labeled with a symbol G in honor of Griffith. This is known as energy release rate or crack driving force. And this is developed based on energy approach.

You know, the energy release rate is a very, very important concept. See, people were struggling to understand the result of ingles solution of an elliptical hole. because what it says is, when it becomes crack, the stresses become very, very large; infinite is what you get analytically. But it will not become infinite in a real sense scenario; the material, immediate vicinity of the crack line would undergo plastic deformation. And it was Griffith’s approach and ingenuity provided a solution which explains what we see in actual practice; because his observation was, if you have a crack, the crack will not grow by itself, but you need a, a force to drive the crack. There is inherent resistant associated with the material; the crack becomes critical, then only fracture occurs.

In fact, we would see a few examples, where you see a crack growth followed by fracture. It is very important conceptually. See, in this course, I focus on the concepts behind fracture mechanics. In the initial phases it may appear to be descriptive, which is also needed. Later we will take up mathematics and mathematics is quite complex in this course.

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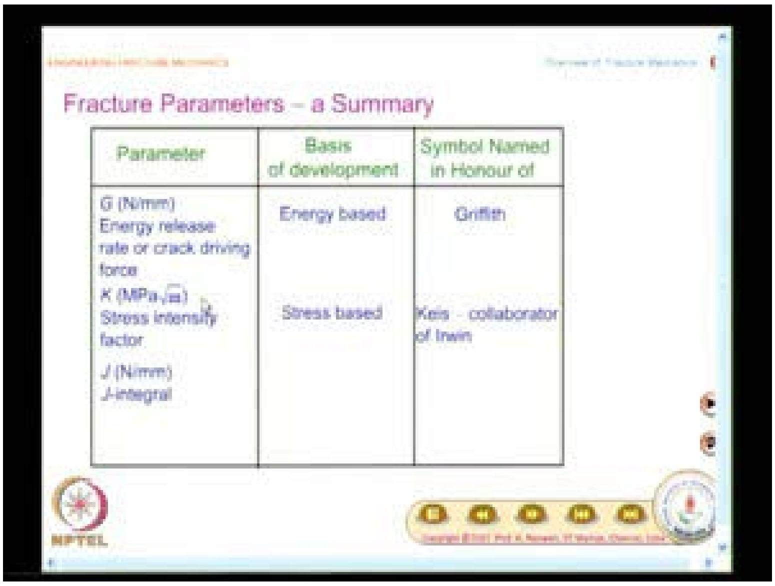

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G (N/mm) Energy release rate or crack driving force	Energy based	Griffith
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J (N/mm) J-integral		

We would see mathematics as well, but you should also appreciate some of the conceptual development. And fracture mechanics was made useful to society, and people realized that people could find out the useful parameter that was initiated by Irwin, who coined a parameter called K, which is labeled as stress intensity factor. A symbol K was given in honor of one of his collaborator Keis. And if you look at the development, the concept of K was based on stress analysis. And you should also note down that K has a unit of MP a root meter. It is a very very funny unit. You know, particularly, when you have to do conversion of units from fps to SI system, you will have to be very careful. The root meter is going to be a nuisance. You have to keep that in mind. And before fracture mechanics, you had only the concept of stress concentration factor.

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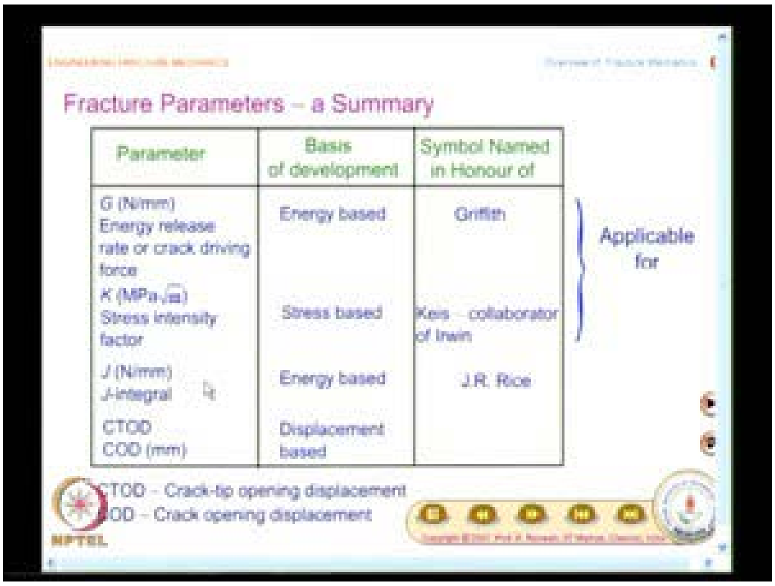

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Parameter	Basis of development	Symbol Named in Honour of
G (N/mm) Energy release rate or crack driving force	Energy based	Griffith
K ($MPa\sqrt{m}$) Stress intensity factor	Stress based	Keis – collaborator of Irwin
J (N/mm) J-integral	Energy based	J.R. Rice
CTOD COD (mm)	Displacement based	

Applicable for

CTOD – Crack-tip opening displacement
OD – Crack opening displacement

The contribution of ingles' was, when you have a crack, simply saying it has stress concentration no longer valid; you need to have new approaches. If you look at stress concentration factor, it had no units. It is only a number. On the other hand, to model a crack, you have a parameter called stress intensity factor, which has a very funny unit MP a root meter. Then you have another concept, which was introduced by Rice, which is known as J integral. This is again energy based and this is in honor of Rice.

