**Engineering Fracture Mechanics**

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**Module No. # 08**

**Lecture No. # 39**

**FAD and Mixed Mode Fracture**

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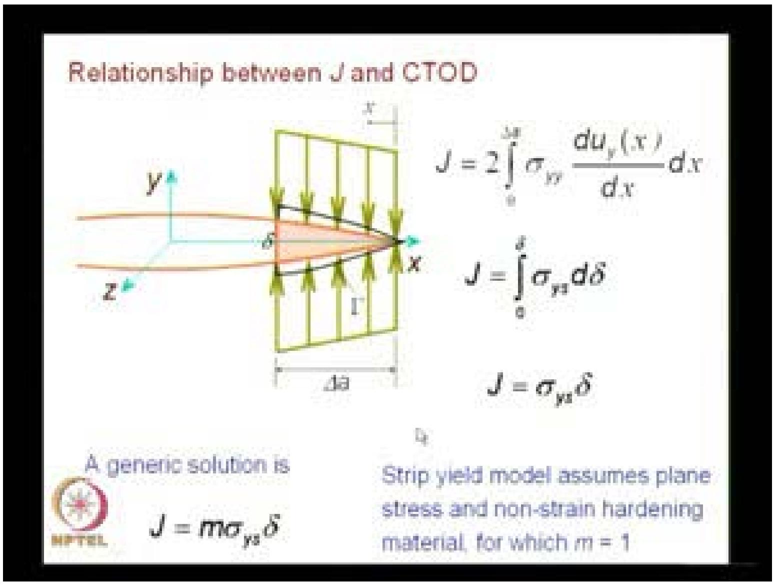


You know in the last class we had looked at a relationship between J integral and CTOD. We have just looked at the final expression; we will see how to get that interrelationship. We go to the Dug dale's yield strip model and this is the fictitious extension of the crack and this is very highly exaggerated. You will have to consider that this height is quite small, it can be almost said that d y equal to 0 for this. If you want to establish the relationship between J and CTOD, take a contour along the boundary of the strip yield zone and the contour is shown here. I have taken a contour for clarity, it is slightly displaced, it is actually very close to the strip yield model.

So, you start from one crack face and end in another crack face. This is the contour that you have and we know the definition of J-integral is given as J equal to integrate over the contour, first term deals with the strain energy density U dy minus T i n dow u i by dow x into ds. So, this gives the work done, you have the stress vector and the corresponding displacement and as I mentioned, if the extent of plastic zone is much larger than CTOD. CTOD is nothing but this distance delta and this is the extent of the plastic zone, in comparison to this delta is very small, it is a highly exaggerated picture, if we have that kind of a situation dy equal to 0. So, when dy is 0 in the J-integral this term goes to 0 and J-integral simply becomes I have traction which is nothing, but sigma yy which is compressive. So, minus of minus has become plus into dow uy divided by dow x into ds.

So, this is what I have to evaluate, mind you, this contour is very close to the crack surface for clarity, it is shown at a little distance away ,the crack face is subjected to the compressive load. In fact, that is taken as traction on the contour. So, this is what I have, I have to evaluate this quantity and in order to do that you define a new coordinate system with origin at the tip of the strip yield zone. I take x equal to 0 here and x goes from 0 to delta a and I define small x as delta a minus capital X this is shown as capital X in the diagram, that is why it is taken in that fashion.

So, I eventually get J equal to 2 times integral 0 to delta a sigma yy d uy by dx into d x and if you look at what is 2 uy ,nothing but delta at any position, delta is actually twice the value of the displacement uy. So, if I use that identity, I get a simplified expression j equal to 0 to delta sigma ys into d delta and you know when you put the limits it reduces to simply J equal to sigma ys into delta. In fact, this is what we had seen as the interrelationship in the last class and now, we have looked at how to get this interrelationship. If you look at the literature a general solution or the general relation between J and delta is given as J equal to M times sigma ys delta and for this particular case you have M equal to 1.

The strip yield model assumes plane stress and non-strain hardening material, that is why you have the traction as sigma ys uniform throughout; we are not bringing in strain hardening at all into the analysis.

See if you really look at Dugdale’s strip yield model, it is a very simple methodology, but it finds very good use in elasto-plastic fracture analysis. People have developed concept related to c t o d from that, you have an interrelationship between J and CTOD based on this; later people also have developed what are known as failure assessment diagram and the starting point was from Dugdale’s model.

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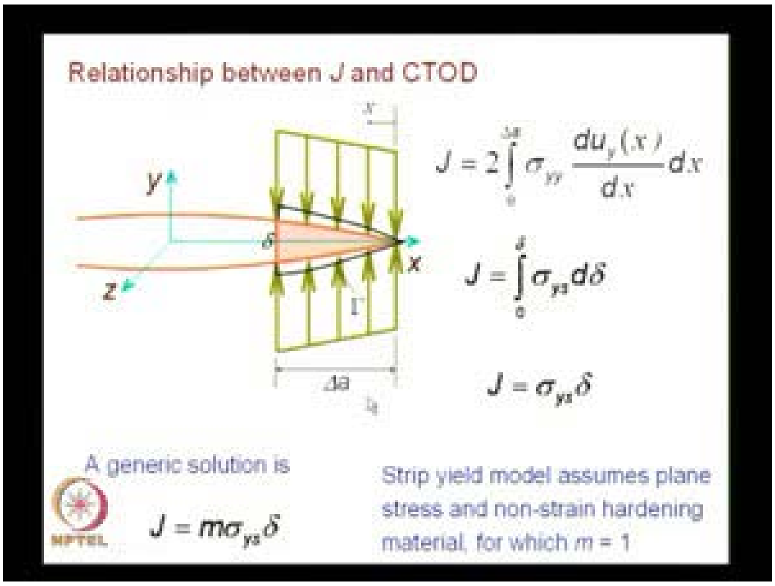


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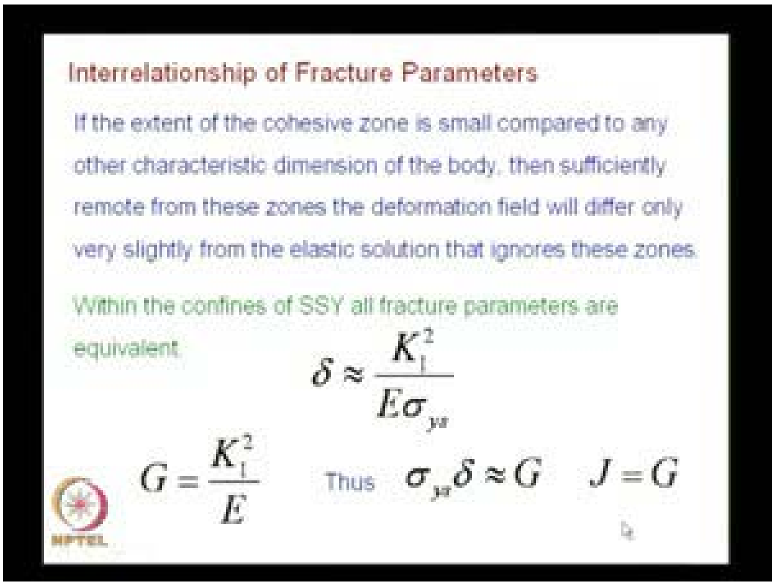


So, what you find here is J equal to sigma ys delta which you get from Dugdale yield strip model, but a generic relationship you could have a factor which could be different from one in real elasto-plastic fracture mechanics and this was also mentioned in the last class this is again discussed to emphasize that there is an interrelationship between various fracture parameters.

So, what you'll have to look at is, within the confines of small scale yielding whatever the fracture parameters that we have looked at earlier like stress intensity factor K is related to the CTOD, that is delta and we have also seen elaborately an interrelationship between energy released rate and stress intensity factor; even from this relationship you can find the identity between g and delta that is sigma ys delta equal to G and we have also recently established sigma ys delta equal to J and even otherwise energy release rate G is equal to J for a linear case.

So, within the confines of linear elastic fracture mechanics although we have seen parameters like J, G, CTOD and stress intensity factor they are all interrelated. So, this gives a sort of confidence that, though the starting point in understanding the fracture behavior is different they all converge to certain kind of interrelationship that gives there is certain internal consistency in our understanding of how the crack behaves within the confines of l e f m the mathematical development l e f m is much more refine then what you see in e p f m we have seen K 1 becomes K1c you have the fracture toughness and then you also have G becomes G 1c or J becomes J 1c on similar lines people also have identified a critical value of CTOD.

