

Engineering Fracture Mechanics
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Module No. # 07

Lecture No. # 34

Paris Law and Sigmoidal Curve

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Questions to be Answered for a Fracture Mechanics Based Design

- One of the mechanisms of crack growth is by fatigue.
- In structures subjected to fatigue loading, if a crack is detected by NDT methods, as a designer one should know how to react to the situation.
- The questions to be answered are
 - Is it safe to operate the component/machine?
 - If safe for how long?
 - Is it possible to monitor crack growth so that one can discard the component or stop the machine before catastrophic failure can occur?

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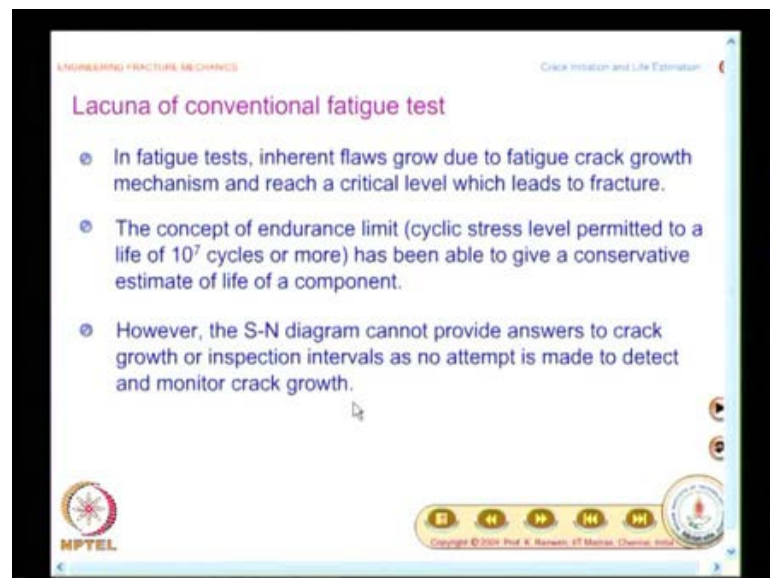
We were looking at questions to be answered, for a fracture mechanics based design. And you have to keep in mind, one of the mechanisms of crack growth is by fatigue. And if you really take a fatigue, there are two categories you can think of - one is a low cycle fatigue; another is a high cycle fatigue. We are going to pay attention on high cycle fatigue, which has a crack initiation, crack growth, and then finally, catastrophic failure. In the case of low cycle fatigue, the stress levels are very high and within 1000 cycles, the component may fail. So, there, fracture mechanics would not be of much use.

In the case of high cycle fatigue, we could find out whether fracture mechanics be used. The issue is, in structures subjected to fatigue loading, if a crack is detected by NDT methods, as a designer, one should know how to react to the situation. And what are the

questions that need to be answered? They are - is it safe to operate the component or machine? This is very important, from safety point of view. If safe, for how long? Is it possible to monitor crack growth, so that, one can discard the component or stop the machine before catastrophic failure can occur.

See, we have seen earlier, when we moved from simple tensile loading to combined loading, for failure analysis, we will still use the result from a simple tension test and used it for combined loading, in our conventional design approaches. Then, we graduated for fatigue loading, when you want to study structure subject to cyclical load. Now, the question is - you also want to know, how the crack would grow.

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The slide is titled "Lacuna of conventional fatigue test" and is part of a presentation on "ENGINEERING FRACTURE MECHANICS" and "Crack Initiation and Life Estimation". It contains three bullet points:

- In fatigue tests, inherent flaws grow due to fatigue crack growth mechanism and reach a critical level which leads to fracture.
- The concept of endurance limit (cyclic stress level permitted to a life of 10^7 cycles or more) has been able to give a conservative estimate of life of a component.
- However, the S-N diagram cannot provide answers to crack growth or inspection intervals as no attempt is made to detect and monitor crack growth.

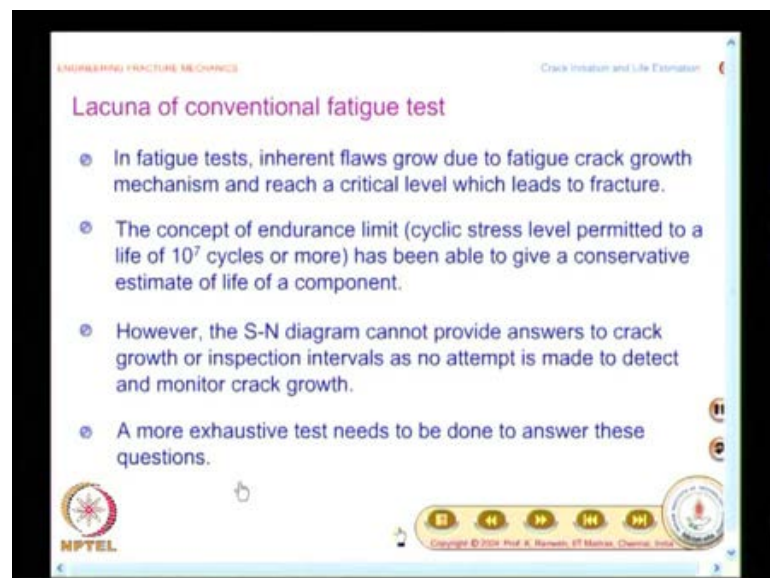
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So, the first issue is, can you extrapolate the results from conventional fatigue test to answer these questions? If you ask that kind of a question the answer is, no. In fatigue tests, inherent flaws grow, due to fatigue crack growth mechanism and reach a critical level, which leads to fracture. This is perfectly alright. But, how the fatigue test is done? You are only noting the endurance limit. The concept of endurance limit was quite useful. We say, what is the cyclic stress level permitted to a life of 10^7 cycles or more. And I had also mentioned, all components, fail eventually. The latest research shows, there is nothing like an endurance limit for any material. It has been experimentally demonstrated. However, from practical point of view, if your life is

beyond 10^7 cycles, it is good enough and you can use the endurance limit for designing your component.

What you have to note is, the S-N diagram cannot provide answers to crack growth or inspection intervals, as no attempt is made to detect and monitor crack growth. So, this is very important. You will know for a given stress level which is cyclically varying, what is the expected life. Because, that is the only information you are collecting it. If you have a crack, with the crack, when you apply the cyclical load, what is the remaining period of life, you do not have a knowledge from a fatigue test.

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The slide is titled "Lacuna of conventional fatigue test" and is part of a presentation on "Engineering Fracture Mechanics" and "Crack Initiation and Life Estimation". It contains four bullet points:

- In fatigue tests, inherent flaws grow due to fatigue crack growth mechanism and reach a critical level which leads to fracture.
- The concept of endurance limit (cyclic stress level permitted to a life of 10^7 cycles or more) has been able to give a conservative estimate of life of a component.
- However, the S-N diagram cannot provide answers to crack growth or inspection intervals as no attempt is made to detect and monitor crack growth.
- A more exhaustive test needs to be done to answer these questions.

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Because such an information is not recorded at all. In order to get such information, a more exhaustive test methods needs to be done, to answer these questions.

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ENGINEERING FRACTURE MECHANICS

Crack Initiation and Life Estimation

Crack growth curve

- One of the standard specimens for fracture testing is fatigue loaded and the crack growth is monitored by NDT methods.
- The fatigue loading is not fully reversed as in fatigue tests but the lower load is kept zero in these tests.
- The experiment is repeated for various loads and a plot of crack length vs number of cycles is obtained.