So, you had this as, labeled as J. It has units of Newton per millimeter. And in fact, for non-linear fracture mechanics as well as elasto-plastic fracture mechanics, J was a very convenient parameter to model the stress field. And parallelly, you know in UK, people developed another concept, which is known as COD, crack opening displacement. It is in terms of millimeters. This is displacement based; this was developed by Wells. And there is also another related concept, which is known as CTOD. This is crack-tip opening displacement. You know, it is a very funny nomenclature. See, when you say a tip, tip is a point; the point cannot open, but you still have in fracture literature, a parameter called CTOD - crack-tip opening displacement. When we take up the issues related to displacement and so on, we will look at how CTOD is defined.

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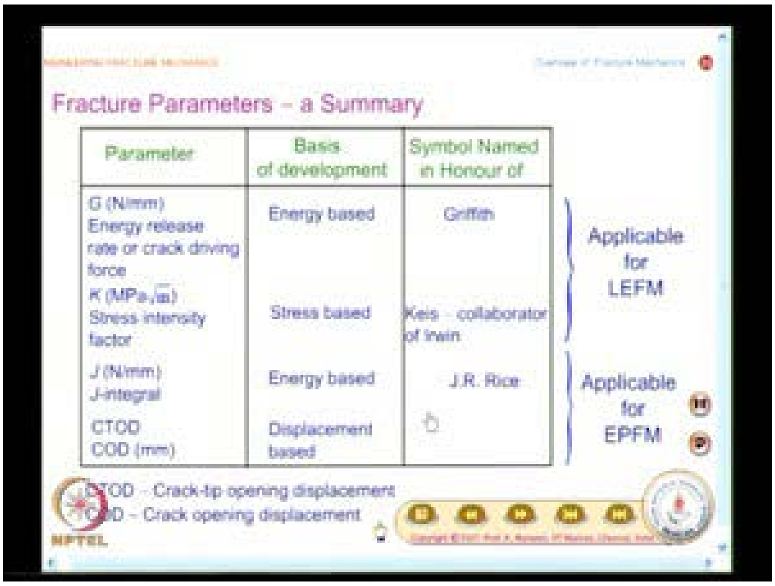

Fracture Parameters – a Summary

Parameter	Basis of development	Symbol Named in Honour of	
G (N/mm) Energy release rate or crack driving force	Energy based	Griffith	Applicable for LEFM
K (MPa$\sqrt{m}$) Stress intensity factor	Stress based	Keis – collaborator of Irwin	
J (N/mm) J-integral	Energy based	J.R. Rice	Applicable for EPFM
CTOD COD (mm)	Displacement based		

CTOD – Crack-tip opening displacement
COD – Crack opening displacement

For the time being, you understand there is CTOD available, as well as COD available. And these parameters can also be looked from different points of view. You know, certain parameters are applicable for LEFM, linear elastic fracture mechanics and the other parameters are applicable for elasto-plastic fracture mechanics. See, finally, we are only dealing with fracture. You have several parameters G, K, J, CTOD and so on. You know, there cannot be many. There have to be interrelationship among them. In fact, we would establish an interrelationship between G and K and in the case of LEFM, G is same as J.

So, we would establish certain interrelationships among this. These were developed, mainly because people approach the fracture problem from various perspectives. That is a reason, why you have different parameters. And when you are able to find out interrelationship, it only reinforces that, the process of fracture has been well understood through various methodologies. This is the way you have to look at it. And certain developments are country specific. In the US, they were mainly focusing on J integral. In the UK, they were talking about COD and CTOD. These are essentially meant for non-linear fracture mechanics.

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Typical Failures Initiated by a Crack

Now, we will take up certain examples, where in you have failures initiated by a crack. My interest is to show, that there is a crack growth phase followed by fracture. You know, this is very, very important. It is a conceptual step, which was beautifully understood by Griffith. And if you look at, he was working on glass. And we would see in detail in the chapter on energy release rate, how he was able to find out that, there is a crack growth followed by fracture, because that is how all structures failed.

So, he was able to provide a rational explanation to this. And before we go into that, if you see postmortem of failures, that would give you a mental picture, yes, there is a growth phase of crack, followed by fracture. Not only this, when you look at the crack, it also has a particular shape. You know, when we want to do a analytical modeling, we would like to have simple geometric shapes and you can really see that, this could be modeled as quarter of an ellipse; it is a corner crack, you have a corner here.

So, this corner crack would be modeled as quarter of an ellipse. And, in fact, we would do this. In all our analytical development, we would take a semi-elliptical crack or an elliptical flaw inside. These are all easy to handle mathematically. The flaw can be modeled in this fashion for analytical development. More or less, it will match with the nature of flaw. And how this crack has been initiated? This is initiated because of corrosion and you definitely see a difference texture in this zone and the texture in this zone. That is what you can make out here.

