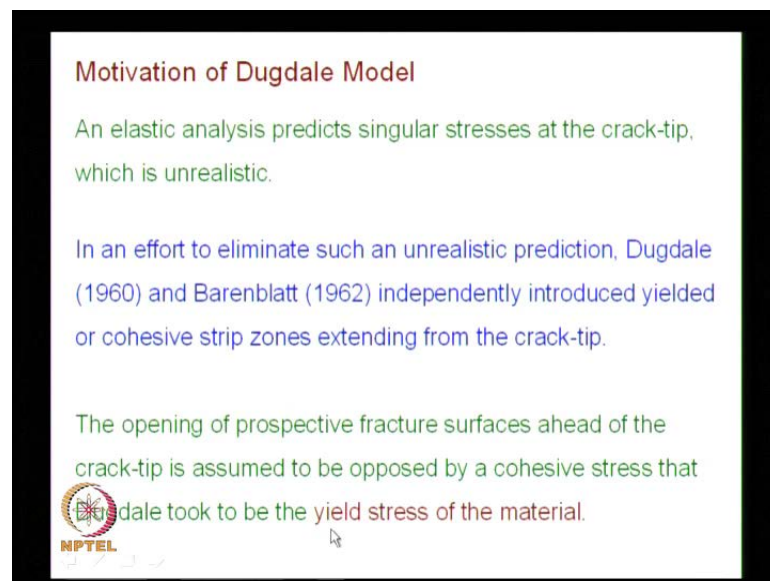


Engineering Fracture Mechanics
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Lecture No # 30
Dugdale Model

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Motivation of Dugdale Model

An elastic analysis predicts singular stresses at the crack-tip, which is unrealistic.

In an effort to eliminate such an unrealistic prediction, Dugdale (1960) and Barenblatt (1962) independently introduced yielded or cohesive strip zones extending from the crack-tip.

The opening of prospective fracture surfaces ahead of the crack-tip is assumed to be opposed by a cohesive stress that Dugdale took to be the yield stress of the material.

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See, in the last class we have looked at what is the motivation of Dugdale model; we will have a brief look at that and proceed further.

An elastic analysis predicts singular stresses at the crack-tip, which is unrealistic. In an effort to eliminate such an unrealistic prediction, Dugdale in 1960 and Barenblatt 1962 independently introduced yielded or cohesive strip zones extending from the crack-tip.

So, you will have to keep in mind these are all models. People were attempting to solve a very complex problem, so they have proposed from their understanding, which way to go about. The opening of prospective fracture surfaces ahead of the crack-tip is assumed to be opposed by a cohesive stress, and what did Dugdale do? Dugdale took that stress to be the yield stress of the material, so it is an assumption. Essentially, he has taken this kind of an assumption and proceeded with the model.

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Motivation of Dugdale Model

The extension of the cohesive zone is determined by the condition that the stresses be nonsingular.

There are obvious similarities between the approach of Dugdale and Barenblatt, which has led researchers to refer it as "Barenblatt-Dugdale crack theory".

However, a distinction has to be made as the physical basis of these approaches are different:

- Dugdale – based on macroscopic plasticity
- Barenblatt – based on molecular cohesion.

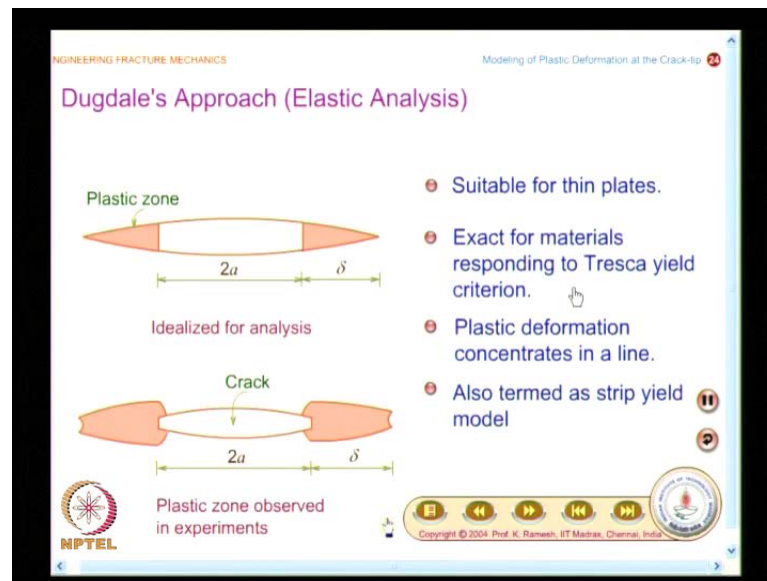


The next step is to find out the cohesive zone length and how this is determined? It is determined by the condition, that the stresses be nonsingular. When you look at the mathematical development, this statement would become quite clear and you will also have to note, that there are obvious similarities between the approach of Dugdale and Barenblatt, which has led researchers to refer it as Barenblatt-Dugdale crack theory.

But you will also have to note, that these have different physical basis. In fact, all of this we had discussed towards the end of last class, this is only for recapitulation.

The physical basis of Dugdale's model is macroscopic plasticity; on the other hand, Barenblatt theory is based on molecular cohesion.

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Now, we will take up the mathematical development of Dugdale's approach and you need to make neat sketches of this, and one is an idealization of the crack, other one is an actual crack with the plastic zone developed.

So, what you find here is, you have a crack of length $2a$; there is an extent of plastic zone, which is depicted as δ . And you would also see the experimental result and convince our self, that this kind of picturization is what is seen in the actual experimentation. A figure like this is taken up for mathematical analysis and you will have to keep in mind, that the Dugdale's approach is suitable for thin plates. This is the restriction number 1 and the theory also assumes, that the materials respond to Tresca yield criterion. So, only for those materials the results will be exact.

And Dugdale also made another assumption. You know, we had earlier looked at what is the shape of the plastic zone near the vicinity of the crack-tip, we had seen a finite shape in front of the crack-tip for the Dugdale's model. He assumed that plastic deformation concentrates in a line. See, this goes with the other two assumptions. When you have a thin plate and the material respond to Tresca yield criterion, this is also valid.

In fact, we would see later, when Dugdale reported the results people did not accept it immediately, people were waiting for the experimental result of Hahn and Rosenfield;

we will see that also. And because you have plastic deformation concentrates in a line, the model is also termed as strip yield model.

So, what you have is, you have an extension of plastic zone, that distance is given as delta and we will have to find out how to get the extension of length delta.