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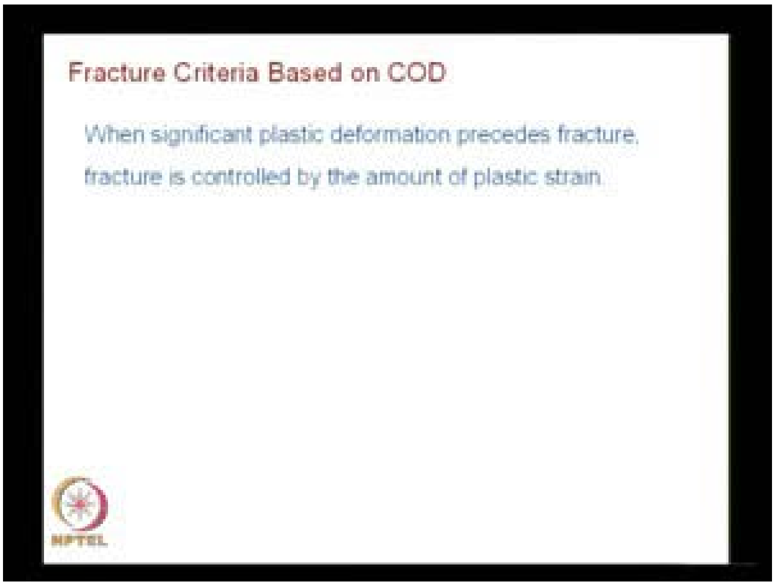


So, you can also have a fracture criteria based on COD because it is used in either way CTOD or COD and, when does this become important, when significant plastic deformation precedes fracture. Fracture is controlled by the amount of plastic strain.

See if you look at the need for CTOD people wanted to extend fracture mechanics concepts when you have plastic strain significantly develop near the vicinity of the crack.

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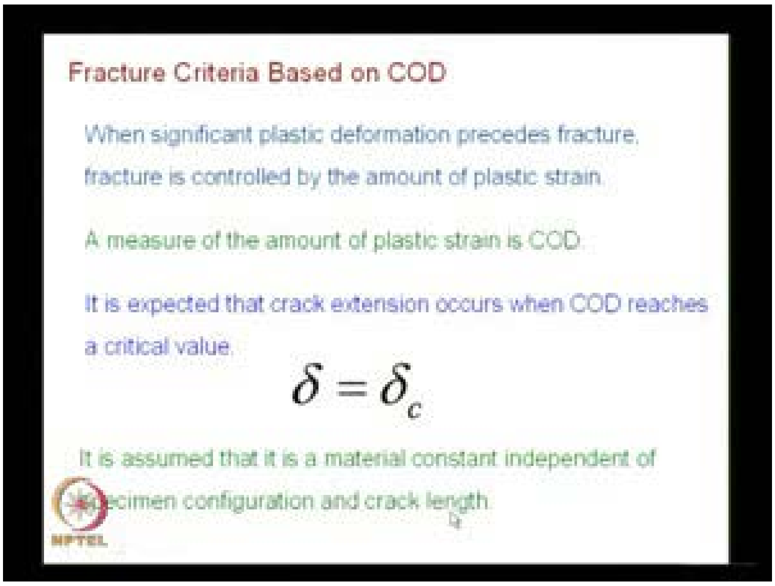


When you have larger plastic zone l e f m will not be applicable, you have to look for other alternate and what is mentioned is, a measure of the amount of plastic strain is nothing but COD, it is actually CTOD because we have looked at crack could be made to have a longer length because of the plastic deformation and the original physical crack would have a displacement because of that and that is actually CTOD.

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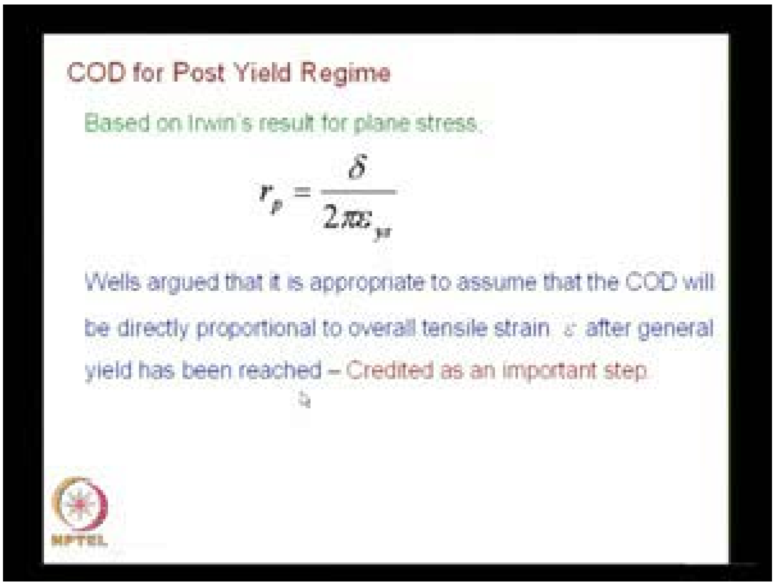

So, similar to other parameters, it is expected that crack extension occurs when COD reaches a critical value so; that means, whatever the delta the delta becomes delta c. Then fracture would commence and it is assumed that it is a material constant independent of specimen configuration and crack length. See if this assumption is not made you cannot take it for practical geometry unless delta c is a material constant you will not be able to investigate the fracture, like we have looked at fracture toughness becomes a material property under plane strain situation under suitable circumstances you can take it as a material constant and proceed with it.

And you know we can also look at certain analytical development in handling COD and this was actually done by Wells based on Irwin's result for plane stress you could get r p equal to delta by 2pi epsilon ys where epsilon ys is the strain at yield point. We have already seen an expression for r p; we have also seen an expression for delta. So, if you combined these two you could find r p equal to delta by 2pi epsilon s, all these expressions you have in your notes.

Wells argued that it is appropriate to assume that the COD will be directly proportional to overall tensile strain epsilon, the very important step, after general yield has been reached. You know if you look at fracture mechanics development people will initially start with a physical basis go to some extend and beyond that the whole analysis will become empirical. We have seen in several earlier occasions. So, here again you find there is a physical basis for CTOD as a criterion, but when it is actually implemented the lot of empirical relations come into fill up the gap.

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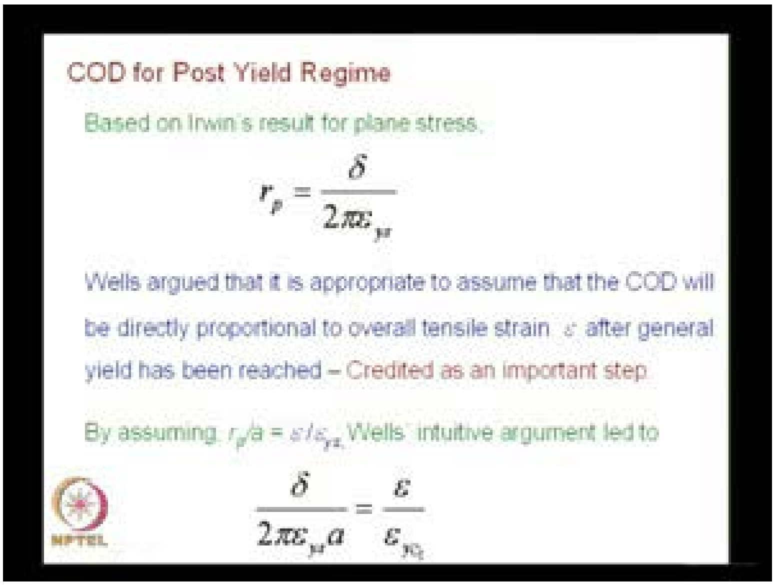


So, Wells argued that COD will be directly proportional to overall tensile strain. So, with that argument what he did was he made an assumption r p by a equal to epsilon by epsilon y s, this provided the final expression delta divided by 2pi epsilon ys a equal to epsilon by epsilon ys; this was further extended by Dews and his co-workers at the U K welding research institute and they developed the COD design curve.