So, this indicates that crack has grown. This is the crack growth phase and this is the fracture surface. And this was actually, a helicopter blade retaining bolt hole fracture. And the courtesy goes to Elsevier, and we will take up another example. I would like you to make a neat sketch of this. The focus is, what is the shape of the flaw? Because later, we are going to model it as a quarter ellipse. At that time you could recall, indeed, the flaws were like this. You know, that is what is important. Because analytical development should match with actual observation; otherwise the analytical development is useless.

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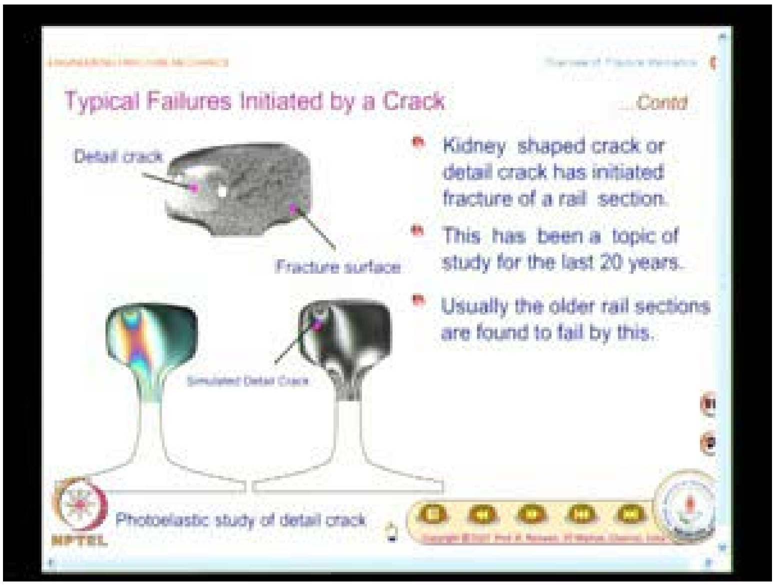

Typical Failures Initiated by a Crack ...Contd

- Kidney shaped crack or detail crack has initiated fracture of a rail section.
- This has been a topic of study for the last 20 years.
- Usually the older rail sections are found to fail by this.

Photoelastic study of detail crack

Another failure is, what you see in the case of rails. And rails have a typical loading scenario. You find the crack has originated below the surface, because you have contact loading here. In the contact loading the maximum stresses occurs beneath the surface. It does not occur on the surface. In most of the problems, maximum stress occurs at the surface. Contact problems are far different. This is one of the issues in gears also, because you have involute profile, that they come in contact, you know flakes of material that come out; because cracks generate below the surface and then flakes of material come out in operation.

So, you need to understand this. And in rail traffic, we have come a long way. I said that we were traveling at 15 miles per hour, now 350 miles per hour. So, the rails have to be understood for its fracture behavior. And because of the shape, this is known as a kidney shaped crack. And you see the typical photo-elastic fringe pattern for the whole system; you are not seeing for the crack. What you need to do is, you need to do an experiment of three dimensional photo-elasticity, where you do the stress freezing of this; then make slices perpendicular to the crack; then you analyze. Such a work was done in our laboratory.

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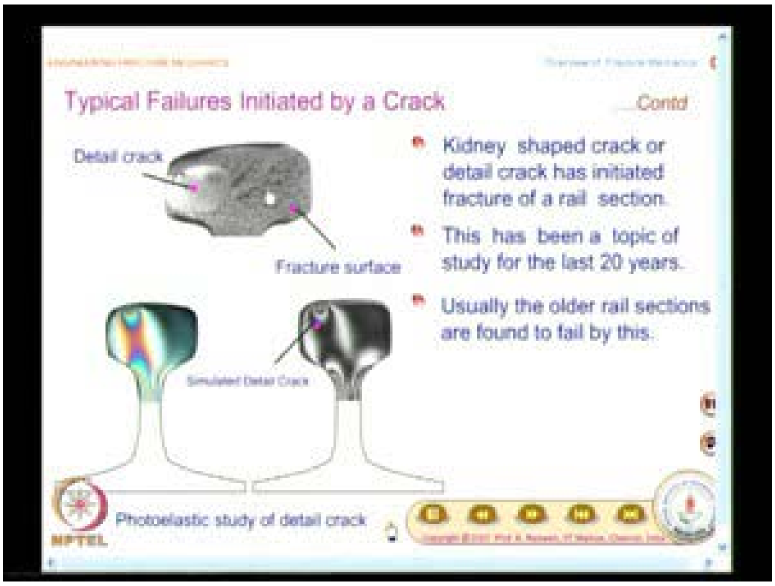

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- This has been a topic of study for the last 20 years.
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We had seen the variation of stress intensity factor on the boundary of the hole. It no longer remains constant. It is a three dimensional problem. The problem is very complex and you need to understand that. And, what is recorded is, the older rail sections are found to fail by this. And this has been the topic of study for the last 20 years, and the important aspect here is, crack has appeared below the surface. Here again, you see the crack surface and fracture surface.

