Engineering Fracture Mechanics
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Module No. # 08
Lecture No. # 40

Crack Arrest and Repair Methodologies

We have looked at in the last class that in the case of a mode one loading, the crack will propagate in a self-similar manner. Then we said when you go to the case of pure mode two, the crack will propagate at an angle which you have to find out from the appropriate criterion and we have looked at the criteria of energy balance and I said it is essentially useful for coplanar crack extension.

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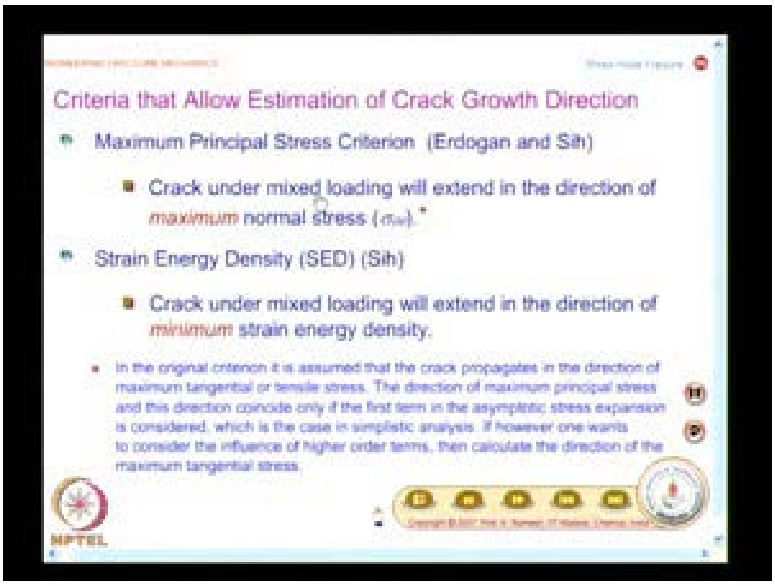

Criteria that Allow Estimation of Crack Growth Direction

- Maximum Principal Stress Criterion (Erdogan and Sih)
  - Crack under mixed loading will extend in the direction of *maximum* normal stress ($\sigma_{\theta\theta}$).*
- Strain Energy Density (SED) (Sih)
  - Crack under mixed loading will extend in the direction of *minimum* strain energy density.

* In the original criterion it is assumed that the crack propagates in the direction of maximum tangential or tensile stress. The direction of maximum principal stress and this direction coincide only if the first term in the asymptotic stress expansion is considered, which is the case in simplistic analysis. If however one wants to consider the influence of higher order terms, then calculate the direction of the maximum tangential stress.

Then we looked at the criteria that allowed estimation of crack growth direction. I had mentioned maximum principal stress criterion; originally when this was introduced, it is known as maximum tangential stress criterion. As long as a use only the first term in the asymptotic series expansion, the maximum principal stress as well as maximum tangential stress sigma theta theta, both the directions coincide. And we had also developed the expression for finding out the crack growth direction based on this theory. The other theory which you could get crack growth direction is from strain energy density theory. And in this crack under mixed loading will extend in the direction of minimum strain energy density. We will go back and review the crack growth direction in m p s criteria; you have to make the shear stress zero.

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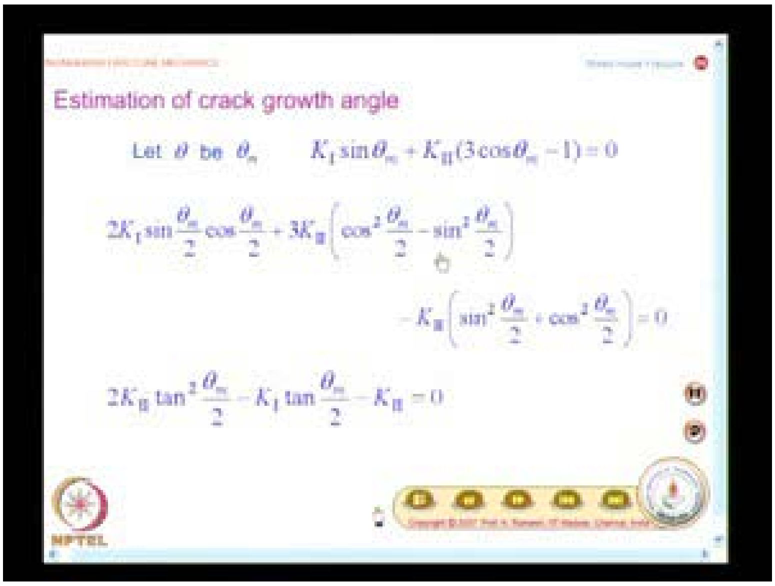

Estimation of crack growth angle

Let θ be θ_m $\quad K_I \sin\theta_m + K_{II}(3\cos\theta_m - 1) = 0$

$$2K_I \sin\frac{\theta_m}{2}\cos\frac{\theta_m}{2} + 3K_{II}\left(\cos^2\frac{\theta_m}{2} - \sin^2\frac{\theta_m}{2}\right)$$

$$-K_{II}\left(\sin^2\frac{\theta_m}{2} + \cos^2\frac{\theta_m}{2}\right) = 0$$

$$2K_{II}\tan^2\frac{\theta_m}{2} - K_I \tan\frac{\theta_m}{2} - K_{II} = 0$$

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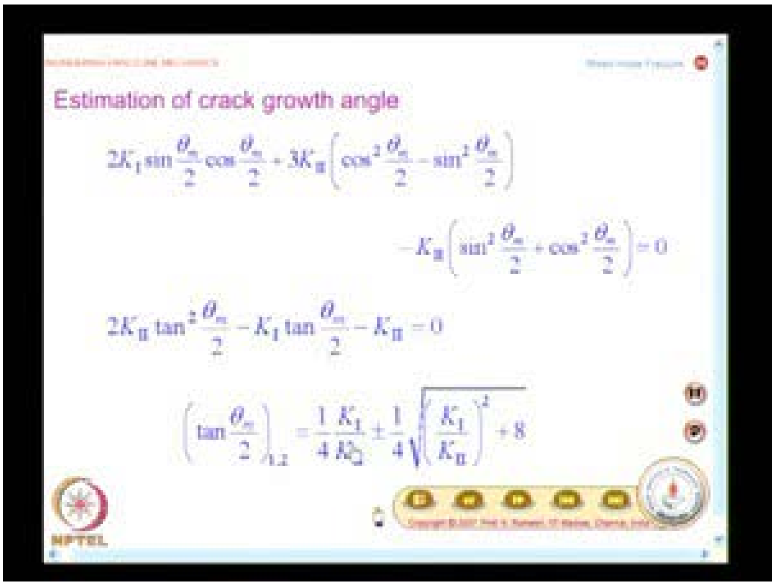

Estimation of crack growth angle

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$$2K_{II}\tan^2\frac{\theta_m}{2} - K_I \tan\frac{\theta_m}{2} - K_{II} = 0$$

$$\left(\tan\frac{\theta_m}{2}\right)_{1,2} = \frac{1}{4}\frac{K_I}{K_{II}} \pm \frac{1}{4}\sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}$$

So, the key equation is this K1 sin theta m plus K2 3 cos theta m minus 1 equal to 0. After some simplification, you could get the expression for crack growth direction, you have this as tan theta m divided by 2 equal to 1 by 4 k1 by K2 plus or minus 1 by 4 square root of K1 by K2 whole square plus 8.

So, you will have to calculate the value of theta m from this, out of the two roots which root is admissible that you'll have to select. So, in this, what you are actually looking at in a given problem for the type of loading estimate the value of K1 and K2. So, this expression would give, in which direction crack can possibly grow, but for crack to proceed by fracture, it has to read the instability condition. What is that instability condition, that also you will have to look at it, but before we go into that we will also see the comparison of results from experiments.

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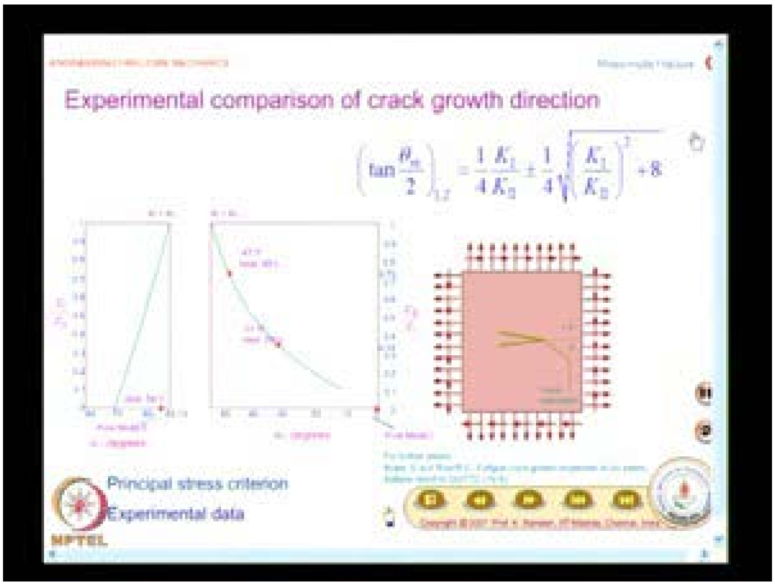

Experimental comparison of crack growth direction

$$\left(\tan\frac{\theta_m}{2}\right)_{1,2} = \frac{1}{4}\frac{K_1}{K_2} \pm \frac{1}{4}\sqrt{\left(\frac{K_1}{K_2}\right)^2 + 8}$$

Principal stress criterion

Experimental data

And what you have here is, on this axis K1 by K2 is plotted, it varies from 0 to 1, so that means, the horizontal axis whatever that you have here this is for a pure mode two situation. At the top of the graph you have this as K1 equal to K2. So, you have one segment where you go from mode two to the situation where the effect of mode one and mode two are equivalent. In the other one, mode one is dominant you go from mode one to the situation where K1 equal to K2. So, this axis is in terms of K1 by K2 ratio, this axis is in terms of K2 by K1 ratio.

