Engineering Fracture Mechanics

Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

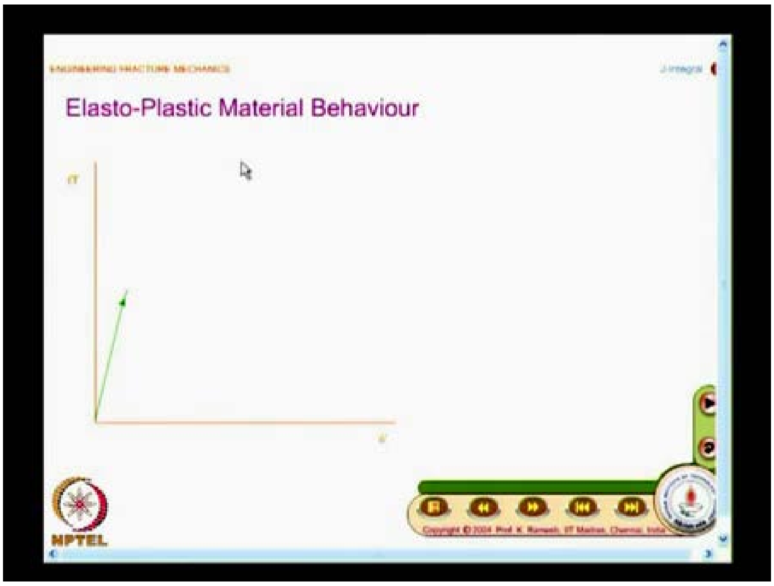
Module No. # 08

Lecture No. # 38

HRR Field and CTOD

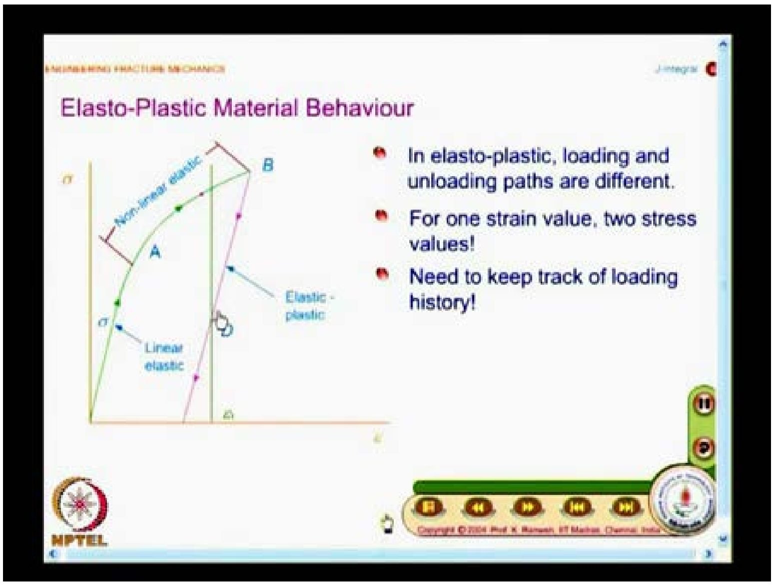
Let us continue our discussion on j-integral and in this class I would also try to cover concepts related to CTOD and if time permits, we will also look at FAD failure assessment diagram which will have to look at when you are talking about elasto-plastic fracture mechanics.

(Refer Slide Time: 00:38)



And one of the very important aspects that you will have to keep in mind is; when you have an elasto-plastic material behavior, you will have to realize, there is a difference between non-linear elastic behavior and elasto-plastic behavior.

(Refer Slide Time: 00:52)



When you are having the system as non-linear, if you load and if you unload, the loading and unloading path would remain same. What makes elasto-plastic analysis difficult is that when you unload, the unloading path is different.

This is one of the difficult aspects. This needs to be addressed and whenever there is a crack propagation, crack growth, you would have definitely unload.

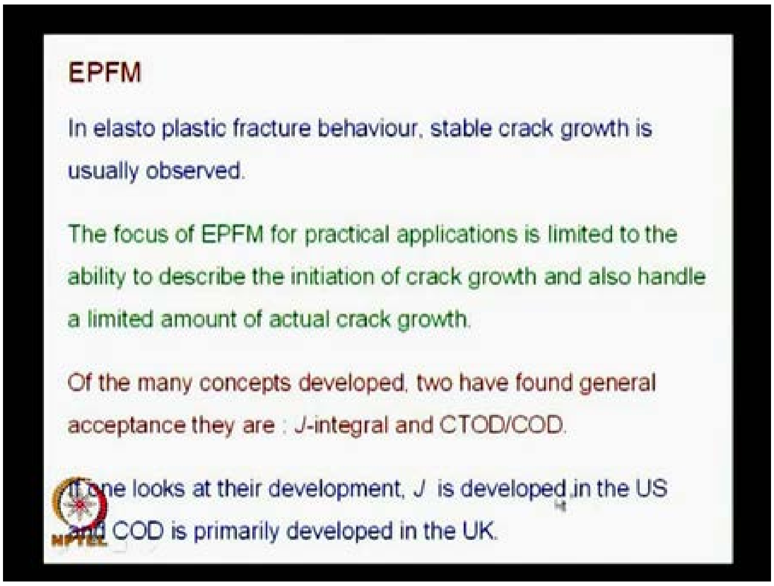
So, what you find here is for a particular strain value you have 2 stress values. So, unless you keep track of the strain history, it is not possible for you to know what is the aspect of the material behavior you are talking about. This makes elasto-plastic analysis difficult.

(Refer Slide Time: 01:59)

In elasto-plastic fracture behavior, stable crack growth is usually observed. I had mentioned, when there is crack growth, there is unloading. So, what people have put as their focus in EPFM was, for practical applications, it is limited to a ability to describe the initiation of crack growth, as long as you are able to achieve it, the purpose is satisfied and also handle a limited amount of actual crack growth.

So, this is what we want to keep that as a focus in EPFM. Even these questions are answered, you are quite happy with it. And if you look at, many concepts have been developed. Of these two have found general acceptance and they are J-integral and CTOD or COD and there is also a history behind it.

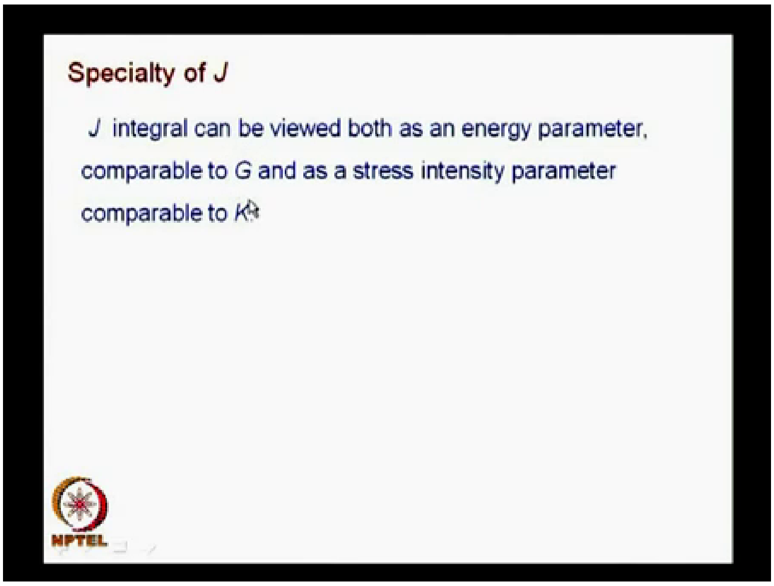
(Refer Slide Time: 03:15)



If you look at that development, J is developed in the US and COD is primarily developed in the UK. So, these concepts are also country specified because the kind of problems people were facing that prompted them to arrive at methodologies to provide answers further questions.

So, people have looked at it differently and we would also see within the confines of a linear elastic fracture mechanics. All these parameters are one and the same. That gives you some kind of a satisfaction that we are proceeding in the right direction.

