# Engineering Fracture Mechanics

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## Lecture No. # 28
## Modeling of Plastic Deformation

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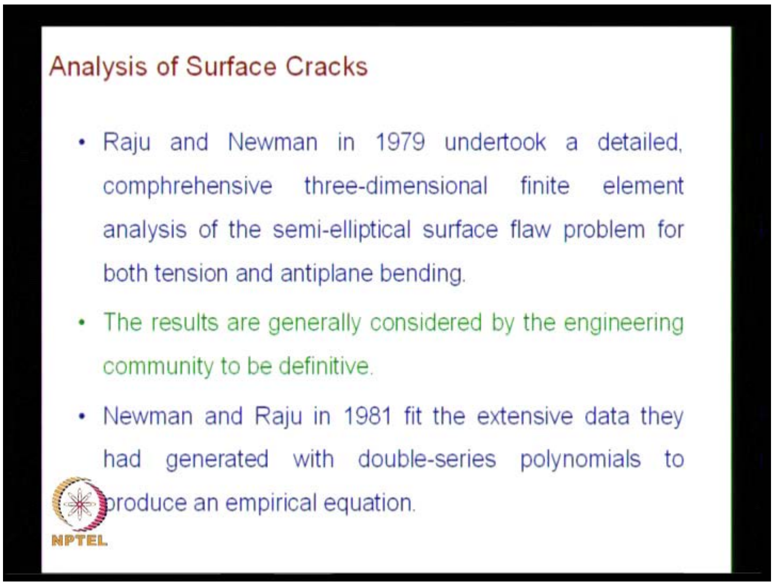


In the last class we had started discussing the solution of Newman and Raju, we were not able to completely cover it, for the sake of continuity we will quickly look at what we have discussed in the last class. And I had mention Raju and Newman in 1979 under took a detailed, comprehensive three dimensional finite element analysis, and based on that result, in 1981 they had come out with an empirical relation consisting of double-series polynomials.

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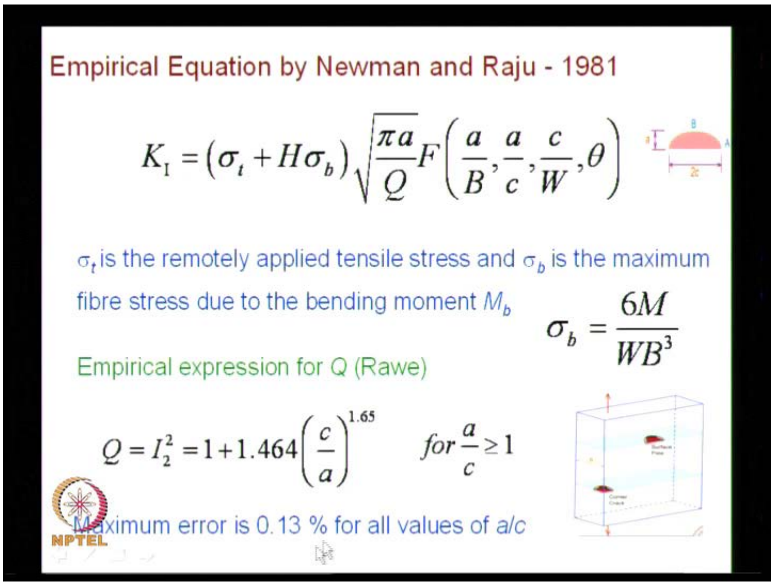


And the relationship looks something like this, this you have determined in the last class. And sigma t refers to the tensile stress, and sigma b refers to the bending stress, and we have also looked at how to calculate Q.

This is given for a by c less than equal to 1 also given for a by c greater than equal to 1, and if you do this, with this kind of an expression the maximum error is about 0.13 percent for all values of a by c.