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ENGINEERING FRACTURE MECHANICS

Modeling of Plastic Deformation at the Crack-tip

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Dugdale's Approach

Dugdale considered effective crack of length $a + \delta$, where δ is the length of plastic zone determined such that

$$K_{\sigma} + K_{\delta} = 0$$

$$K_{\sigma} = \sigma \sqrt{\pi(a + \delta)}$$

K_{σ} is the singularity due to pressure σ_{ys}

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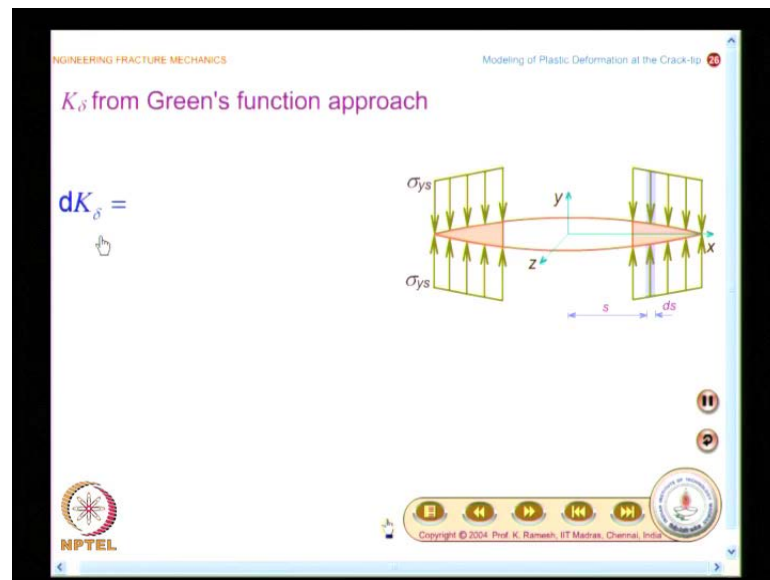
And make a neat sketch of this, you have a center crack in an infinite strip and what you have here is, the length of the crack is $2a$ and for this zone, which is plastically deformed. The stress level is taken as σ_{ys} , this is an assumption made by Dugdale in developing the model. And what is the effective crack length now becomes? It becomes $a + \delta$.

And from the discussion we have done earlier, what we are really looking at is, we want to see $K_{\sigma} + K_{\delta}$ should go to 0. You have a center crack in an infinite strip, because of the applied stress σ you will have a singularity developed. And what Dugdale assumed is, ahead of the crack-tip, there would be a plastic zone and the associated stress level, he had taken that as yield. The strength of the material σ_{ys} , this will also introduce stress singularity. And we have to find out the length delta in such a fashion, the summation of these two goes to 0. And what is K_{σ} ? K_{σ} is $\sigma \sqrt{\pi(a + \delta)}$.

We are taking a fictitious extent of crack length by a distance delta and based on that only you have to calculate the stress intensity factor K_{σ} . And as I mentioned earlier, K_{σ} is a singularity due to pressure σ_{ys} .

See, the singularity, because of the applied stress, is given in a very simple fashion and can you find some similarity here to find out how to evaluate the stress intensity factor in the presence of the σ_{ys} ? The crack phases are symmetrically loaded, that you can recognize and in fact, we have done evaluation of stress intensity factor for symmetric concentrated load acting on the crack surface. Earlier, in fact, you could invoke that and write an expression.

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The difference is you are having a distributed loading where we had looked at a concentrated loading. So, I can take a distance s from the center of the crack and look at an incremental distance, ds . Please make a neat sketch of this graph, the crack loaded with stresses. I can also look at simultaneously on the negative x -axis at distance s , another strip of load.

And what we are going to write is we will write dK_{σ} . We would just invoke the expressions that we have developed for a symmetric load and write this for an incremental value of stress intensity factor. Then, you integrate for the extent of the load σ_{ys} applied; you will be able to calculate the stress intensity factor.

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ENGINEERING FRACTURE MECHANICS SIF for Various Geometries and Loading

Symmetric wedge load

Wedge load on cracked surfaces

Stress function

$$Z_1 = \frac{2Pz}{\pi(z^2 - s^2)} \left(\frac{a^2 - s^2}{z^2 - a^2} \right)^{\frac{1}{2}}$$

$$K_1 = \frac{2P\sqrt{\pi a}}{\pi\sqrt{a^2 - s^2}}$$

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So, if you look at the form of the stress intensity factor for a symmetric wedge load, you have this load P per unit length, you have this as 2 by pi P into root of pi a divided by root of a squared minus s squared. So, I can invoke this and write the incremental value of stress intensity factor in the current problem.

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ENGINEERING FRACTURE MECHANICS Modeling of Plastic Deformation at the Crack-Tip

K_δ from Green's function approach

$$dK_\delta = -\frac{2\sigma_{ys} ds \sqrt{\pi(a+\delta)}}{\pi \sqrt{(a+\delta)^2 - s^2}}$$

On integration

$$K_\delta = -\frac{2\sigma_{ys} \sqrt{\pi(a+\delta)}}{\pi} \int_a^{a+\delta} \frac{1}{\sqrt{(a+\delta)^2 - s^2}} ds$$

$$= -\frac{2\sigma_{ys} \sqrt{\pi(a+\delta)}}{\pi} \left[\cos^{-1} \frac{a}{a+\delta} \right]$$

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So, you have dK delta equal to minus 2 by pi. That expression was for a load trying to pull the crack surface; here, the load is trying to close the crack surface. So, I have a minus sign here and the stress level is known, this is given as sigma ys and incremental

distance is ds . So, $\sigma_y ds$ denotes the load P and $\sqrt{a + \Delta}$ and $\sqrt{a - \Delta}$ is taken as $\sqrt{a + \Delta}$ and you have divided by $\sqrt{a + \Delta^2 - s^2}$. And rest of it is simple mathematics, you know, we have been handling integration of such quantities and we would get those expressions.

So, what I have here is, it becomes K_{II} equal to $-\frac{2\sigma_y}{\pi} \int_{\sqrt{a-\Delta}}^{\sqrt{a+\Delta}} \frac{ds}{\sqrt{a+\Delta^2-s^2}}$.

You know, we have solved such expressions earlier; you can look at your notes a few pages back and then, see, what would be the integrated value? You are essentially using the table of integrals, so you can easily find out what would be the integrated value, then apply the limits. In fact, I am going to skip that step; I will apply the limits and write the final expression. The intermediate step, I would like you to do it, we have already seen it earlier while discussing evaluation of stress intensity factors, so you can use that part of the notes. So, I am skipping the intermediate step of putting the limits.

After putting the limits, whatever the expression I get, could be simplified to $\cos^{-1} \frac{a - \Delta}{a + \Delta}$ and this is pre multiplied by $-\frac{2\sigma_y}{\pi} \sqrt{a + \Delta}$.

So, what we have done? We have used our knowledge of stress intensity factor for symmetric load acting on the crack phase, extended it for finding out the stress intensity factor when I have σ_y on either side of the crack-tip, and we have got the expression for K_{II} . What we have to do next? We have to write the expression $K_I + K_{II} = 0$, that is what I am going to do.