(Refer Slide Time: 03:59)

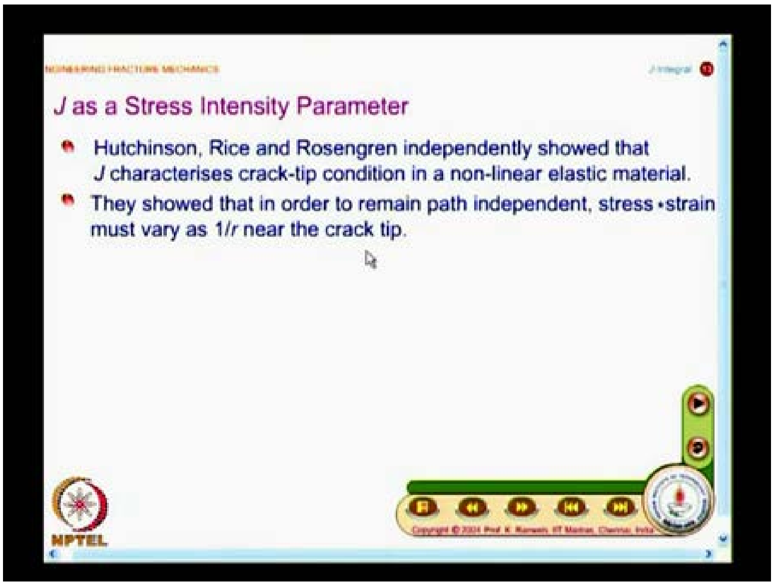


Those concepts also we look at in this class. And what is the specialty of J. See if you look at linear elastic fracture mechanics, you had a concept called G and you had a concept called K. G was look that as an energy release rate and K was a stress intensity parameter, stress intensity factor.

The specialty of J is that it can be viewed both as an energy parameter comparable to G and as a stress intensity parameter comparable to K and in fact, we had looked at in the last class; the energy definition of what is J.

We are looked at in the case of a non-linear elastic solid. It is very similar to what we see as g, and in the case of linear elastic solid G and J are identical. When we go for elasto-plastic analysis, we are very careful that unloading does not take place or you bring in certain approximations to use it. So, now what we will do is, we will go and see how J can be used as a stress intensity parameter.

(Refer Slide Time: 05:24)



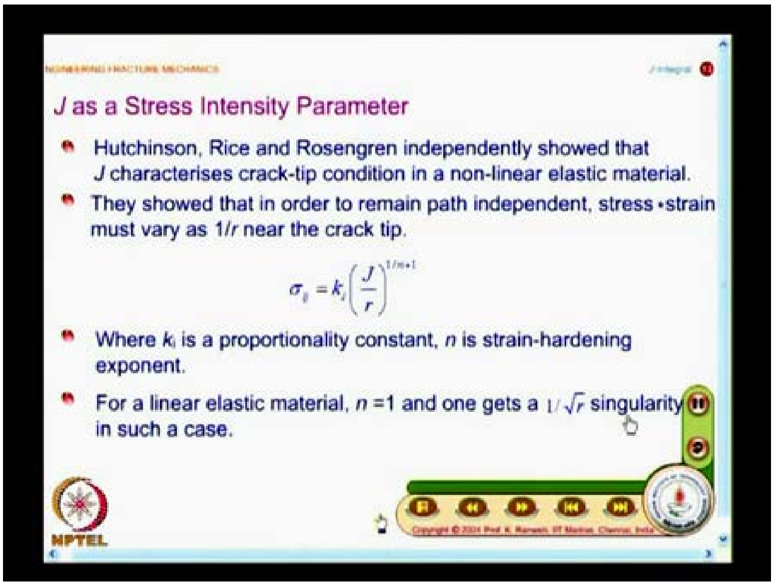
And here again you know, people have started from non-linear elastic solids. So, you have papers by Hutchinson and there was also a paper by Rice and Rosengren. They have independently showed that J characterizes crack-tip condition in a non-linear elastic material. You know it appears Hutchinson and Rice were UG classmates.

So, they came out with similar theories for handling fracture problems, such incidences or such information also adds life to the discussion on fracture mechanics. And we have already seen that J is path independent.

They show that in order to remain path independent, the product of stress and strain, it is put as strain star strain; it is a product of stress and strain must vary as 1 by r near the crack-tip. So, the kind of singularity in the case of elasto-plastic fracture mechanics is different from linear elastic fracture mechanics.

In linear elastic fracture mechanics, you had the famous root r singularity Here, the strain hardening index also will play a role.

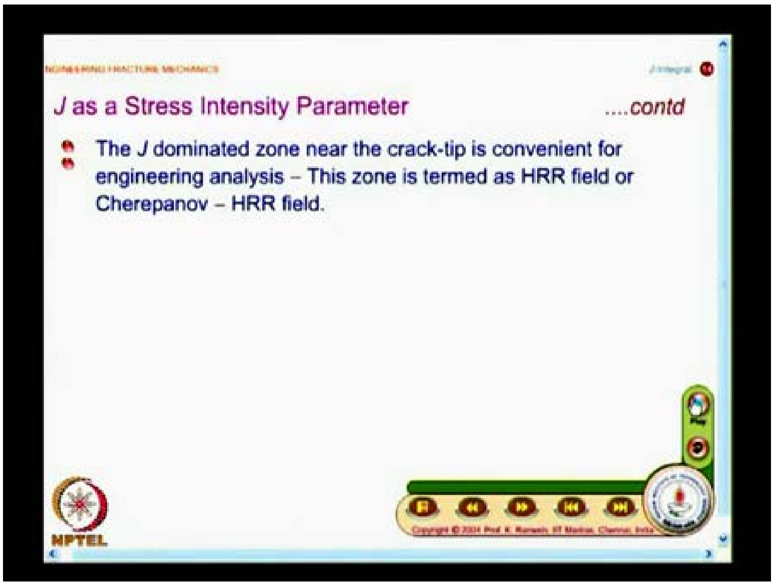
(Refer Slide Time: 07:01)



We will see how it is. Based on this premise, they have arrived at an expression for stress component. It is sigma i j; it is given as kj multiplied by J by r whole power 1 by n plus 1. And Ki is proportionality constant; n is strain-hardening exponent. See if you recall, when we had looked at Dugdale's model, Dugdale model considered elastic perfectly plastic. It was not considering strain-hardening at all.

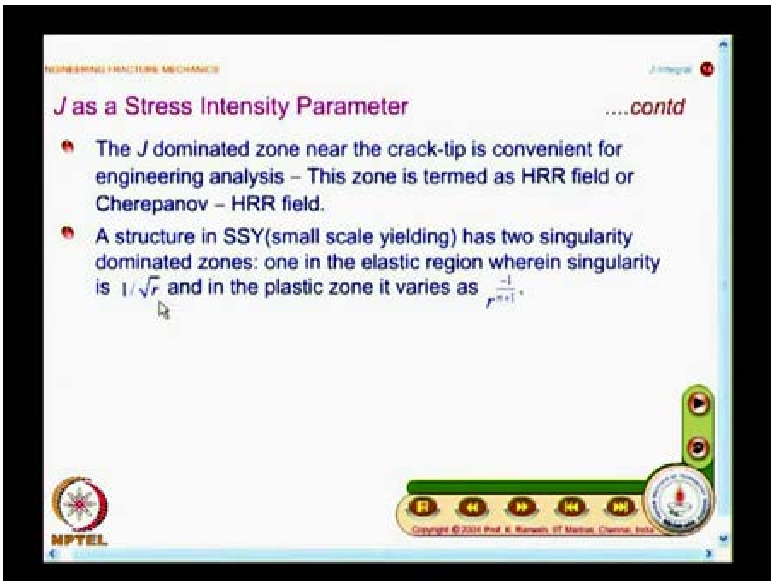
Once you come to J, people have looked at a different type of material model and you do take care of the strain-hardening aspect. And for a linear elastic material, n is equal to 1 and one gets a 1 by root r singularity, in such a case, because, whenever you come out with a new result you want to go back and see whether the earlier results are obtainable from the generalized form. That way you accept that generalization is proceeding in the right direction. It is an indirect check that the mathematics what you have developed is consistent, there are no contradictions.

(Refer Slide Time: 08:42)



And you know, like we had seen K dominated zone in a LEFM in EPFM people have looked at J dominated zone and this is found to be convenient for engineering analysis because there are approximations involved and this zone is termed as HRR field or Cherepanov HRR field. That is, because Hutchinson Rice and Rosengren you have this HRR. Later Cherepanov also arrived at similar relations. So, in order to give credit to all these investigators, people calling it as HRR field or Cherepanov HRR field, but famously know as HRR field.

(Refer Slide Time: 09:48).