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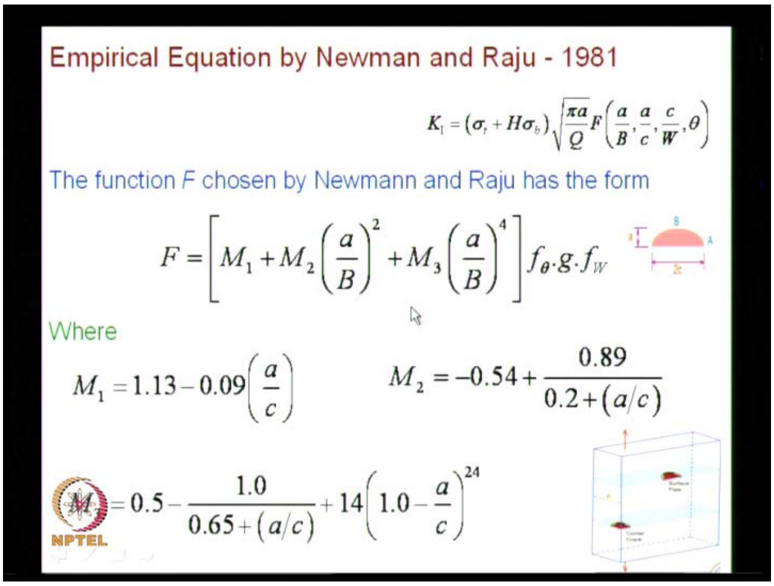

Then, we moved on to look at what are these functions F. This has component of M 1, M 2, M 3.

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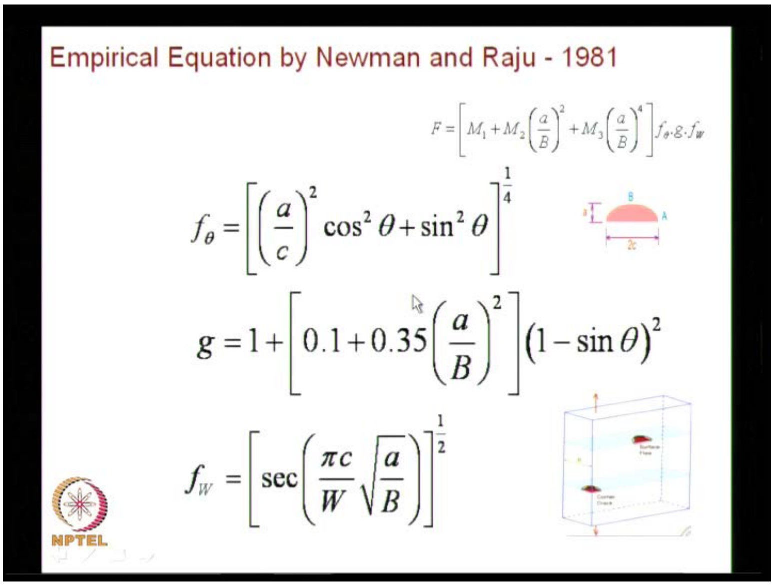


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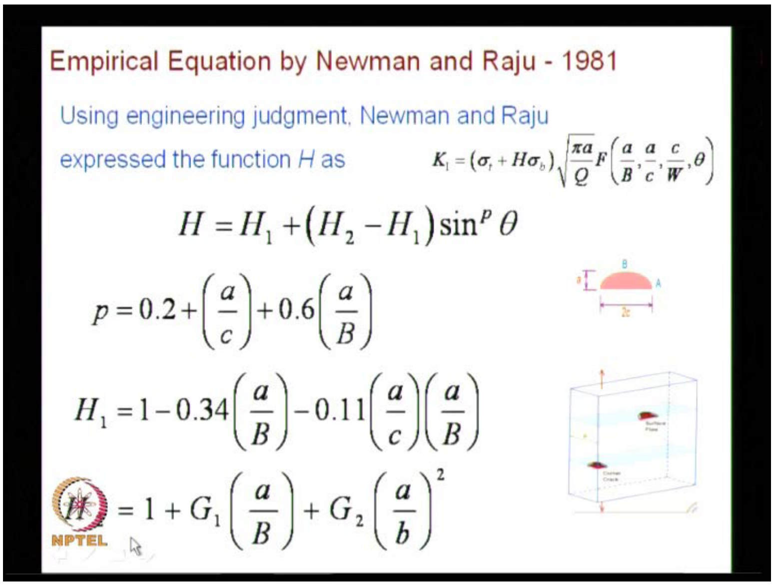


And these were also defined, all these equations were determined in the last class. We have also looked at the functions F theta, G, as well as F w. Then, we moved on to look at what is the function H? And you have to note, that using engineering judgment Newman and Raju express the function H as in this form, H equal to H 1 plus H 2 minus H 1, sin power p theta, and p is defined as 0.2 plus a by c, plus 0.6 into a by B. H 1 is defined as 1 minus 0.34 a by B, minus 0.11 a by c, multiplied by a by B, and H 2 is given as 1 plus G 1 multiplied by a by B plus G 2 a by B whole square, to this extent, I think we have seen it in the last class and we have to know the functions G 1 and G 2, these are defined next.