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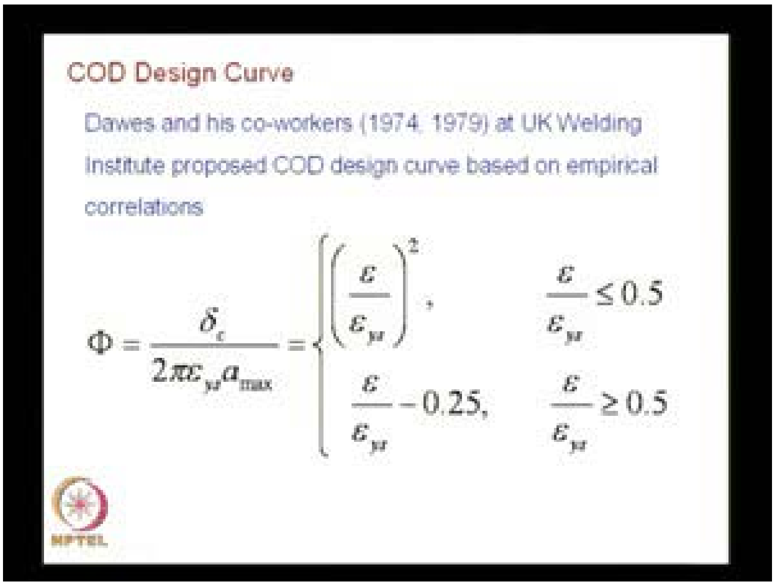


So, if you look at the physical basis the foundation was laid by Wells, but that was not sufficient you know people have to take in experimental results. So, Dews and his co-workers between the years 1974 and 1979 at UK Welding Institute proposed COD design curve based on empirical correlations.

So, you have capital phi equal to delta c divided by 2pi epsilon y sa max that is equal to epsilon by epsilon ys whole squared when epsilon by epsilon ys is less than or equal to 0.5 this quantity, is equal to epsilon by epsilon ys minus 0.25 when epsilon by epsilon ys is greater than or equal to 0.5.

See, you have to take it that this is an empirical relation. How was Paris law? It was an empirical relation, it is very useful based on the experimental results people have been able to identify, you can do a curve fitting procedure and they have obtained Paris law that was an empirical relation, on similar lines COD design curve is again an empirical relationship, but it has a physical basis. So, people also call it as several semi-empirical improvements have been made. So, partly it is a physics based then you have to depend on actual material data and then find out a empirical relationship to explain the fracture behavior.

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And you also keep in mind where J is applicable. In view of costly production techniques in nuclear industries, the behavior of the actual structures may not deviate much from material behavior of laboratory specimens. So, this luxury was there in nuclear establishment where there is close monitoring of production methods. So, if I take a sample and test, whatever the results I get that could be dependably used for actual structures. So, J is developed in the USA as a fracture criterion for materials used in nuclear installations.

On the other hand, if you look at COD, it is mainly developed in the UK at the Welding Institute which accounts for the local differences of the welded joint, thus more directed to the design of welded parts.

See, if you look at welding as a manufacturing process, it has matured now. But in the initial stages, people did not really understand the variations introduced because of the welding process because you need to have proper heat dissipation so that residual stresses are not built up after welding is done.

So, people had to learn by costly accidents. And failures have been there. So, there could be local variations because of welding that is reasonably accounted for in the COD design approach. And we have also seen that this is country based, one is US based and another is UK based, but within the confines of l e f m both are similar. In e p f m, there utility is different, but within the confines of l e f m, an interrelationship exists between all the fracture parameters.

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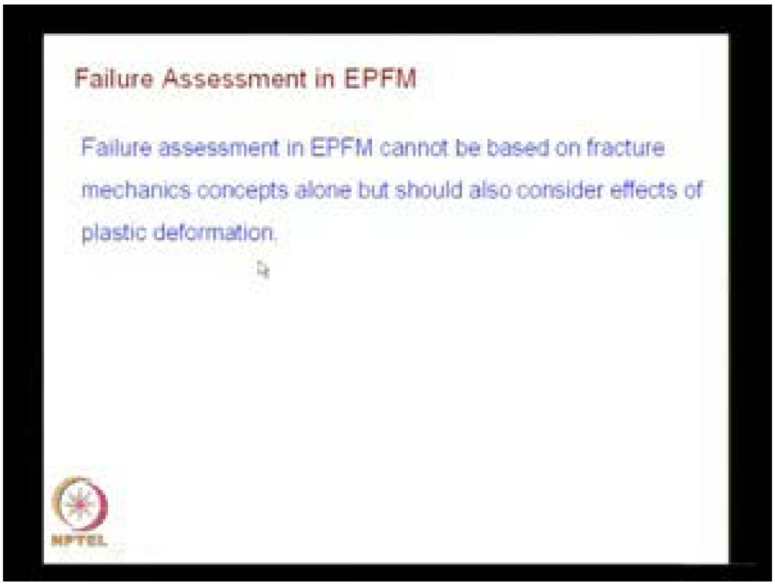


See now we move on to another important topic- how do you do failure assessment in elasto-plastic fracture mechanics. We say that we want to do fracture mechanics analysis, but when you come to failure assessment in elasto-plastic fracture mechanics, it cannot be based on fracture mechanics concepts alone. One should also consider effects of plastic deformation. This is very important. See, you should not simply apply only fracture mechanics and miss out failure due to plastic collapse.

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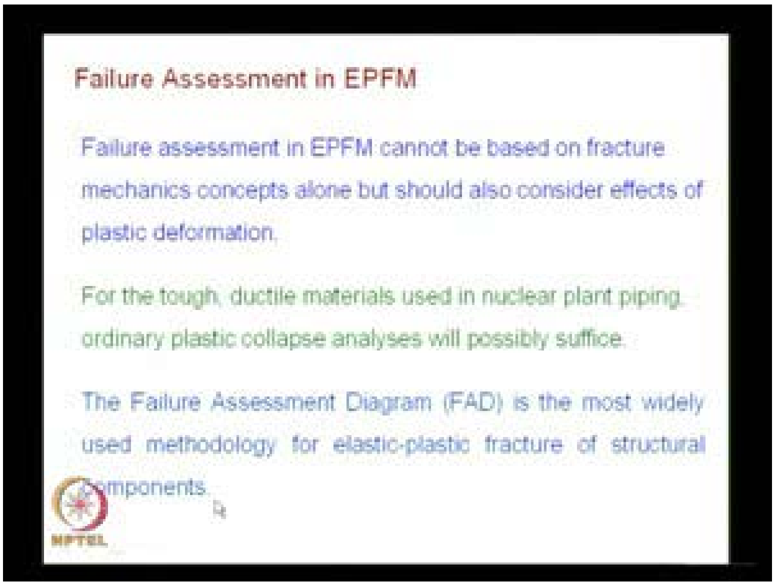


So, if you look at the nuclear plant piping which is made of tough ductile materials, ordinary plastic collapse analysis will possibly suffice. And if you look at the failure assessment diagram abbreviated as FAD is the most widely used methodology for elastic-plastic fracture of structural components.

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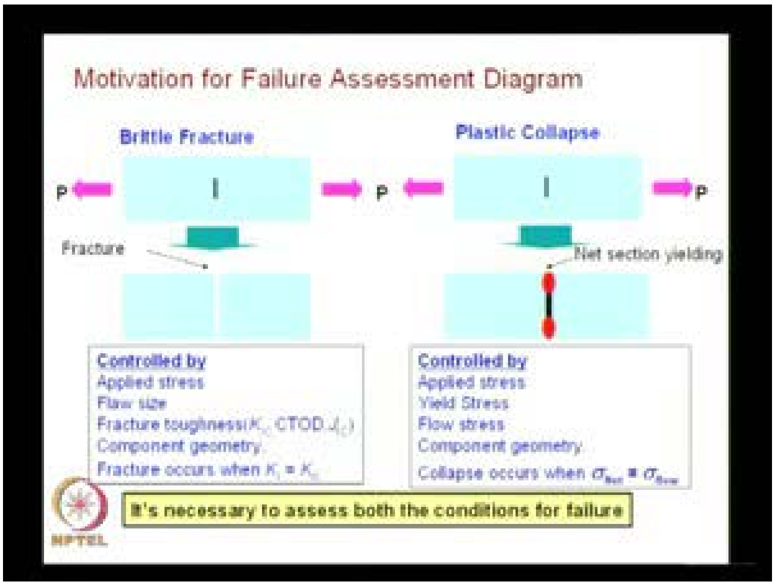


In fact, in this class, I am going to give you a flavor of it. I will give you what is the basic methodology without getting into much of the derivations. We will look at final results and apprise our self what is the utility of FAD. And what is the motivation? Suppose, I have a plate with a crack, I pull it if the conditions are such, it leads to brittle fracture and the components separates into two. There is also another important concept which is depicted in this picture that crack has grown in this direction in a self similar manner. If I have a mode one loading, that is how the crack propagates and that is what is shown. So, this is definitely a failure.