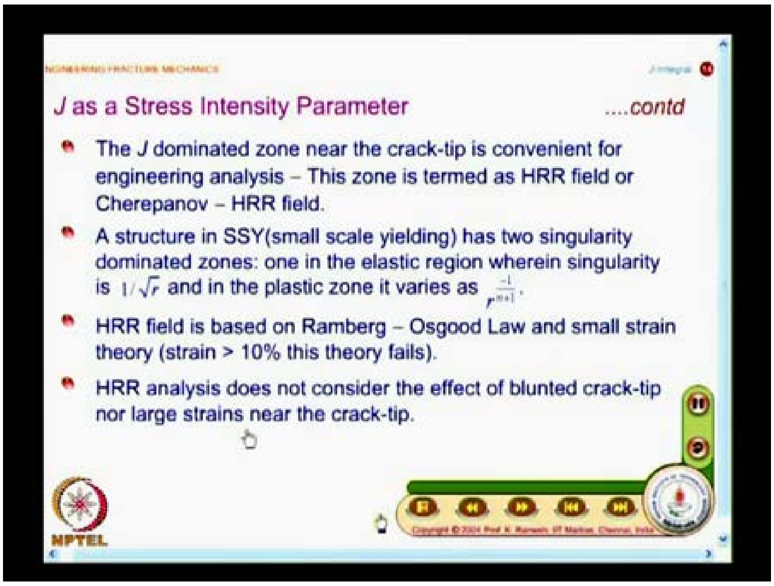


If we look at a structure in SSY (small scale yielding) now has two singularity dominated zones, one in the elastic region we would also see a pictorial representation of how HRR field looks like, there you would be able to see a K dominated zone and a J dominated zone.

So, in the elastic region singularity is 1 by root r and in the plastic zone it varies as r power minus 1 by n plus 1. So, this is the first learning, the moment you come to elasto-plastic structure analysis, singularity also has a new representation, the strain-hardening exponent comes into play. So, it is not only root r, root r goes up to some extent, then you have a zone dominated by J and there is still a fracture process zone where we know very little information.

It is not that EPFM as solved up to the crack-tip, it’s only slightly away from the crack-tip you are able to account for little more plasticity than what you had encountered in linear elastic fracture mechanics.

(Refer Slide Time: 11:30)



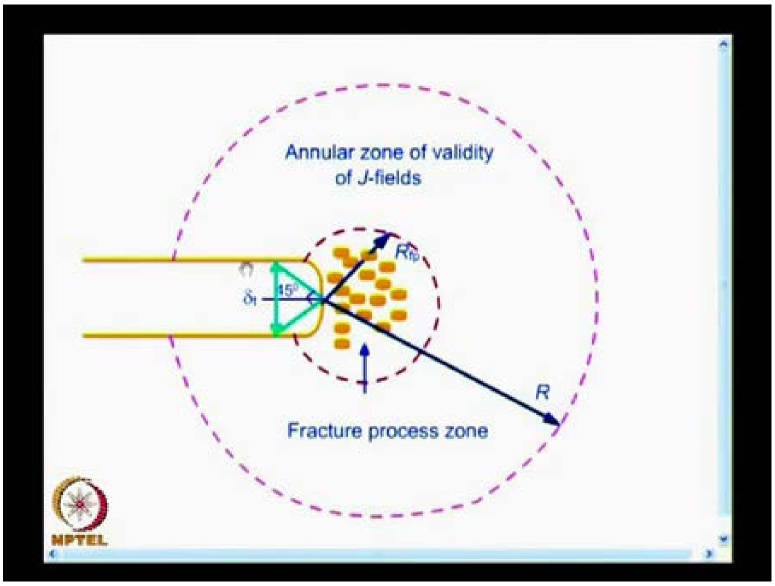
And I had mentioned that they had taken a constitutive model to represent the plastic behavior and HRR field is based on Ramberg -Osgood law and small strain theory.

Strain greater than 10 percent this theory fails; see we will have to keep in mind when you have 0.2 percent strain the material has tilted. So, 10 percent is still large though we say it is a small strain theory 10 percent is not a small value when you are looking at strain.

But you have to know at this is limited, you are not talking about larger strains than 10 percent that is a way you have to look at it still accounts for larger plastic zone .But restricted to small strain approximation. see while we look at linear plastic fracture mechanics we had this root r singularity and we said because of very high stresses near the crack-tip, the crack would invariably blunt that was not taken care of in LEFM, the moment you come to elasto-plastic fracture analysis also the blunting of the crack-tip is not accounted for.

So, even in the HRR analysis the effect of blunted crack-tip nor large stains near the crack-tip is accounted for, but nevertheless it is a very useful approach people also compare this to boundary layer theory in fluid mechanics.

(Refer Slide Time: 13:37)

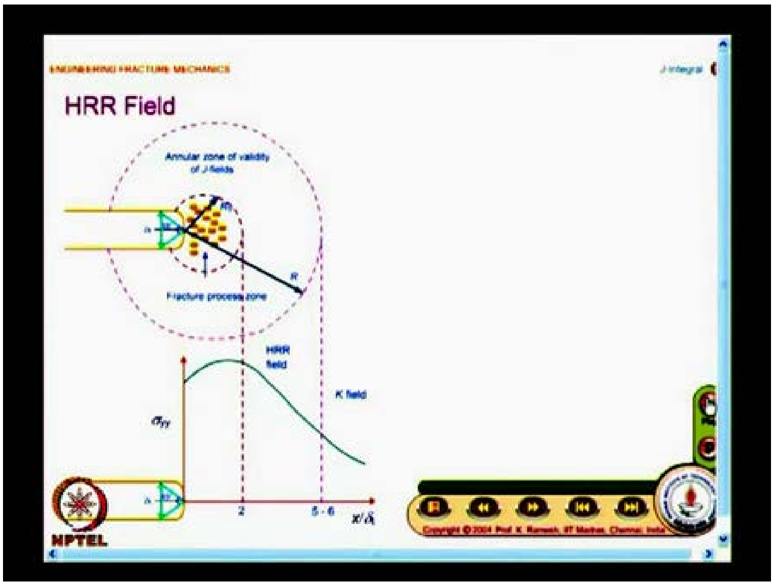


And we would see how the HRR field looks like. A pictorial representation and you know when you are talking about EPFM, the concepts related to J and CTOD will go hand in hand and what you have here is a blunted crack-tip shown in very large magnification. immediately next to the crack-tip is a zone where we have a very little knowledge it is labeled as fracture process zone, after this zone you have a annular zone of validity of J fields, this is a schematic, this is conceptually trying to show there could be different zones identified near the crack tip, do not conclude that this is circular in shape it may vary from problem to problem and you have to do sophisticated EPFM calculations to get the size and shape of the zones.

And within the crack-tip you find some lines are drawn and if you look at from the center of this you have lines drawn at 45 degrees they are mutually perpendicular they hit the crack and this height is taken as delta t. You would also see that when we look at what is CTOD and what is represented here, is the crack-tip opening displacement because once you come to EPFM these concepts go together, you know you have to go back and forth we are not seen CTOD in greater detail and if we look at historically it was developed first, followed by J.

But my presentation I am discussing J first and go to CTOD. So, now, you have looked at a simple definition of what is CTOD from the picture and we would see later expressions for it then you come back.

(Refer Slide Time: 16:06)



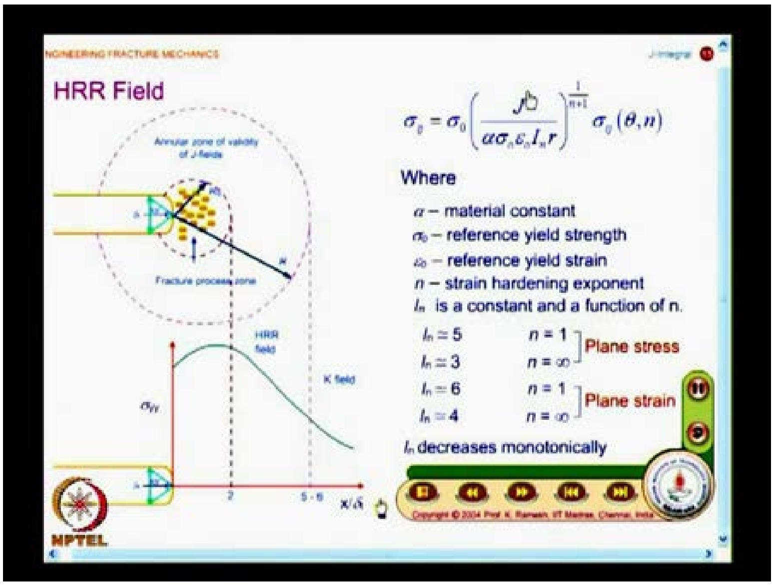
So, J and CTOD they go together you cannot discuss them independently and you also have another graph associated with it how does sigma y; y varies in this zone and what is put here is, along the x axis you have x divided by delta t.

So, it is written in terms of the CTOD. So, for 2 times CTOD you have the fracture process zone that is what is given as 2. So, between 2 and about 5 to 6 is the HRR field, after HRR field you have the k field after K field you will have a general stress field.