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ENGINEERING FRACTURE MECHANICS Modeling of Plastic Deformation at the Crack-Tip

Dugdale's ApproachContd

$$\sigma\sqrt{\pi(a+\delta)} - \frac{2\sigma_{ys}\sqrt{\pi(a+\delta)}}{\pi} \left[\cos^{-1} \frac{a}{a+\delta} \right] = 0$$

$$\sqrt{\pi(a+\delta)} \left(\sigma - \frac{2\sigma_{ys}}{\pi} \left[\cos^{-1} \frac{a}{a+\delta} \right] \right) = 0$$

$$\sigma - \frac{2\sigma_{ys}}{\pi} \left[\cos^{-1} \frac{a}{a+\delta} \right] = 0$$

$$\frac{a}{a+\delta} = \cos \left(\frac{\pi\sigma}{2\sigma_{ys}} \right)$$

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And that is what is written here. So, what I have here is, sigma square root of pi a plus delta, this is the K sigma expression and K delta expression we have recently calculated, that is, minus 2 sigma ys square root of pi a plus delta divided by pi multiplied by cos inverse a by a plus delta equal to 0.

See, there is a subtle difference in the development of the expressions itself. See, if we have looked at what way we developed Irwin's model, we had a plastic zone, the plastic zone length was given as r p, but the value of delta to modify the crack length is taken as 1 half of r p that was taken as delta. But here, in the development itself, we always take the complete length of the plastic zone as a correction required for the crack length. This is seen at every step of Dugdale's development, so keep a note of this.

So, the next simplification is, if I take out square root of pi a plus delta, the expression turns out to be sigma minus 2 sigma ys divided by pi multiplied by cos inverse a by a plus delta equal to 0. At every stage you are correcting the crack length with the expression delta.

So, finally, when we look at the plastic zone length and there correction necessary for crack length, we have to handle it carefully in Dugdale's model as well as Irwin's model, there is a subtle difference between the two.

And I can take this to right hand side, I can knock off this. So, essentially, I get sigma minus 2 sigma ys divided by pi multiplied by cos inverse a by a plus delta equal to 0. So, this will help me to get an expression for delta. Let us see what we get? So, I get a by a plus delta. On simplification you can take it on the right hand side and you will get this as pi sigma 2 sigma ys, so you get this a by a plus delta as cos of pi sigma divided by 2 sigma ys. We have to solve this further.

See, whenever we have an expression like this, as engineers we have to bring in certain approximations. Only by bringing certain approximations you would be able to get a simplified expression and what is approximation we will do? When you have sigma and sigma ys appearing here in most of the fracture problems, the value of sigma will be far below the value of yield strength of the material. See, that was the worry; structures were failing as stress is far below the yield strength value.

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ENGINEERING FRACTURE MECHANICS

Modeling of Plastic Deformation at the Crack-tip

Dugdale's Approach

....Contd

- An approximate but simpler relation may be obtained for the cases $\sigma \ll \sigma_{ys}$ and $\delta \ll a$ as

$$1 - \frac{\delta}{a} = 1 - \frac{\pi^2 \sigma^2}{8 \sigma_{ys}^2}$$

$$\delta = \frac{\pi \sigma^2 \pi a}{8 \sigma_{ys}^2} = \frac{\pi}{8} \left(\frac{K_I}{\sigma_{ys}} \right)^2$$

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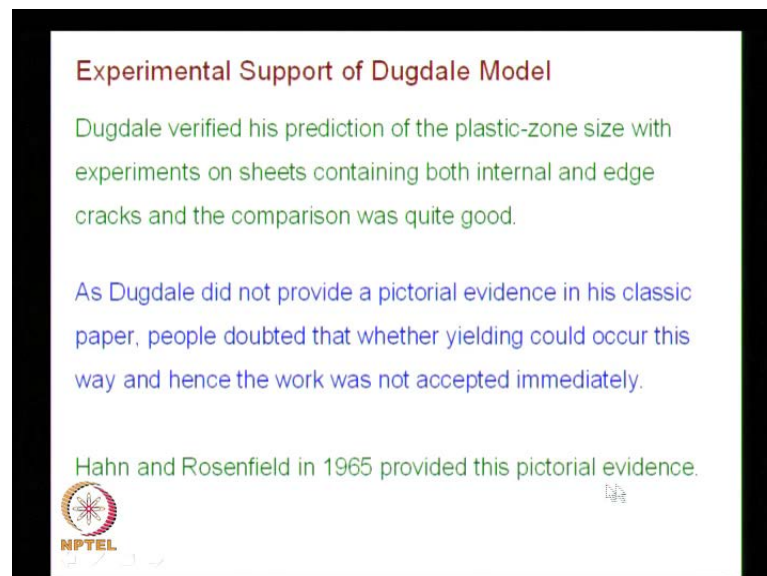
So, I can invoke this kind of simplification by saying sigma is far below sigma ys and also make another observation, that the extension of plastic zone is much smaller compared to the crack length. You know, these are very important, only when you make approximations like this we would be able to simplify and proceed further.

So, what you have is 1 minus delta by a equal to 1 minus pi square sigma square divided by 8 sigma squared ys. So, from this I would be able to calculate what is the expression

for delta and delta turns out to be $\pi \sigma^2 \pi a$ divided by $8 \sigma^2 y_s$. Since we are handling a center crack in an infinite plate, I can bundle this and write this as π by $8 K_1$ by σy_s whole squared. In fact, in all problems dealing with plastic zone length you will have an expression K_1 by σy_s whole square. It will be pre multiplied by a factor here. If the factor is π by 8 , we will also have a summary of these quantities and see how this can be utilized.

What is the kind of comparisons that we can make from simplistic model, Irwin's model and Dugdale's model? See, you will have to note, when Dugdale developed his model, he has got the extent of plastic zone length. In fact, in his original paper, he had just compared from certain experiments, what is the value of this extent of plastic zone experimentally, as well as predicted by his theory this matched quite well, but what he had not done is, he had not provided the pictorial support for his observations. We will also have to note that.

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Experimental Support of Dugdale Model

Dugdale verified his prediction of the plastic-zone size with experiments on sheets containing both internal and edge cracks and the comparison was quite good.

As Dugdale did not provide a pictorial evidence in his classic paper, people doubted that whether yielding could occur this way and hence the work was not accepted immediately.

Hahn and Rosenfield in 1965 provided this pictorial evidence.



So, Dugdale verified his prediction of the plastic zone size with experiments on sheets. So, he was working on thin specimens containing both, internal and edge cracks, and the comparison was quite good because it is only a model. See, you are charting a difficult problem and you have to idealize it, so that you get some useful result for you to proceed further. So, his idea was, there would be a plastic zone, it will be in the form of a strip and he was also able to verify his prediction based on experiments.

As Dugdale did not provide pictorial evidence in his classic paper, people doubted that whether yielding could occur this way and hence, the work was not accepted immediately. See, we will have to keep it in mind, you know, we had done a circus to find out, approximately, what is a shape of the plastic zone near the crack-tip. I said, only in the case of **mode three situation**, you have, this plastic zone is like a circle, in all other cases it has some shape. And someone comes and says, in the case of thin plate you have this as a strip, you know, people just could not digest his contributions. So, people are doubting, is there anything wrong in his approach?