So, when I have an actual component, I would like to ensure brittle fracture does not occur. Another aspect is, if the material is very tough and also if a loading is quite significant, you may find the net section is experiencing plastic deformation. And this is known as net section yielding. Because of this, the structure may collapse. So, we will have to investigate whether fracture would occur or plastic collapse would occur. And what are the parameters that control brittle fracture; applied stress is a factor size of the flaw and what is the fracture toughness which could be expressed in terms of K1c, could be expressed in terms of CTOD, could be expressed in terms of J1c and definitely the component geometry and you would say, fracture occurs when K 1 becomes equal to K1c.

On the other hand, we go to plastic collapse, this is controlled by again applied stress, yield stress as well as flow stress .You have to look at a plasticity literature to understand what the flow stress is. Component geometry also plays a role and collapse of the structure would happen when the net section stress equal to the flow stress of the material.

So, the idea is, you have to assess both the conditions for failure. You cannot leave out one over the other. You have to investigate whether that could be plastic collapse as well as brittle fracture possible. So, keeping these issues in mind, failure assessment diagram has been created.

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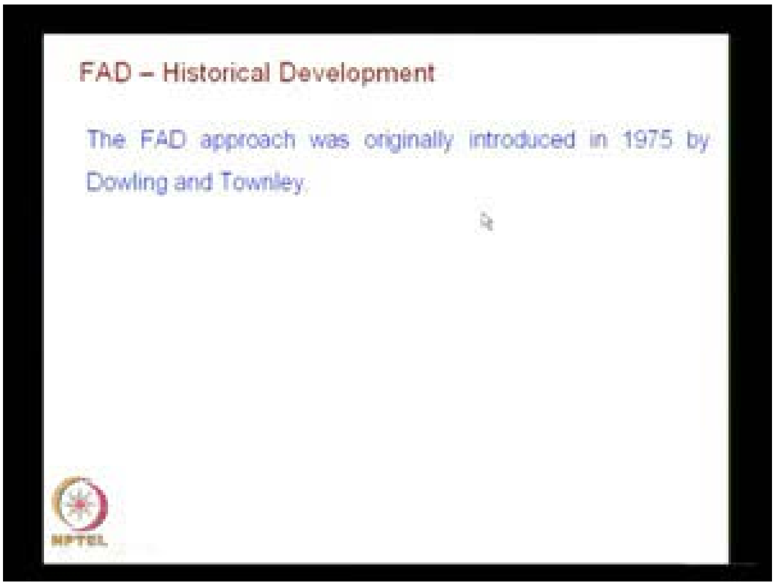


So, you have to look at, investigate and find out brittle fracture as well as possibility of plastic collapse. And the approach is like this. Before we get into the approach, we will also see the historical development. The FAD approach was originally introduced in 1975 by Dowling and Townlay. You know year is also very important. Around sixty's, you find the plasticity correction by Irwin as well as Dug dale’s yield strip model all of them came. Around sixty eight you had the J-integral concept, around seventy five they had sufficient understanding on what is a elasto-plastic fracture mechanics.

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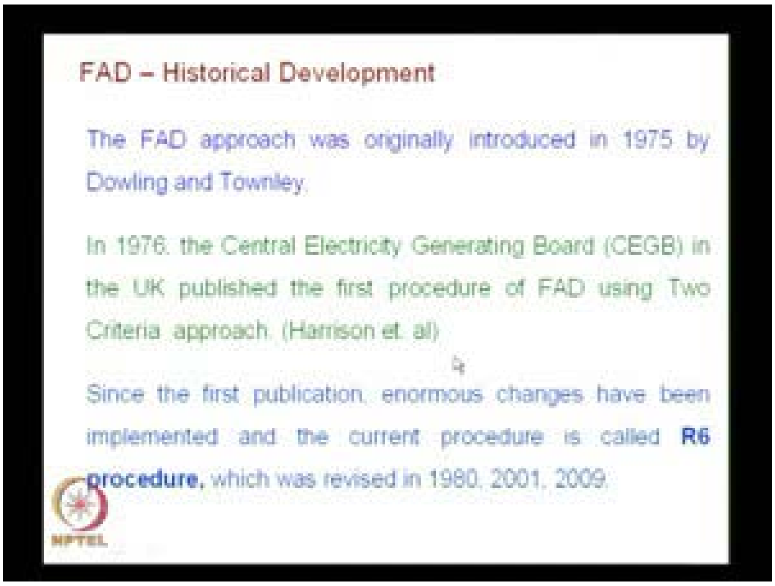


So, people looked at how to assess failure in e p f m. It is essentially driven by industries you know, in 1976 the central electricity generating board abbreviated as CGEB in the United Kingdom published the first procedure of FAD using two criteria approach. That work is credited to Harrison et al and the first procedure was based on the Dug dale yield strip model. And they have also named the procedure as R6 procedure.

Since the first publication, enormous changes have been implemented and the current procedure is called R6 procedure which was revised in 1980, 2001 as well as 2009 as more and more experimental information force in people into the procedure. So, this FAD approach helps to investigate whether brittle fracture would occur or plastic collapse would occur or will it be controlled by e p f m concepts.

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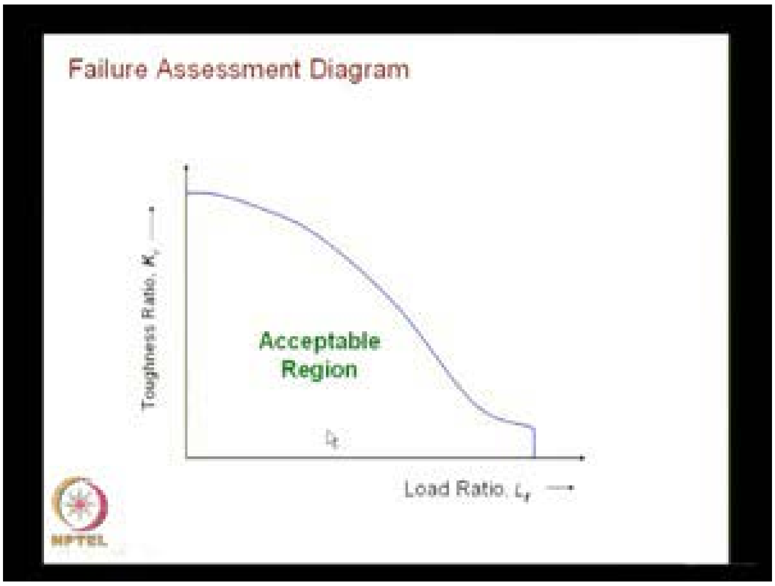


So, this whole spectrum is looked at, in this approach. And the basic procedure is as follows: You have a graph drawn between load ratio which is put as L r in the x axis and toughness ratio K r in the y axis. And you have a failure envelope which is obtained like this and if your actual structure lies in this zone below the curve into((it is an)) the acceptable region. So, that would be determined by your L r and K r values. We will see how L r is obtained and how K r is obtained. And the failure envelope is based on either yield strip model or sophisticated J-integral analysis or by empirical correlation all three you find in the literature.