So, there is a growth phase followed by fracture. You know, this is what is emphasized here. You know, if you look at beam theory, we develop it in about half an hour in a class of strength of materials. If you really look at the history, it took brilliant minds to solve this problem for 200 years. Only, after 200 years of concentrated effort by scientists across the world, could solve the problem of stresses developed in a beam under bending. Imagine we take just half an hour in a class.

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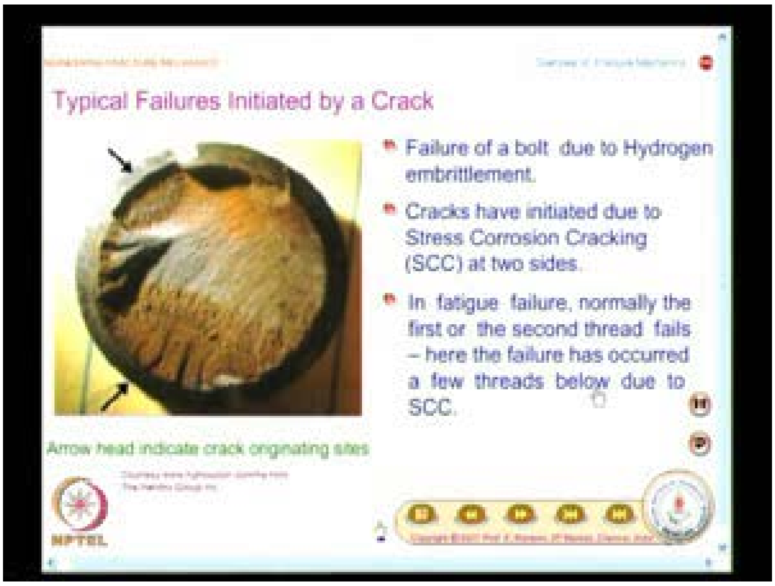

Typical Failures Initiated by a Crack

- Failure of a bolt due to Hydrogen embrittlement.
- Cracks have initiated due to Stress Corrosion Cracking (SCC) at two sides.
- In fatigue failure, normally the first or the second thread fails – here the failure has occurred a few threads below due to SCC.

Arrow head indicate crack originating sites

So, similarly I am emphasizing again and again, there is a growth phase of crack followed by fracture was a very important concept. And, only with this Griffith was able to find out a suitable way of explaining and coming out with a model for fracture. We take another example. See, if you look at the examples which we had seen, they were essentially subjected to fatigue loading. And now, you have another example, which is predominantly corrosion and also, another aspect is seen in this failure. You look at this broken bolt. I would enlarge this and you find there is a crack here. There is another crack here. So, this is what is natural. You know, in real situations, when you say structures can have internal flaws, the internal flaws need not be one. They can be many. And we would focus our attention on the flaw which is critical. And you can see that in the picture. Right now, you have seen different surfaces.

So, this has a different texture and this has a different texture. And you could guess that, this is the growth phase of the crack. And this crack has grown bigger than this. So, eventually, this would have precipitated the failure of it. This is one aspect of this. You can have multiple cracks. On this, one of the may be critical. There is also another aspect that is sought to be explained through this example. Here, the failure has occurred a few threads below. This is a threaded connection; it has not happened in the first two threads. Usually if it is by fatigue, it would fail only in the first two threads. And this has failed a few threads below, and the cause is related to stress corrosion cracking, abbreviated as SCC. There are many many abbreviations in fracture mechanics.

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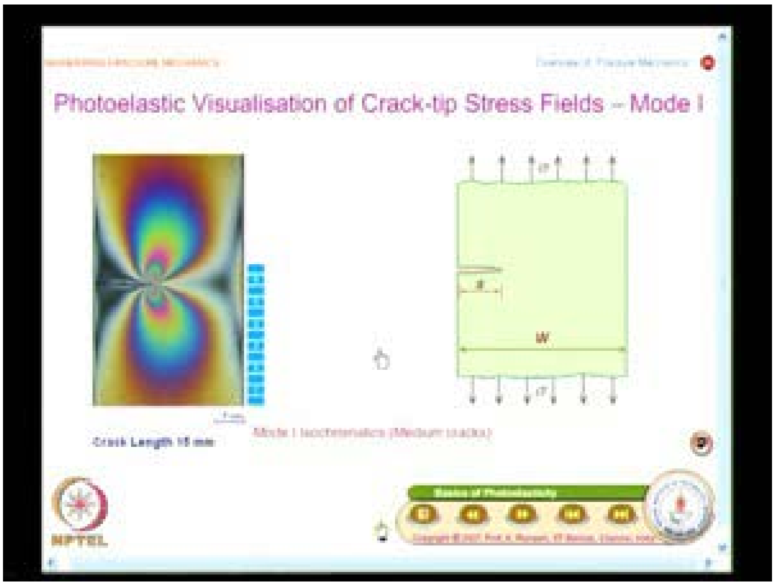