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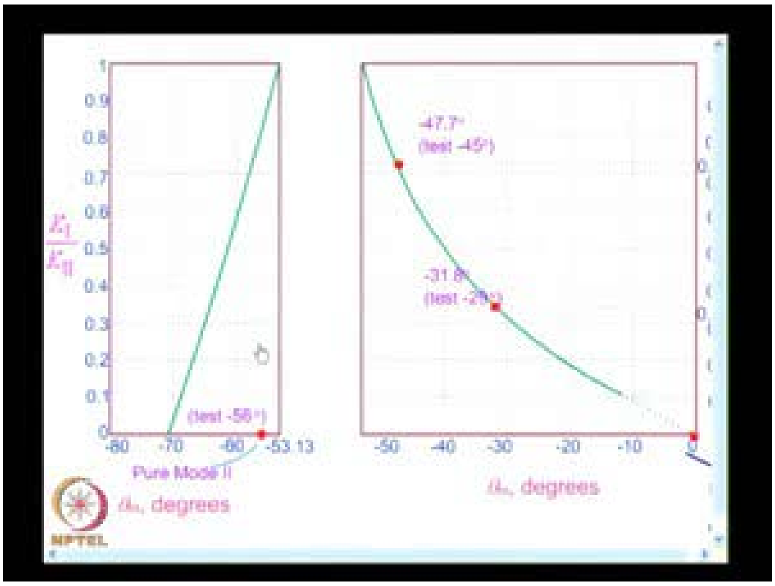

And these graphs are drawn based on this expression and what you see as red dots they are from experiment. And what I would do is I would enlarge this and you could see clearly when K1 is dominant, the experimental data point reasonably matches with the maximum tangential stress criteria. When K2 is dominant, you do not have that kind of good comparison and the maximum tangential stress criterion gives that angle as somewhere around minus 70 whereas, stress result is minus 56 this way of from the predicted value.

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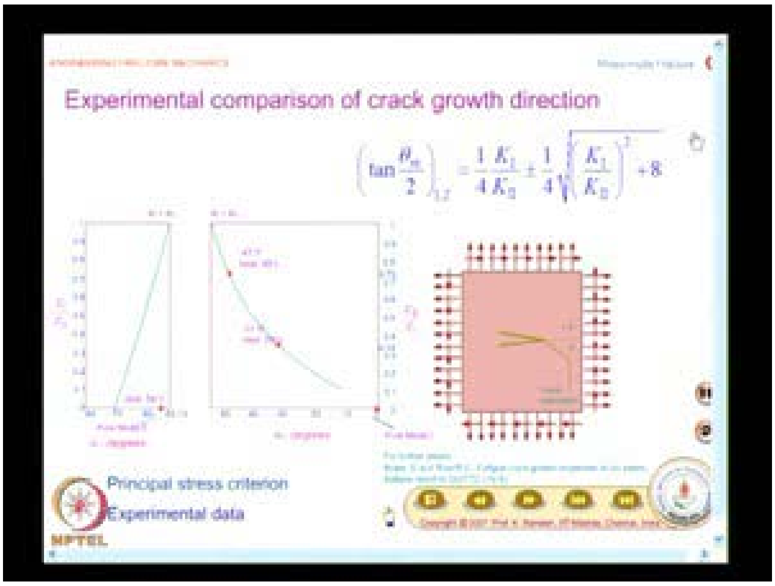

Experimental comparison of crack growth direction

$$\left(\tan\frac{\theta_m}{2}\right)_{1,2} = \frac{1}{4}\frac{K_1}{K_2} \pm \frac{1}{4}\sqrt{\left(\frac{K_1}{K_2}\right)^2 + 8}$$

Principal stress criterion

Experimental data

And you could see that a crack will extend in a direction as shown here. So, you have this angle as minus of theta m and we will also see what the other theories give when you have pure mode two situation. When you have pure mode one the crack grows in a self-similar manner. When mode one is dominant the theoretical as well as experimental predictions reasonably match, that also gives you some kind of a comfort that whatever the expression that we have got is explaining certain observed phenomena.

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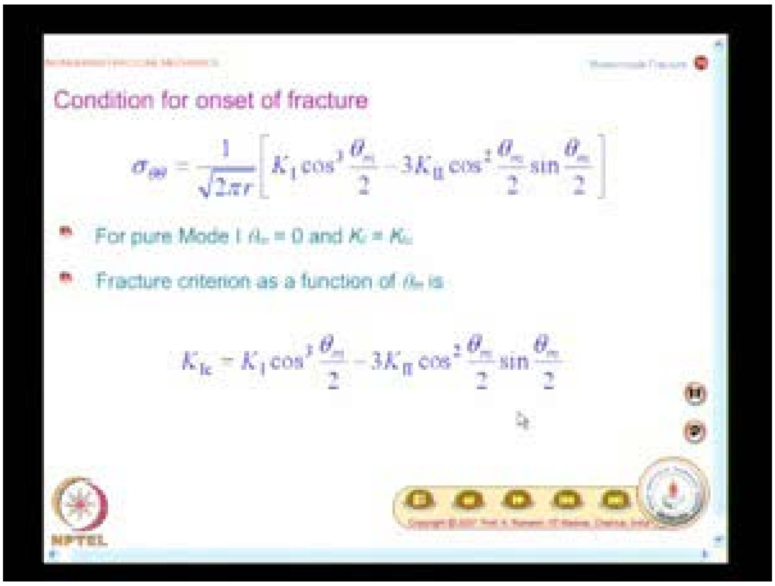

Condition for onset of fracture

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}}\left[K_{\mathrm{I}}\cos^3\frac{\theta_m}{2} - 3K_{\mathrm{II}}\cos^2\frac{\theta_m}{2}\sin\frac{\theta_m}{2}\right]$$

- For pure Mode I $\theta_m = 0$ and $K_I = K_{Ic}$
- Fracture criterion as a function of θ_m is

$$K_{\mathrm{Ic}} = K_{\mathrm{I}}\cos^3\frac{\theta_m}{2} - 3K_{\mathrm{II}}\cos^2\frac{\theta_m}{2}\sin\frac{\theta_m}{2}$$

Next we move on to what is the condition for onset of fracture. I think this is where we had stopped in the last class and the idea which I want to convey is you know we are used to conventional design approach. In conventional design approach what we were doing in a real situation you may have combine loading; however, to assess whether yielding would occur or not we would take the result from a simple tension test, whatever the value of yield strength you get, use it in a appropriate way for your combine loading situation also. You also have multiple theories you had Tresca yield criteria as well as Von Mises yield criteria. You did not have one theory, but the focus is from the material point of view whatever the parameter that you want you do it from one single test.

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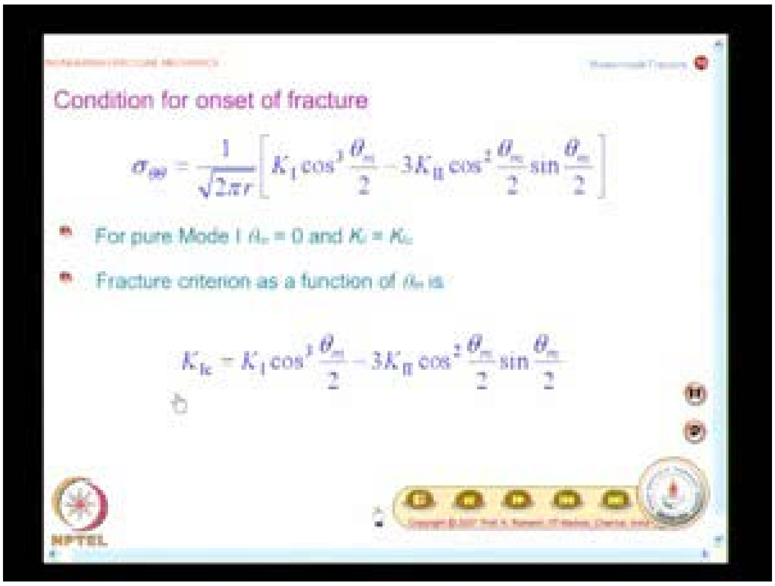

Condition for onset of fracture

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So, in this fracture instability analysis also, you look at what is the value of sigma theta theta at any value of theta m. And in a mode one situation this value should go to the K1 should go to K1 c. So, based on that, you use this for any value of theta m also. So that means, for a given problem of situation find out K1 and K2, use the fracture toughness which is obtained for a mode one situation to assess whether the crack will propagate or not. Very similar to what we were used to in conventional design approach, conventional analysis you take the yield strength from simple tension test; here you focus only on mode one fracture toughness even though it is a combine loading situation.

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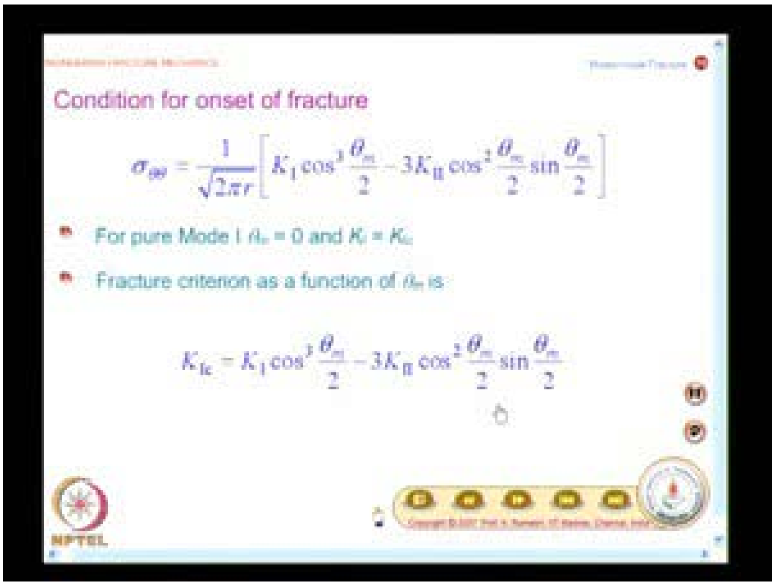

Condition for onset of fracture

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}}\left[K_I \cos^3\frac{\theta_m}{2} - 3K_{II}\cos^2\frac{\theta_m}{2}\sin\frac{\theta_m}{2}\right]$$

- For pure Mode I $\theta_m = 0$ and $K_I = K_{Ic}$
- Fracture criterion as a function of θ_m is

$$K_{Ic} = K_I \cos^3\frac{\theta_m}{2} - 3K_{II}\cos^2\frac{\theta_m}{2}\sin\frac{\theta_m}{2}$$