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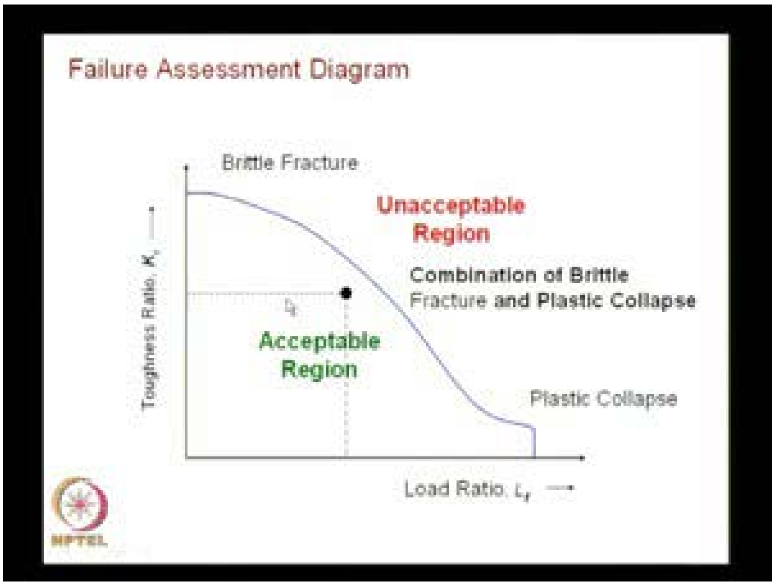


And if you are actual structure point alloys outside this region, failure would occur, that is an unacceptable region. And on this graph, you could also identified failure basically controlled by brittle fracture in this zone. When K r is quite high, brittle fracture predominates. When L r is quite high, you will have plastic collapse and in between it would be controlled by both brittle fracture and plastic collapse.

So, when L r is very high, you will have a plastic collapse and for your structure, you will like to find out what is the kind of loading that it is experience. That will be judged by your L r and K r value and if the point lies below the curve, then it is safe. Now the question is how to get the failure envelop. How to define L r, how to define K r.

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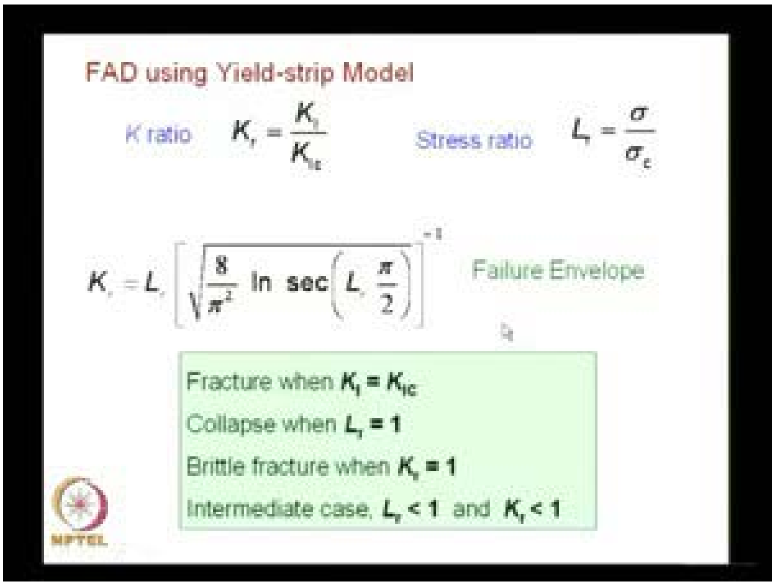


Simplest approach is FAD using yield strip model. You define K ratio as k1by k1c is purely l e f m based. And you have stress ratio L r as sigma by sigma c; sigma c is the critical stress it could be a flow stress. And the failure envelope is basically obtained from your yield strip model; this provides interrelationship between K r and L r.

So, K r equal to L r multiplied by square root of 8 by pi square you have the natural logarithm and secant L r pi by 2 and the whole power minus 1. And what happens here is, fracture when K 1 equal to K1c that is obvious; that means, brittle fracture would occur when K r equal to 1 and collapse would occur when L r equal to 1. Here we are not really considering strain hardening when I am using the yield strip model. And you have intermediate case when L r is less than 1 and K r is less than 1.

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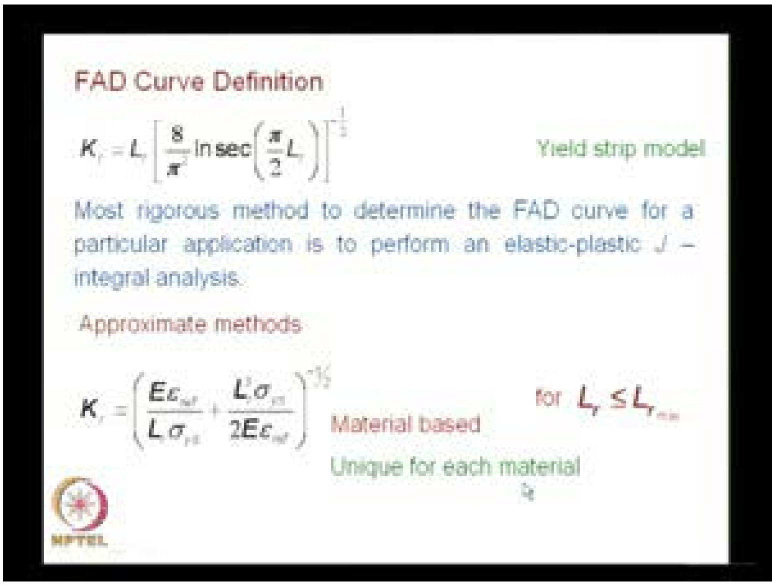


And people have really looked at various types of failure envelopes that is the FAD curve definition. You have this as K r equal to L r 8 by pi square natural logarithm of secant pi by 2 L r whole power minus half based on the yield strip model. The most rigorous method to determine the FAD curves for a particular application is to perform an elastic-plastic J-integral analysis. You know that would be very expensive. So, because it is going to be very expensive, people wanted to have certain generic approaches though they started with the yield strip model, people also have reported approximate methods and this definition of the failure envelope where K r equal to e into epsilon reference divided by L r multiplied by sigma ys plus L r cube sigma ys divided by 2 times young's modulus epsilon reference whole power minus half. And this curve is unique for each material; that means, for different materials, you need to have different FAD curve.

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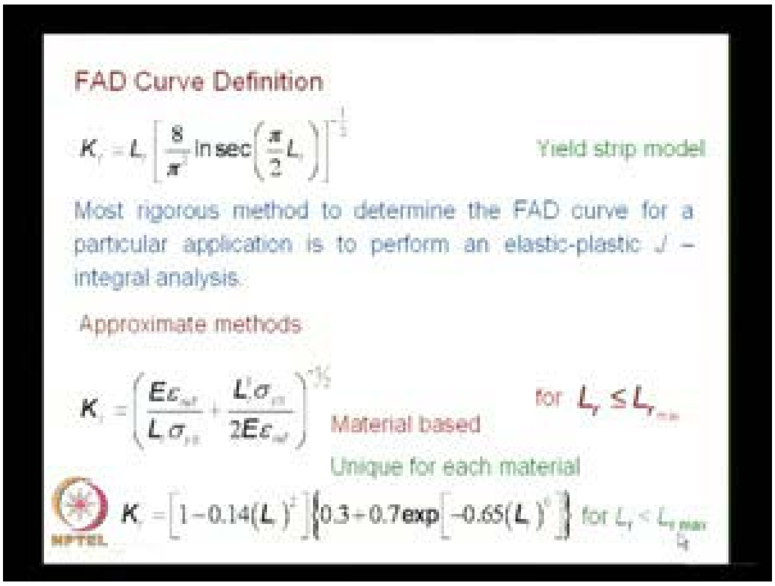


And there is also another model which is purely empirical which gives K r equal to one minus 0.14 L r square multiplied by 0.3 plus 0.7 exponential minus 0.65 L r whole power 6; for L r is less than L r max and likewise you have other empirical relations quoted in the literature. See the empirical approaches have found rated acceptance in applying fracture mechanics to actual structure components. Because the problem domain is so complex people have found by performing suitable experiments and collecting data doing a curve fitting exercise is lot more profitable and that is seen in fad approach also.