So, this is the way that you have been able to picturise the field information in elasto-plastic analysis. At the crack-tip the value of sigma y is not as what you would get when it is a short crack because if you look at HRR field also this also asymptotically goes to infinity at the crack-tip like you have K field going asymptotically infinity at the crack-tip if you look at mathematical expression for HRR field this also predicts only infinite stresses at the crack-tip.

But what is shown here is because of blunting this would be a typical variation of sigma y and sigma y reaches a peak value at small distance away from the crack-tip and what we had seen in the case of LEFM, the stress field is completely dictated by the value of K.

(Refer Slide Time: 18:09)



Whatever is a strength of the stress field is dictated by the value of K, in similar way the stress field in the case of a elasto-plastic fracture mechanics you have sigma i j is given as sigma naught multiplied by J by alpha sigma into naught epsilon naught there is I n multiplied by r whole power 1 by n plus 1, plus a function of theta and strain-hardening exponent n.

And the strength of this is dictated by the value of J and you have definition for all this terms. Alpha is a material constant, sigma naught is a reference yield strength, epsilon naught is a reference yield strain, n is a strain hardening exponent and I n is the constant and function of n you know people have given expressions for this.

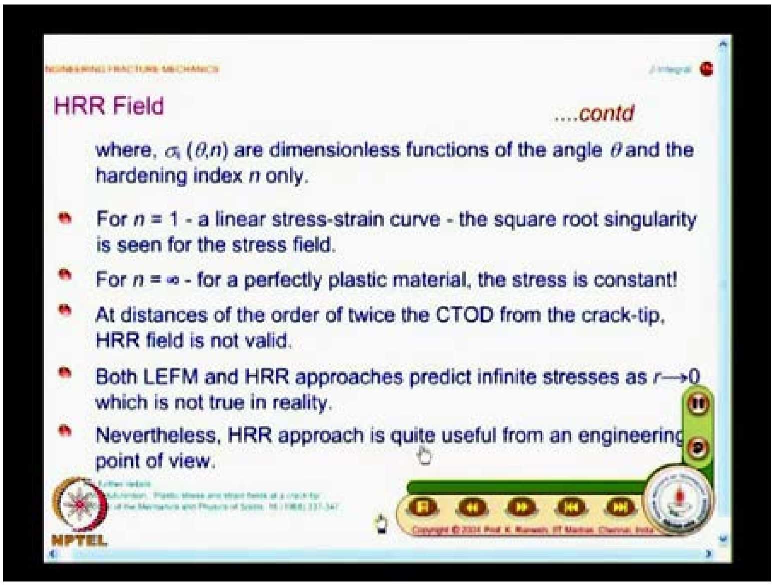
So, when you substitute the value of n you will be able to get the value of I n some representative values are shown. So, I n is approximately equal to phi when n equal to 1 and when n equal to 1, if we look at the Ramberg-Osgood law it gives a linear elastic analysis.

When n is infinity, if n is 1 it will go straight when n is infinity you will have a straight portion followed by a fully plastic linear elastic and fully plastic n which lies between 1 and infinity would be like a suitable curve that is how you modeled a material behavior.

So, this is for linear elastic analysis, I n is approximately three when you go for elastic perfectly plastic that is n equal to infinity and I n equal to approximately 6 when n equal to 1 in the case of plain strain and I n equal to 4 when n is infinity. So, we are really talking about perfectly elastic and elastic plastic, and one observation is n decreases monotonically when you change the I n decreases monotonically when you change the values of n.

So, what you’ll have to recognize is you have a blunted crack-tip then you have a fracture process zone this is engulfed by HRR field which is J dominated zone which would be surrounded by K field and then you have a general stress field.

(Refer Slide Time: 21:19)



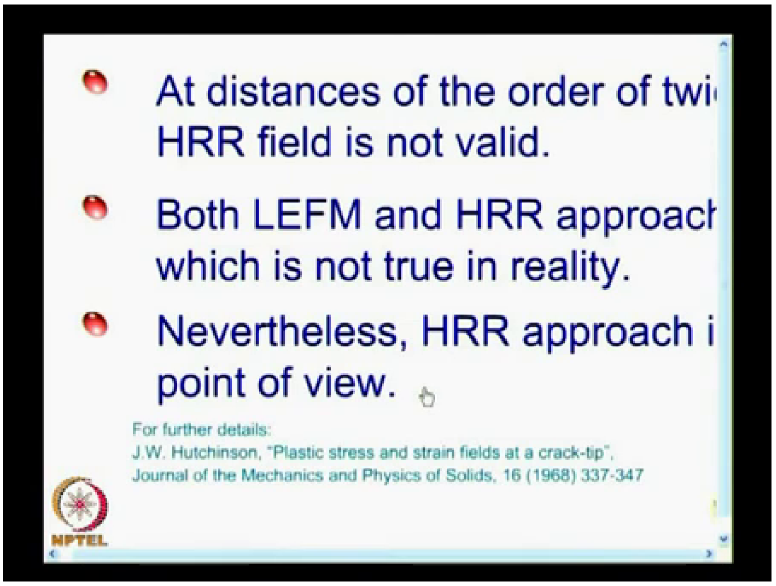
And this only summarizes what I had mentioned that sigma i j theta comma n are dimensionless functions of the angle theta and the hardening index n only. When n equal to 1 it represents the linear stress strain curve square root singularity is seen for the stress field.

So, this is the indirect verification that what we have pictured as the possible values of stresses is indeed correct. For n equal to infinity, it is for a perfectly plastic material the stress is constant and we had also noted down when we have drawn the picture at distances of the order of twice the crack tip opening displacement that is CTOD from the crack-tip HRR field is not valid.

You should have to keep in mind HRR field is also represents a zone slightly away from the crack-tip where it is dominated by J and this is again emphasized both linear elastic fracture mechanics and HRR approaches predict infinite stresses as r tends to 0.

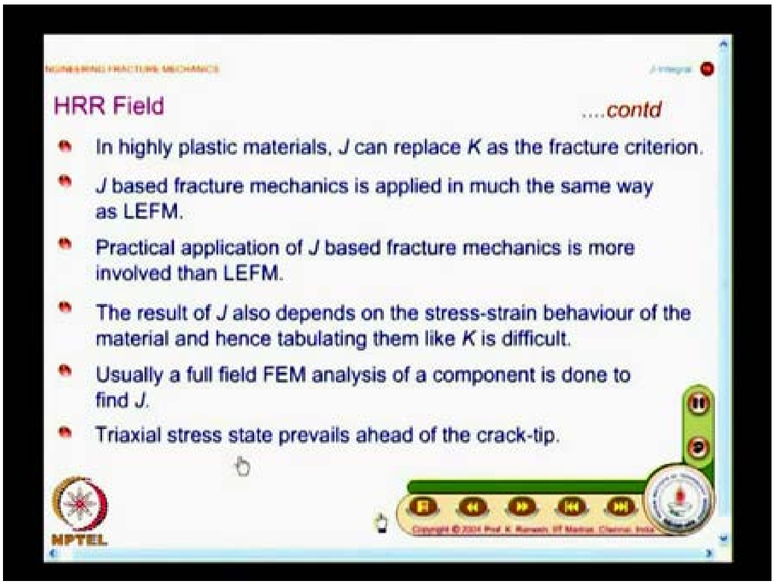
Which is definitely not true in reality nevertheless HRR approach is quite useful from an engineering point of view now this is what you have to keep in mind.

(Refer Slide Time: 23:14)



And you have a paper by Hutchinson plastic stress and strain fields at a crack-tip published in journal of the mechanics and physics of solids. This was in the year 1968 even Rice reported J-integral in 1968.

(Refer Slide Time: 23:39)



So, 1968 is a very important year from fractural mechanics point of view, we have important contributions that have been, made and let us look at, in highly plastic materials J can replace K as the fracture criterion.

So, what we have done in linear elastic fracture mechanics we would have a critical value of the stress intensity factor, we called it as fracture toughness.

We had elaborate test methods to find out what is K 1 c on similar lines you can also have J 1 c you know we are already seen the elastic plastic fracture is very complex. So, we would take out a simpler approach. So, we looked at a non-linear elastic solid whatever that concepts developed for non-linear elasticity solid we extended and we always fall back on LA of approach in applying fracture mechanics similar things we extend by using J. So, that simplifies your methodology purely from an engineering analysis point of view.

So, that is what is mentioned here, J based fracture mechanics is apply in much the same way as linear elastic fracture mechanics and you will have to look at that practical application of J based fracture mechanics is more involve than LEFM.