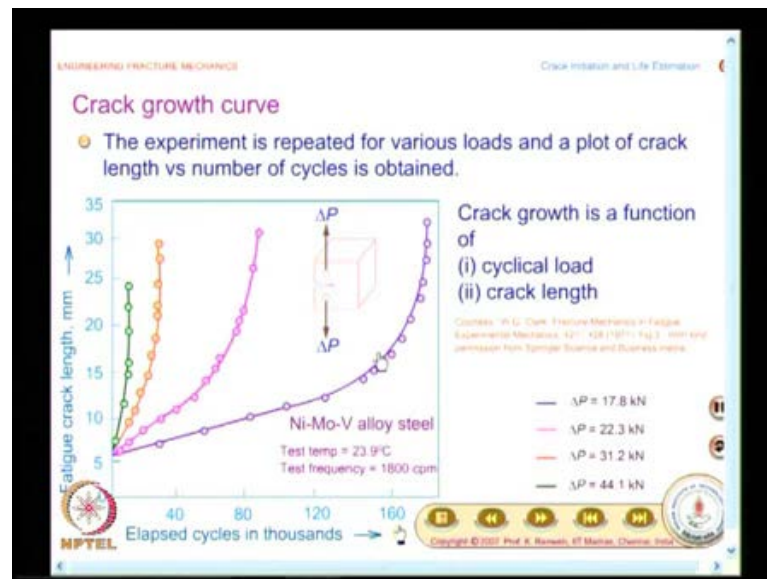
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And what is the kind of test people have attempted? People have attempted to get what are known as crack growth curve. And in this, what they do is, they take one of the standard specimens for fracture testing, which is fatigue loaded and the crack is monitored by NDT methods.

So, the key issue here is, you are also making an attempt, to see how the crack grows, when the test is being performed. So, you are collecting additional data. And how is the fatigue loading done? The fatigue loading is not fully reversed, as in fatigue tests, but the lower load is kept 0 in these tests. That also, you have to keep in mind. And what do you do? The experiment is repeated for various loads and a plot of crack length versus number of cycles is obtained. See, essentially, you will be collecting a voluminous data; for a single material, you will have to keep changing the load amplitude and for each one of the load amplitude, you will have a different type of crack growth curve.

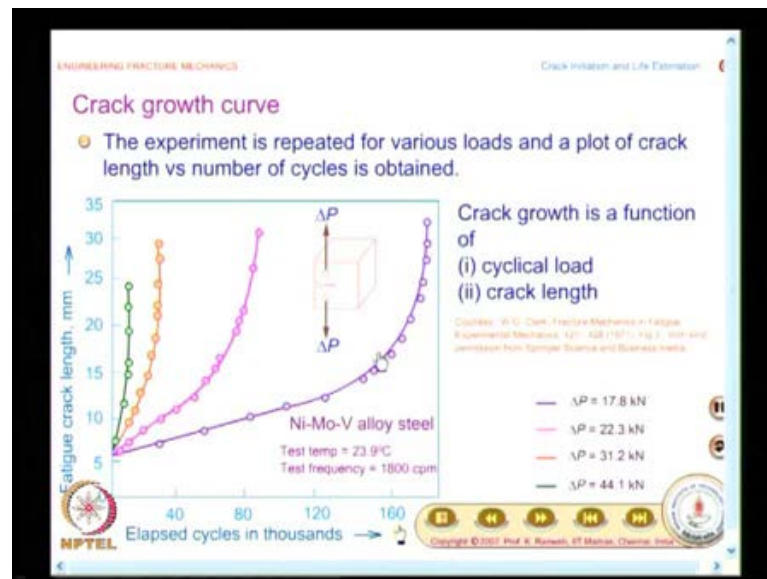
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Now, the challenge is, how you will utilize this data in a meaningful way. And, please draw this graph as well as possible. And what you have here is, on the x axis, you have elapsed cycles in thousands; on the y axis, you have the fatigue crack length in millimeters. And if you look at here, within a few cycles the crack has grown to a large extent, this is, obviously, for a high load and that is given as delta P equal to 44.1 kilo Newtons.

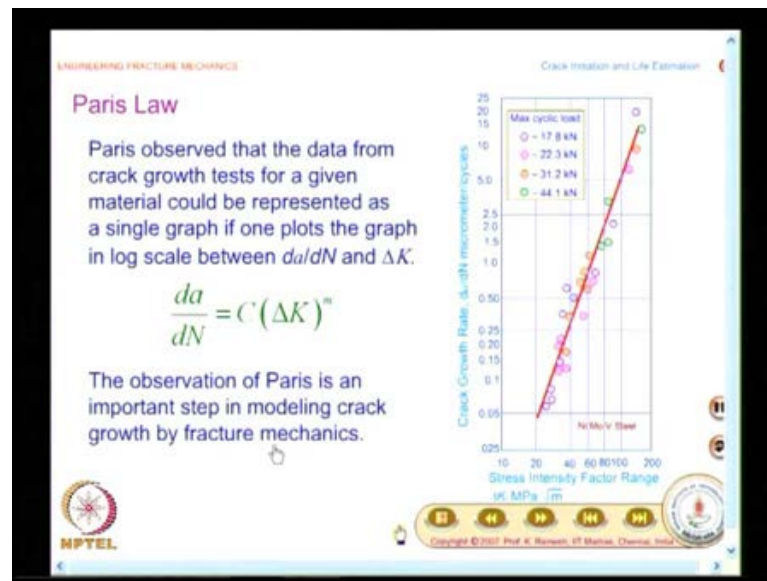
The second graph is for 31.2 kilo Newtons. So, it takes more time for the crack to grow when the delta P value is smaller. And you will have also have to keep in mind, that I have drawn these data points with different colors. I have a reason for it, which shall become clear, once we see the next set of graphs. You have graphs for smaller loads, 22.3 kilo Newtons as well as 17.8 kilo Newtons. So, what you will have to keep in mind is, for different values of delta P, you get different crack growth curve for one single material. You know, this is what you have to keep in mind. For one single material you get so many graphs. I have shown only four of them. you can get many of them, if you change the delta P value.

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And what do you see here? Crack growth is a function of cyclical load and also crack length. Now, the question is if you have so many graphs for one material, how will you be able to use this, in your design calculation? It becomes extremely difficult. You know, if you really recall, how the concept of stress and strain was initially developed, people took rods of isotropic material; when they plotted force elongation graph for a single material, they got number of graphs, depending on the length of the specimen, as well as the cross section area. When you change them, you will get one additional graph. Then people thought, instead of plotting force versus displacement, if you plot stress versus strain, all of this data bundled in a single curve. Something similar to that, also happened in fracture mechanics.

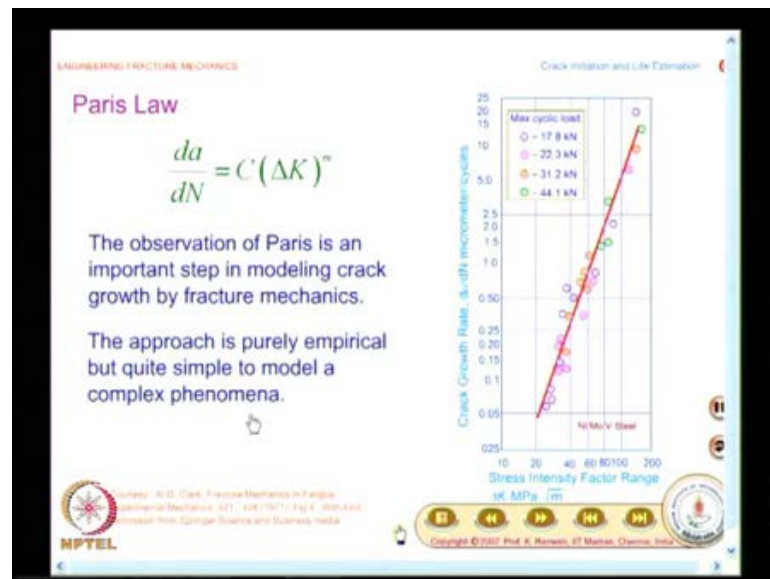
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And this credit goes to Paris and you just observe the graph. So, what you find here is, I have a straight line; it is actually a log-log plot and you find data point of all colors lie very close to this line. So, whatever the data of individual crack growth curve, all of them could be fitted, if you choose the x axis as delta K, which is the stress intensity factor range and the y axis as crack growth rate given by d a by d N.