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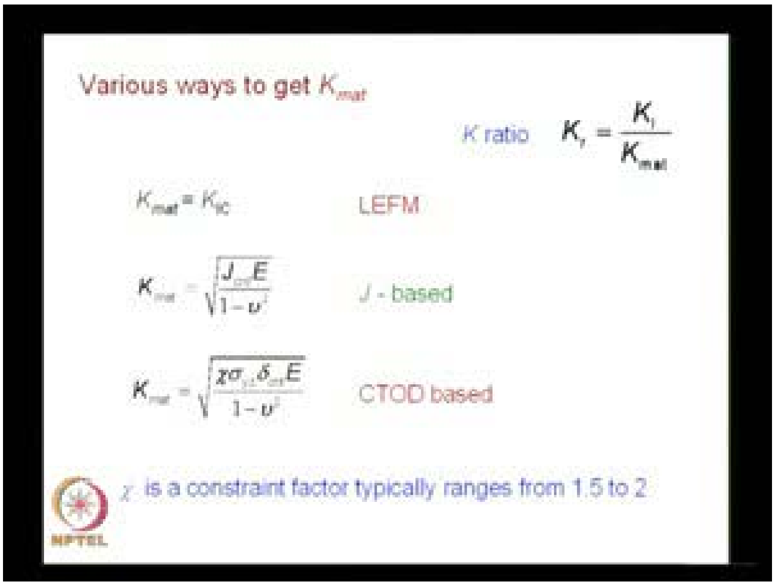

And to refine the analysis, the K r and L r definitions are suitably taken. There are many ways to get what is K r. K r is generally represented as K 1 divided by K of material. We had seen earlier, the denominator is K1c that is only when you're applying concepts of l e f m. So, the whole analysis could be refined if you replace the definition of K material; if you have a J critical for the material from J based, you put calculate the K material. You use the interrelationship and then find out K and you could also find out K material based on CTOD.

So, K material is given as square root of chi into sigma ys delta critical multiplied by e divided by 1 minus nu square, where chi is a constraint factor typically ranges from 1.52. So, what they do is if you're doing a l e f m analysis, you simply say take K material as K1c. So, if you want to refine your analysis, modify this definition based on e p f m parameters like J 1 c or delta K.

So, what they find is, if you are able to get the failure envelope and if your assessment point lies below that, then it is safe if it is above that, then you will have to find out how to improve your design. So, this gives a flavor of what is the kind of failure assessment that people do in elasto-plastic fracture mechanics analysis.

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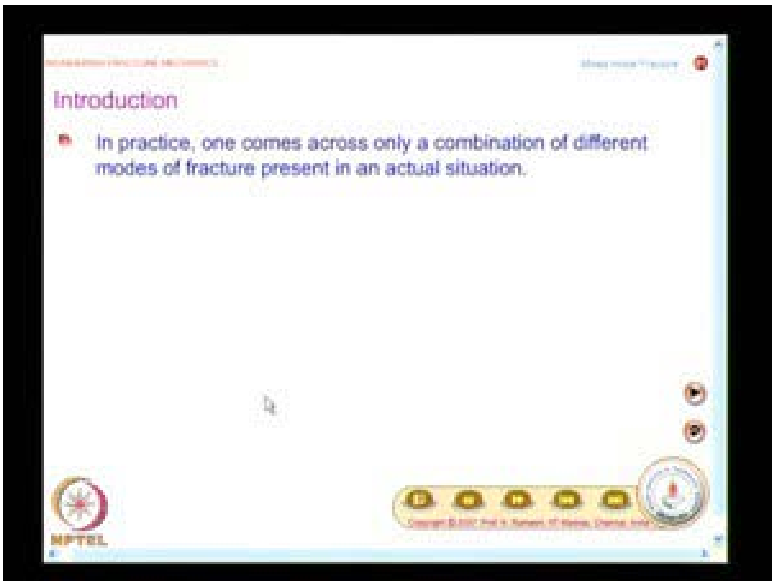

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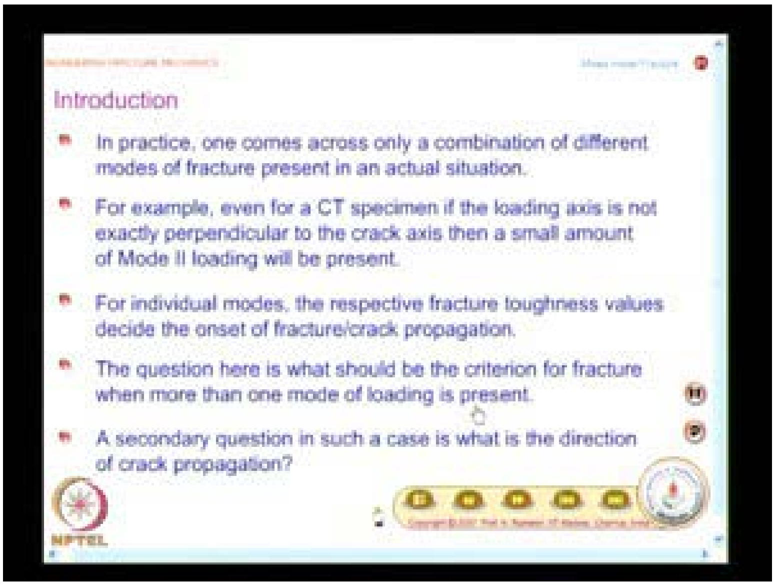


Now we move on to our next topic mixed mode fracture. See we have paid quite a bit of attention on mode one analysis, but that is not the way actual problems co[me]- that you come across have only one mode one loading. You will always have combination of different modes of fracture in an actual situation. For example, even for a compact tension specimen, if the loading axis is not exactly perpendicular to the crack axis, there are small amount of mode two loading will be present.

And we have seen in the case of mode one when K 1 becomes K1c catastrophic failure would occur. Similarly for individual modes, the respective fracture toughness values decide the onset of fracture or crack propagation. But the question is, if I have a combine loading what should be the criterion for fracture when more than one mode of loading is present.

So, this is the first question. The second question is in which direction the crack would propagate; that is also equally important. Only if you understand all of these, then you can confidently say we have understood what happens near the vicinity of the crack. And here again we will go back and borrow what we have been doing it in conventional design approaches. We simply perform one tension test, find out the yield strain on that you device the failure criteria. You have the Tresca yield model and Von Misses yield model. And you had even for simple problems, you had different failure theories.

So, when you come to fracture mechanics; obviously, the failure theory is will be meaning you have to anticipate that. And here again what we will look at is, initial methodologies would be based on a physics, but later empirical analysis have come to stay.

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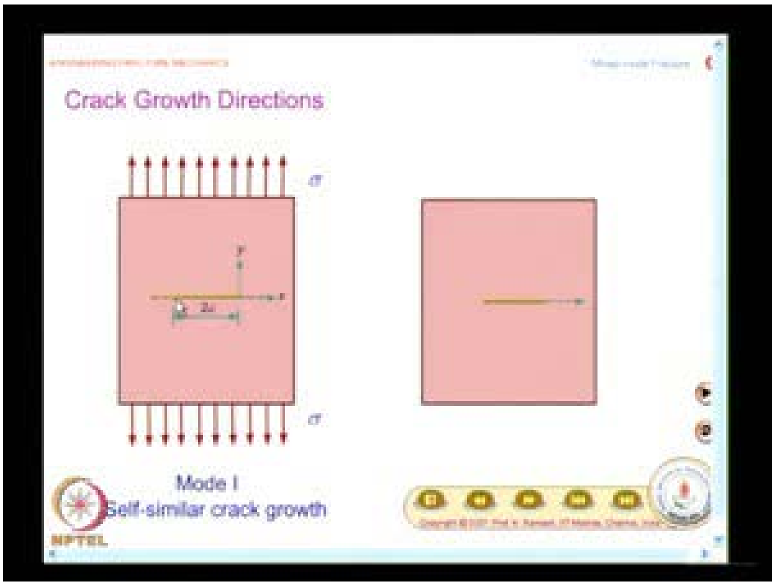


So, that is the way we will look at it. So, we will try to find out what should be the criterion that I should use. And the secondary question is what is the direction of crack propagation. And we have to understand how crack grows when it is subjected to different types of loading. I have a crack, center crack subjected to uniaxial tension here and you find when the loading is sufficiently increased, the crack grows which was like this horizontally like this. This is known as self similar crack growth. You need to understand this terminology. When a crack is subjected to mode one loading, you have self similar crack growth.