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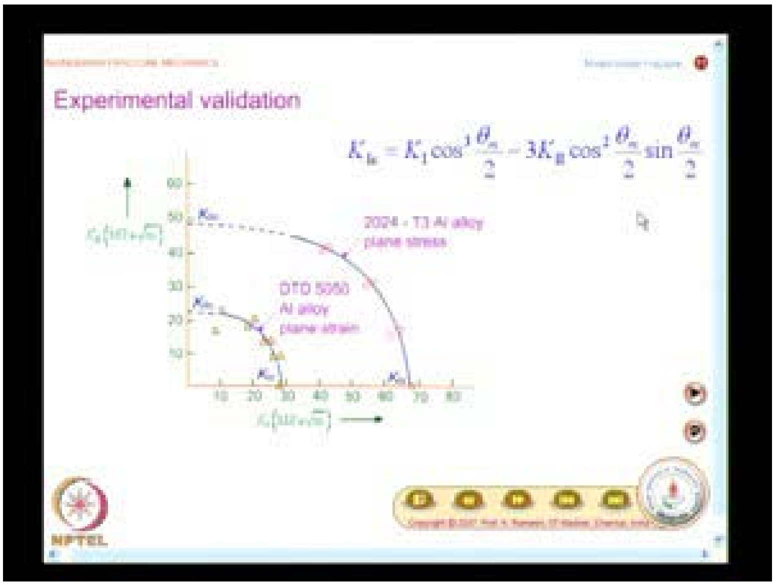

Experimental validation

$$K_{Ic} = K_I \cos^3\frac{\theta_m}{2} - 3K_{II}\cos^2\frac{\theta_m}{2}\sin\frac{\theta_m}{2}$$

And we have to see whether the expression, whatever we have got that is, K1 c equal to K1 cos cube theta m divided by 2 minus 3 K2 cos square theta m divided by 2 into sign theta m by 2 reasonably models the experimental situation. People have found that it matches with the experiments and that is what you see here. You have data of K1 and K2 plotted on the x and y axis and you have this done for two different materials. The circles and triangles are from experiments and the line is from the condition that you have K1 c equal to K1 cos cube theta m by 2 minus 3 K2 cos square theta m by 2 sin theta m by 2.

So, what you will have to do is, in a given problem find out the value of K1 and K2 for the given loading. Also determine the direction in which crack can propagate. So, use that theta m and find out whether this quantity is equal to K1 c or not. If it is equal to K1 c then fracture would occur in that particular direction. This is found to be satisfied for a class of materials because like Von Mises and Tresca, they are not applicable for all materials, some materials obey Von Mises criteria better than Tresca.

So, in fracture also you will have multiple theories people are still working on. A reliable comprehensive theory is not yet developed, you have to find theory for your own application. And you also have what is the limiting value of K2 c which could be expressed in terms of K1 c. We would see such expressions a little while later.

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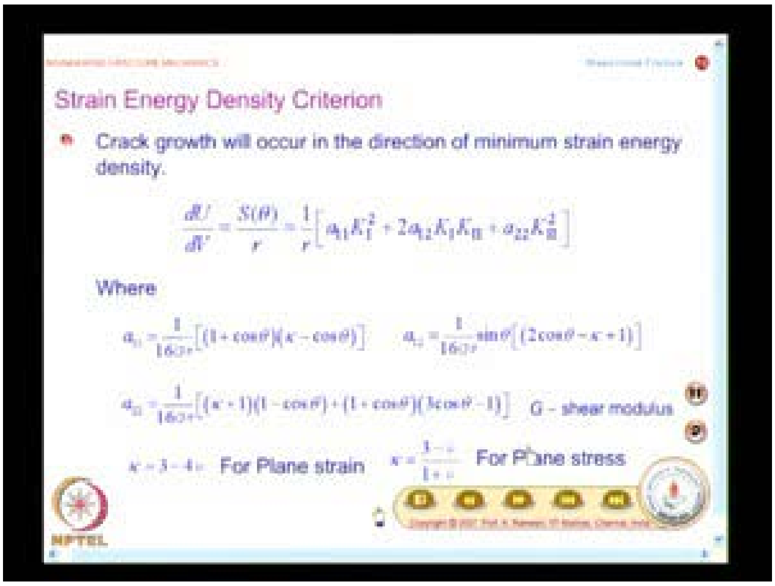

Strain Energy Density Criterion

- Crack growth will occur in the direction of minimum strain energy density.

$$\frac{dU}{dV} = \frac{S(\theta)}{r} = \frac{1}{r}\left[a_{11}K_I^2 + 2a_{12}K_IK_{II} + a_{22}K_{II}^2\right]$$

Where

$$a_{11} = \frac{1}{16G\pi}\left[(1+\cos\theta)(\kappa-\cos\theta)\right] \qquad a_{12} = \frac{1}{16G\pi}\sin\theta\left[(2\cos\theta-\kappa+1)\right]$$

$$a_{22} = \frac{1}{16G\pi}\left[(\kappa+1)(1-\cos\theta)+(1+\cos\theta)(3\cos\theta-1)\right]$$ G - shear modulus

$\kappa = 3-4\nu$ For Plane strain $\kappa = \frac{3-\nu}{1+\nu}$ For Plane stress

Now, we move on to the next theory which is strain energy density criterion. And what you are saying here is crack growth will occur in the direction of minimum strain energy density. So, you have S which is the function of theta divided by r equal to 1 by r a11 K1 square plus 2 a12 K1 K2 plus a22 K2 square. And these coefficients are define, these coefficients are defined with a value of kappa by choosing kappa appropriately, you could get the expression for plane stress as well as plane strain.

See, what you will have to note is in the case of maximum tangential stress criterion, you never discussed about plane stress or plane strain whereas, when you come for strain energy density criterion you have separate expressions available. And a11 is given as 1 by 16 g into pi where g is the shear modulus multiplied by 1 plus cos theta kappa minus cos theta. a12 equal to 1 by 16 g pi sin theta 2 cos theta minus kappa plus 1. a22 is 1 by 16 g pi kappa plus 1 multiplied by 1 minus cos theta plus 1 plus cos theta multiplied by 3 cos theta minus 1. When I use kappa equal to 3 minus 4 nu, the expressions are for plane strain. When kappa is replace by 3 minus nu divided by 1 plus nu it is for plane stress.

So, here again you will find out how to calculate the direction at which crack will initiate and the condition for what combination of k1 and k2 fracture may initial that we would again go back to simple mode one situation. So, we would again use only k 1 c, we will not use k 1 c and k 2 c to asses onset of crack growth. So, it is very similar to your conventional procedure.

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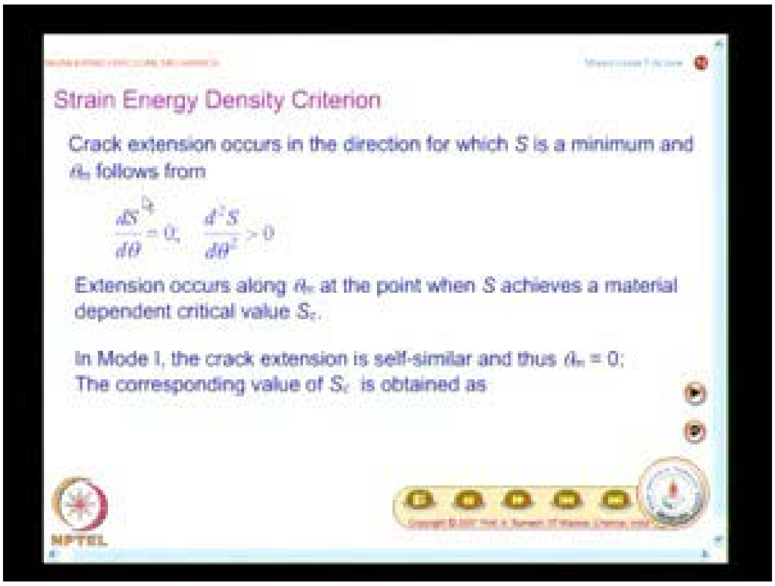

Strain Energy Density Criterion

Crack extension occurs in the direction for which S is a minimum and θ_m follows from

$$\frac{dS}{d\theta} = 0, \quad \frac{d^2S}{d\theta^2} > 0$$

Extension occurs along θ_m at the point when S achieves a material dependent critical value S_c.

In Mode I, the crack extension is self-similar and thus $\theta_m = 0$; The corresponding value of S_c is obtained as

So, we have already mentioned crack extension occurs in the direction for which S is a minimum and theta m follows from d S by d theta equal to 0 and your d squared S by d theta squared should be greater than 0.

So, if you have multiple roots, check the second condition and identify only one root. So, extension of crack occurs along theta m at the point when S achieves a material dependent critical value S c; S critical. And here again mode one comes to the rescue. In mode one, the crack growth is self-similar. See, what people have found is if you have a crack in a mixed mode loading also the crack will tend to propagate in such a manner the stress is it will become a mode one type of crack, the role of mode two and mode three is to deflect the crack path in such a manner the crack will experience essentially mode one loading. That is how crack aligns itself and one of the reasons we have been paying more attention on mode one was it is the most significant modes of failure. So, what you find here is, in mode one the crack extension is self-similar and thus theta m equal to 0.

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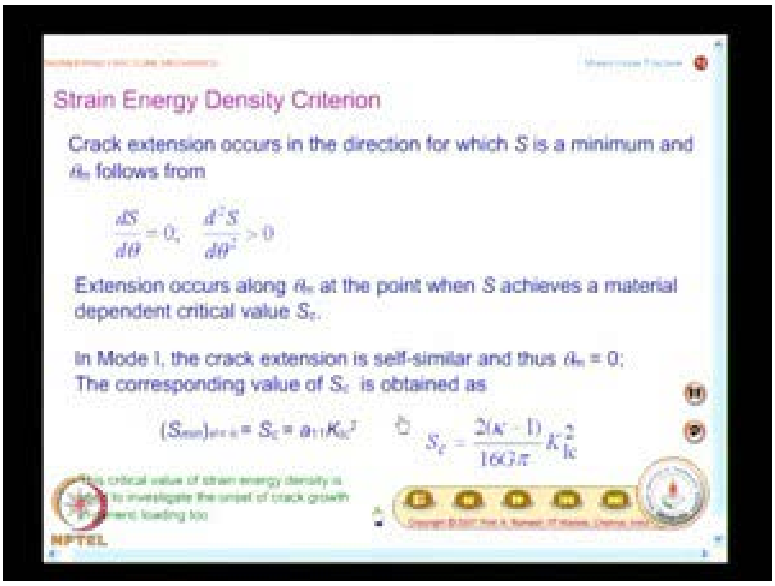

Strain Energy Density Criterion

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$$\frac{dS}{d\theta} = 0, \quad \frac{d^2 S}{d\theta^2} > 0$$

Extension occurs along θ_m at the point when S achieves a material dependent critical value S_c.