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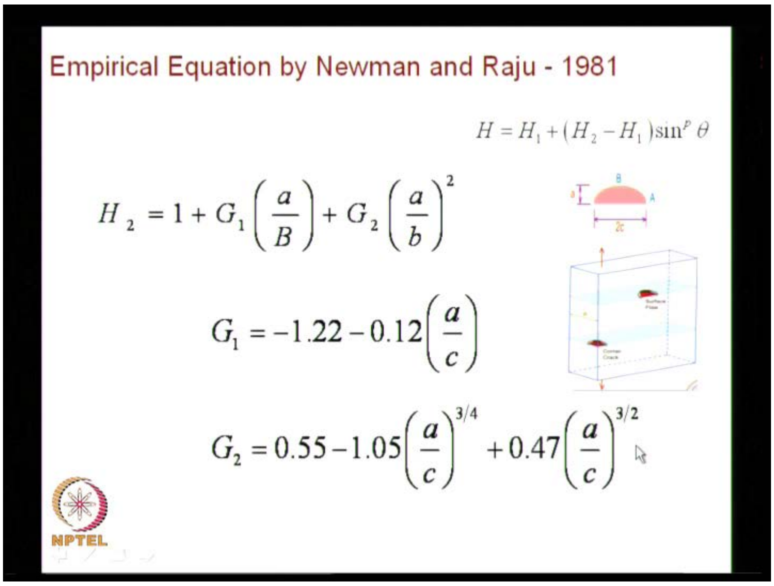


The function G 1 is given as minus 1.22, minus 0.12 multiplied by a by c, then the function G 2 is given as 0.55 minus 1.05 multiplied by a by c whole power 3 by 4 plus 0.47 a by c whole power 3 by 2. You know this completes the definition of the components consisting of the empirical relation, and you should note that he had done a three dimensional final read element analysis, based on that at fit a empirical relationship and the relationship is as follows.

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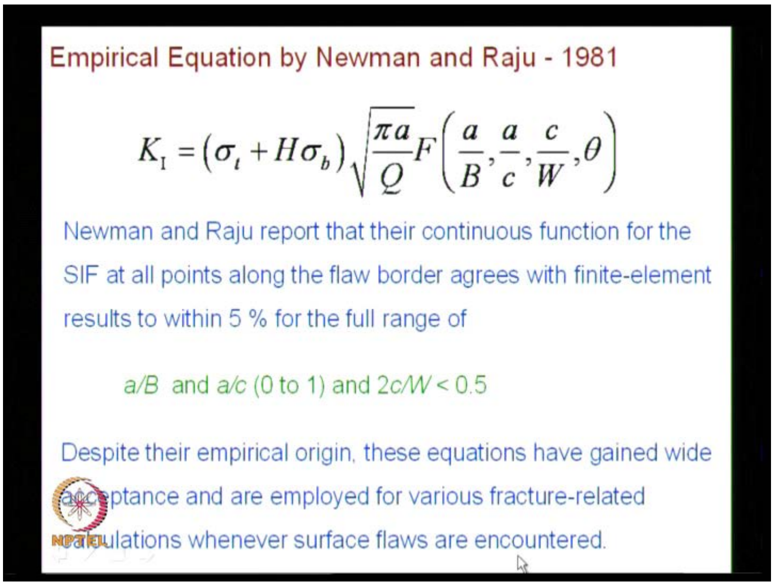


K 1 equal to sigma t plus H sigma b multiplied by root of pi a divided by Q, and a function which is dictated by a by B ratio, a by c ratio, c by W ratio and also the position theta. And what you'll have to note is, Newman and Raju have reported that the empirical relation is within 5 percent of the finite element calculation for the full range of a by B, and a by c, from 0 to 1, and 2 c by W less than 0.5, we will also have to note down the width of the specimen, that also playing a role, And you have to note, despite their empirical origin these equations have gained wide acceptance, and are employed for various fracture-related calculations whenever surface flaws are encountered.