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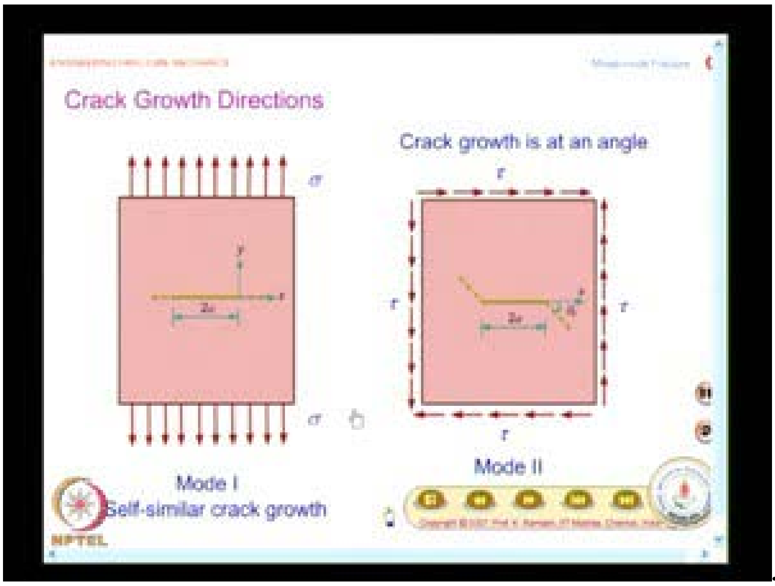


Suppose I take the same panel and apply pure shear how does the crack grow. The crack grows at an angle; it does not propagate in a self similar manner. So, you have to realize depending on the kind of loading, the crack growth direction can define.

So, in the case of pure mode one and if I have an isotropic material, I have self similar crack growth. When I have a mode two situation, the crack grow certain angle. So, my theory should be able to predict this angle precisely. Then I will accept the theory. That is the way you have to look at it. The question what we are going to look at is, suppose I have a combination of mode one and mode two how will I judge when the crack would become catastrophic, in which direction the crack will grow, what way I can get this kind of an answer? One of the simplest approach is we will go to the energy method.

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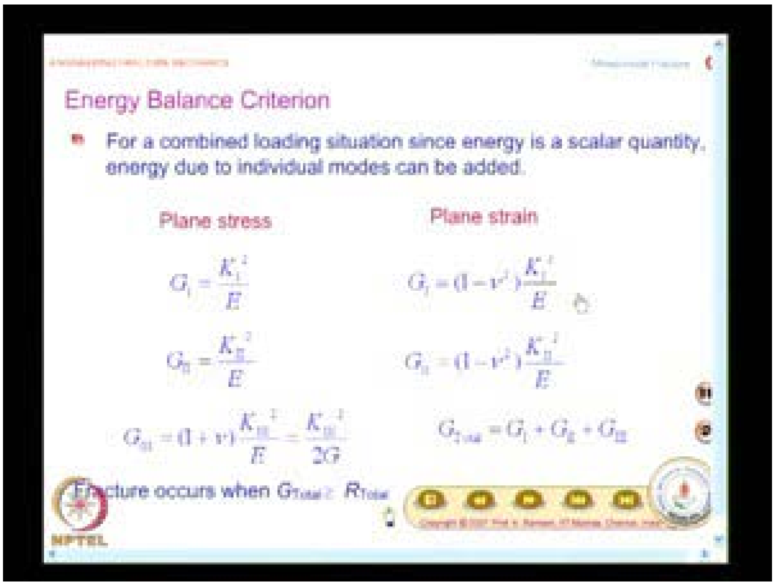


So, you do the energy balance criterion. For the combine loading situation, since energy is a scalar quantity, energy due to individual modes can be added. We have all these expressions. You have expressions for G 1, G 2 and G 3. It is separately written for plane stress, this separately written for plane strain.

So, what I could do is I could simply say some all these individual energies. There should be a critical value for the total energy and that could decide when the fracture would occur. And if you look at even for this interrelationship which we have obtained, it assumes coplanar crack extension. So, even Griffith when he propounded this energy release rate, he found when you have a combine modes, this may not be sufficient to explain the next mode situation.

As long as the crack growth is coplanar this theory is  so. When the crack growth is going to be coplanar there is no question of which direction the crack will grow. The direction of crack growth is already known. In certain specific applications, you can precipitate coplanar crack extension even in mixed mode. Suppose I have a adhesively bonded join so, that join decides the crack propagating direction, but you may have combined mode loading or I have side groups like what we had seen in the case of chevron notch, you have made the particular plane we care.

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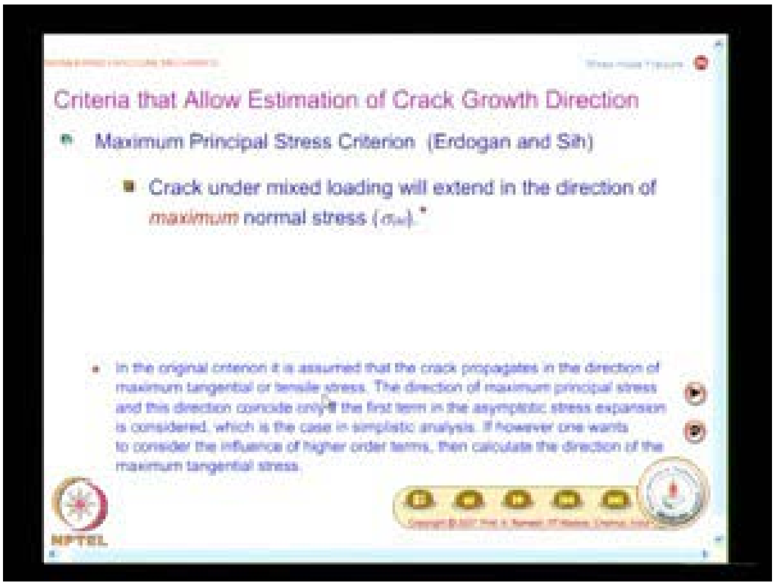


So, the crack growth will happen in that plane and you also have criteria that allow estimation of crack growth direction from your calculations. And one of the theories is advanced by Endogen and Sih. It is listed as maximum principle stress criterion, but if you really look at the original criterion, it is assumed that the crack propagates in the direction of maximum tangential or tensile stress. See we have looked at Westergaard stress field equations. We also looked at William stress field equations. From the Williams approach, we have directly got sigma r r sigma theta theta and tau r theta. And what this criterion says is, it is also famously known as maximum tangential stress direct criterion. So, that is called as m t s criterion. The direction of maximum principle stress and this direction coincide only if the first term in the asymptotic stress expansion is considering.

We have already seen, though we have developed the multi parameter solution, we set for all our fracture discussion; we would confine our attention to the first term in the series. When I consult only the first term, this boils down to principle stress, maximum principle stress. So, that is why this is put as maximum principle stress criterion. But a generic terminology is maximum tangential stress criterion. And in the case of simplistic analysis, the maximum principles of directions and the direction of maximum value of sigma theta theta coincides.

If however, one wants to consider the influence of higher order terms, then calculate the direction of the maximum tangential stress. So, you have to calculate that. So, you will have dow sigma theta theta divided by dow theta equal to 0. From that you find out what is the direction of maximum tangential stress and then do the calculation, but we would see for first term it boils down to maximum principle stress direction.

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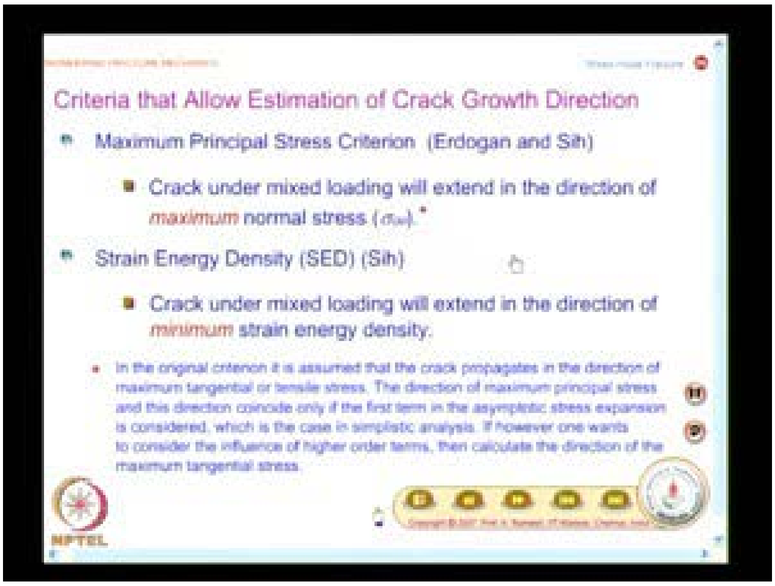


So, this is one of the criterions. The other criterion is known as strain energy density which is abbreviated as SED and its due to Sih. And this criterion says, crack under mixed loading will extend in the direction of minimum strain energy density. In the case of m t s or maximum principle stress criterion, crack under mixed loading will extend in the direction of maximum tangential stress or the principle stress in the simplistic case and it is very easy to see. You will be able to get an expression for theta so elegantly.