The difficulty here is the result of J also depends on the stress-strain behavior of the material and hence tabulating them like K is difficult. We had seen for variety of geometry what is the value of K very similar to stress concentration factor you are able to do it.

But here we have to evaluate J material behavior also place a role. So, tabulating them like K is difficult, you have to do exhaustive numerical computation and then obtain for each of your configuration.

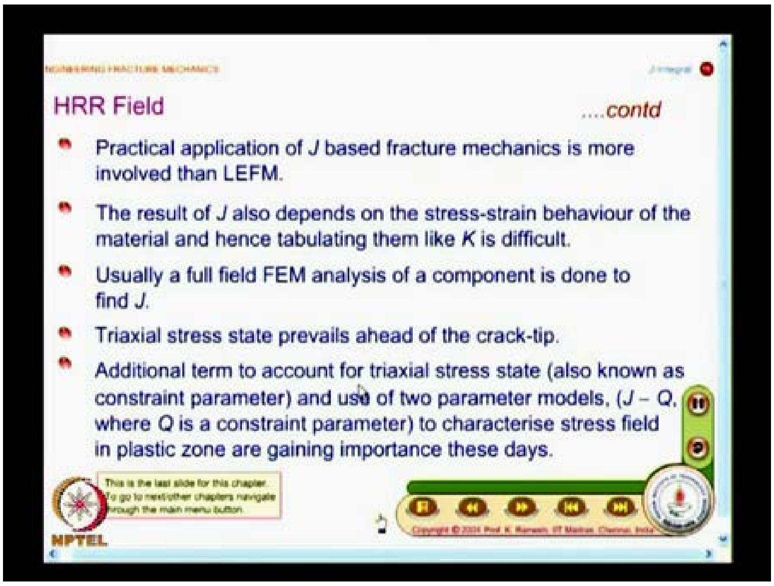
So, that is what is mentioned here usually a full field finite element analysis of a component is done to find J though from a conceptual point of view J can replace K as a fracture criterion; evaluation of J itself is a challenge.

Similar to k one c you will do have test methods to find out J 1 c, that apart, for a given configuration finding out J itself is challenging. Many times what you find is you know people to find out K they find out J and then use the identity and get the value of K from finite element computation. This is only in the domain of linear elastic fracture mechanics you know many times people use J more as finding out K rather than elasto-plastic fracture mechanics parameter.

Because the elasto-plastic fracture analysis are costly and time consuming not many people get involved into that. So, you have to find out a differentiation, are you doing elasto-plastic fracture analysis and find out J or are you calculating J to find out K in linear elastic fracture mechanics.

So, keep these two things different and understand. You know you will have to look at triaxial stress state prevails ahead of the crack-tip we have the blunted crack-tip. So, you have a triaxial state slightly ahead of the crack-tip and people also have try to model this in some form.

(Refer Slide Time: 27:44)

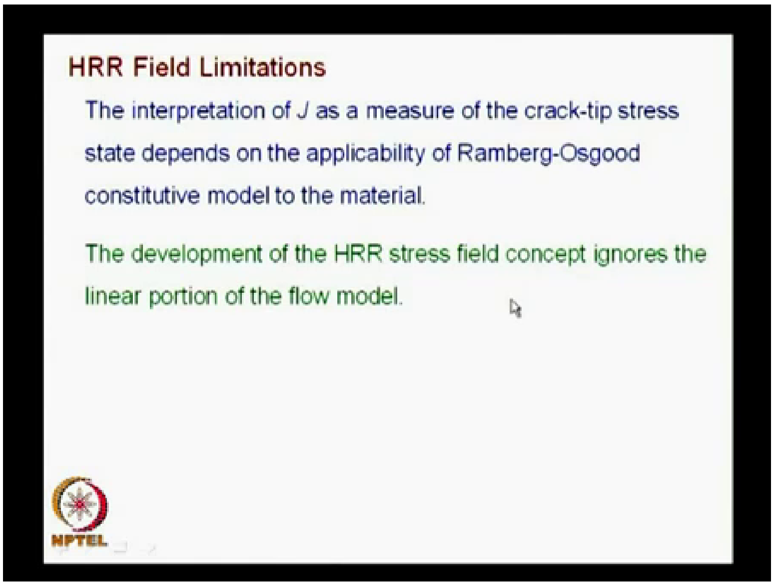


So, they have tried to find an additional term to account for triaxial stress state which is also known as constraint parameter and you have used of two parameter models known as J Q where Q is a constraint parameter. You know, in the case of linear elastic fracture mechanics what we saw, photo elastician said that you need to account for the second term in the sigma x stress term which is on a sigma naught x which was later understood by analytical people of as t stress, that is in the domain of linear elastic fracture mechanics in the domain of elasto-plastic mechanics they have arrived at another parameter called Q.

Which is known as a constrained parameter and this is used to characterize a stress field in plastic zone and such approaches are gaining importance these days.

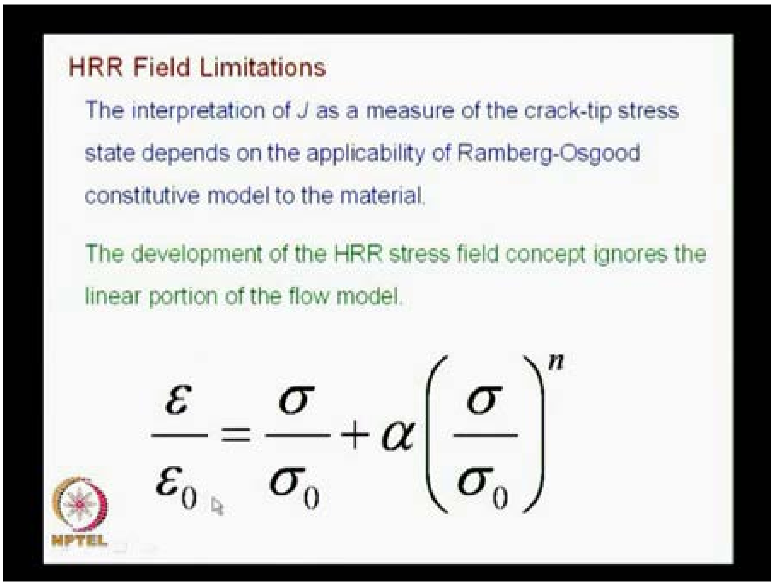
So, you will have to know what is the current approach people are looking at, like we had looked at t stress influence in linear elastic fracture mechanics, people are also talking about the constrained parameter Q in elasto-plastic fracture analysis and you know having said that HRR field is useful, there are also limitations, you will have to know those limitations clearly.

(Refer Slide Time: 29:15)



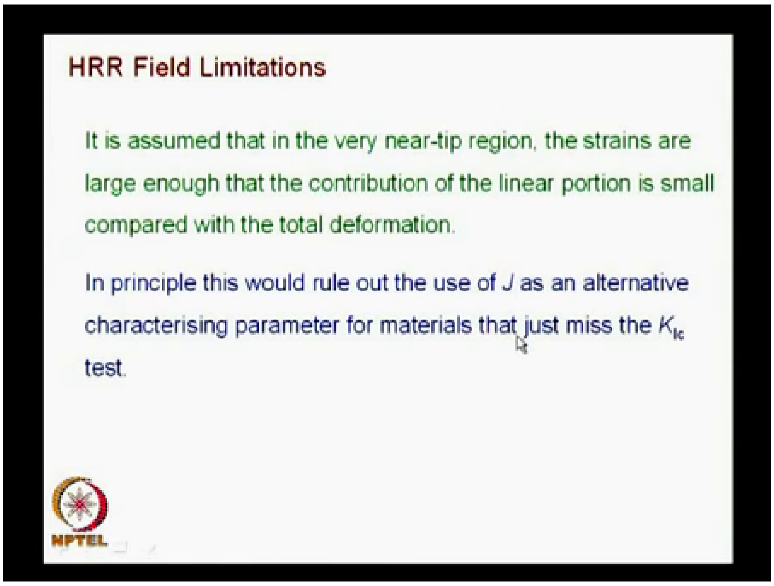
And if you look at the interpretation of J as a measure of the crack-tip stress state, depends on the applicability of Ramberg-Osgood constitutive model to the material and even in that what people do. In the development of HRR stress field concept the linear portion of the flow model is conveniently ignored.

(Refer Slide Time: 29:57)



When you look at Ramberg-Osgood law, if you take any book in plasticity, we will have this expression, epsilon divided by epsilon naught equal to sigma by sigma naught plus alpha into sigma by sigma naught whole power n and in the development of HRR stress field the linear portion of the flow model is ignored, only in the non-linear part is considered and what is the consequence of, it has an advantage as well as disadvantage you will have to know both.