Actually, if you look at the result, that he had quoted in the paper, the experimental and theoretical prediction, the comparison was quite good, he had not provided the pictorial representation. This was done by Hahn and Rosenfield in 1965, who provided this pictorial evidence, then people started believing, yes Dugdale's model is a valid approach for evaluating the plastic zone length.

You know, this is very important, like, as students you get doubts scientists also got those doubts in those days, so it had to be verified by experiments. So, whatever the theory, that you develop, the predictions have to be substantiated by experimental observation only, then the theoretical approach can be taken on its face value. It is very important.

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The slide is titled "ENGINEERING FRACTURE MECHANICS" and "Modeling of Plastic Deformation at the Crack-Tip". The main heading is "Experimental work of Hahn and Rosenfield". It contains three bullet points: 1. Hahn and Rosenfield demonstrated the existence of plastic zone as postulated by Dugdale's model. They used 3% silicon steel for the specimen and the surface of the test pieces were electro-polished and etched. 2. The method of etching ensured preferential attack of individual dislocation. This resulted in gradual darkening of the surface as the strain is increased from 1-2% (Note: Strain at yield is just 0.2%). 3. Beyond 2% strain the etching response has diminished and above 5% strain no attack at all! The slide includes a navigation bar at the bottom with icons for back, forward, and search, and a copyright notice: "Copyright © 2007 Prof. K. Ramesh, IIT Madras, Chennai, India".

- Hahn and Rosenfield demonstrated the existence of plastic zone as postulated by Dugdale's model. They used 3% silicon steel for the specimen and the surface of the test pieces were electro-polished and etched.
- The method of etching ensured preferential attack of individual dislocation. This resulted in gradual darkening of the surface as the strain is increased from 1-2% (Note: Strain at yield is just 0.2%)
- Beyond 2% strain the etching response has diminished and above 5% strain no attack at all!

So, what is the experimental work of Hahn and Rosenfield? They demonstrated the existence of plastic zone as postulated by Dugdale's model. And what they did was, they used 3 percent silicon steel for the specimen and the surface of the test pieces were electro-polished and etched, and this etching is a very special process. This has helped to reveal the shape of the plastic zone and essentially, they were metallurgists, they were handling these kinds of issues earlier, so they had used this for crack problem.

The method of etching ensured preferential attack of individual dislocation. So, whatever is the plastic zone, it is because of dislocation pile up and the etching process ensures, that there is a preferential attack and it makes the plastic zone reveal in some form. The etching has resulted in gradual darkening of the surface as the strain is increased from 1 to 2 percent.

See, you will have to note down, in the context of strain, 1 to 2 percent is very high because if you really look at material testing, the offset strain, that you take when the yield strength is not sharply defined, you take that as 0.2 percent offset. So, 0.2 percent itself is sufficient for the material to yield. So, when you have 1 to 2 percent, the material has definitely deformed plastically.

Beyond 2 percent strain the etching response has diminished and above 5 percent strain, no attack at all. These are certain minor details on how the experiment was done. What is important to us is by the process of etching, because of preferential attack, you are in a position to obtain the shape of the plastic zone that is what you have to look at. You are in a position to get the shape of the plastic zone by a particular procedure adopted in the experiment.

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ENGINEERING FRACTURE MECHANICS

Modeling of Plastic Deformation at the Crack-Tip

Experimental work of Hahn and Rosenfield

The etching technique thus revealed both the extent of the plastic zone and to some degree, the distribution of strain within the zone.

- To reveal plastic zone at various depths, the specimen is reground to various depths, polished and re-etched.
- Plastic zone shape of the surface and mid section of a sample specimen are as follows.

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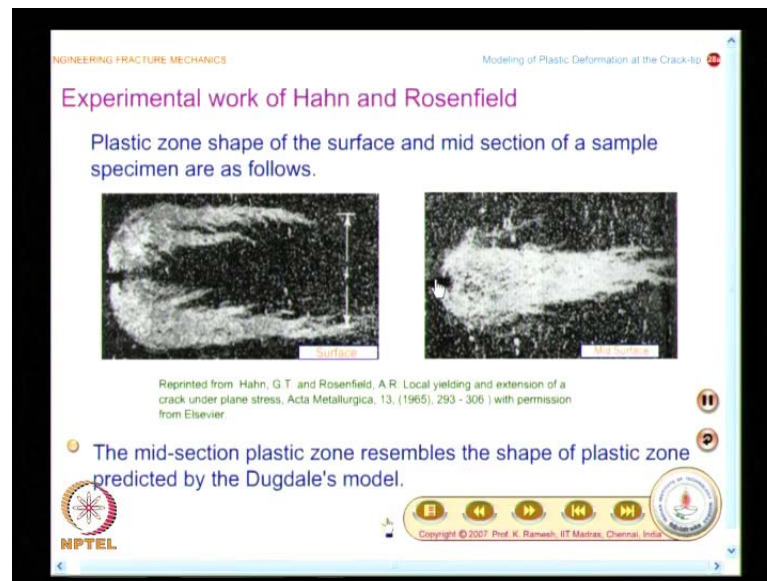
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The etching technique thus revealed both, the extent of the plastic zone and to some degree the distribution of strain within the zone because I have already mentioned, that 1 to 2 percent, it is able to do well; beyond 5 percent, there is nothing has happened. So, you also get some crude values on what is a level of strain, shape is very interesting; level of strain is additional information, though it is crude, it provides some such additional data.

And what they had done was to reveal plastic zone at various depths, the specimen is reground to various depths, polished and re-etched. So, you have a via media, whether the plastic zone remained constant over the thickness of the sheet of the specimen, all that you have to investigate because we are going to see, how the plastic zone changes across the thickness. And we will also see, how the plastic zone, say, changes as the crack propagates, both we will have to look at it.

So, whatever the experimental method of Hahn and Rosenfield, also ensured, not only you see the plastic zone on the surface of the specimen, but also at various depths, you have that kind of an advantage. And you have a sample of the plastic zone shape of the surface and mid-section of a sample and that is given as follows.

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On the surface the plastic zone is like this, at a mid-section the plastic zone is like this. See, this is a very highly magnified picture, you can for all practical purposes consult this as a line.

So, this happens on a surface, I would like you to make a sketch of this, you know, you need to have, these are all very valuable information and only after seeing such a plastic zone people believe Dugdale's model. So, from that point of view this is very important result, and make a neat sketch of this. This happens on the surface and we would also spend sufficient time on this a few slides later, so make a neat sketch.

So, this is from Hahn and Rosenfield from the paper on local yielding and extension of a crack under plane stress, published in Acta Metallurgica and what you have to notice is the mid-section plastic zone resembles the shape of plastic zone predicted by the Dugdale's model. So, this provided a comfort in accepting Dugdale's model as a valid approach.