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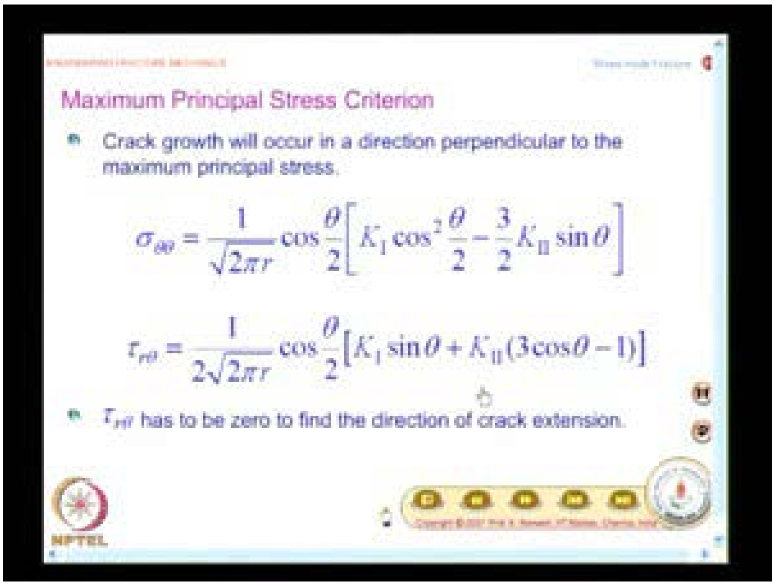


And we look at the details of it, you have this expressions already in your note book nevertheless you can write it again. In terms of polar coordinates, you have the stress components and I have sigma theta theta is given as 1 by root of 2pi r cos theta by 2 multiplied by K 1 cos square theta by 2 minus 3 by 2 K 2 sin theta.

And your shear stress tau r theta equal to 1 by 2 into root of 2 pi r cos theta by 2 K1 sin theta plus K2 3 cos theta minus 1 and since I say that I am going to consider only the first term in the series, I am really looking for tau r theta has to be 0 to find the direction of crack extension. Otherwise, I will take dow sigma theta theta divided by dow theta equal to 0 from that you proceed. This gives me a very important relationship for me to find out the crack growth angle.

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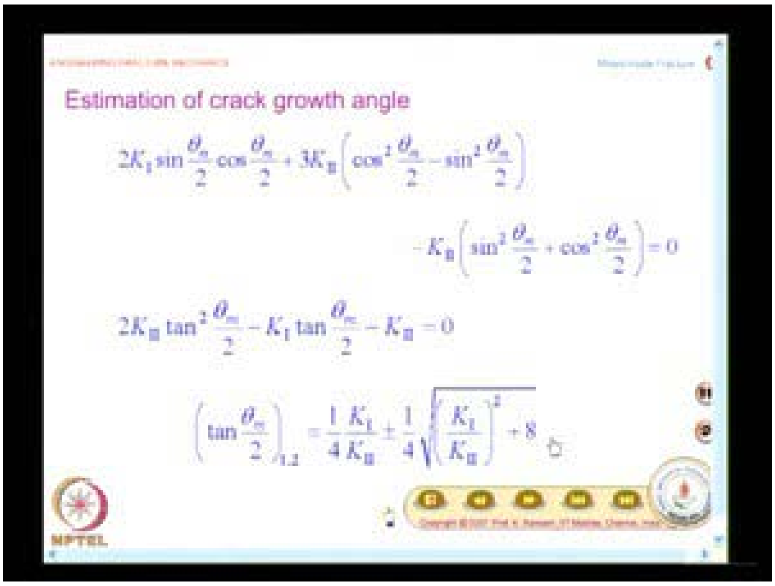


So, what I have here is, let theta b theta m at which crack grows for me to have shear stress to 0 I get a equation K 1 sin theta m plus K 2 3 cos theta m minus 1 should be equal to 0. You could solve for theta m directly and this is done in a slightly different fashion. We express sin theta m as 2 sin theta m by 2 into cos m by 2, likewise you expand cos theta m, you get an expression involving these terms. You have this as 3 K 2 cos square theta m divided by 2 minus sin square theta m by 2 minus K 2 sin square theta m by 2 plus cos square theta m by 2 equal to 0 which could be simplified to 2K2 tan square theta m by 2 minus K1 tan theta m by 2 minus K 2 equal to 0. And this gives me an expression to get the value of theta m. I have tan theta m divided by 2 equal to 1 by 4 K1 by K2 plus or minus 1 by 4 square root of K1 by k 2 whole square plus 8.

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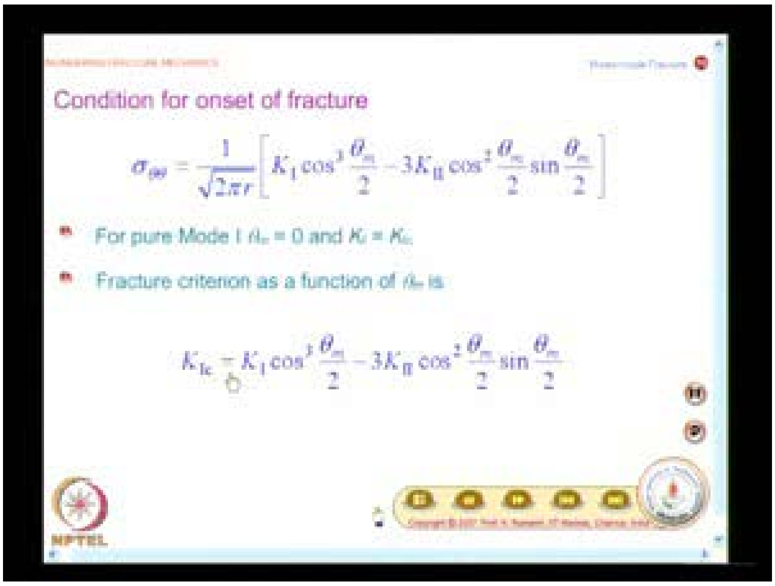


So, in a given problem, if the individual stress intensity factors are known, the m t s criterion provides you how to calculate the possible crack growth angle. The second question is, when the fracture will occur because I need to know how to find out the onset of fracture.

So, that comes from substituting in your expression for sigma theta theta, theta equal to theta m. For a pure mode 1 case, theta m equal to 0 and K 1equal to K1c. So, in a given problem when this quantity that is K1cos cube theta m divided by 2 minus 3 K 2 cos square theta m by 2 into sin theta m by 2 reaches the fracture toughness in mode one, fracture would occur. do you find a similarity between mixed mode loading in the case of simple analysis suppose I have a combine loading situation, tension, torsion and as well as bending present, you go to the concept of principle stresses and you combined them in an appropriate fashion in Tresca or Von Misses yield criteria and investigate what happens in a simple tension test and find out the failure value and then access whether yielding would occur or not in conventional analysis.

Similar idea is extended here. You have to find out K 1 and K 2 for a given problem, but the fracture toughness for K1c that is mode one loading is taken. So, in this class what we had looked at is, we looked at the interrelationship between J and CTOD, some details of the derivation, we have looked at then we have looked at the COD design curve I said it is purely based on empirical relationship. Then we moved on to how to assets failure in elasto-plastic fracture analysis; in that we are looked at rudiments of FAD, failure assessment diagrams. Then we moved on to mixed mode fracture, we have looked at what is the self similar crack growth and how does the crack grow under mode two situations. Then we looked at two theories; one is based on energy balance criterion, another is on maximum tangential stress criteria. We look at the other theory in the next class.

Thank you.