So, as far as surface flaws are concerned you can always go and look at the empirical relations of Newman and Raju, and use it for your calculations that level of confidence in the fracture community has placed on the axis. So, we have seen surface cracks are very important from practical point of view, and we have looked at 3 different methodologies.

One was the very simplistic approach by Irwin then, there was improvement of a front free-surface correction factor. Then, there was also correction factor, because of plastic zone length and you had graphs of flaw shape parameter. Then we had also looked at direct analysis of surface cracks, where people have provided separate graphs for tension as well as bending, and finally, we have looked at the empirical relations of Newman and Raju.

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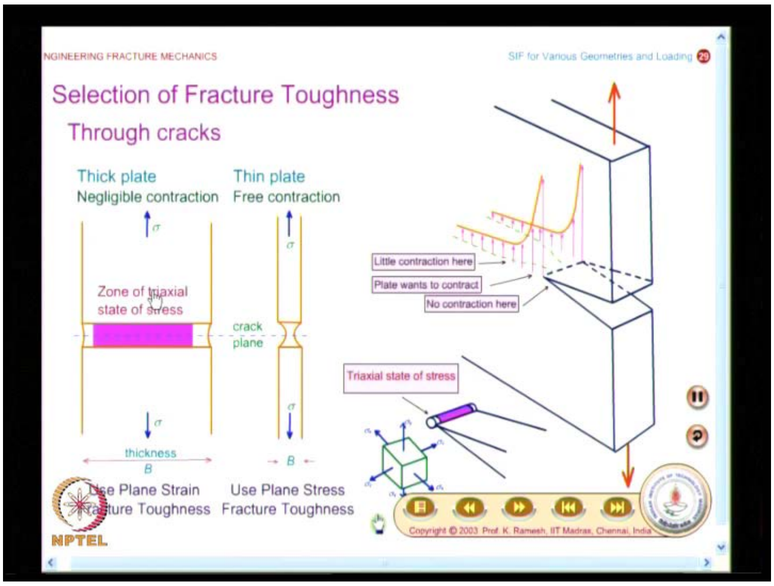


Now, another important aspect that we will have to discuss in relation to stress intensity factor and the role, you should also look at what way we will select fracture toughness for radius problems.

Suppose, I have a through the thickness crack, and for the purpose of discussion; the crack is shown as a very sharp corner, and you have the crack front straight, with the simplest problem to take. And let us look at what way the material responds, because of high localized stresses near the crack-tip region.

So, you have very high levels of stresses, this your elastic solution provides, in view of these very high localized stresses, what happens? There is no contraction in this zone, in this zone the material wants to contract, and we will have to see to what extent in this, contract contraction is possible. And ahead of this zone there is little contraction, and what happens in this zone?

So, what I would do is, I would repeat the animation and you can recapture the various sequence of ideas that we have represented.

So, I have the plate with a crack pull, you see a very high level of stresses then, because of a consequence of the stresses the material wants to contract, and you consider a cylinder of material in this zone. And we will also have to qualify this result for plates of various thicknesses.

What happens in a very thin plate? And what happens in a thick plate? These are two extremes that we will look at.

What you can always notice is, when you have a stress concentration zone normally you come across a uniactive field changing into biaxial field but, in the case of crack problems for thick plates you have to also consider one more aspect, we have already seen, when you consider the crack-tip is very sharp sigma x equal to sigma y, if the crack is blunt, at the crack-tip you will have sigma x as 0, for the purpose of discussion we, let us consider crack-tip is sharp.

So, you have sigma x equal to sigma y, and if the plate is sufficiently take, you will also have stresses developed in the thickness direction of the plate, and that is given as nu times sigma x plus sigma y. And if you look at the strain, strain will be 0, if you look at the stress between nu times sigma x plus sigma y and, because of very high localized stresses you will have essentially plastic deformation at the crack-tip.