In Mode I, the crack extension is self-similar and thus $\theta_m = 0$; The corresponding value of S_c is obtained as

$$(S_{min})_{\theta=0} = S_c = a_{11} K_{Ic}^2 \qquad S_c = \frac{2(\kappa - 1)}{16 G \pi} K_{Ic}^2$$

critical value of strain energy density is
to investigate the onset of crack growth
loading too

And find out what is the corresponding value of strain energy density critical. And that is given as by substituting in the appropriate expression, you get this as S c equal to a11 K1 c squared and if you simplify that it has only K1 role, there is no role of K2 then S c becomes 2 into kappa minus 1 divided by 16 g pi multiplied by K1 c square. And what you find here is this critical value of strain energy density is used to investigate the onset of crack growth in generic loading too. And if you look at the expression would be like this.

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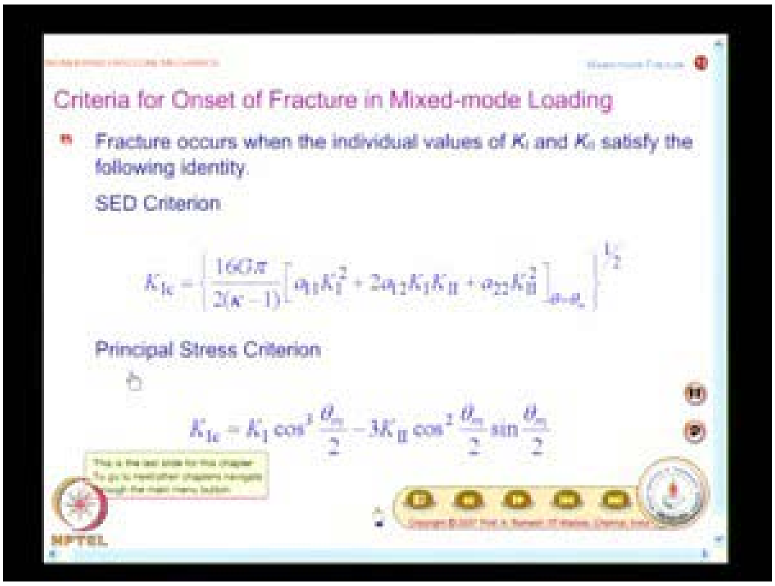

Criteria for Onset of Fracture in Mixed-mode Loading

- Fracture occurs when the individual values of K_I and K_{II} satisfy the following identity.

SED Criterion

$$K_{Ic} = \left\{ \frac{16G\pi}{2(\kappa - 1)} \left[a_{11}K_I^2 + 2a_{12}K_I K_{II} + a_{22}K_{II}^2 \right]_{\theta=\theta_m} \right\}^{1/2}$$

Principal Stress Criterion

$$K_{Ic} = K_I \cos^3 \frac{\theta_m}{2} - 3K_{II} \cos^2 \frac{\theta_m}{2} \sin \frac{\theta_m}{2}$$

So, in the case of strain energy density criterion you find out for a given loading the values of K1 as well as K2, then find out this value ,16 g pi divided by 2 into K minus 1 kappa minus 1 a11 K1 square plus 2 a12 K1 K2 plus a22 K2 square in such a manner that you substitute theta equal to theta m in the expressions whole power half if it is equal to K1 c then crack would initiate at an angle theta m. This is given by strain energy density criterion. The principle stress criterion or maximum tangential stress criterion you have K1 cos cube theta m divided by 2 minus 3 K2 cos square theta m divided by 2 sin theta m divided by 2 if it is equal to K1 c, then crack initiation would occur.

So, you have at least two theories I have shown and you have to anticipate that they will not give same result when you go for a real life situation.

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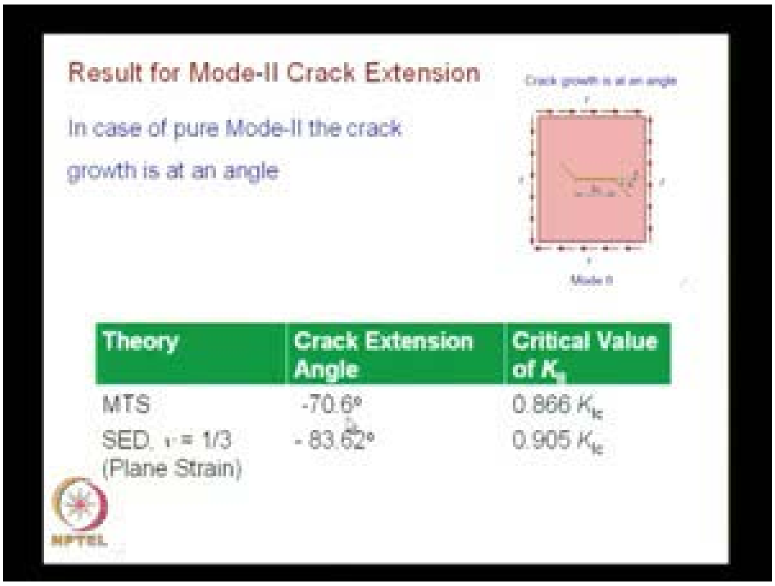

Result for Mode-II Crack Extension

In case of pure Mode-II the crack growth is at an angle

Theory	Crack Extension Angle	Critical Value of K_{II}
MTS	-70.6°	0.866 K_{Ic}
SED, ν = 1/3 (Plane Strain)	- 83.62°	0.905 K_{Ic}

So, we will see for a pure mode two case what way these theories give the values. In the case of pure mode two, the crack growth is at an angle and we have already seen that the crack is expected to grow like this and if you look at the results from the two theories which we have recently looked at, the maximum tangential stress criterion predicts this angle as minus 70.6 degrees and the corresponding value of critical value of K2 is 0.866 times K1 c. On the other hand, if you go for strain energy density criterion and if you consider poisson ratio equal to 1 by 3 and if you take the plane strain situation, the crack extension angle is minus 83.62 degrees. Obviously, they do not match from one point of view. Another point of view you can say both give crack growth angle in the negative direction they are not way off, you know this is the way that you will have to look at it. And what is the value at which the crack initiation occurs; in one case it is 0.866 times K1 c, in s e d it is .905 K1 c here at least the different is not that much.

So, this you will have to accept the problem is so complex and you do not have a valid theory which is useful when mode one is dominant, but they are not really good when mode two is dominant, that is what the result pictures.

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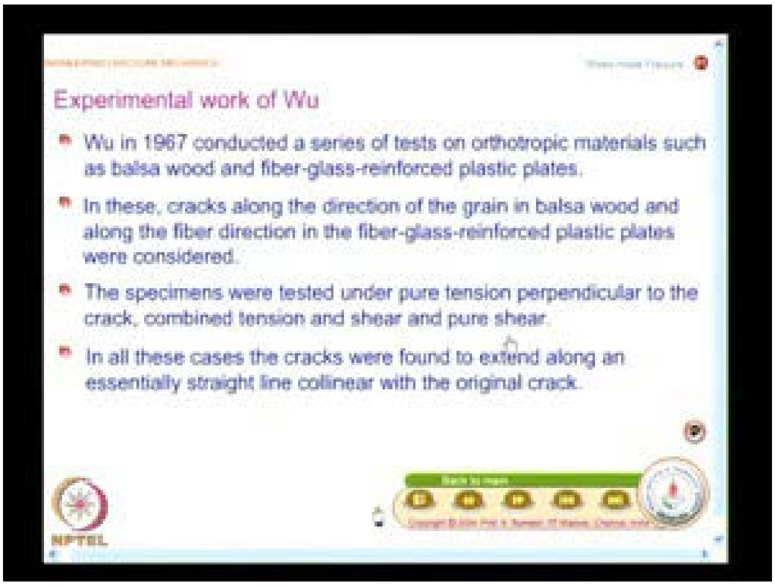

Experimental work of Wu

- Wu in 1967 conducted a series of tests on orthotropic materials such as balsa wood and fiber-glass-reinforced plastic plates.
- In these, cracks along the direction of the grain in balsa wood and along the fiber direction in the fiber-glass-reinforced plastic plates were considered.
- The specimens were tested under pure tension perpendicular to the crack, combined tension and shear and pure shear.
- In all these cases the cracks were found to extend along an essentially straight line collinear with the original crack.

So, what people have thought? People have gone for developing theories based on empirical approaches we would also see that. You know there was an experimental work by Wu in 1967. He conducted a series of test on orthotropic materials such as balsa wood and fiber-glass-reinforced plastic plates. In these, cracks along the direction of the grain in balsa wood and along the fiber direction in the fiber-glass-reinforced plastic plates were considered. The specimens were tested under pure tension perpendicular to the crack, combine tension and shear and pure shear. So, they it is done for mode one, mixed mode as well as pure mode. In all these cases, the cracks were found to extend along an essentially straight line collinear with the original crack.

So, you will call this as coplanar crack extension or self-similar crack growth. This is what Wu was observed, but he handled orthotropic materials, it is not an isotropic material. He found that an empirical relationship of the form K1 by K1 c power a plus K2 by K2 c power b equal to 1 reasonably models the behavior and for the cases he considered a equal to 1 and b equal to 2 was found to be sufficient. See you have to note here, the final expression is obtain in an empirical fashion. This empirical relation uses the fracture toughness in mode one as well as mode two in arriving at the result. In the earlier cases we had use only K1 c. Here both K1 c K2 c are used and the form is given like this: K1 by K1 c power a plus K2 by K2 c power b equal to 1. This comes from an empirical approach and this is the paper that he has publish this was in 1967 application of fracture mechanics to anisotropic plates a s m e journal of applied mechanics in the pages 967 to 974.