And really, what is the use of finding out the plastic zone? See, we have seen earlier when we looked at SSY approximation. You have to take the effective crack length as a plus the extent of plastic zone in Dugdale's model. In the case of Irwin's model, it was a plus 1 half of plastic zone; that is what we had looked at. And we will also have to find out what is the resulting stress intensity factor that would be useful information because

we want to graduate from high strength alloys behave in a brittle fashion to alloys, which also have some small trace of plastic zone. You could apply fracture mechanics to that, so you have a simple via media of modifying the crack length.

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The slide displays a table summarizing plastic zone lengths for three models under two conditions: Plane stress and Plane strain (ν=1/3). The models are the Simplistic model, Irwin's model, and Dugdale's model. The table is as follows:

	Values of r_p	
	Plane stress	Plane strain ($\nu=1/3$)
Simplistic model	$\frac{1}{2\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2$	$\frac{1}{18\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2$
Irwin's model	$\frac{1}{\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2$	$\frac{1}{3\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2$
Dugdale's model	$\frac{\pi}{8} \left(\frac{K_I}{\sigma_{ys}} \right)^2$	—

And before we proceed to the calculation of stress intensity factor, we would have a look at the plastic zone length that we have calculated by various methods. We will also look at what way the results of plane stress and how the results change for plane strain. You had the simplistic model, where you did not consider redistribution of load, the plastic zone length is $\frac{1}{2\pi} K_I^2 / \sigma_{ys}^2$ and in the case of plane strain this is much smaller, you have this as $\frac{1}{18\pi} K_I^2 / \sigma_{ys}^2$.

Then, you have the Irwin's model, for plane stress it is $\frac{1}{\pi} K_I^2 / \sigma_{ys}^2$ and when you go to plane strain, it is $\frac{1}{3\pi} K_I^2 / \sigma_{ys}^2$; see, this is a very important value. Later on, we are going to study what should be the selection of the specimen thickness for fracture toughness testing; there you would see the use of such expressions.

Then, finally, we had seen the Dugdale's model and we will have to keep in mind this is applicable only for a plane stress situation. He had taken only thin specimens, essentially thin plates; he had not calculated the plastic zone size for plane strain situation. And in

Dugdale's model, the plane stress plastic zone length is π by 8 multiplied by K_{I}^2 by σ_{ys} whole squared.

And we had noted that Irwin's model is an elasto-plastic analysis and this, we have set this as an elastic analysis. He had considered redistribution of load, whereas Dugdale considered a strip of plastic zone. You have no singularity because of that, on that premise he was able to calculate the plastic zone length.

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The slide displays a table summarizing the values of δ for different models and conditions. The table is structured as follows:

	Values of δ	
	Plane stress	Plane strain ($\nu=1/3$)
Simplistic model	r_p	r_p
Irwin's model	$r_p/2$ (Crack-tip is at the centre of the plastic zone.)	$r_p/2$
Dugdale's model	r_p	—

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And we will also look at what is a correction that we have to do for crack lengths.

In the case of simplistic model, you take that as, length as r_p for plane stress; for plane strain, you know, the calculations we have done for ν equal to 1 by 3, you take that as r_p . However, when you go to Irwin's model, you take that length as r_p by 2 because only on this basis, you know, Irwin has arrived at even the estimation of the plastic zone length, it is embedded in the development of the equation itself. Finally, when you come to Dugdale's model, you take the total length of the plastic zone as the correction for crack length.

You know, these are all subtle differences. When people handle complex problems, they do by different approaches. So, when you go to actual application, if you find it satisfies your experimental observation, you carry on with it.

Now, with the corrected crack length we should also improve the value of stress intensity factor because the whole idea is to get the stress intensity factor. When you have small amounts of plastic zone, an intermediate step is, consider the crack as slightly longer than the actual crack length; that is physics here.

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The slide is titled "Estimation of SIF Considering Plastic Zone Size" and is a sub-section of "Modeling of Plastic Deformation at the Crack-tip". It discusses the use of Irwin's model in plane stress for an infinite plate. The corrected SIF for an infinite plate is given by the equation $K_I = \sigma [\pi(a + \delta)]^{1/2}$. The slide notes that K_I is based on the effective crack length a_{eff} and that a closed form expression is possible. This is shown as $K_I = \sigma \pi^{1/2} \left[a + \frac{K_I^2}{2\pi\sigma^2_{ys}} \right]^{1/2}$. A final simplified equation is $K_I = \sigma \sqrt{\pi a} + \frac{\sigma \sqrt{\pi} K_I}{\sqrt{2} \sqrt{\pi} \sigma_{ys}}$. The slide includes NPTEL logos and a copyright notice for Prof. K. Ramesh, IIT Madras, Chennai, India.

What we will do is we will calculate the stress intensity factor for an infinite plate considering the plastic zone size.

K_I is $\sigma \sqrt{\pi(a + \delta)}$. See, you have to note, that δ is dependent on K_I , but because the problem is simple, I do not have to do iteration, I can find out an expression for K_I . For this specific example, a closed form expression is possible.

So, I will substitute the expression for δ from Irwin's model. I have this as $\sigma \sqrt{\pi}$ multiplied by $a + \frac{K_I^2}{2\pi\sigma^2_{ys}}$ whole power half. And this could be further simplified as $\sigma \sqrt{\pi a} + \frac{\sigma \sqrt{\pi} K_I}{\sqrt{2} \sqrt{\pi} \sigma_{ys}}$, these are all intermediate steps.

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The slide is titled "Estimation of SIF Considering Plastic Zone Size" and includes the subtitle "Use of Irwin's model in plane stress (Infinite plate)". It contains two equations for the stress intensity factor K_I . The first equation is $K_I \left(1 - \frac{\sigma}{\sqrt{2}\sigma_{ys}} \right) = \sigma\sqrt{\pi a}$. The second equation is $K_I = \frac{\sigma\sqrt{\pi a}}{\left[1 - \frac{1}{2} \left(\frac{\sigma}{\sigma_{ys}} \right)^2 \right]^{1/2}}$. The slide also features the NPTEL logo, a copyright notice for Prof. K. Ramesh, IIT Madras, Chennai, India, and navigation controls.

And we would finally get the expression for K_I as, in this fashion, K_I equal to σ root of πa divided by $1 - \frac{1}{2} \left(\frac{\sigma}{\sigma_{ys}} \right)^2$ whole power half.

See, you note down, the ratio σ by σ_{ys} appears in all these calculations. So, we can also comment, when the stress levels are far below the σ_{ys} , the correction is almost negligible. We can also estimate, when σ is closer to σ_{ys} , to what extent the values of K_I can change. We would see that also in the case of an infinite plate with a center crack.

If you consider the plastic zone, the K_I becomes an expression, other than σ root πa you have a denominator, which will be less than 1. So, this value will be eventually higher.