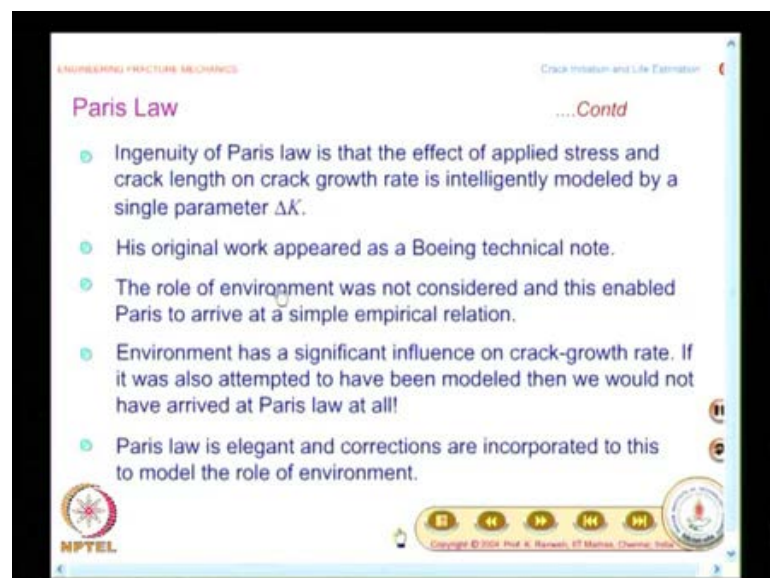
And, this is a challenging task. This is the log-log plot and you are able to fit all the crack growth curve data for a single material into a single graph. So, that means, this could be utilized in your design calculation. And this was obtained by Paris and this is known as Paris law. So, the Paris law states, d a by d N equal to C into delta K power m. So, here C and m are material constants. They have to be determined from material testing. And what you will have to note is, the observation of Paris is an important step in modeling crack growth by fracture mechanics. It is a very significant step, never forget that. There is no point in collecting voluminous data and you find, you do not know what to do with it. The voluminous data has to be properly represented for you to use in design calculation.

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And let us see, what was the history behind this Paris law. Before we go into that, it was also recognized that, the approach is purely empirical; there is no theoretical basis for you to arrive at this equation. It is actually curve fitting exercise of the data that you have collected. And that is how you would see, when they collect more data from the initiation stage to catastrophic failure state, people have developed more complicated versions of Paris law. Nevertheless, it is quite simple to model a complex phenomenon. So, the origin is purely empirical, but very useful.

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What is the ingenuity of Paris law? The effect of applied stress and crack length on crack growth rate is intelligently modeled by a single parameter ΔK . That is the advantage of... His original work appeared as a Boeing technical note. And what you will have to keep in mind is, in many of these fatigue failures, environment plays a very important role. While developing the Paris law, Paris did not consider the effect of environment and just because he had not considered the environment to start with, he was able to arrive at a simple empirical relation. See, in engineering, this is what we do. We try to model a particular phenomenon by a simpler expression. From experience, whatever the field results that they give, you bring in correction factors.

So, this kind of approach is viable, from a design practice point of view. So, Paris looked at the kernel and arrived at a relationship between $\frac{da}{dN}$ and ΔK , without bringing in factors like role of environment etcetera. And you will have to keep it note, that environment has a significant influence on crack growth rate. People have later modified the Paris law to accommodate for the environment. And in summary, Paris law is elegant and corrections are incorporated to this, to model the role of environment. We would also see, how does the environment changes the crack growth rate. What kind of aggressive environments that we have to look at? We will see all that, after a few slides. And before we proceed further, you know, we will have to ask a question, is Paris law valid?

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ENGINEERING FRACTURE MECHANICS

Crack Initiation and Life Estimation

Validation and existence of Paris law

Does loading configuration affect the Paris Law?

- Donaldson and Anderson collected crack growth data for two entirely different configurations.
- One is a center crack subjected to a wedge load.
The SIF decreases as a function of crack length.
- The other is a center cracked panel subjected to a far field stress.
The SIF increases as a function of crack length.

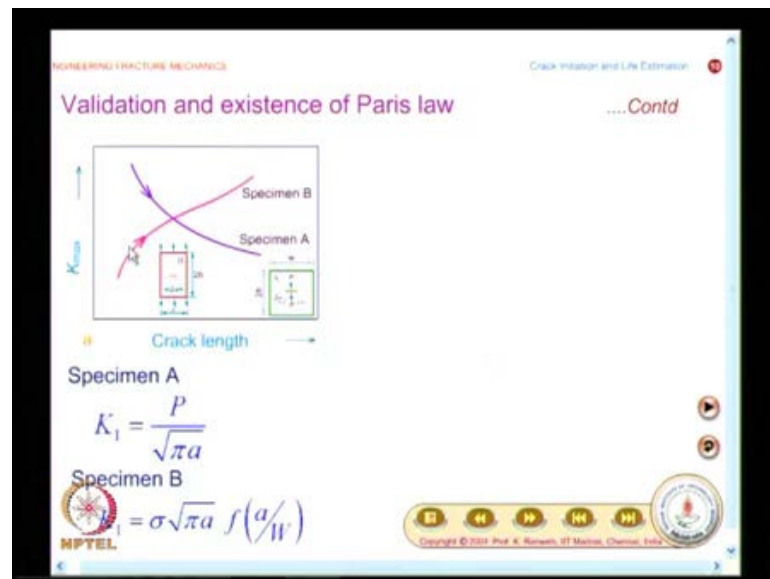
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In fact, this question has been answered in the literature and they have taken two different specimens. This was done by Donaldson and Anderson; they collected crack growth data for two entirely different configurations. See, you are really talking about, at what rate the crack would grow. And what way they choose the configurations? In one case, stress intensity factor decreases as the function of crack length. In fact, we had seen that, when we had developed stress intensity factors. If a center crack is subjected to a wedge load, we saw that SIF decreases as a function of crack length.

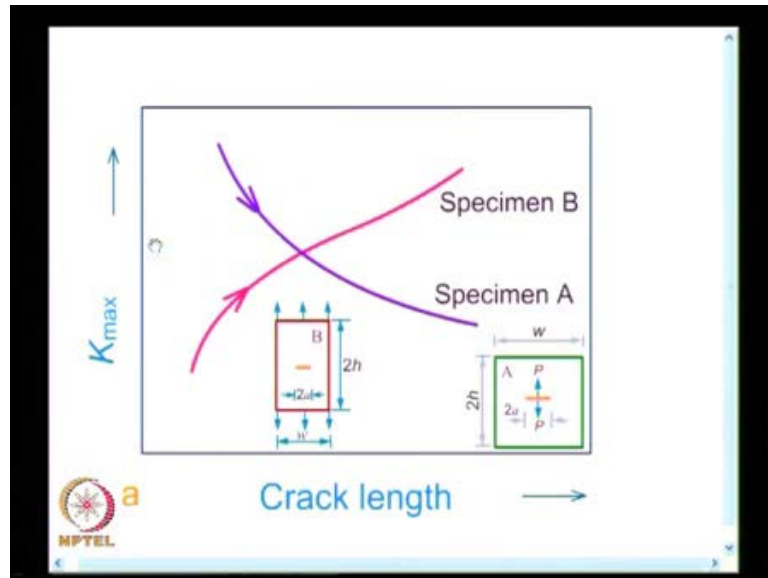
The other example they chose was, the SIF increases as a function of crack length. And this is obvious. This happens in most of the problems. So, if you take a center cracked panel, subjected to a far field stress, SIF would increase as the function of crack length. Suppose, Paris law correctly predicts, in these two opposing type of SIF behavior, that da by dN could be related to ΔK , then there is substance in the development of Paris law. This is the way people raised the question and answered the question also.