(Refer Slide Time: 30:36)



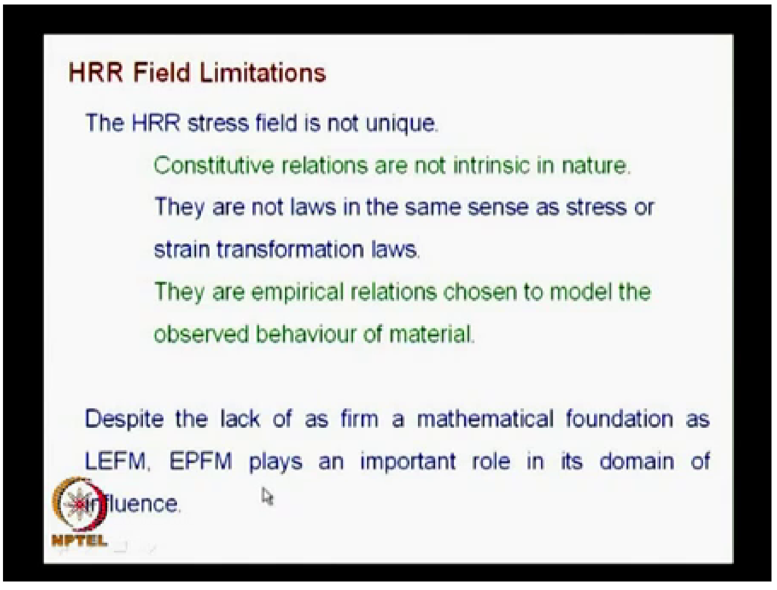
It is assumed that in the very near-tip region the strains are large enough that the contribution of the linear portion is small compared with the total deformation. Such contradictions come in several fields you know you will say something is large in comparison to something else, but large quantity is still small compared to something else now this kind of contradictions exist without such approximations you cannot perform any engineering analysis.

But it is better to know that such approximations have been made in the development of HRR field that is what I am trying to point out. In principle this would rule out the use of J as an alternative characterizing parameter for materials that just miss the K 1 c test. You know while we discussed fracture stiffness testing I also said this is very unique type of test you are not guaranteed at the start of the test whether the value would be acceptable or not.

You do so much complicated test and at the end you say it may be accepted or not, instead of leaving out with the doubt people also developed another type of codes where you have to do little more exhausting measurement if K 1 c fails then at least you put report CTOD or J if you have to do that then this kind of assumptions does not go with it.

Then in the near tip region the stains are large enough that the contribution of the linear portion is small if you say that then this is strictly not applicable, but we keep doing that.

(Refer Slide Time: 32:33)



And what you will have to keep in mind is the HRR stress field is not unique and this is again the discussion is very fundamental. It applies to all empirical relations, constitutive relations are not intrinsic in nature and a comparison is also set, the constitute relations are not like loss in the same sense as stress or strain transformation laws.

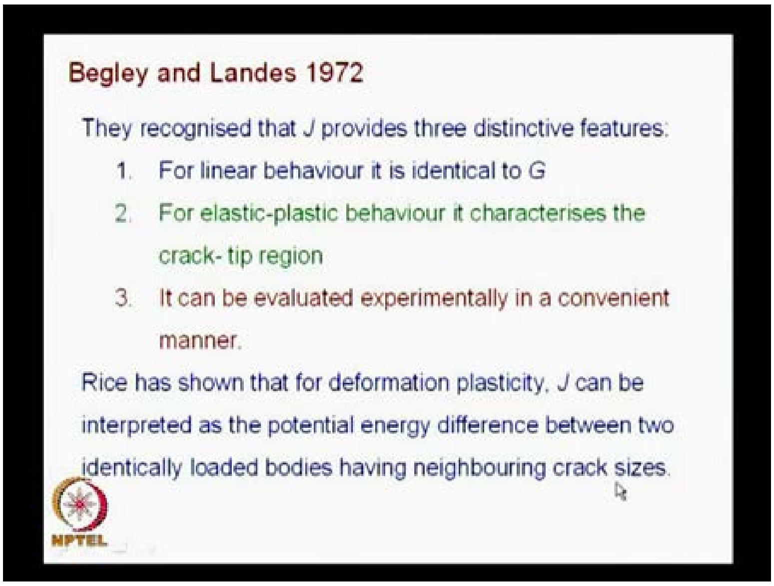
See the stress or transformation law is definite, if we have tensor of rank two from one coordinate system to another coordinate system it will transform only in this way. In contrast constitutive relation are not like this, they are empirical relations, you have to keep in mind, chosen to model the observed behavior of material, the moment I say empirical relations what could arrive at another set of empirical relations for the given type of data.

So, from that point of view HRR stress field is not unique and whatever the discussion we do here it applies to all empirical relations. In fact, in EPFM you see only empirical relations because that is how engineering community has found utilization of EPFM possible.

Even in CTOD you would come across only empirical relations. So, you have to keep in mind despite the lack of as firm a mathematical foundation as LEFM EPFM plays an important role in its domain of influence and if you really look at who has advanced the use of J for EPFM it was actually the experimental work done by Begley and Landes in 1972.

They recognized that J provides 3 distinctive features, mind you, it was developed in 1968 the paper by Rice was in 1968 its full utilization in understanding came only in 1972.

(Refer Slide Time: 35:13)



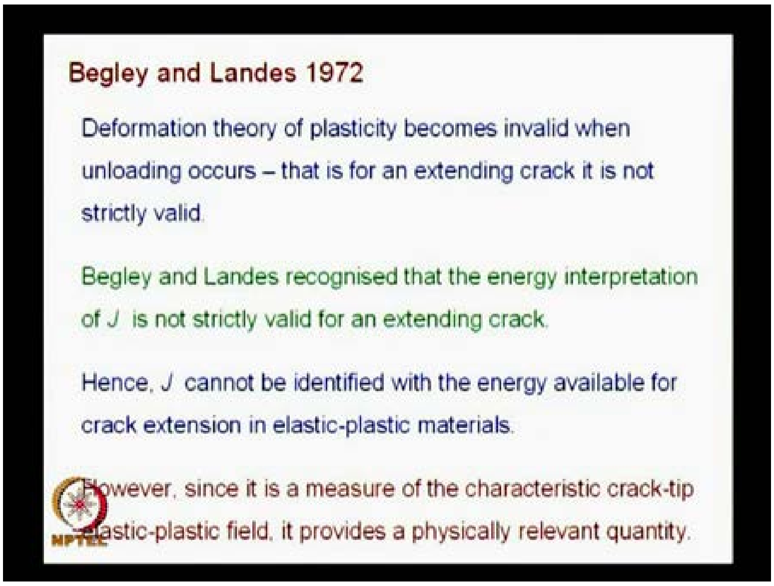
For linear behavior J is identical to G. So, that gives the comfort that we are going in for generalizations. For elastic plastic behavior it characteristics the crack-tip region like K determines the strength of the stress field linear elastic fracture mechanics in elasto-plastic fracture mechanics J determines that the strength of the field is determine by J and another observation they have made it can be evaluated experimentally in a convenient manner.

Because they had done experiments to find out J they have also summarized their observation like this and even if we look at G, people were able to find out G experimentally only then analytical computation became very popular and you also keep in mind that J is developed based on deformation plasticity.

Rice has shown that for deformation plasticity J can be interpreted as a potential energy difference between two identically loaded bodies having neighboring crack sizes. See the statement is settle in the case of linear elastic fracture mechanics a crack can extend by itself and then the analysis is still valid. In the case of elasto-plastic analysis the moment crack grows, unloading takes place, your analysis becomes not exact because you are in the domain of using deformation plasticity theories.

So, that is why he says you take 2 specimens which are of different crack lengths. In fact, Begley and Landes did that kind of experiment.

(Refer Slide Time: 37:24)



And that is what summarized here deformation theory of plasticity becomes invalid when unloading occurs, that is, for an extending crack it is not strictly valid. In view of that, Begly and Landes recognized that the energy interpretation of J is not strictly valid for an extending crack. In the case of non-linear elastic solid this definition is alright, but once you take it to elasto-plastic analysis the interpretation that J is energy released rate is not strictly valid for an extending crack.

Hence J cannot be identified with the energy available for crack extension in elastic-plastic materials. However, since it is a measure of characteristic crack-tip elastic plastic field, it provides a physically relevant quantity, that is, all the importance that you can give, you cannot give the importance beyond that.