Photoelastic Visualisation of Crack-tip Stress Fields – Mode I

Crack Length 15 mm

Mode I Isochromatics (Medium cracks)

You know, you should be able to quickly associate the abbreviation to your mental picture, what it converts. SCC means stress corrosion cracking; that means, I do not have alternating stress; I have a constant value of stress; nevertheless, because of corrosion, the crack has grown. This is another crack growth mechanism. In fact, we would see different crack growth mechanisms in a separate chapter. And now, you see a failure which is precipitated by SCC. And, another information is also available, that the failure of the bolt is due to hydrogen embrittlement. And the photo courtesies from Hendrix Groups, and this is a very nice picture that was available, that brings in the role of SCC as a growth mechanism for crack followed by fracture. And you can have multiple cracks, one of them may be more dangerous. You know, all along, we have seen the role of photo-elasticity in fracture mechanics. In fact, I am going to train your eyes for looking at geometric features of photo-elastic fringes. In fact, we had seen earlier, a picture similar to this, while comparing stress concentration of a plate with a circular hole, stress concentration of a plate with an elliptical hole and a crack.

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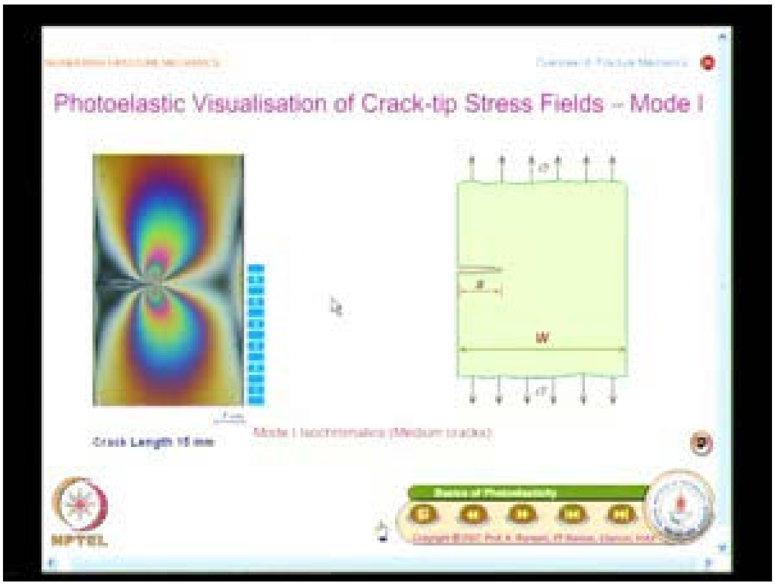

Photoelastic Visualisation of Crack-tip Stress Fields – Mode I

Crack Length 15 mm

Mode I Isochromatics (Medium cracks)

Now, you have learned that, the crack modes could be classified as mode 1, mode 2 and mode 3, depending on how the crack phases move. If the crack phases open up like this, you call it as mode 1 and this is what you have here. And, I would put this animation again.

So, you have, load is applied. As the load is gradually increased, you find more and more fringes come and make a sketch of this. You know, you have a crack axis; it is symmetrical about the crack axis and the significant aspect is, where the fringes are forward tilted. And you know, if you look at, photo-elasticity has the luxury of showing different geometric features when we go from mode 1 to mode 2 or a combination of mode 1 and mode 2. Because I had mentioned, when I started discussing on different modes of loading at the crack-tip, one of the challenges is, in a given practical situation, how to identify what is the mode of loading existing at the crack-tip? See, if I have to employ a fracture mechanics for design purposes, I need to have two quantities. For the given configuration, I should know, what are the stress intensity factors. For the given material, what is a fracture toughness. That is determined by a material testing experiment. From an analytical point of view or numerical point of view or experimental point of view, I will have to evaluate, for a given problem, what are the stress intensity factors K 1, K 2 and K 3. If I take an experimental root, my algorithm for data reduction could be tailor-made, if I know if it is mode 1, what is the mode 1 evaluation of experimental data. If I know it is the mode 2, I can go for appropriate equations and evaluate. It makes my life lot more simple. In a numerical approach, many times, you have algorithms, which give you, from energy point of view, only the bundle of all this. And you have to find out special efforts to segregate.

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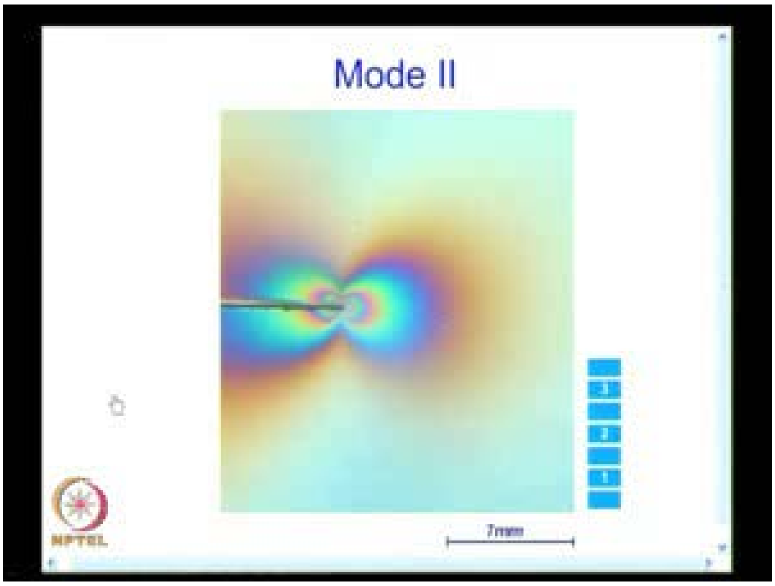