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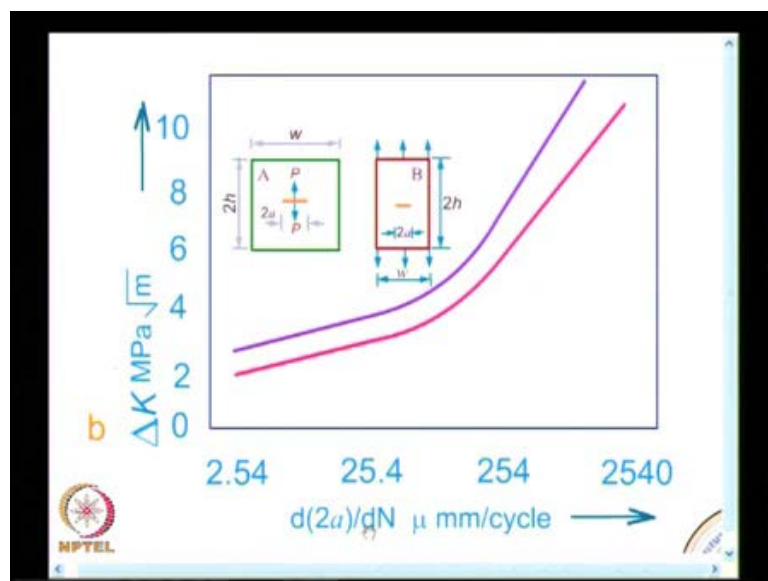
So, they took a specimen A, for which K_I equal to P by root of πa . So, as the crack length increases, the stress intensity factor decreases. They considered specimen B, wherein, as the crack length increases, the stress intensity factor also increases.

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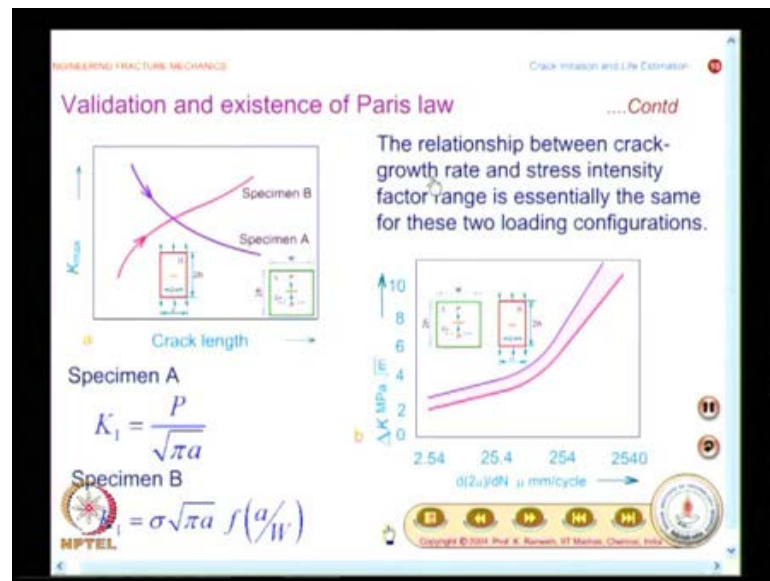
You can have a clear view in this. Two specimen configurations have been taken to validate the existence of Paris law. In the case of specimen A, stress intensity factor decreases as the function of crack length. In the case of specimen B, stress intensity factor increases as a function of crack length. So, when they performed the test, what they found was, for both of these, you could get a meaningful graph, wherein, the x axis is $d(2a)/dN$.

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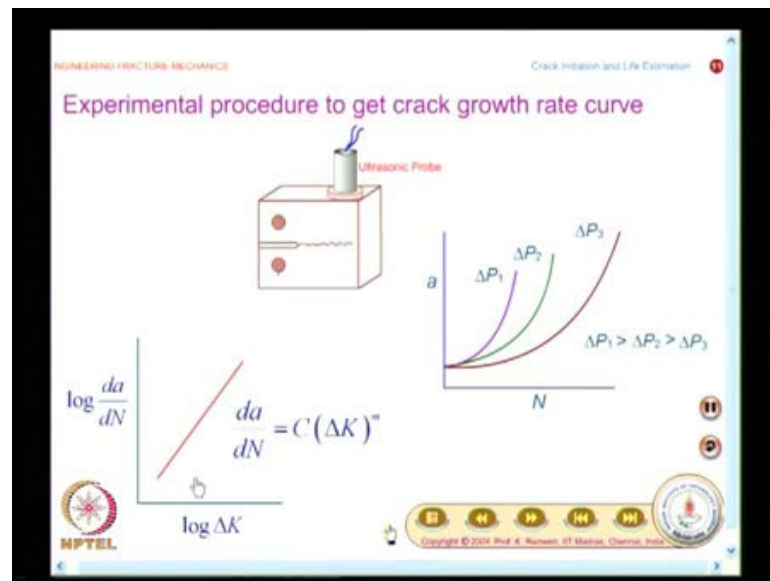
I will magnify it for you. You have $d^2 a$ divided by $d N$ and the y axis is ΔK . So, in both these conflicting waves the S I F changes as a function of crack length. You could establish a relationship between the crack growth rate with respect to ΔK . It is plotted differently. It is put in the x axis $d^2 a$ by $d N$ rather than the y axis, but that is how the literature has originally reported the results. So, this gives a confidence that Paris law could be utilized for your design calculation. That is way you have to look at it.

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So, the relationship between crack growth rate and stress intensity factor range is essentially the same for these two loading configurations. So, that provided enough confidence for people to follow and use the Paris law. Many modifications have come on this. That we shall see.

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And this is just to show a summary, how to get the crack growth rate curve and end with your Paris law. You take a specimen which is subjected to cyclical loading; it goes to 0; it does not reverse; and you monitor the rate of crack growth. So, from this data, find out the constant C and m based on the experimental result, and represent the crack growth rate in this form, da by dN curve.

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ENGINEERING FRACTURE MECHANICS

Crack Initiation and Life Estimation

Crack Initiation and Catastrophic Failure

- It is well known that many ferrous materials exhibit unlimited fatigue life, provided that the stresses are within the endurance limit.
- This indicates that very short cracks may not propagate for small values of ΔK .
- The smallest value of ΔK for crack propagation is known as threshold stress intensity ΔK_{th} .
- At the other extreme, when ΔK becomes comparable to fracture toughness of the material, the crack growth rate becomes very large.

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We will also have to note, about crack initiation; it is a very important phase. In fact, if you look at, many ferrous materials exhibit unlimited fatigue life, provided that the

stresses are within the endurance limit. You know, you will have to keep in mind, once you have a crack, how the crack would grow by fatigue is well represented by Paris law. There is a crack initiation phase. And here, we also bring in our understanding on fatigue. In the case of fatigue, you have a concept of endurance limit. So, that shows, that cracks which are very smaller and also you have loading, which is quite small, under such conditions crack may not grow in service. This is the conclusion that we can arrive at. So, that means, there has to be a threshold stress intensity factor, below which crack may not initiate. So, this concept was advanced. That is what is summarized here. So, on one part of the spectrum, below a value of delta K threshold, the crack would not initiate. We will have to find out what is the value of delta K threshold, for a given material.

The basic idea is, very short cracks may not propagate for small values of delta K. This is one end of the spectrum; at the other end of the spectrum, delta K would become comparable to fracture toughness of the material; the crack growth rate becomes very large. So, both exists. If you want to understand the complete story of the crack, there would be a crack initiation phase, there would be a crack growth phase, followed by catastrophic failure.