So, when we have plastic deformation at the crack-tip, it is prudent to take the Poisson ratio as 0.5. So, in a sense, in thick plates you have triaxial state of stress near the crack and that zone is indicated here.

So, in the case of crack problems near the vicinity of the stress concentration, you could have under suitable circumstances a triaxial state of stress. And that is what is depicted for a thick plate and for a thin plate. And you can make a neat sketch of this so, in a thick plate leaving the surfaces, the zone interior to that will be in a triaxial state of stress. And you find there is negligible contraction in the case of a thick plate.

So, in such problems from fracture instability point of view, what you will have to do? You will have to calculate stress intensity factor when you have a crack in a thick plate, from fracture instability point of view, you will have to know what value of fracture toughness that you will have to take, though, we will have a separate chapter on fracture toughness testing, certain concepts related to that we may have to discuss even in earlier chapters.

So, in such a problem where you have a thick plate, you have to use the plane strain fracture toughness. And I have already mentioned, the plane strain fracture toughness is lower than the plane stress fracture toughness. So, the combination of stress intensity factor value, and the selection of fracture toughness ultimately dictates the fracture instability phenomenon.

On the other hand in the case of a thin plate you have a free contraction, and what is recommended is, you have to use plane stress fracture toughness for your fracture instability calculations. After having looked at through the thicknesses cracks, we will have to go and look at how you are going to handle these surface cracks?

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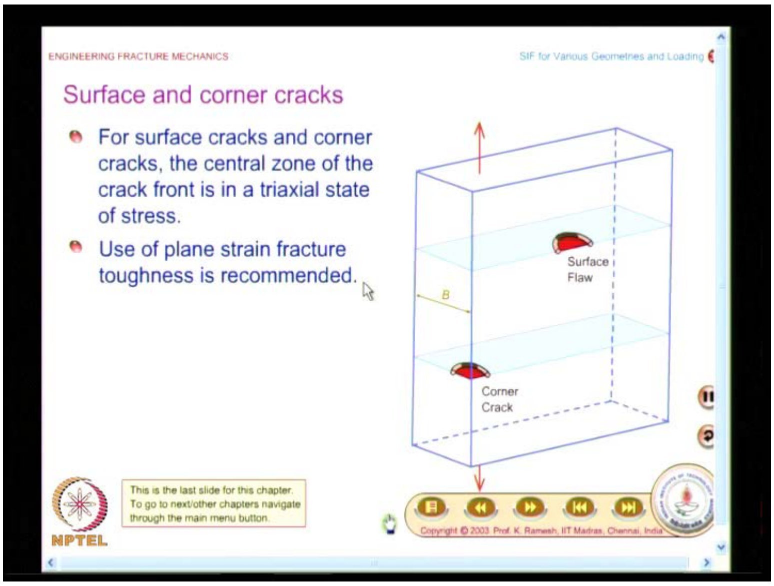


What happens in the case of a surface and corner cracks? We all know that, it has a curved crack front and a stress intensity factor varies along the curved crack front, and if you consider a cylinder of material close to the curved crack front the inner zone is experiencing a triaxial state of stress.

You know this is depicted by a shaded region so, whenever you have a triaxiality constrain the recommendation is, you have to use plane strain fracture toughness for your instability calculations, and when you have to do the use of plane strain fracture toughness, you have to recognize that surface flaws are always dangerous, because you are always comparing the stress intensity factor of a surface flaw for a through the thicknesses edge crack but, for an edge crack in a reasonably thin plate you may use the plane stress fracture toughness but, if you have a surface crack, then you have to use a plane strain fracture toughness.

So, because of that you have to keep in mind, the surface cracks are always dangerous. See, now we have look that stress intensity factor for a variety of problems, and it is better that, we also find out a sort of a thumb rule in react into different crack situations.