Mode II

So, it is always necessary, that you have to go in for identifying the mode of loading. It gives you additional help in understanding the problem. Now, you have seen mode 1; we would see what is the kind of fringes appearing in mode 2. I am sure the class would entirely agree with me that, the fringes are so different. I will make a zoom of it. It is so different from what is appearing in mode 1. I would like you to make a neat sketch of it and we will look at these features. You know, what you will have to look at is, I have the crack here. In an experiment, you essentially make a very, very thin slit, sharp enough at the end. This is what you do it. And analytical solutions would be matched for the region I had of the crack-tip. Even a child can say, if it has seen this kind of a fringe pattern. If different situations are presented, whether it is mode 1 or mode 2, by looking at the geometric feature, you can easily say the external loading causes a mode 2 loading near the crack-tip. You have fringes, are horizontal like this, loops like this. You can also look at it like this. It is like a figure of eight, horizontally; you can imagine it that way.

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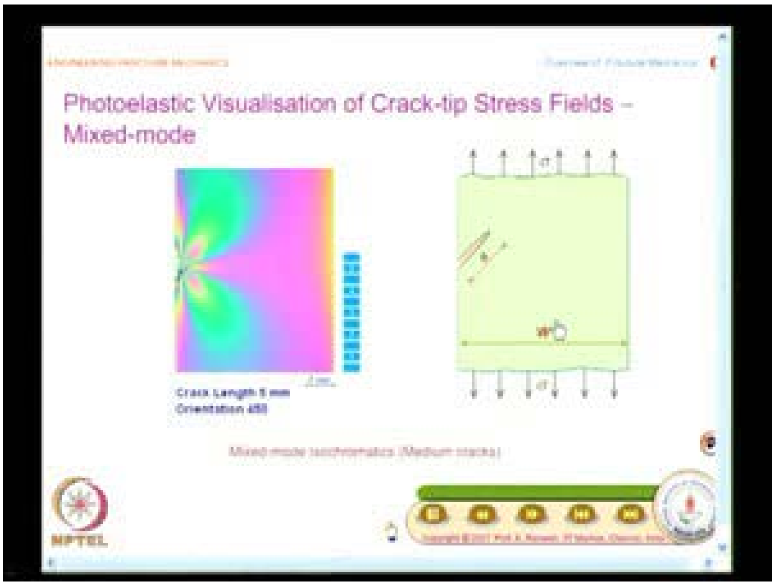

Photoelastic Visualisation of Crack-tip Stress Fields – Mixed-mode

Crack Length 5 mm
Orientation 45°

Mixed-mode isochromatics (Medium cracks)

So, strikingly different in comparison to what I saw as mode 1. Now, we would see what is seen in a mixed-mode. And look at how the configuration in the crack, in relation to the external loading. The crack is inclined and you have a axial loading applied; and you should also a note down a few things; you know, I have put the length of the crack as a; this is the symbol that we will be using in this course. And another important parameter is, you represent the width by the symbol W. In all our fracture mechanics approach, the ratio of a by W will become very very important. That is one of the very important parameters when we want to develop the fracture theory soon and so forth.

Because short cracks will behave in a particular manner; longer cracks will behave in a different fashion. So, a by W ratio is very important. There is an analytical solution. The analytical solution may be valid until a particular value of a by W. So, a by W is another important parameter. And what I will do is, I will also enlarge this fringe pattern. And I would also apply the load one after another, and you would see how the fringes develop.

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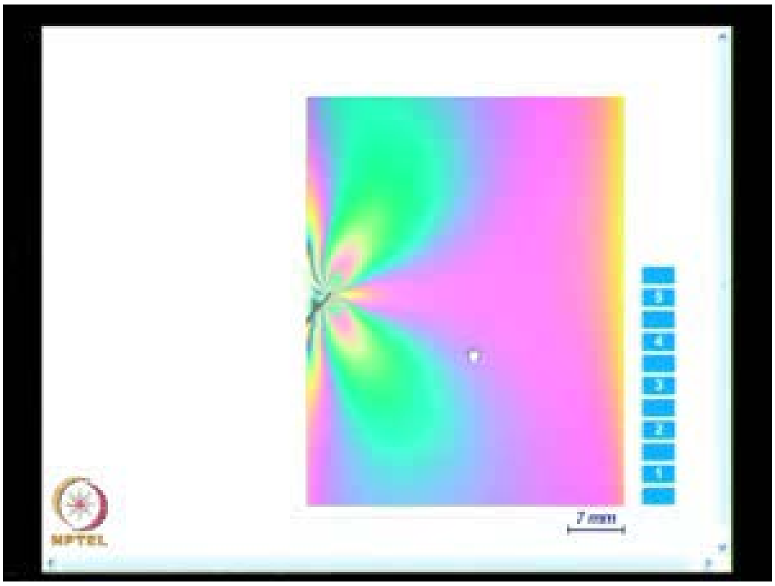