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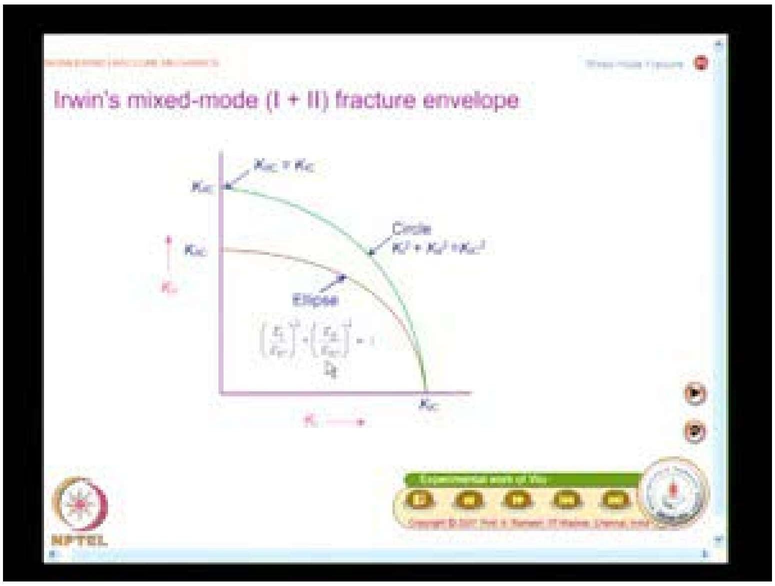

Irwin's mixed-mode (I + II) fracture envelope

And if you look at, people have taken advantage of this and you also have graphs which depict the type of variation as a function of K1 and K2 and what you have here is, I have one graph drawn which is essentially a circle, I get this as K1 squared plus K2 squared equal to K1 c squared this essentially comes from your energy balance criteria. This is valid only for coplanar crack extension. Here when you take the graph as K1 by K1 c whole squared plus K2 by K2 c whole squared equal to 1, you have taken a equal to 2 and b equal to 2. This is an equation of an ellipse which uses K1 c as well as K2 c and what the literature says is, such a expression is also useful for non-coplanar extension. Because it is a empirical formulation if you come from energy balance criterion you will have only K1 squared plus K2 squared equal to K1 c squared. If you divided by K1 c squared and K1 c squared appropriately, the formulation is empirical in nature and people all have also done it for combination of mode one mode two and mode three. There the expressions looks like for a plane strain situation K1 by K1 c whole square plus K2 by K2 c whole square plus 1 by 1 minus nu multiplied by K3 by K3 c whole square equal to 1.

See the crack growth direction does not stop here. People have observed king king as one of the observations near the crack tip then people also have observed crack curving. For all these, you need to go for better models. People are now looking at influence of higher order terms only with that these directions could be predicted satisfactory. So, this area is open for research. So, I would appreciate that you consult the current literature to learn further.

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Need for Crack Arrest/Delay in Crack Re-initiation

- One of the goals of fracture mechanics is to extend the life of a component in the presence of inherent flaws.
- Once a crack has been detected, its growth needs to be monitored.
- If possible, steps are to be taken to stop the advancing crack and delay the crack reinitiation time.
  - Introduce crack stiffeners/patches.
  - Other measures could be to heal a crack.
  - Necessary repair technologies be applied to reuse the cracked component.

Now, we move on to the very important chapter on crack arrest and repair methodologies. What is the need for crack arrest? Obviously, by arresting the crack, you would be able to enhance the life of the component.

So, one of the goals of fracture mechanics is to extend the life of a component in the presence of inherent flaws. Because if you look at many of the bridges built fifty to sixty years back, now they are all develop cracks and if you do not salvage it you have to rebuild up. So, people are developing repair technology so that they could extend the life for some more time saving enormous amount of cost. So, it is a very important area and fracture mechanics understanding helps in doing this. And we have always been looking at once a crack has been detected, it is growth needs to be monitored. They also have theories that say how the crack will grow. So, will also know how to inspect at what intervals and so on and so forth.

So, once you know the crack is growing, you take steps to stop an advancing crack. This is one approach. Other approach is delay the crack re-initiation time. So, how do you stop the crack? Introduce crack stiffeners and patches. That is very obvious, you know if you have a car and if you have a plastic cover to that, it will invariably develop a crack. So, you should know how to stop it. So, the question is which way you will put the tape along the crack direction or perpendicular to it. We will see an answer. Other measures could be to heal a crack; you know this is patching up now the influence is from biological systems people have also developed self healing composite. You know in certain applications where it is not load bearing member, necessary repair technologies can be applied to reuse the cracked component. That is also needed, you know like use stitch your cloth you also have metallic stitching. There are companies ready to do this for a price.

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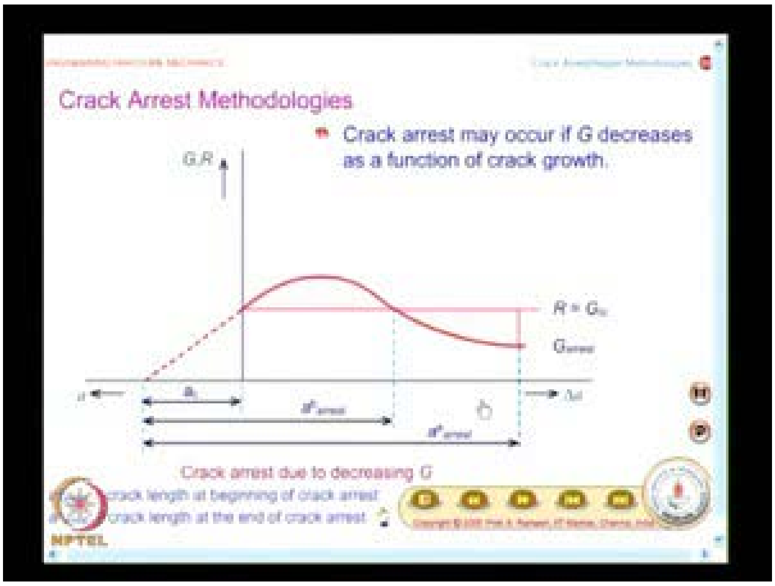

Crack Arrest Methodologies

- Crack arrest may occur if G decreases as a function of crack growth.

Crack arrest due to decreasing G

crack length at beginning of crack arrest

crack length at the end of crack arrest

So, we will also have a brief look at that and what is the physics in crack arrest methodologies. So will go back to our understanding of energy release rate and resistance. We take an ideal brittle solid and what you are having here is, for a given loading, the G increases. So, crack advances and by some mechanism you are able to bring down G. So, what you will find is at some value of G, the crack would start arresting and it would completely arrest at some other higher value. So, G and R concept is very useful to discuss the possibility of arrest. The energy availability is much below the resistance. So, the crack has to get arrested. So, you have initial critical crack as a c. At the start of arresting, the length of the crack is a superscript b. When it becomes a superscript e, the crack is fully arrest.

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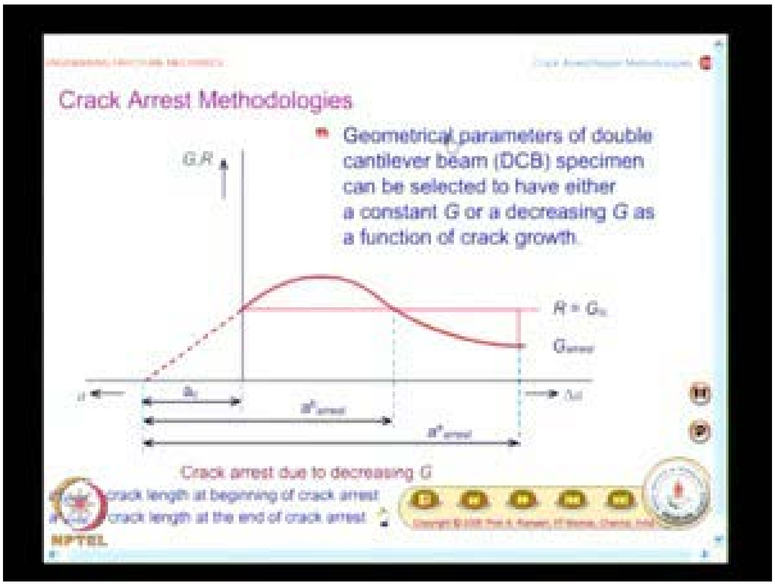

Crack Arrest Methodologies

- Geometrical parameters of double cantilever beam (DCB) specimen can be selected to have either a constant G or a decreasing G as a function of crack growth.

Crack arrest due to decreasing G

crack length at beginning of crack arrest

crack length at the end of crack arrest

So, you should have a mechanism by which bring down the G. That is done effectively by putting a patch. You put a patch, G can be brought down. Now the question is what should be the size, what should be the location, what should be the material. There are so many parameters attached to it. This basically illustrates the principle in crack arrest. I must bring down the value of G; that is physics. And we will see what way we can do, if you really go back to your knowledge of the double cantilever beam specimen the parameters, the geometrical parameters of the specimen can be suitably selected to have either a constant g or a decreasing g as a function of crack growth, on that basis you could perform an experiment and satisfy that crack arrest is possible, the other approach is put a patch.

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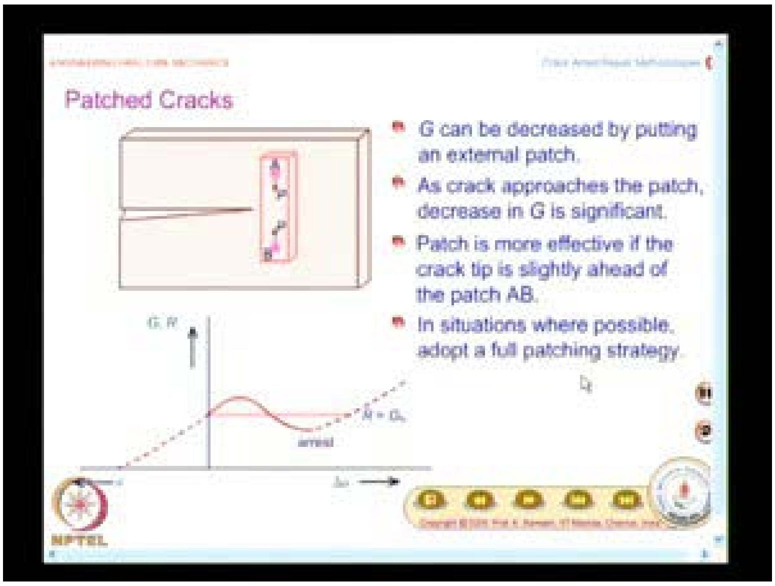

Patched Cracks

- G can be decreased by putting an external patch.
- As crack approaches the patch, decrease in G is significant.
- Patch is more effective if the crack tip is slightly ahead of the patch AB.
- In situations where possible, adopt a full patching strategy.