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Correction of SIF for a finite plate

- Since K_I is based on the effective crack length a_{eff} , in general it has to be determined iteratively.

$$K_I = \sigma [\pi(a + \delta)]^{1/2} f\left(\frac{a + \delta}{w}\right)$$
$$K_I = \sigma \pi^{1/2} \left[a + \frac{K_I^2}{2\pi\sigma_{ys}^2} \right]^{1/2} f\left[\left(a + \frac{K_I^2}{2\pi\sigma_{ys}^2} \right) / w \right]$$

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And you can also extend this kind of a correction for a finite plate. The moment you go to your finite plate you cannot avoid iteration; in general, it has to be determined iteratively, there is no other go. So, I would write this expression as K_I equal to sigma multiplied by pi a plus delta whole power half and a function of a plus delta by w. So, this function also will change when you do the iterative process and it is possible to write software to do this job.

So, I will get K_I equal to sigma pi power half multiplied by a plus K_I squared divided by 2 pi sigma squared σ_{ys} . You are essentially replacing the delta in terms of Irwin's result. I have the function rewritten as a plus K_I squared divided by 2 pi sigma squared σ_{ys} whole divided by w.

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Steps for an iterative evaluation

- In the first round of iteration, K_I on the right hand side is taken based on the actual crack length a .
- Evaluated value of K_I is then fed on the right hand side in the second round.
- The iteration procedure is repeated until two successive values of K_I are within a prescribed percentage difference.

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And what are the steps for an iterative evaluation? In the 1st round of iteration, K_I on the right hand side is taken based on the actual crack length a .

Evaluated value of K_I is then fed on the right hand side in the 2nd round. It is very simple; it is not complicated, straight forward.

The iteration procedure is repeated until two successive values of K_I are within a prescribed percentage difference, then you stop.

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Plasticity Correction of Crack Length

Plasticity correction is negligible when $\sigma \ll \sigma_{ys}$ but K will increase by 40% in plane stress when the applied stresses are of yield stress magnitude and 10% in plane strain.

Based on Dugdale's Model

$$r_p = \frac{\pi}{8} \left(\frac{K_I}{\sigma_{ys}} \right)^2 \qquad r_p = 0.393 \left(\frac{K_I}{\sigma_{ys}} \right)^2$$

Based on Irwin's Model

$$r_p = \frac{1}{\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2 \qquad r_p = 0.318 \left(\frac{K_I}{\sigma_{ys}} \right)^2$$

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See, what is important at this stage is what is the influence of correction of crack length? I have already mentioned, when the applied stress is far below the yield strength of the material, plasticity correction is negligible because we have seen in all the expressions, a ratio of σ by σ_{ys} was appearing.

Now, what we will have to know is what way K can change? If the value of applied stress is closer to σ_{ys} , K will increase by 40 percent in plane stress and 10 percent in plane strain, so that relative increase of K is significant. What happens in plane stress and what happens in plane strain? And it is also worthwhile to look at the expression for Dugdale's model written out in a different fashion. When you rewrite, you can write this as $\frac{1}{8}$ as 0.393. In the case of Irwin's model, the extension is given as $\frac{1}{\pi} K$ divided by σ_{ys} whole square; this $\frac{1}{\pi}$ is actually 0.318.

So, if you compare, they are not really very different. Irwin has approached the problem from one methodology, Dugdale has approached in another methodology; both seem to have got the extension of plastic zone within limits. But the way how it is used in modifying in the crack length is slightly different in Irwin's model and Dugdale's model. And you have to keep in mind, you know, the problem is very complex, people wanted to proceed further, so they wanted to have simplifying assumptions and go ahead. From that point of view, the contribution of Irwin is very significant.

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Variation of Plastic Zone Shape Over the Thickness of the Specimen

- The shape of the plastic zone is not same over the thickness of the specimen! A variation exists.
- An approximate variation can be obtained as follows.
- The surface of the specimen is ready to contract and the plastic zone shape can be approximated to be as that for plane stress case at the surface.
- In the interior of the specimen, the shape can be approximated to be that for the plane strain case.

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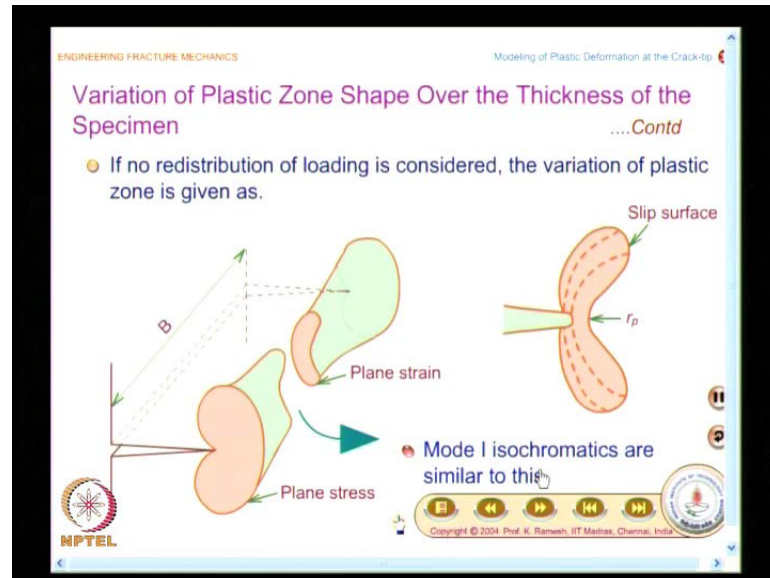
Now, having looked at all this, what we will look at is what is a variation of plastic zone shape over the thickness of the specimen? See, you have to look at the mathematical development. We try to proceed in a way as simple as possible, so you evaluate at approximate shape, but approximate shape is not the one, which is going to be seen in an experiment, that was for specimens of different thickness.

On the other hand, Dugdale came and said, for thin plates it is like a strip. That is other extreme, but for a specimen of given thickness, you have to know how the plastic zone actually is. So, whatever the discussion I am going to present is based on what is seen from finite element solution or from experiment. Whatever the discussion we had earlier was approximate analytical approaches and you have to keep in mind the shape of the plastic zone is not same over the thickness of the specimen, a variation exists.

You had that evidence even in your verification of Dugdale's approach. You had seen how the plastic zone was on the surface, how it was there in the middle, even for a thin plate. Now, if you are having a thickness specimen, you are going to have... And we also had a discussion, what way we call plane stress and plane strain in the case of fracture mechanics. If I have a thick specimen, I will consider the surface to behave like a plane stress and interior when you go, it behaves like a plane strain. So, that kind of an idea I would use in looking at the plastic zone shape also. So, what you will have to look at is the plastic zone shape differs over the depth of the specimen; so, that is what is summarized.

The surface of the specimen is ready to contract and the plastic zone shape can be approximated to be as that for plane stress case at the surface. And I have already mentioned the plane stress and plane strain are loosely defined in the case of fracture mechanics. So, on the surface, the shape is what is seen for plane stress case; in the interior of the specimen the shape can be approximated to be that for the plane strain case.