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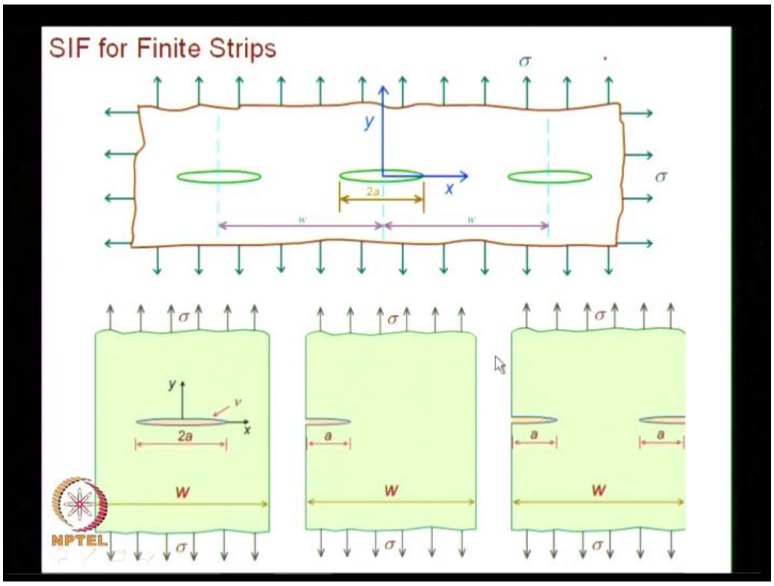


So, we will go and review those results. And we have started the discussion with equally spaced cracks in an infinite strip, and we had discussed that from this, you get the center crack specimen, from this solution you are also able to get the single edge notch specimen, and also a double edger notched specimen or crack specimen.

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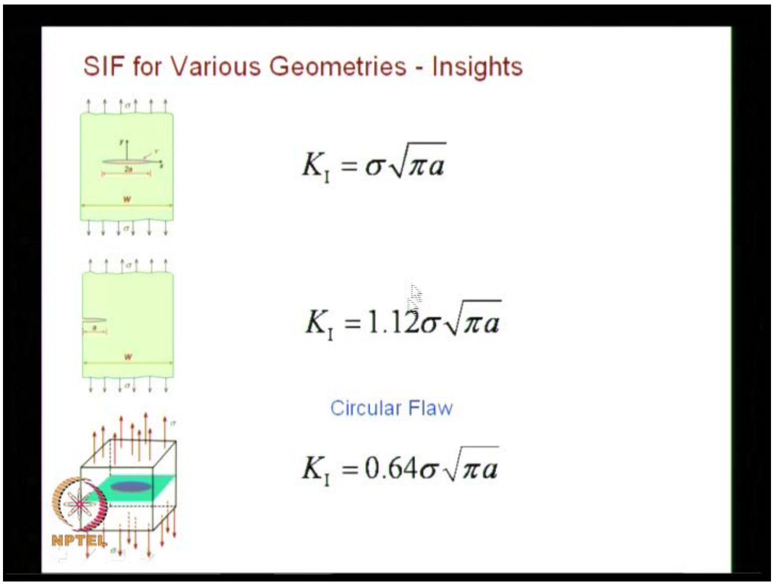


And when you have a center crack, we are looked at in an infinite plate it is K 1 equal to sigma root pi a, for a finite plate you will have a function of a by W, it may be a by W or 2 a by W, depending on how the result is reported.

The moment you go to an edge crack, we have noted from our discussion, edge cracks have a higher stress intensity factor than a center crack. We had a factor of 1.12, for an infinite plate it will be like this, for a finite plate you will have a function related to a by W or 2 a by W, depending on how the result is reported. On the other hand, when I have an embedded circular flaw, what we noted that stress intensity factor remain constant on the crack front, and in comparison to a center crack in an infinite plate the stress intensity factor is lower, it is 0.64 times sigma root pi a.

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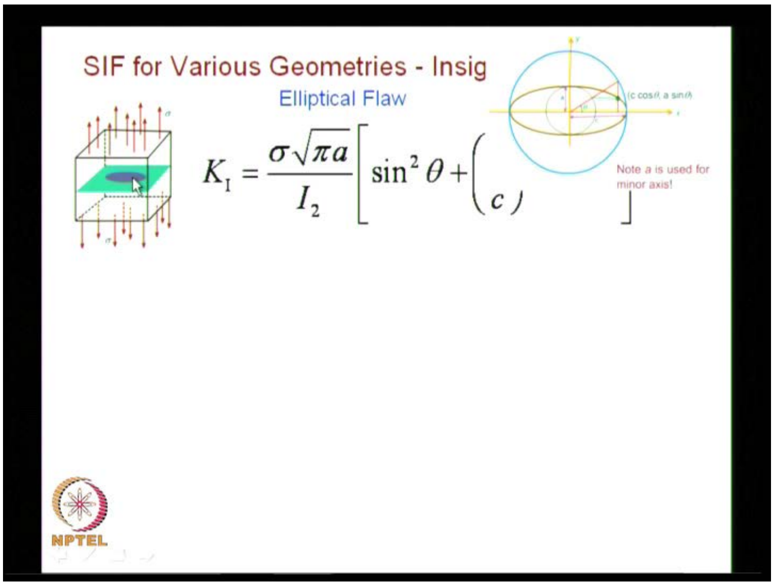