So, I want you to make an important observation. First, look at the fringes developed for different loads. First thing is, I have this as a crack axis; the fringes are tilted like what we saw in the mode 1 scenario, but they are not symmetrical about the crack axis. This is one observation. Another observation is, the fringes, which are seen on one side of the crack and another side of the crack are not of same size. In the case of mode 1 loading, about the crack axis, the fringes were exactly symmetrical. Not only that, they are tilted symmetrically. The moment you come to a combination of mode 1 and mode 2, you find, it is no longer symmetrical about the crack axis - observation number one; and I would again load the specimen sequentially. Then you will observe the fringes on one side of the crack and other side of the crack, are not of same size, which you could observe. I slowly increase and you could see how the fringes are developed.

So, you have fringe on this side and you have fringe on the other side; they are of different sizes. So, in an actual structure, when you do a photo-elastic testing, if you find the fringes are of different sizes and they are not symmetrical, you can immediately conclude that, this is a mixed mode situation. And, this is all done for medium length crack. I have not taken a longer crack. In fact, we would see very interesting practical examples, in which, how the situation changes, when the crack length is longer.

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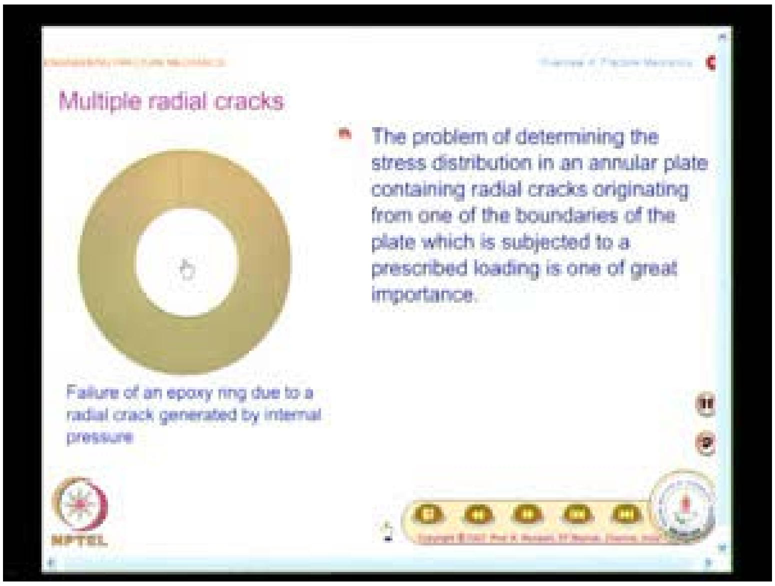

Multiple radial cracks

- The problem of determining the stress distribution in an annular plate containing radial cracks originating from one of the boundaries of the plate which is subjected to a prescribed loading is one of great importance.

Failure of an epoxy ring due to a radial crack generated by internal pressure

We would take up another important problem, which you come across in engineering, where you have essentially an annular plate; and in this plate, what is done is, you insert a oversized rod, by that you are introducing a pressure on the inner surface, and what was observed was, you find a crack progressing in a radial direction and the whole specimen got fractured.

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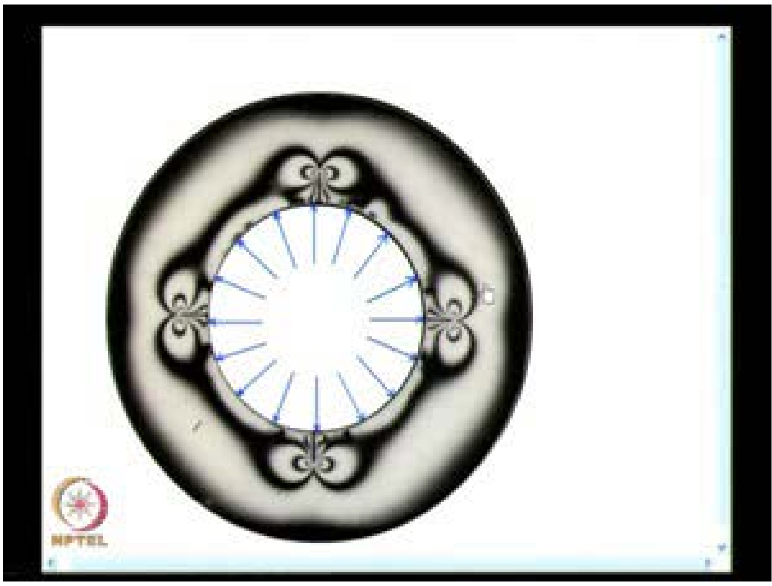

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So, this gives a confirmation that, when you have internal pressure, you could have cracks developing radially. In fact, a study was conducted to see, when you have multiple cracks, how the cracks interact, in a pressure vessel. We would see that. So, in the case of pressure vessels, radial cracks can emanate from the inner boundary. Why? From your Lanis problem you know that, hoop stress is maximum at the inner boundary.