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And what I would like you to do is make a reasonable sketch of this. So, the 1st set of result, what is presented here is if no redistribution of loading is considered, the variation of plastic zone is as follows. So, you have to note, that the crack is like this, you have a thickness of the specimen as B . On the surface, the shape is something like this; interior to the surface, the shape is like as a butterfly. I have this slipping takes place, I have this length of the plastic zone, along the crack axis is given as like this and whatever you see here, is similar to your mode one isochromatics.

See, people will have also done finite element calculation, including the second term, that is, t stress. They have plotted a very similar to this, so this gives you an understanding over the depth of the specimen. Over the thickness of the specimen the plastic zone varies and views the approximation, that surface behaves like a plane stress situation. On interior it acts like plane strain situation and this is very similar to your mode one isochromatics.

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Variation of Plastic Zone Shape Over the Thickness of the Specimen

....Contd

- If redistribution of loading is considered, then the inner region will be slightly larger, the outer ones on the surface would be smaller.

The diagram illustrates the plastic zone shapes for plane stress and plane strain conditions. On the left, a 3D view shows a crack in a specimen of thickness B . The plastic zone is shown as a butterfly shape. The inner region is labeled 'Plane stress' and the outer regions are labeled 'Plane strain'. On the right, a 2D view shows the plastic zone shape with a dashed line indicating the redistribution of loading. A text box states: 'Mode I isochromatics are similar to this'. The NPTEL logo is visible in the bottom left corner.

Suppose, I consider the redistribution, what way these zones change? You just observe this, then copy it. What happens is, the inner places, it increases and the surface shrinks. So, what I will do is, I will just redo the whole thing.

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Variation of Plastic Zone Shape Over the Thickness of the Specimen

....Contd

- If no redistribution of loading is considered, the variation of plastic zone is given as.

The diagram illustrates the butterfly shape plastic zone. On the left, a 3D view shows a crack in a specimen of thickness B . The plastic zone is shown as a butterfly shape. The inner region is labeled 'Plane stress' and the outer regions are labeled 'Plane strain'. On the right, a 2D view shows the plastic zone shape with a dashed line indicating the slip surface and a radius r_p . A text box states: 'Butterfly shape plastic zone'. The NPTEL logo is visible in the bottom left corner.

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Variation of Plastic Zone Shape Over the Thickness of the Specimen

....Contd

- If no redistribution of loading is considered, the variation of plastic zone is given as.

Slip surface

Plane strain

Plane stress

Corroborated by experiments

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The slide features a diagram of a crack in a specimen. On the left, a 3D view shows the plastic zone shape for 'Plane stress' (a larger, more rounded shape) and 'Plane strain' (a smaller, more elongated shape). On the right, a 2D view shows the plastic zone shape for 'Plane strain' with a 'Slip surface' and a radius r_p . The text 'Corroborated by experiments' is highlighted. The NPTEL logo and copyright information are at the bottom.

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Variation of Plastic Zone Shape Over the Thickness of the Specimen

....Contd

- If redistribution of loading is considered, then the inner region will be slightly larger, the outer ones on the surface would be smaller.

Plane strain

Plane stress

Mode I isochromatics are similar to this

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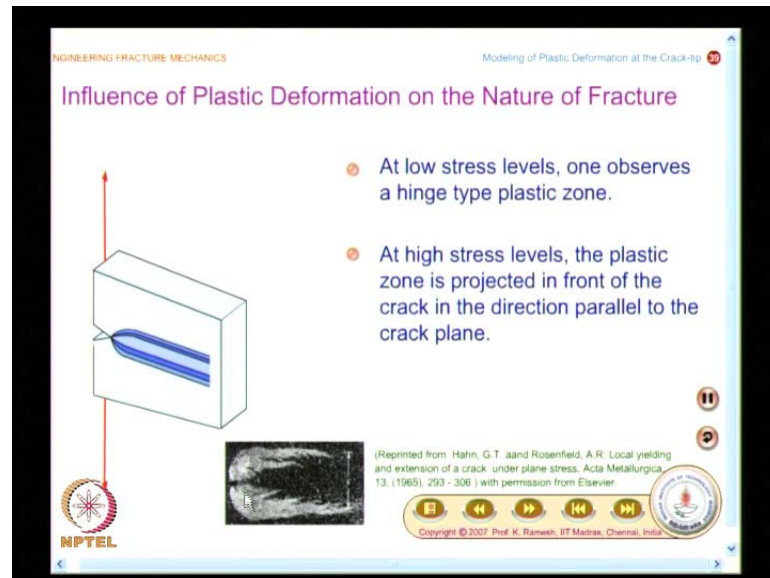
The slide features a diagram similar to the previous one, but with a note: 'If redistribution of loading is considered, then the inner region will be slightly larger, the outer ones on the surface would be smaller.' The text 'Mode I isochromatics are similar to this' is highlighted. The NPTEL logo and copyright information are at the bottom.

If no redistribution of loading is considered, you have this and this is what happens at the slip surface and you should also note, that these are known as butterfly shape plastic zone because it looks like this, corroborated by experiments, that is very important, similar to mode one isochromatics. And if you consider redistribution, just observe the animation, this will shrink and that will increase.

So, drawing it on your notes maybe quite difficult because the variation, it may not be easy for you to show, but nevertheless, you have seen, how this changes when you have

redistribution. So, we have seen over the thickness of the specimen how the plastic zone changes, now we will have to see, when the crack propagates how does the plastic zone change? That is also people have studied.

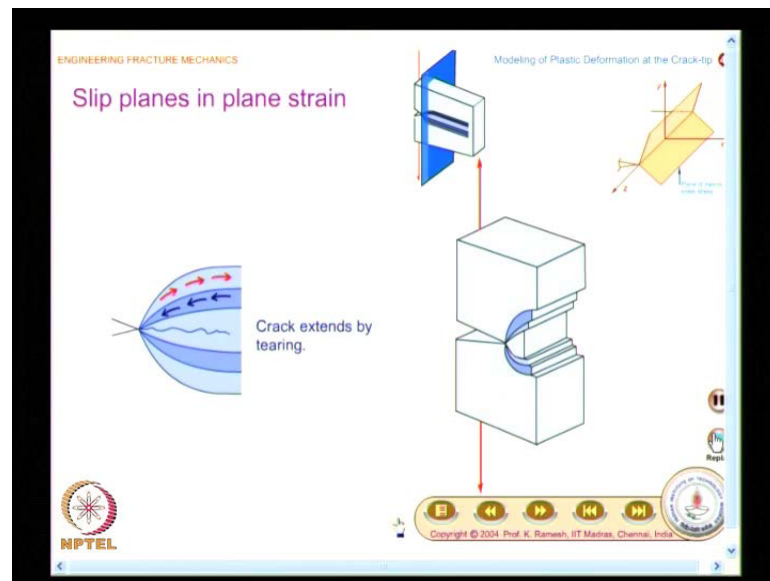
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You know, in the case of Hahn and Rosenfield, we have seen the plastic zone in this shape. This people have given explanation, so we will have to go and see how the plastic zone is dictated near the crack-tip and how it is dictated at distances away. So, what I would appreciate is, you observe this animation today and in the next class, may be you can make a sketch.