So, you have an external patch that is put and what you find here is the patch is more effective if the crack-tip is slightly ahead of the patch a b; that means do not put the patch here, put it somewhere here. Essentially you have increase in the stiffness in that location so that the energy release rate comes down and this is the same graph that you had seen earlier. So, because of the patch you know when the crack has come to this you will have a crack getting arrested. Then as the load is increased, that crack can again reinitiate and grow further.

And you know for all this we have earlier looked at a severity of crack from photo elastic fringe pattern. You would also investigate the effective inners of a patch from photo elastic fringe pattern and once I say a patch, I can put it on both sides or one side; all this variations are possible. So, we would see how photo elastic fringes will look like.

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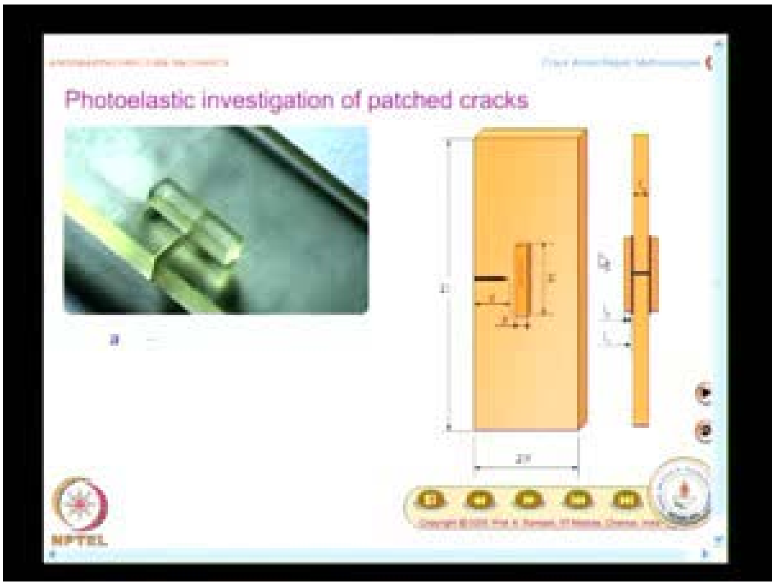

Photoelastic investigation of patched cracks

So, what you have here is you do not have to sketch this. This gives a indication I have a crack, how the patch is put? The patch is put perpendicular to the crack; this is the configuration which will help in arresting the crack. We will see how the fringe patterns look like. And all the parameters you know whether you can put patch on both sides, for example, if you are working on the aircraft components only one side is visible to you other side you cannot access. So, I can put patch only on one direction.

So, we will have to investigate whether putting a patch on one direction is sufficient enough for your given application, if possible you put patch on both sides you should have accessibility for that and in fact, the aircraft manufacturers have really paid attention on several aspects of it whether the patch should be of the same material as the base material or it should be made of a composite because when you have a aluminum frame they would try to put a patch made of a glass fiber or caviler fiber or carbon fiber type of a patch and see how the effective effectiveness of it, what should be the length, what should be the width; these are all parameters.

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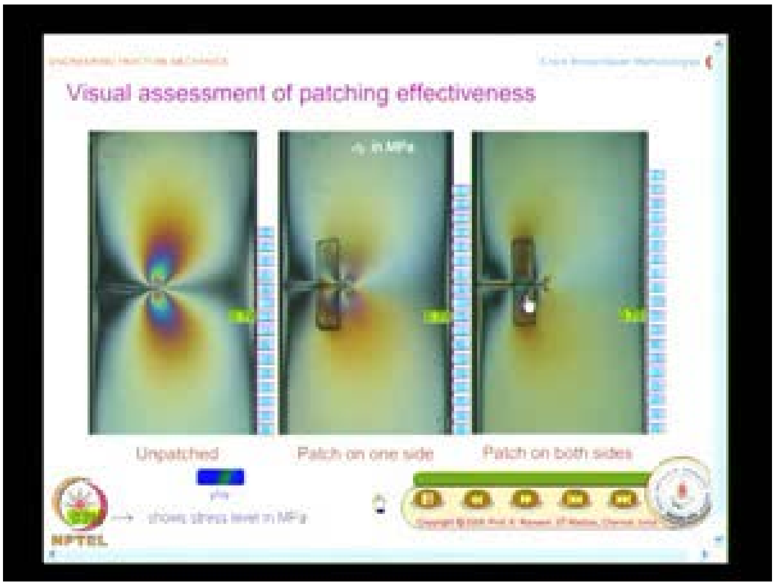

Visual assessment of patching effectiveness

So, what we will see is, we would just have a look at the fringe pattern that gives you enormous amount of information is, I will take a fringe patterns somewhere here, I will put the similar load here. So, you could see from the size of the fringes, this is the un patched specimen this, you could sketch, I would like you to sketch and this is the patched specimen on one direction, this is the patch specimen on both the directions. You could see the fringes drastically come down; obviously, it indicates the stress intensity is low when you have two patches, goes with your common sense; when you say that it is fully patched it has to be well protected. So, that is what you see here and this gives you dramatically is a very carefully done experiment; this was done by my student Madhu, a very carefully experimentalist and you have one patch here you have another patch and I could also increase the load and show how it is.

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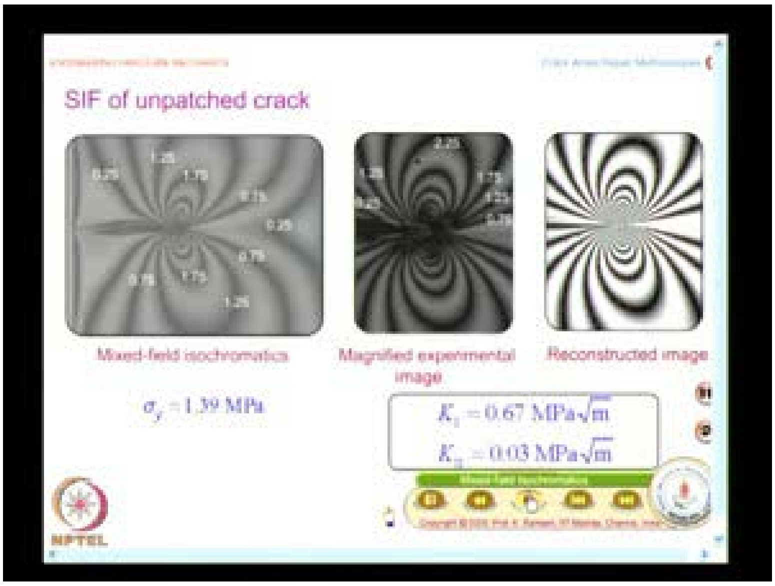

SIF of unpatched crack

Mixed-field isochromatics

Magnified experimental image

Reconstructed image

$\sigma_y = 1.39$ MPa

$K_I = 0.67\ \mathrm{MPa}\sqrt{\mathrm{m}}$

$K_{II} = 0.03\ \mathrm{MPa}\sqrt{\mathrm{m}}$

So, we have gone out to 0.79 m p a in this also you can see. So, at 0.79 m p a, fringe pattern is of particular size whereas it is very small when you have double patch. And you know these were calculated by photo elastic analysis; you could see for one case, you collect data from the field. This is reconstructed fringe pattern and the value of k1 is 0.67 mpa root meter and K2 because of small deviation of a crack as well as the loading, you have a very small value of K2 present in the 0.030 mp a root meter.

So, it is possible by processing the photo elastic fringe pattern extracting the value of K1 and K2 from the field.

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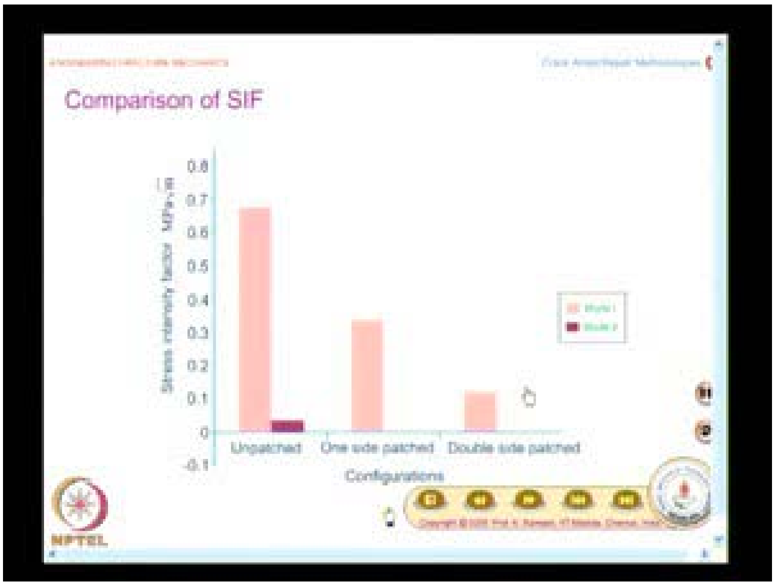

Comparison of SIF

What I would like to show is a comparison in the form of a graph, this is more important, you make a sketch of this. You have configurations listed on one axis, stress intensity factor is rested on the y axis. So, when you have a unpatched crack, it is about 0.79 MP a root meter one side patch it has come down to around 0.36 or so when it is patched on both sides it is 0.15 or so. So, this shows crack arrest by patching is affective patching brings down the value of stress intensity factor. So, it is a useful methodology. Another approach what people do is people use hole as a arrester.