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ENGINEERING FRACTURE MECHANICS Crack Initiation and Life Estimation

Fatigue Crack Growth Threshold, ΔK_{th} for Selected Engineering Alloys

Material	σ_s MPa (ksi)	$R = K_{min}/K_{max}$	ΔK_{th} MPa√m	ΔK_{th} ksi√in
Mild steel	430 (62)	0.13	6.6	6.0
		0.35	5.2	4.7
		0.49	4.3	3.9
		0.64	3.2	2.9
		0.75	3.8	3.5
A533B		0.1	8.0	7.3
		0.3	5.7	5.2
		0.5	4.8	4.4
		0.7	3.1	2.8
		0.8	3.1	2.8

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Move Material Properties

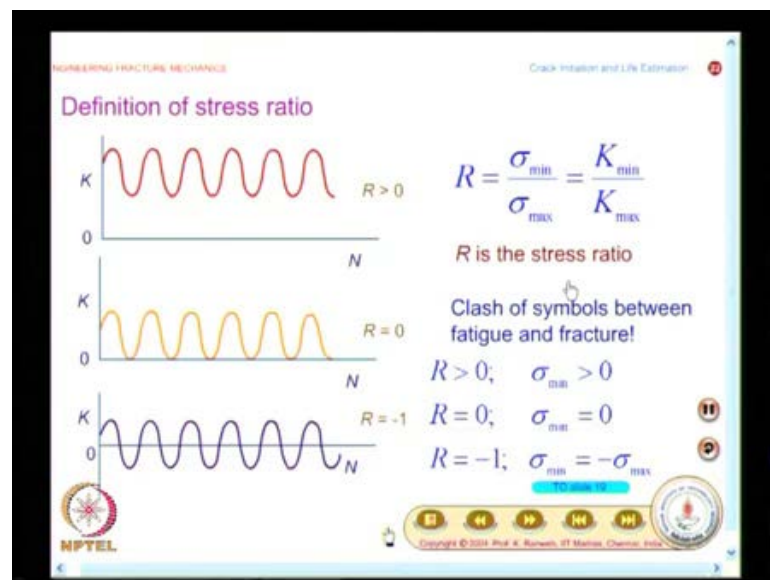
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And you also have in the literature, what is the value of delta K threshold for many of the alloys. We have, we will just see a sample of it. And you have the steel and there are two columns that, you have to pay attention. There is a column, where you have R. We will

quickly see it is definition. R is a stress ratio. This is also equal to K minimum divided by K max. This is not your R curve. There is a mixture of symbols from fracture and fatigue.

So, if you go to fatigue literature, when the mean stress increases, R also would increase. It changes from 0.13 to 0.75 and if you look at, the delta K threshold value decreases as the mean stress is increased. It goes from 6.6 Mpa root meter, drops down to 3.8 Mpa root meter. That means, if the mean stress is increased, the delta K threshold comes down. So, the crack would initiate earlier than what it would occur when R equal to 0. You see the same situation in the other material also. When R is changed from 0.1 to 0.8, the delta K threshold changes from 8 Mpa root meter to 3.1.

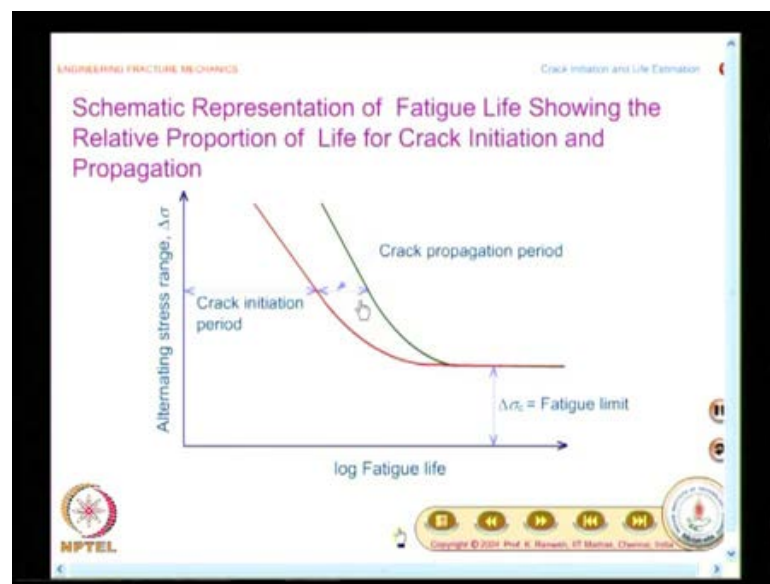
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So, we will see, what is R. This comes from the fatigue literature. R is nothing but stress ratio. So, I have a graph between N and K, and you have cyclically varying load. So, in this case R is greater than 0, and R is defined as sigma minimum divided by sigma max, which is equal to K minimum divided by K max, where R is the stress ratio. This is not your R curve. So, depending on the context, you should attach a meaning to the symbol R. And when you are talking about crack initiation and life estimation, we will have to look at R, which is the stress ratio in fatigue literature. So, when I have R equal to 0, the minimum load touches 0, and R equal to minus 1 is the kind of loading that is given to your rotating bending specimen in the case of fatigue.

In fracture, we will not do this; we would have either R equal to 0 or R as greater than 0. The tests are done for these kinds of situations. So, when R is greater than 0, σ minimum is greater than 0; when R equal to 0, σ minimum equal to 0; when R equal to minus 1, σ minimum is minus σ max. So, you have to keep in mind, there is clash of symbols between fatigue and fracture. In this chapter, we would use R as stress ratio. We will have to discuss some of the issues and we would use R in this manner.

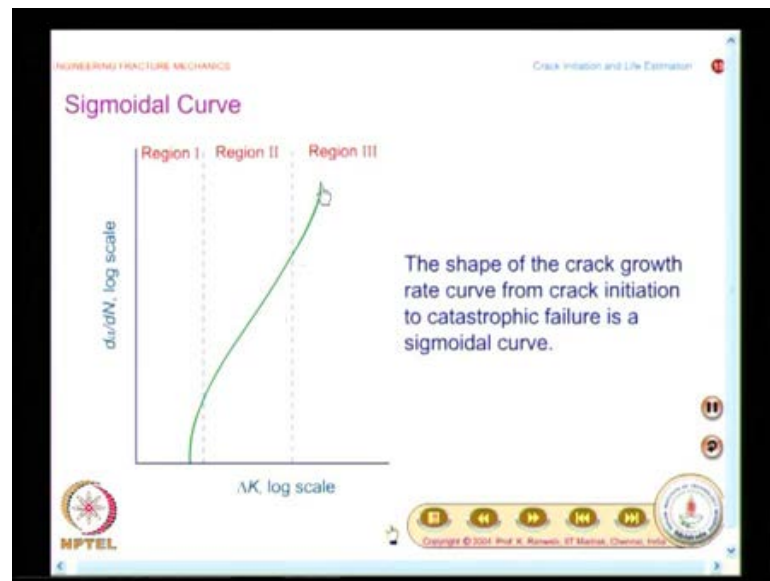
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And it is also important, what is the relative proportion of crack initiation versus crack growth. We had seen this slide long time back, in the over view of fracture mechanics. Depending on which stress level that you operate, crack initiation and crack propagation could be 50-50, or crack initiation could be longer and crack propagation could be smaller. The crack initiation phase, it is the metallurgists, who really make a contribution. If you delay the crack initiation, it is all the more better. So, you gain advantage in the design life, if you have mechanisms to delay crack initiation. But once the crack starts growing, we could develop NDT **shell** schedules based on fracture mechanics and handle the situation.