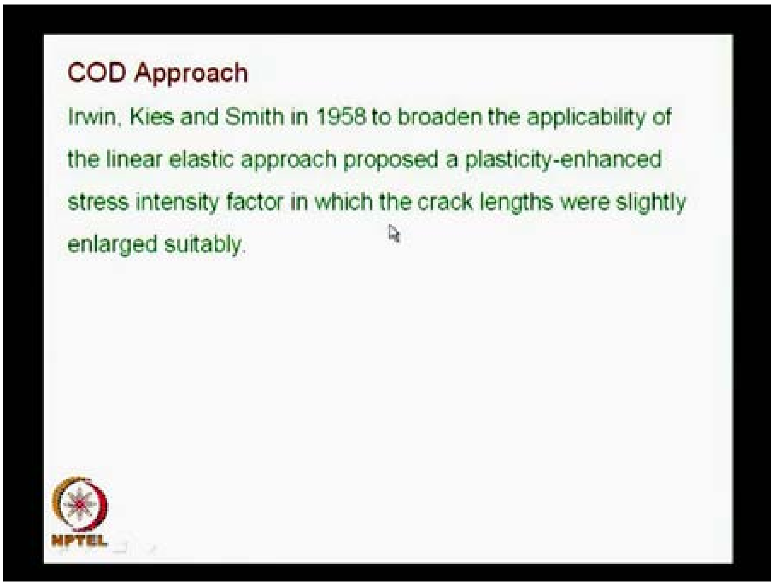
(Refer Slide Time: 38:47)

And we will see what are the application areas of EPFM, because people have gone for EPFM there must be a definitely need for it. In power generating and chemical processing industries most cracks occur in high pressure parts which are thick wall vessels and pipes.

In nuclear pressure vessel industry material toughness is very important- despite high initial toughness, subcritical flaws developed due to fatigue and stress corrosion cracking and the material also degrades due to neutron bombardment. This is something peculiar to nuclear installations. In all those cases EPFM concepts are essential. It is to be remembered that LEFM is principally applied in aerospace industry.

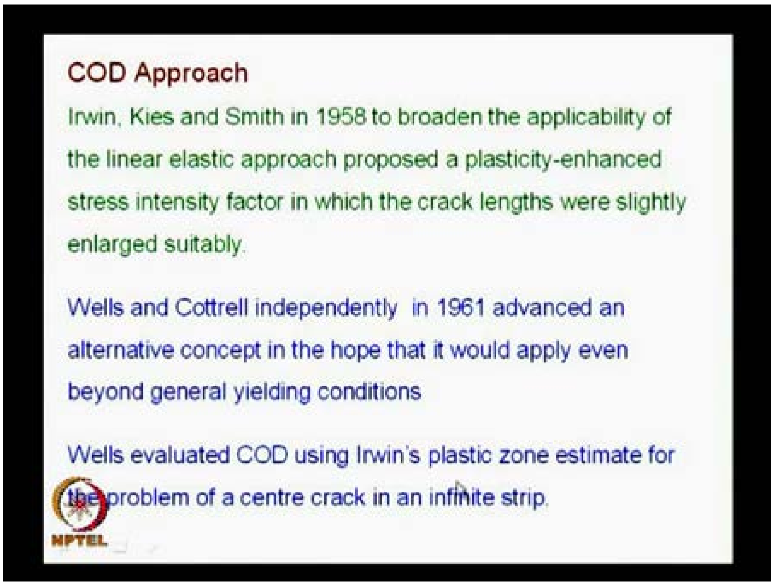
So, the application areas are different, LEFM is confined to thin structures what we come across in aerospace when you have very thick material and highly tough materials you have to go in for J and I had also mentioned long time back that K once in determination fails for a nuclear reactor steel material because if you determine the thickness, a person can sit on it, is about 1 meter thick very large, on the other hand for nuclear reactor steel if we go and find out what is the thickness of the specimen that you require for determination of J 1 c, it is about 15 millimeters. So, it is doable. So, depending on the application area you have to choose the methodology of fracture mechanics.

(Refer Slide Time: 41:01)



We will also have a brief introduction to the COD approach and if you recall the first attempts to account for plastic zone near the crack-tip was initiated by Irwin, Kies and Smith in 1958. When they wanted to broaden the applicability of the linear elastic approach proposed a plasticity enhanced stress intensity factor in which the crack lengths were slightly enlarged suitably. This we had seen, you had looked at Irwin's model, you had looked at Dugdale's model, and you also evaluated what should be the incremental length that you have to add to the crack length and make that crack length as effective crack length.

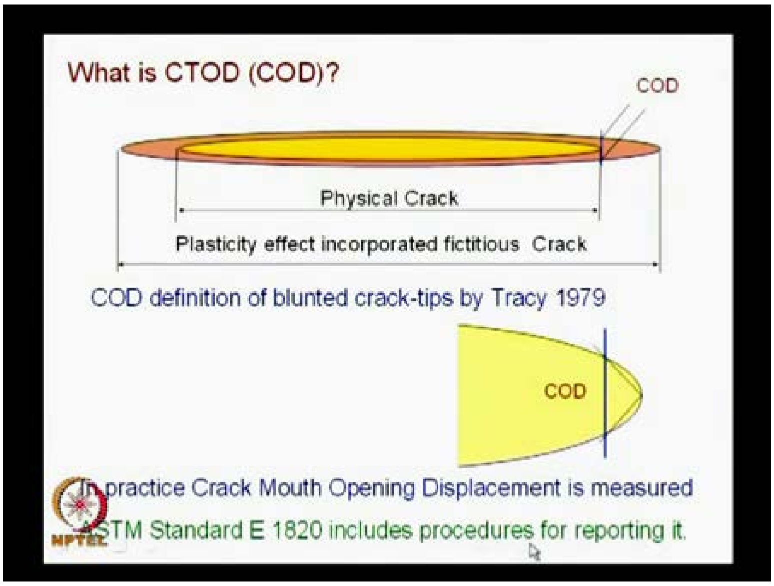
(Refer Slide Time: 42:00)



So, this was limited, this was the first type of approximation. Wells and Cottrell independently in 1961 advanced an alternative concept in the hope that it would apply even beyond general yielding condition.

So, that was the focus and historically the concepts related to COD or CTOD was before the development of J and when Wells proposed he had the expressions of Irwin's plastic zone. So, he evaluated COD using that and that was for a problem of a center crack in an infinite strip.

(Refer Slide Time: 42:53)



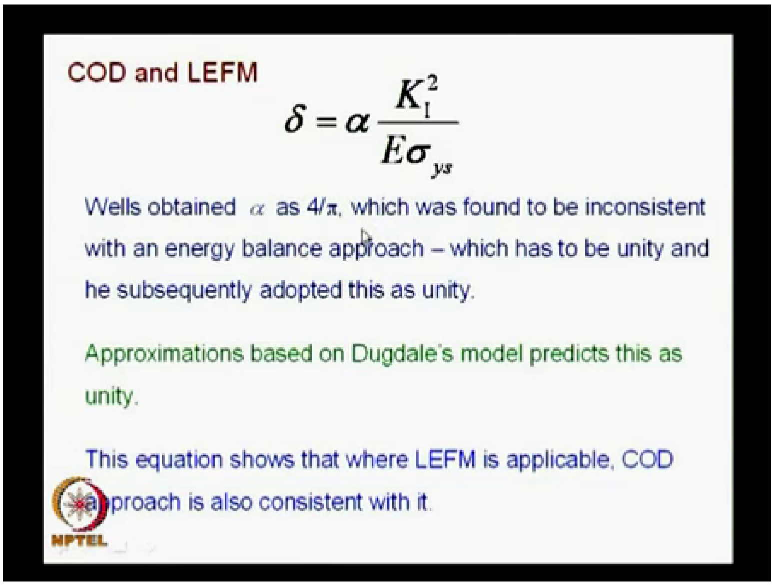
And before we proceed further we need to understand what is this CTOD and you carefully make a sketch of this. So, you look at plasticity effect incorporated fictitious crack, this is only fictitious, and you have an actual crack because the idea is, when you have a crack-tip they cannot be displacement there, then what is crack-tip opening displacement that has to be defined, that comes from your plasticity corrected fictitious crack-tip.

So, this opens up like this, and you have the actual crack like this, and whatever the opening at the original crack length is considered as CTOD or COD. We had looked at COD as a expression for the shape of the open crack we have looked at linear elastic fracture mechanics. In EPFM they use CTOD or COD and this definition is taken from your plasticity enhanced crack-tip. So, this is a fictitious crack and this is the actual crack and whatever the opening here is known as CTOD or COD.