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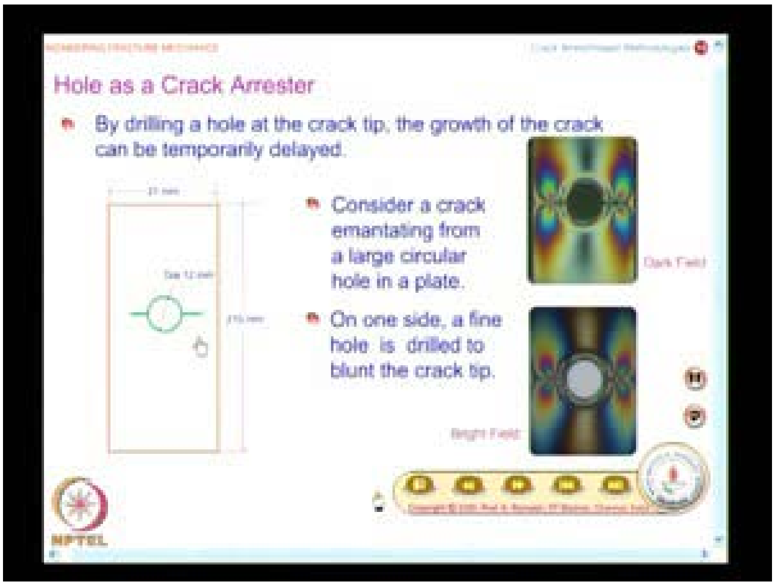

Hole as a Crack Arrester

- By drilling a hole at the crack tip, the growth of the crack can be temporarily delayed.
- Consider a crack emantating from a large circular hole in a plate.
- On one side, a fine hole is drilled to blunt the crack tip.

So, what you see here is I have a plate with a hole from which cracks have emanated. On one side you have a crack on the other side, the crack is stopped by a hole, a small hole is put and why do you go for a hole? When you put a hole, the stress intensity factor comes drastically down to around 3 because the finite body it will be greater than 3. It is not infinite any more theoretically because at the crack-tip you have infinite value of stresses the moment you put a hole and this is the reason you know when you have a reverted joints the reverted hole serves like a hole. So, it acts like a crack arrester, the reinitiate time is delay it may not have much change in the stress intensity factor, but the crack for you to grow further it has to again initiate and then go.

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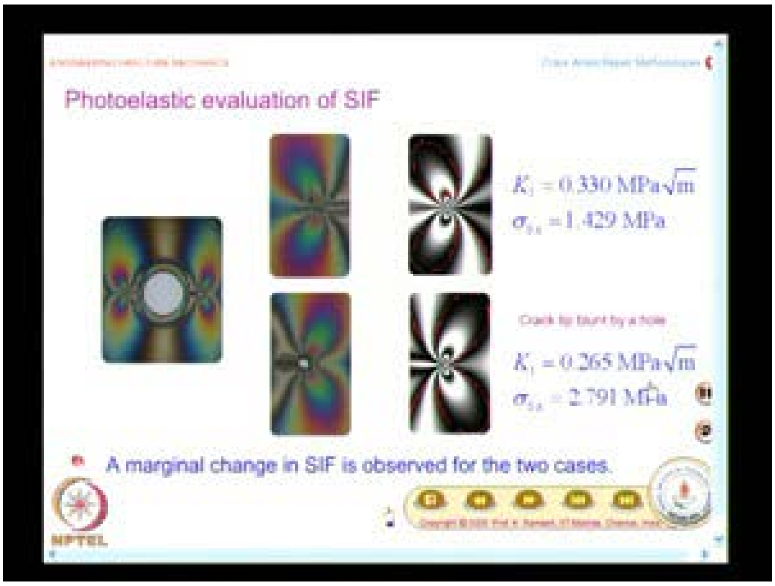

Photoelastic evaluation of SIF

$K_I = 0.330\ \text{MPa}\sqrt{\text{m}}$

$\sigma_{s,s} = 1.429\ \text{MPa}$

Crack tip blunt by a hole

$K_I = 0.265\ \text{MPa}\sqrt{\text{m}}$

$\sigma_{s,s} = 2.791\ \text{MPa}$

- A marginal change in SIF is observed for the two cases.

So, you delay the re-initiation time. That is advantage of hole as a crack arrester. This is again then by photo elastic analysis. You have the fringe patterns in dark field as well as white field. You would not see much difference between the fringe patterns because the SIF value is not significantly altered. And you have taken this close up use are shown and these are the reconstructed fringe pattern from the data process, one is for the actual crack another is crack-tip blunt by a hole you do not find major difference in the SIF value, but what is the advantage of a hole is it delays the re-initiation theory.

You have k1 as 0.330 MPa root meter when it is blunt by a hole it is 0.265 MP a root meter. See these are all done on epoxy that is why you see such small values of stress intensity factor. It is not done on aluminum or steel there the values would be total different.

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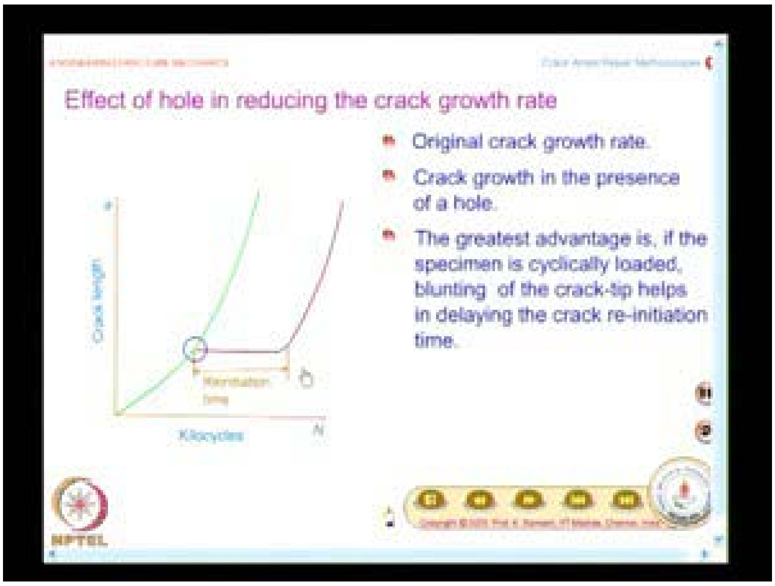

Effect of hole in reducing the crack growth rate

- Original crack growth rate.
- Crack growth in the presence of a hole.
- The greatest advantage is, if the specimen is cyclically loaded, blunting of the crack-tip helps in delaying the crack re-initiation time.

So, the effect of hole is to delay the re-initiation time originally crack growth rate is like this. When you have a hole it I will delay and then crack growth will be like this. So, this is the advantage you have, it delays the re-initiation time and this naturally happens when you have reverted joint whatever the crack that is initiated by one hole it will come and stop at the other hole then it will take some time for it to go further.

So, this was the difference when you have a welded joint once the crack starts propagating there is nothing to arrest. So, cracks will simply zip through. So, in the later ship designs people have provided crack arresters at appropriate locations. Then (( )) ship will also become very safe. So, this is what you have keep in mind, the re-initiation time is what is the gain that you get when you have a hole put at the end of the crack. And the next concept is self healing, it is all precipitated by biological systems.

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Self Healing – a Methodology for Crack Arrest

- Structural polymers are susceptible to damage in the form of cracks, which form deep within the structure where detection is difficult and repair is almost impossible.
- In nature, damage to an organism initiates a healing response.
- This concept has been applied to synthetic material design and a self healing polymer has been developed.

See what happens is when you have a fall you get a bruise and your system understands that this is where something has happened. So, you have skin is reformed only in that location how does this happen? Can this be mimic in real structures particularly when you have a space exploration device which is made of composite, you cannot go to space and repair. It has to repair by itself. So, in such exotic applications these concepts are tried. Structural polymers are susceptible to damage in the form of cracks, which form deep within the structure where detection is difficult and repair is almost impossible.

In nature, damage to an organism initiates a healing response. So, very similar to that when the damage occurs, that should imitate a healing response; that is a way people have looked at, that is they have learned from leaving organism. So, this concept has been applied to synthetic material design and a self healing polymer has been developed. And I could magnify it and then show the composite looks like this.

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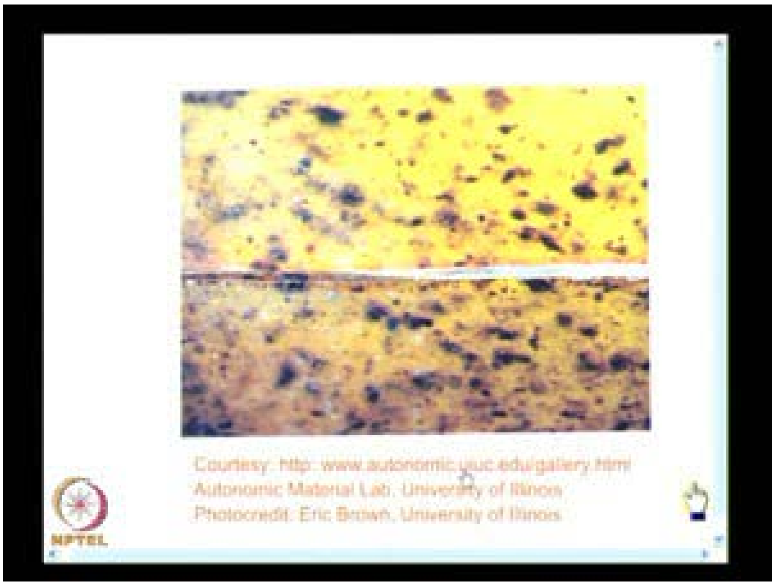
Courtesy: http: www.autonomic.uiuc.edu/gallery.html
Autonomic Material Lab, University of Illinois
Photocredit: Eric Brown, University of Illinois

It is done at University of lllinois. Professor Sotos is credited with this contribution, you have a crack and the crack also gets healed for which they have to fabricate the polymer in a particular fashion. They have to embed micro bubbles the micro bubbles carry resin as well as hardener. So, when the crack goes and pierces the catalyst as well as the resin, they get released and then healing takes place. So, you have to calculate whether you are able to statically distribute this bubble satisfactory all these are manufacturing issues, the concept is like this.

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The autonomic healing concept

- A micro-encapsulated healing agent is embedded in a structural composite matrix containing a catalyst capable of polymerising the healing agent.
- Cracks form in the matrix wherever damage occurs.
- The crack ruptures the microcapsules, releasing the healing agent into the crack plane through capillary action.
- The healing agent contacts the catalyst and triggers polymerization that bonds the crack faces to be closed.

So, will just quickly see the concept. So, this is known as autonomic healing concept and what happens here is you have the polymer system. You have bubbles of various sizes the microcapsules carry resin as well as the catalyst. So, what happens is wherever there is crack growth, the crack growth invariable bump a catalyst as well as the resin system. So, they come out and then sealed it. That is a concept behind it and it is patented and people are developing it further because to make it commercial it has to work, the concept is simple to appreciate.