After the circular flaw, we had moved on to an elliptical flaw and only in an elliptical flaw we noted that, the stress intensity factor can change from point to point on the crack front. And that definition is given here, you have a sigma root by a divided by I 2, and you have a function related to theta, and we have also noted when you specify theta, how to locate the point on the ellipse.

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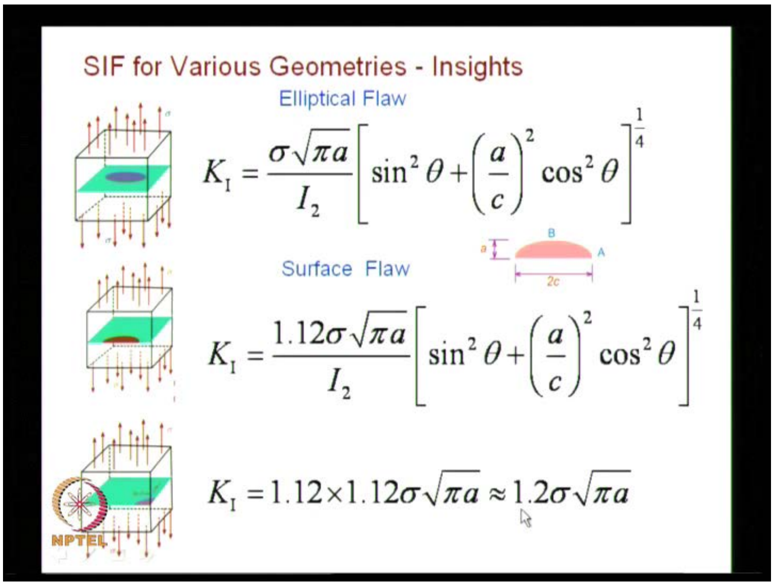


If I have theta like this, you drop a normal and this hits this, ellipse at this point so, for this point the stress intensity factor is given. From an embedded elliptical flaw we moved on to a surface crack, and here we had extrapolated what all the understanding of that elliptical flaw, and also for an edge crack and the simplest solution was simply change the solution by 1.12 times sigma. Later, on we improved upon a definition of crack length modified for plastic correction, and then we had also looked at empirical relation of Newman and Raju.

But basically you have to recognize, compare to an embedded elliptical flaw a surface flaw is little more dangerous. And then we moved on to a corner crack, and for the corner crack we recognize you have a free surface on this side, as well as another free surface and we realize that, you will have K 1 is much higher for this and you have this as simplified to 1.2 times sigma root pi a.

So, this discussion in a sense brings out a relative appreciation of how you should react to cracks of various types; whether they are through the thickness crack, or interior to the object, or on the surface, or it is an embedded crack, you have a rough approach on how to look at the influence of the stress intensity factor on the overall structural behavior.

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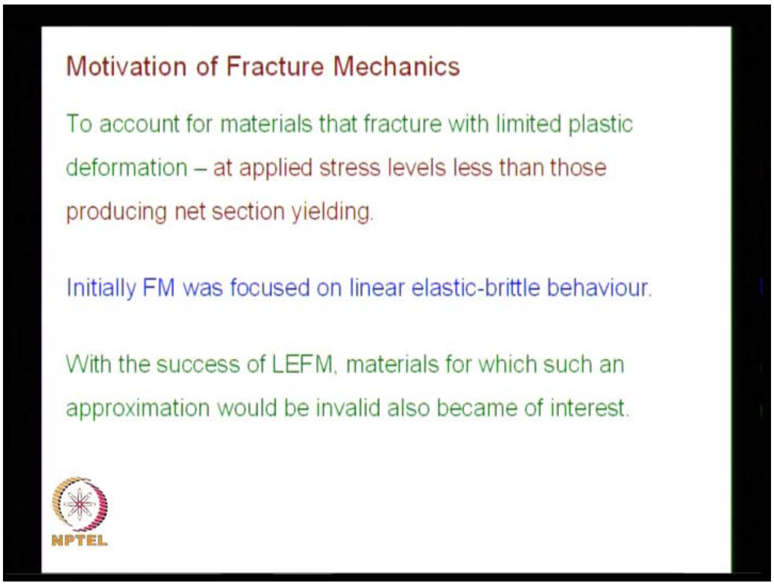