So, when you have hoop stress is maximum at the inner boundary, there is a possibility of crack developing at the inner boundary. And hoop stress diminishes when you go out. And by looking at this feature, what do you find? What is the kind of loading that exists at the crack-tip? It is essentially mode 1, because you have symmetry about the crack axis; this is observation number one.

Now, what I will do is, I will magnify this further and I want you to observe a few more things. I think it is reasonably big enough; let me see, one more. And can you see here, I have this as a crack. In the earlier examples, we saw the fringes were tilted forward. Here how the fringes are? Fringes are tilted backwards. We have a very complicated combination here. The first fringe is backward tilted; the second fringe is also backward tilted; the third fringe is forward tilted and the inner fringe is almost straight. If your eye is tuned, you will be able to see this.

You know, I would use all of this in our later development of crack-tip stress and displacement field equations. If you take higher order terms in the stress field, which would explain, all these geometric features. In fact, it is so fortuitous that photo-elasticity has shown difference between different modes of loading. That, probably you could expect in other experiments also. We would see that. But the geometric features change as a function of number of terms in the series, very unusual. You know, God really wanted people to understand fracture mechanics and he had taught you through photo-elasticity. You would see all these features. Because once I develop Westergaard stress field equation, you would plot them and see how the fringes are. You wait for that surprise. because I need to have an analytical modeling, which explains, what I observed in experiment. If I am unable to explain it, then analytical development has to be relooked.

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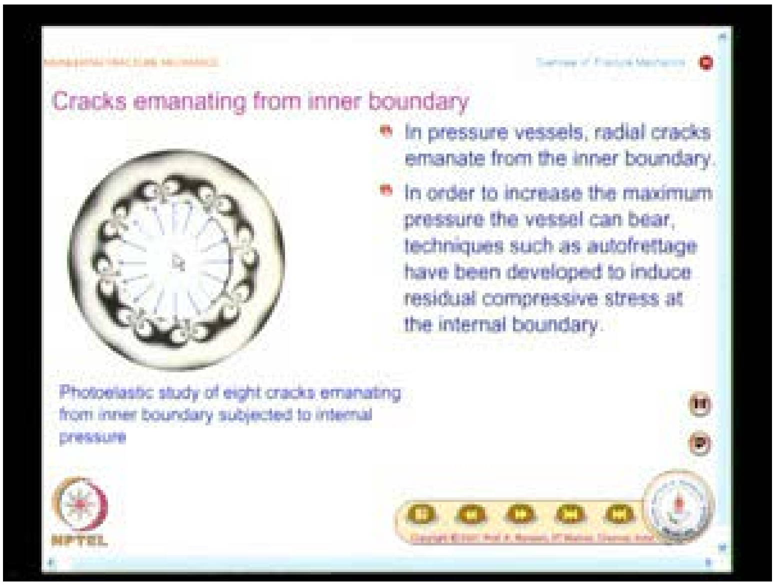

Cracks emanating from inner boundary

- In pressure vessels, radial cracks emanate from the inner boundary.
- In order to increase the maximum pressure the vessel can bear, techniques such as autofrettage have been developed to induce residual compressive stress at the internal boundary.

Photoelastic study of eight cracks emanating from inner boundary subjected to internal pressure

You have to find out, what approximations we have done. That is very very important. You have another example, where, the number of cracks have increased to 8; and here again, you find the fringes are very nice. In fact, it is a very difficult experiment. You know, you observe symmetry. In an experiment, symmetry will be lost, even if there is some small deviation. You should congratulate my student, who had taken the trouble of making all these beautiful radial cracks. It was done by my student V. P. S. Chauhan, way back and you find the fringes are very nicely formed; and you also find that, there is no major interaction between the cracks here. So, this is another conceptual understanding that, multiple cracks can exist and only when they remain very close, we would see another example in the next class, that, there could be some kind of crack interaction. So, people initially started with one crack, then, they went to multiple cracks. Now, there are studies, if there are thousands of cracks, how you can handle it. Particularly, the pressure vessel problem is very, very important and people have tried to model this in various ways.

So, in this class, what we had looked at was, we have raised certain questions that fracture mechanics should be able to answer; in order to answer such questions, you need to conduct more tests. Anyway, you will have to do a test on fracture toughness that is a material test. Apart from that, you need to record crack growth as a function of number of cycles. Then, we moved on to, saying that fracture mechanics is holistic nets approach. You have different recommendations for thin and thick specimens. Then, we looked at series of service failures, which brought out, definitely, there is a crack growth phase followed by fracture. Finally, we have looked at some examples, wherein photo-elastic results show, the geometric features of the fringe are indicative of whether it is a mode 1 loading, mode 2 loading or a combination of mode 1 and mode 2. Thank you.