So, when the crack grows, you have release of resin from this and the catalyst interacts with this and you have polymerization takes place and then healing takes place. So, the damage even precipitates healing action also. There several technological questions if you really look at if there is a gap here how this gap will behave. So, these are all issues people have to look at and what way these micro bubbles have to be statistically distributed. So, in fabrication how they can ensure this these all very difficult issues that you have to address while fabrication, but the concept is very similar to how the biological system would respond healing takes place at the place where the damage has occurred.

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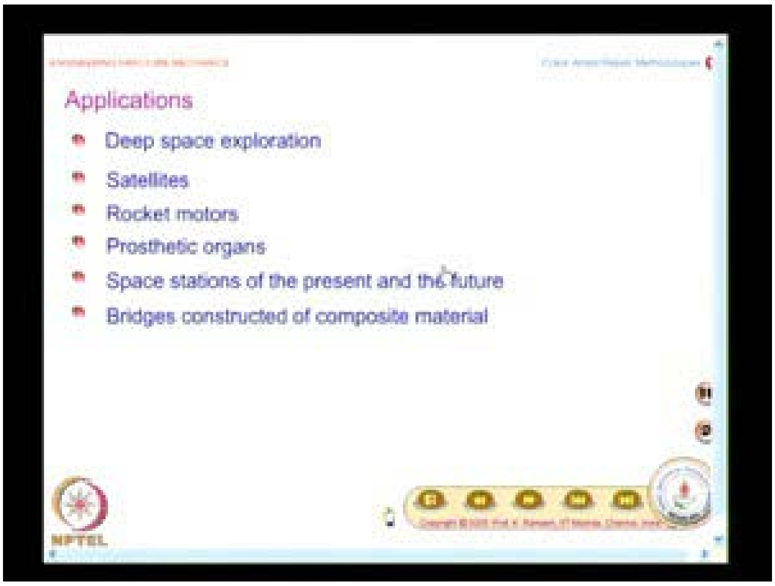

Applications

- Deep space exploration
- Satellites
- Rocket motors
- Prosthetic organs
- Space stations of the present and the future
- Bridges constructed of composite material

So, this finds application in deep space exploration, satellites, rocket motors, prosthetic organs, space stations of the present and the future, bridges constructed of composite material you can list, but the idea is this in places where you cannot go and inspect and do rectification, if healing can take place and delay that itself is good enough.

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Crack Repair by Metal Stitching

- Metal-stitching is a highly skilled, reliable and cold work mechanical process of repair by which cracked, broken or blown-out pieces of cast-iron, cast steel and aluminium housings are repaired.
- This process involves insertion of specially designed locks across the cracks in pre-made slots.
    - These alloy locks have a resistance to tensile strain, which is 3 to 5 times that of cast-iron.
    - Crack faces are brought into close proximity by specially designed threaded pins.

We also move on to the next concept of crack repair by metal-stitching and what you will have to note is it is a highly skilled reliable and cold work mechanical process of repair by which cracked, broken or blown-out pieces of cast-iron, cast steel and aluminum housings are repaired.

This is a useful aspect do it does not come under the preview of fracture mechanics per say because you are really looking at the housing, but this is very useful from practical application point of view because many people do not know that metal-stitching is possible. We all know only welding we have no bracing and so on. Metal-stitching which is the cold operation is also possible and what this process involves is insertion of specially designed locks across the cracks in pre-made slots these alloy locks have a resistance to tensile strain which is 3 to 5 times that of cast-iron.

So, you use a different material high strength material. Crack faces are brought into close proximity by specially designed threaded pins. So, you have locks as well as threaded pins that serve the purpose. And here again you will find you will have a crack and you do the locks perpendicular to the crack. So, that you learn from fracture mechanics, if you find a crack in your car cover put your tape perpendicular to the crack, it will stay for longer duration than putting it parallel.

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Configuration of metallic stitch

Stitching is perpendicular to the crack

- Metal stitching is not a welded repair. It is cold repair of fractured components. So it is called as cold pinning.

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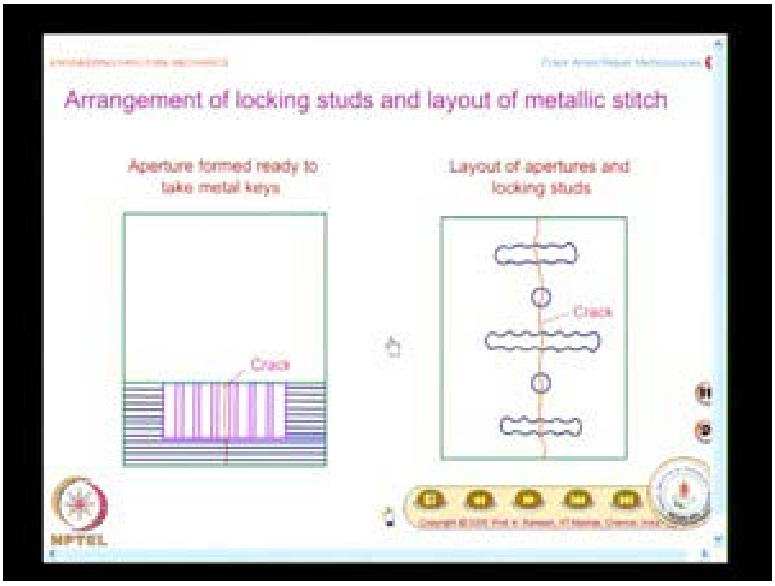

Arrangement of locking studs and layout of metallic stitch

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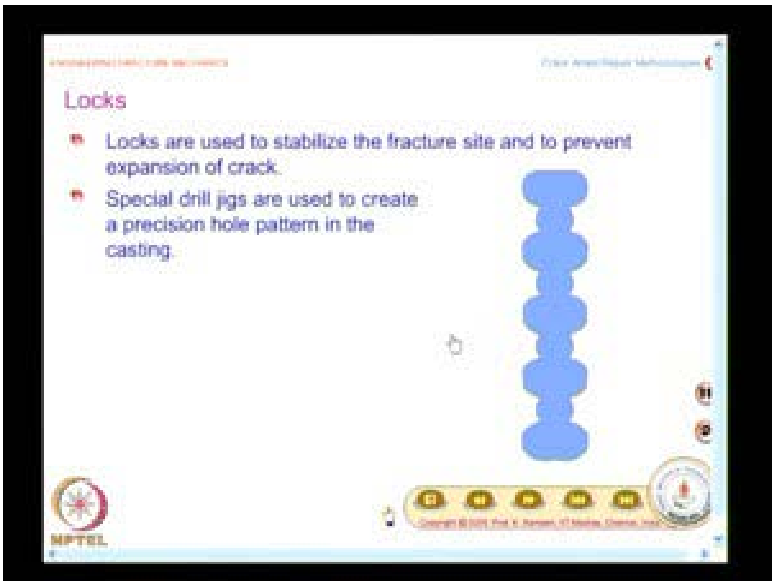

Locks

- Locks are used to stabilize the fracture site and to prevent expansion of crack.
- Special drill jigs are used to create a precision hole pattern in the casting.

So, this is what the configuration of metal stitch is shown, I have the crack I put it perpendicular to that and these locks have very special shape, we will have a look at it very special shape. This is a top view you have a crack that is going through and you have this special lock. It is not simple straight piece, it is a measuring itself is skilled, you need to have this kind of measuring and the typical lock would be like this; shape is given here. You have a lock that is a design and you have to make a slit by appropriate tools on the component like this. That is why you have to specialized companies which involve in this you make a sketch of this sketch of this a lock. So, this measuring itself is going to be challenging.

And what they do is they put layers of this. They have to drill it put one layer after the other and once they repair polish it and then paint it you will find as if it is the good component. You will not be able to distinguish between broken and unbroken parts. So, you have special drill jigs are used to create a precision hole pattern in the casting and let us see the sequence of operation. You have to make the aperture because you have to drill it appropriately, then insert these locks, then you have to fix studs special studs are required to keep them in place, then you do rough grinding, smooth grinding and finishing.

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Sequence of metal stitching operation

Cleaning

1. Rough Grinding
   - to remove protruding keys
2. Smooth Grinding
   - for smooth finish
3. Painting

So, you have making aperture fixing lock locking studs and cleaning I will redo the animation we just observe the sequence of operation. You have to make special holes to put the locks; they are called as special keys. So, you have to fix that then you have the locking studs, then repair it and the courtesy goes to metal lock code u k.

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Application of metal stitching operation

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Salient features of metal stitching

- This process ensures 100% leak free repair with high degree of rigidity in equipments which are operated under higher pressure.
- The minimum thickness of the material to be stitched is 9 mm and no upper limit for thickness.
- As the repairs are carried in cold process, no distortion or thermal stress is induced.

You have the housing of the gear boxes shown here it is broken and this is stitched. It is not painted. If it is painted you will not be able to find out any difference and what you will have to keep in mind is, this ensures 100 percent leak free repair with high degree of rigidity in equipments which are operated under higher pressure. The requirement is the minimum thickness of the material to be stitched is nine millimeters and no upper limit for thickness. As the repairs are carried in cold process, no distortion or thermal stress is induced. That is a advantage; if you go for welding you have distortion. So, that kind of defect is not seen here.

You know this is the last class in this course we have come a long way and we had in detail seen the development of linear elastic fracture mechanics; even the mathematical basis was very thoroughly studied, even the derivations we have done to the last step, then we moved on to concepts related to elasto plastic fracture mechanics, we looked at briefly j integral as well as c t o d, then we also looked at failure assessment diagram, then we took up important aspect of which way the crack will grow in mixed mode fracture, we saw how mode one fracture toughness could be used in physical concept based fracture theories, later on people switched on to empirical approaches in which they had used both k 1 c as well as k 2 c. And in this class essentially we looked at crack arrest and repair methodologies. Crack arrest is possible by putting patches; this has found wide application in aerospace structure. What is the effect of putting patch on one side, putting patch on both sides, then how does the hole helps in delaying the crack re-initiation. Then we also saw metallic stitching. So, we have looked at history on what prompted fracture mechanics to develop mathematics and detailed derivations on several aspects of it, we have also gone to the extent of looking at application aspects of it and this a field which is developing, there is scope for research and you have to consult the recent articles as well as books and there is lot more for you to read. I am sure whatever we have discussed will provide you enough basic understanding for you to read the correct literature. Thank you.