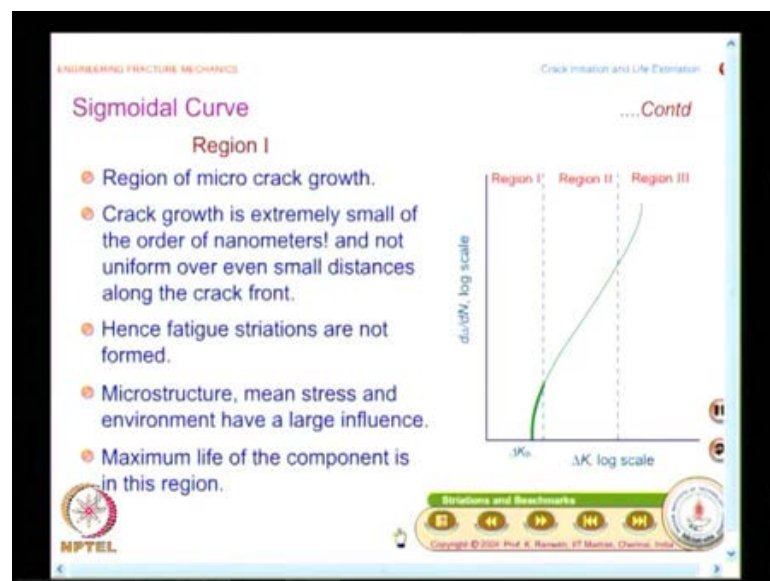
And we would see later, what are the mechanisms for crack initiation. After discussing the crack propagation phase, towards the end of this chapter, you would also see models that tell you how a fatigue crack gets initiated.

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And you know, you have a phase, wherein ΔK threshold is required for a crack to get initiated. This could be called as region 1. There is a growth phase which is called the region 2. There is a catastrophic failure, which is called region 3. So, if you look at the complete graph of a crack growth, this is known as a sigmoidal curve. Currently, we know how the crack grows in the region 2. So, we would see region-wise, what are the issues that are important. What happens in region 1, what happens in region 2, what happens in region 3. This is the way we will look at it.

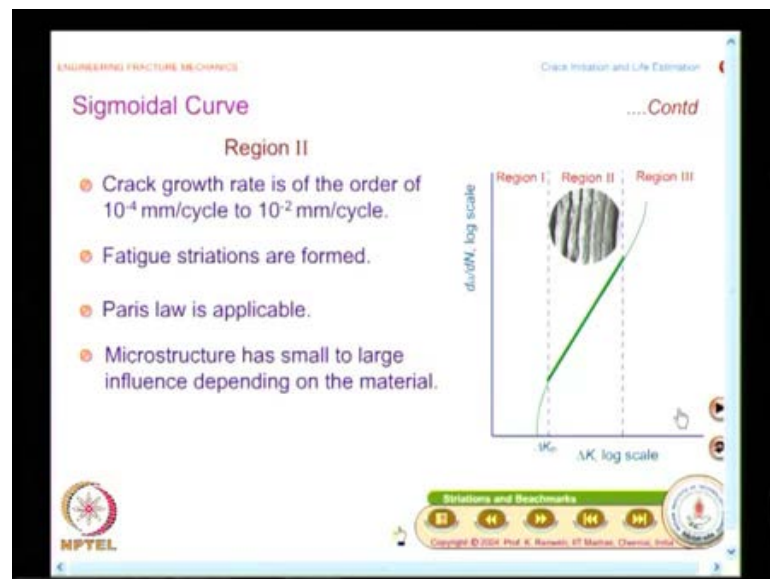
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What happens in region 1? This is the region of micro crack growth. And if you look at this graph, only this portion is highlighted. And in this case, what you find is, the crack growth is extremely small, of the order of nanometers. And further, they are not uniform over even small distances along the crack front.

So, what is the result? You would not have formation of striations, which is what you see in a fatigue crack growth. In the case of region 1, you will not see clear cut striations. So, that is what is summarized here. And what are the aspects that influence - the micro structure of the material, mean stress and environment have a large influence. So, as I mentioned, the metallurgists play with the micro structure, so that, they are in a position to delay the crack initiation phase. And mean stress thus play a role. You have to know, whether you are operating at higher values of R. And environment also has a large influence. And as I mentioned, and you also see in the graph, maximum life of the component is in this region. It depends on the operating load. When you are going to operate at lower stress levels, you will have a longer initiation phase, followed by growth phase.

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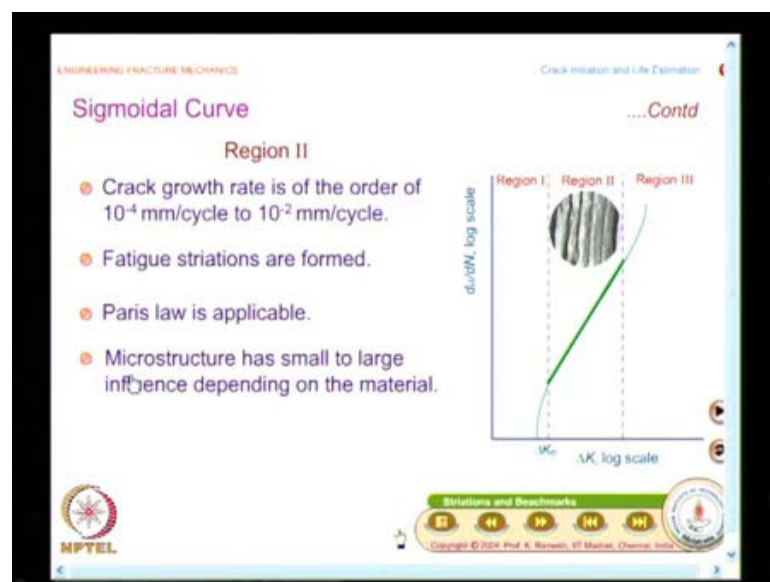


Then we will move on to region 2. In the case of region 2, crack growth rate is of the order of 10^{-4} millimeter per cycle to 10^{-2} millimeter per cycle; that means, one hundredth of a millimeter. So, it is still small. So, what you will find is, initially the crack growth rate will be smaller; as the crack length increases, the

crack growth rate also will increase. In fact, in one of your assignment problems, you will have striations recorded from a metallurgy test and based on that striations and that distances, you will have to calculate the crack growth rate. It is possible. And because you have larger crack growth rate, in this region, striations are possible.

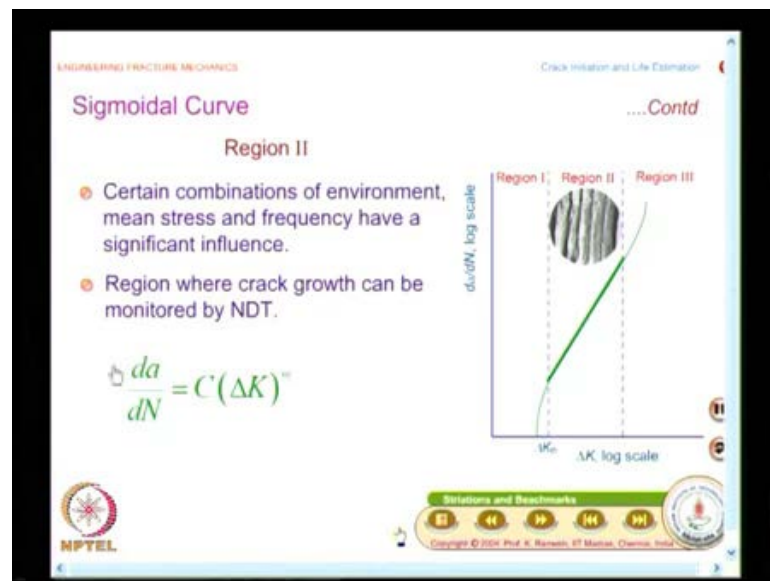
We have very clearly seen, in those materials where striations are common. So, that is what is shown here. In region 2, you will be able to see striations, if the material has the ability to show them. Because while discussing striations I said, in some materials striations are not seen. That does not mean fatigue crack growth has not occurred in that. It is also a function of the composition of the material. So, in those materials where striations can be seen it would be seen in the region 2.

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And if you look at, the micro structure has small to large influence depending on the material. It depends on what material you are considering. And the next aspect is, in what way the environment has a role?