And there was also a definition given by Tracy in 1979 which is, what you had seen when we had looked at HRR field also, you draw lines which are mutually perpendicular at angle 45 degrees, it cuts the blunted crack and this height is taken as CTOD or COD. In practice crack mouth opening displacement is measured and you're ASTM standard E 1820 includes procedures for reporting it.

So, you measure only crack mouth opening displacement by your appropriate gauge, we had seen it in the context of fracture testing, from that you find out how to calculate the crack-tip opening displacement.

(Refer Slide Time: 45:31)



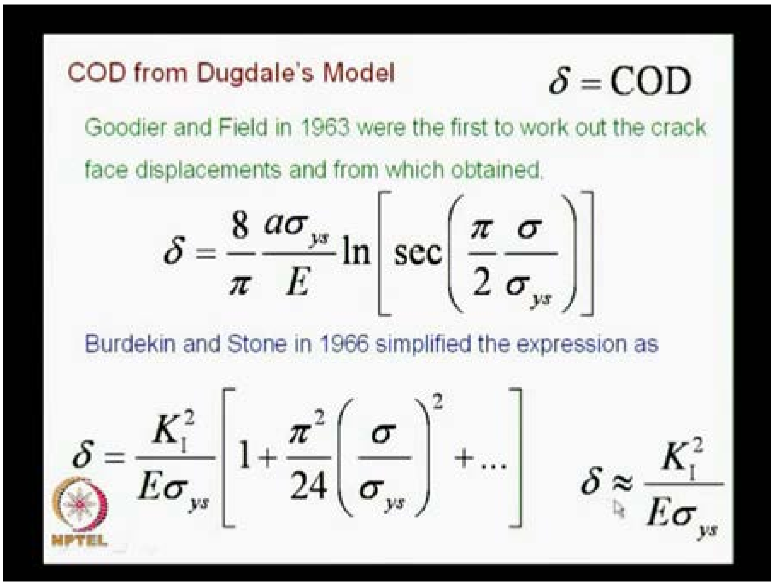
And you know I am not shown the derivations steps from Irwin's modified the value of the incremental extent of crack length it is possible to calculate an expression for the crack-tip opening displacement.

So, what you do is, you have the expression for COD like 4 sigma by E into you have crack length minus the distance. So, you put that as a effective crack length; effective crack length is a plus the plastic zone length divided by 2 r p by 2. If you substitute those expressions, you can find the relationship between the crack-tip opening displacement and K. Wells obtained alpha as 4 by pi from Irwin's approach which has to be unity from energy balance approach and you subsequently adopted this as unity. See the difficulties you know you have one set of expressions you get one result and you find that this is no consistent with energy analysis, then you modify it, then you take a comfort.

If I do not go through the Irwin's model, if I go through the Dugdale's model, I get this as unity. Approximations based on Dugdale's model predicts this as unity. So, you have a confirmation from Dugdale's model plus energy balance approach..

So, you take delta equal to K 1 squared by E into sigma y s and what does this show? It is able to show within the confines of LEFM, COD and K 1 are related. And you have this is CTOD or COD; this is related to K 1 as long as LEFM is applicable.

(Refer Slide Time: 47:39)

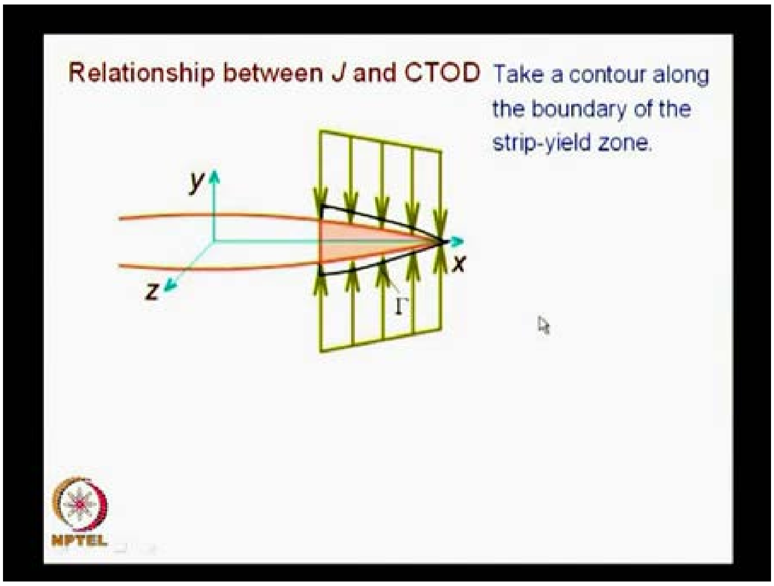


So, they are not different parameters and you also have the expression for COD from Dugdale's model. And if we look at Dugdale reported its model 1961 or so. He had only got the extent of plastic zone. From that Goodier and Field in 1963 where the first to work out the crack face displacements and from which obtained a long expression. COD is given as 8 by pi multiplied by a into sigma y s divided by E natural logarithm of secant of pi by 2 sigma by sigma y s. and mind you, the symbolism. Earlier, we had used delta as a extent of plastic zone in Dugdale's model. In the context of EPFM, delta refers to CTOD.

We had already seen what the diagram is for CTOD. This expression was simplified in 1966, 3 years later, by Burdekin and Stone and expressed this as Taylor series and looked at delta equal to K 1 squared divided by E into sigma y s multiplied by 1 plus pi squared by 24 into sigma by sigma y s whole square plus so on and so forth.

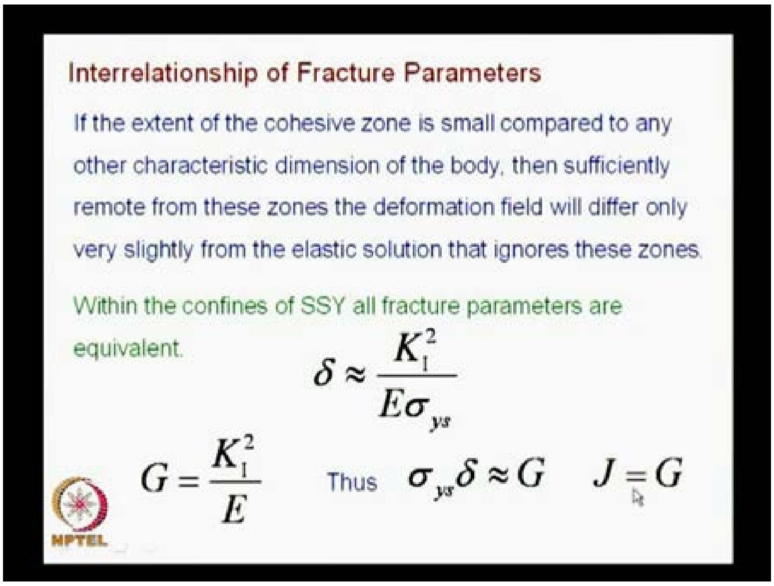
So, if you consider that the second term is small, second and higher order terms are small. Delta can be approximately related to K 1 squared divided by E into sigma y s. And I will just show one more result without proof; I would look at it one can also find out a relationship between J and CTOD.

(Refer Slide Time: 49:44)



So, what you do is, you take the Dugdale’s yield strip model, take a contour along the boundary of the strip yield zone like what is shown here for clarity it is shown as big, but you consider that this is very close to this. And if you apply J-integral concept, it is possible for you to relate J equal to sigma y s into the CTOD. I will not getting into the derivation part of it, you will be able to find out based on J-integral there is a relationship between J and crack-tip opening displacement.

(Refer Slide Time: 50:41)



And in summary what you find is, if the extent of the cohesive zone is small compared the any other characteristic dimension of the body, then sufficiently removed from these zones the deformation field will differ only very slightly from the elastic solution that ignores these zones.

So, you find within the confines of small scale yielding, all fracture parameters are equal. CTOD is related to K 1 squared divided by E into sigma y s, G is K 1 squared by E, and you have sigma y s delta equal to G or equal to J and J equal to G.

So, this brings in a set of comfort that within the confines of linear elastic fracture mechanics, all these seemingly different parameters or interrelated.So, that is what is shown in this slide. So, this gives a comfort that we are proceeding in the right direction. So, in this class, what we have looked at was, we had looked at J as the stress intensity parameter, we had looked at what is HRR field, then what are the application areas of EPFM, then we had looked at expressions for the CTOD; it can be obtained from Dugdale's model or Irwin's model and finally, we had looked at the interrelation ship between fracture parameters.

Thank you.