At low stress levels, one observes a hinge type plastic zone; that is what is seen here. At high stress levels, the plastic zone is projected in front of the crack in the direction parallel to the crack plane; that is what happens. And what you will have to now remember is the plane where the maximum shear stress occurs, is different in plane strain, as well as plane stress. See, this is very important, that influences.

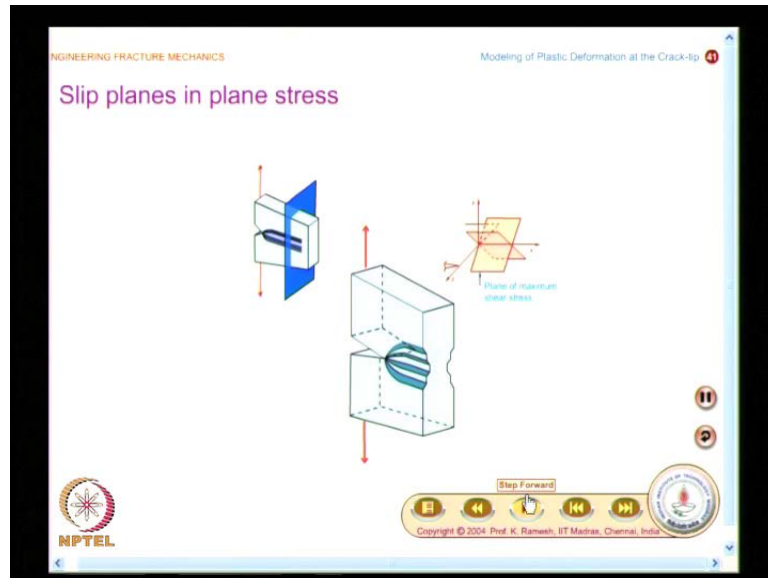
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Now, I take a situation, which is very close to the crack-tip and this is the plane of maximum shear, it is different. And because of this, what happens is, you have a slip occurs like this, you have a slip occurs like this, you have planes like this, it occurs like this. So, you have the steps shown here and slip occurs between these planes and a tearing action can take place and the crack will propagate.

So, what I will do is, I will just repeat the animation sequence, you just observe. The plane is different, it is in the x-y plane itself and you will have slip occurring like this. This is magnified here, you have slip occurs here and because of this you will have crack propagation, crack extends by tearing.

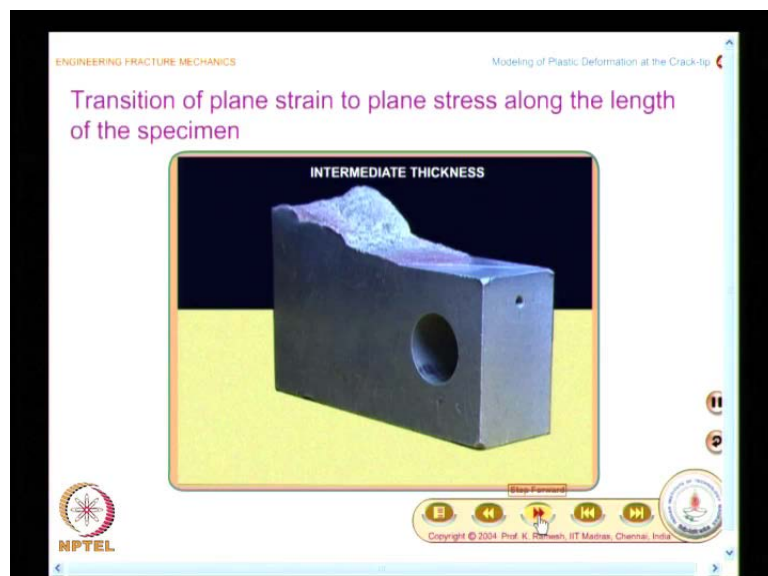
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Now, let us see what happens at distances away. I am looking at a plane like this, where I have essentially plane stress. The plane of maximum shear is really out of plane. So, do not think when I say plane stress everything remains in the plane, it is not so. That is why we have looked at stress tensor as well as strain tensor.

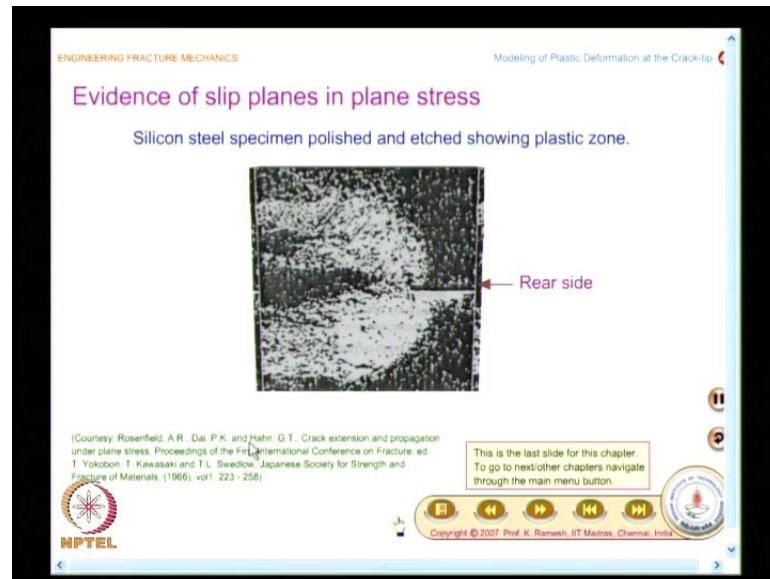
And you can see the animation here and when you look at the side of the specimen, you have slip takes place at 45 degrees because this is the plane where you have the maximum shear stress and this is how you anticipate plastic zone to occur.

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And in fact, if you go ahead and then see how the specimen looks like, this you had seen it earlier from the context of delineating, the crack surface and fracture surface. So, initially a crack propagates by tearing action, finally you have shear lip. Now, the question is, do we have an experimental evidence for this? I have to show, at the end of the specimen, you should have slip taking place at 45 degrees.

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So, if you have that as an objective, you see the experimental result, you find, on the surface it is like this, close to the crack-tip. At the ends, you indeed see slip planes at 45 degrees. This was again, reported by Rosenfield, Dai and Hahn. So, very important result, you know, this gives you an understanding, that people have been able to find out what happens at the crack-tip.

So, in this class we have essentially looked at Dugdale's model followed by what is an extension of crack length you have to take. Then, we evaluated stress intensity factor with the modified crack-length, then we had looked at how does the plastic zone changes over the thickness of the specimen and as crack propagates, initially it is dictated by plane strain followed by plane stress.

The plane of maximum shear stress is different in plane stress and plane strain; that is a reason why, the plastic zone appears like this.

Thank you.