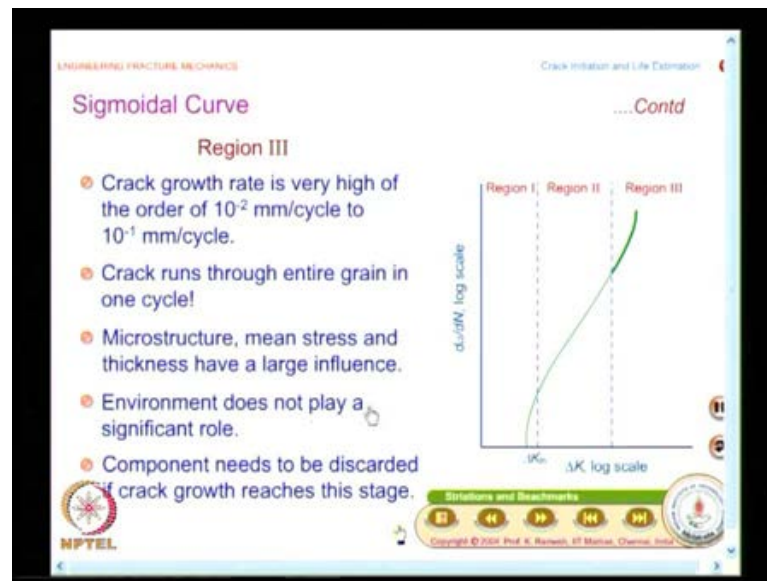
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Certain combinations of environment, mean stress and frequency have a significant influence. See, you have random vibration problem. So, you will have to know what is the frequency with which load is being applied. So, all that matters. The problem becomes more and more complex. Paris law is the simplest representation for you to understand the crack growth rate. And this is the region where crack growth can be monitored by non destructive techniques and we have already seen Paris law and Paris law is applicable in this region.

And if you look at C and m, these are like generic symbols. For applying Paris law, we would have selected data from... On that data you can process and then calculate the value of C and m. If you have another law which you would see sooner, it will also have C and m, but those symbols are not same as these symbols. It is only generic symbol. You have to collect that data from the material, on which you have been able to fit the curve.

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So, it goes with that. And region 3, the crack growth rate is very high, of the order of 10^{-2} millimeter per cycle to 10^{-1} millimeter per cycle; that means, 0.1 millimeter per cycle. Crack runs through entire grain in 1 cycle. So, obviously, when the component reaches this stage, you have to discard it. You cannot use this component at all. And in region 3, micro structure, mean stress and thickness have a large influence. See, we have seen environment has played a role, in the case of region 1 as well as in region 2. For region 3, environment does not play a significant role.

Component needs to be discarded, if crack growth reaches this stage; that is quite obvious. So, if, **if** your Paris law is applied properly, you would be able to say, when does the crack growth rate increase, and Paris law is valid only up to the end of region 2. So, this is the region people wanted to incorporate, what happens to predict region 3, they have tried to incorporate this as well as the threshold. So, you have many laws that have been proposed, which follow the basis of Paris.

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ENGINEERING FRACTURE MECHANICS

Crack Initiation and Life Estimation

Donahue et al law - to account Region I better

$$\frac{da}{dN} = C (\Delta K - \Delta K_{th})^m$$
$$\Delta K_{th} = (1 - R)^\gamma \Delta K_{th(0)}$$

$\Delta K_{th(0)}$ is the threshold value at $R = 0$

R is the stress ratio

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I have $\frac{da}{dN}$ equal to C multiplied by ΔK minus ΔK threshold whole power m . This is what I had mentioned earlier. This C and m , you cannot take from Paris law, fitted the material, those value of C and m here, and then use it; you will have to fit the data for the kind of materials that they have studied and only for those materials, you would be able to use this law. It is essentially a curve fitting exercise.

Whatever the crack growth rate curve, the form that we get, it is essentially a curve fitting exercise. Purely empirical. And, if it is developed based on a particular data set, you should not use this empirical law outside this data set. So, it goes with the data set, that you have focused upon and ΔK threshold is expressed as the function of R . We would see later. And, when R changes, ΔK threshold also changes. So, this is given as $1 - R$ whole power γ multiplied by ΔK threshold, when R equal to 0 . So, when the mean stress is changed, the crack initiation stress intensity factor also changes. So, that is worrisome. So, you have this law to account region 1 better. So, now, we have region 1 as well as region 2. So, the natural extension is, how region 3 can be handled.

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ENGINEERING FRACTURE MECHANICS

Crack Initiation and Life Estimation

Forman law to account Region III better

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1-R)K_{Ic} - \Delta K}$$

R is the stress ratio

Efforts have also been made to model the entire sigmoidal curve through empirical equations.

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So, this is done by Forman law and this is very popular, among the laws that you have for crack growth. Here you have this as $d a$ by $d N$ equal to C into ΔK whole power n divided by 1 minus R K_{Ic} minus ΔK . So, this accounts for the stress ratio R and it also accounts for the region 3, where K_{Ic} is the fracture toughness. And, in fact, if you look at the literature, people have also developed computer software to give the expected life, to give the crack growth and what are the periodic NDT schedules based on these crack growth laws. And you have, what are known as NAS growth and A F growth and so on and so forth. In all those computational softwares, people have used variation of this Forman law, as the basis. It forms the basis, for such softwares. We have seen separately region 1, region 2, region 3. Efforts have also been made to model the entire sigmoidal curve through empirical equations.

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ENGINEERING FRACTURE MECHANICS

Crack Initiation and Life Estimation

Mathematical representation of complete sigmoidal curve (Erdogan and Ratwani)

$$\frac{da}{dN} = \frac{C(1+\beta)^m (\Delta K - \Delta K_{th})^n}{K_c - (1+\beta)\Delta K}$$
$$\beta = \frac{K_{max} + K_{min}}{K_{max} - K_{min}}$$

$C(1+\beta)^m$ → Accounts for mean stress

$(\Delta K - \Delta K_{th})^n$ → Experimental data at low stress levels

$(1+\beta)\Delta K$ → Experimental data at higher stress levels

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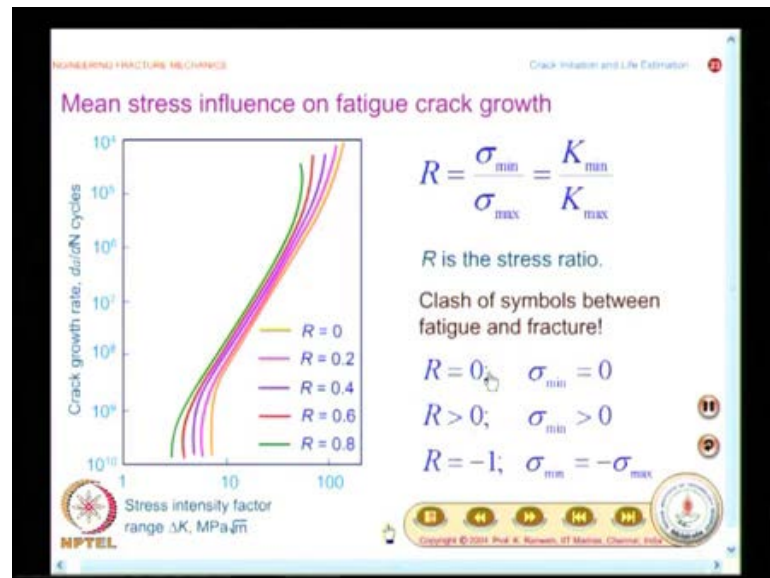
So, you have another law. It is given like this. This is by Erdogan and Ratwani. And they have given da by dN as capital C into $1 + \beta$ whole power m multiplied by ΔK minus ΔK threshold whole power n divided by K_c minus $1 + \beta$ multiplied by ΔK .

See, you have to keep in mind, these are all empirical relations. So, you have to find out many material constants and very expensive test that, also you have to keep in mind. And if you really look at the history, people did not jump on to LEFM until Paris law was invented. Because in Paris law they found, they could monitor the crack growth, so, there is some utility of fracture mechanics concepts. Otherwise fracture mechanics only says, when catastrophic failure will occur. Though it is important information, that information alone, is not sufficient. If you are able to predict the crack growth, then designers felt, this is the right way that I can go and use linear elastic fracture mechanics. So, β is equal to $K_{max} + K_{min}$ divided by $K_{max} - K_{min}$ and you have, all these factors are defined like this.

$C(1 + \beta)^m$, accounts for mean stress, because β has this kind of a ratio. And $(\Delta K - \Delta K_{th})^n$ whole power, whole power n it accounts for experimental data at low stress levels. Finally, the term $K_c - (1 + \beta)\Delta K$, accounts for experimental data at higher stress levels. But for all these laws, what is the basis? Paris law is the basis. That is why you have to give him credit. If he had

combined the environmental effects, you would not have arrived at da/dN versus ΔK . Now, everybody in fracture mechanics literature knows, that you have to play with da/dN and ΔK . Then, they have added, you add ΔK threshold, you add K_c you add R . So, the basic kernel was provided by Paris.

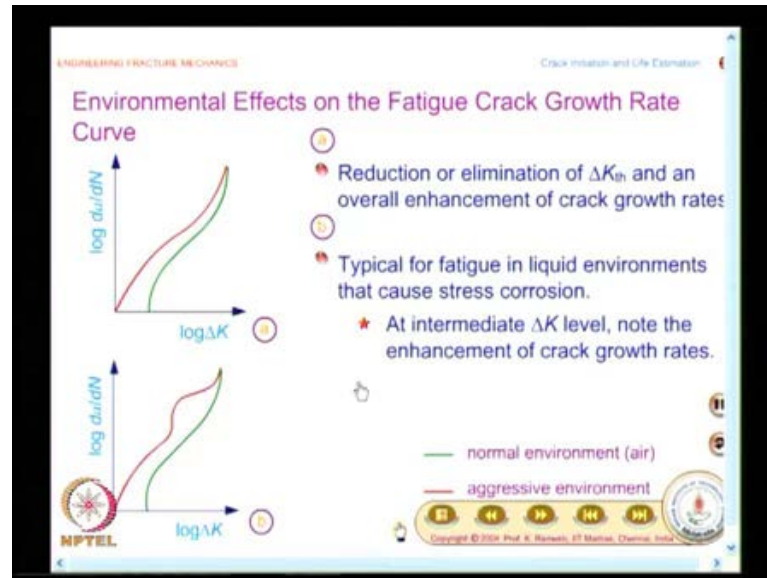
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So, that is why his law is very important. And we will have to see, what is the effect of mean stress on fatigue crack growth. We have already looked at the definition of stress ratio R . And you have a graph. In the x axis, it is ΔK and the y axis, it is da/dN . And this is for a particular value of R ; I think it is for R equal to 0. And when R is changed, how does the graph change. It gets shifted to the left; so, that means, K threshold keeps decreasing. There is no major influence in this zone.

The crack growth rates slightly increases, and your catastrophic failure is represented by this. The significant observation is, the ΔK threshold decreases as the mean stress is increased. There is, obviously, increasing crack growth, but that influence is reasonably small. That is the way you have to look at it. So, try to get a neat picture as much as you can try to do. And I have already mentioned R is not the R curve; it is actually σ_{min} by σ_{max} or K_{min} by K_{max} . And we work on $R = 0$ or R greater than 0.

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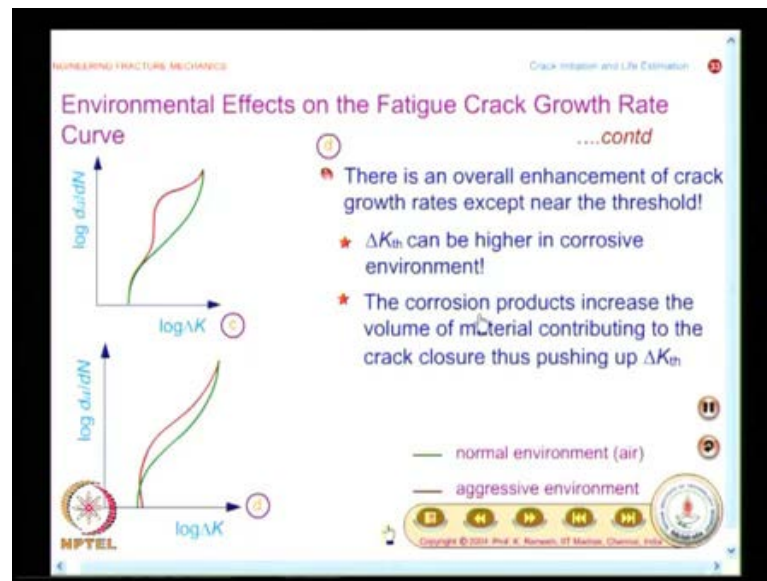


So, we will now look at, how does the environment modifies the sigmoidal curve. If you have an aggressive environment, what you find here is, the delta K threshold is 0. It is very **very** dangerous. In a normal environment, you may have a finite value of delta K threshold; in an aggressive environment, it can be 0. And there is significant increase in crack growth rate, given in the crack growth phase; that is region 2 also, you have a higher crack growth rate. And, this kind of a graph, is seen for a particular kind of environment.

In the case of graph b, it is observed in liquid environments, that cause stress corrosion. There are two things happening. Here, the delta K threshold is 0. Not only this. In comparison to this, there is a sudden increase in crack growth rate in the intermediate stage, which is seen in cases, where you have liquid environments, that causes stress corrosion.

So, the graph is different. See, this graph is simplest one you can model; that was given by Paris. And these are all corrections to it. You could see it is highly non-linear. You know, if somebody tries to fit a law for this, it would be extremely difficult. On the other hand, in the case of Paris, there is a straight line portion followed by one curve at the initial start and one curve at the end.

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And, we will also see, two more situations. This is another aggressive environment phase. The delta K threshold is not modified, but the crack growth increases. So, this happens, if you have a gaseous environment, usually in the hydrogen embrittlement case, this kind of situation happens. And, what do you see here? There is an overall enhancement of crack growth rate, except near the threshold. And you have another case, which is very interesting. See, we have seen cases where delta k threshold is not altered or delta K threshold becomes 0. You have a interesting situation where delta K threshold is pushed up; that is the situation we have here. And you have higher crack growth rate in this section. And this happens, when the corrosion products increase the volume of material, contributing to the crack closure, thus pushing up delta K threshold.

We would spend sufficient time on what is crack closure. Once you understand this, then you can appreciate that delta K threshold can increase under suitable conditions. So, people have found these variations, in the case of crack growth data. So, Paris did not consider the environment and developed a very elegant law, which is purely empirical. Later, people have modified to account for delta K threshold as well as fracture toughness. So, in this class, essentially we looked at what is Paris law, and it was actually reported as the Boeing technical note. When initially reported, nobody believed it. This is what has happened for (()); because he did not provide the experimental result, people did not believe it. Hahn and Rosenfield had to come to his rescue and

demonstrated the experimental patterns. Here, you find, in experiments, where people have done it for two different ways SIF can change, as the function of crack length.

In one case, SIF decreases as a function of crack length; in another case, SIF increase, increases as the function of crack length. In both the cases, the graph between $d a$ by $d N$ versus ΔK was meaningful. So, that established Paris law is acceptable. And we also saw sigmoidal curve. We saw different regions and each region, what are the factors that would influence; and particularly, the micro structure plays a very important role in crack initiation. And this is the domain of material scientists, and they really come out with new materials, where the crack initiation is delayed. That is very **very** advantageous, from fracture mechanics point of view. Thank you.