Now, what we will do is, we will move on to modeling of plastic zone near the vicinity of the crack-tip. And we will have to look at what is the motivation for all this? You know, essentially we have discuss fracture mechanics in the context of brittle materials, that is how Griffith started, later on it was extended by Irwin and Orowan for handling ductile materials, and when you go and look at that what you will have to learn is, we also want to look at materials that fracture with limited plastic deformation, at applied stress levels less than those producing net section yielding.

This is how we have started the extension of fracture mechanism, brittle materials to ductile materials, and you have to recognize initially fracture mechanics was focused on linear elastic material behavior. And we had sufficient success in this, the theory is matched with experimental observation so, with the success of linear elastic fracture mechanics people also thought, for materials for which such an approximation would be invalid also became of interest so, that is the motivation.

See, you cannot keep away from your understanding of what happens at the crack-tip, because of un-elastic deformation, some kind of modeling is always needed, because you have to graduate from brittle materials to high strength alloys, from high strength alloys to even intermediate alloys which exhibit reasonable levels of plasticity.

So, here what you will look at is, what are the tricks that, they develop within the domain of linear elastic fracture mechanics? To extent this analysis to certain kind of a ductile materials, later on if there is extensive plastic deformation, you have to bring in the concepts of elastoplastic fracture mechanics. What we will now recognize is, what is the motivation for finding out all these aspects?

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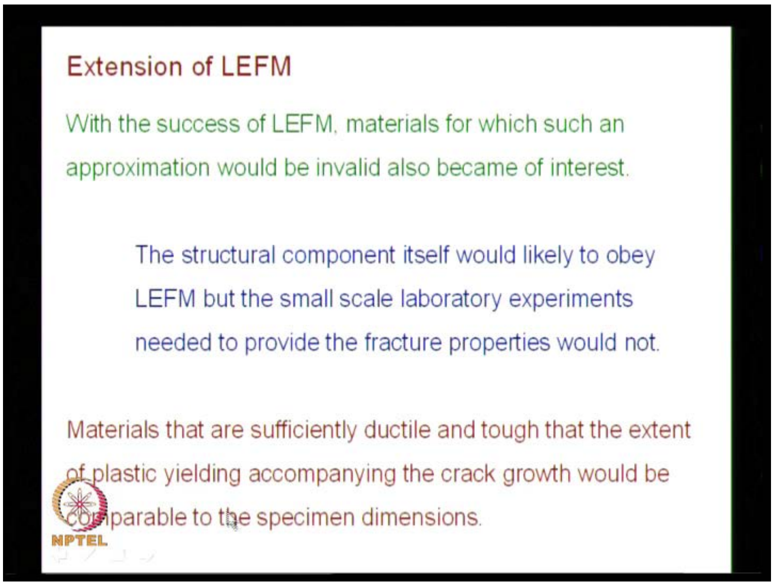

So, the motivation is extension of linear elastic fracture mechanics. And as I mention with the success of LEFM, materials for which such as approximation would be invalid also became of interest. And there is on the another observation, this also has prompted people to go in that direction. See the structural component itself would likely to obey LEFM but, the small scale laboratory experiments needed to provide the fracture properties, would not behave in such a fashion.

So, this is another angle to it, people want to find out the relevant fracture parameters to characterize the particular material in service.

So, this is another dimension why people were looking for newer ways of handling the plastic zone near the crack-tip. What we will have to note is materials that are sufficiently ductile and tough, that the extent of plastic yielding accompanying the crack growth would be comparable to the specimen dimensions.

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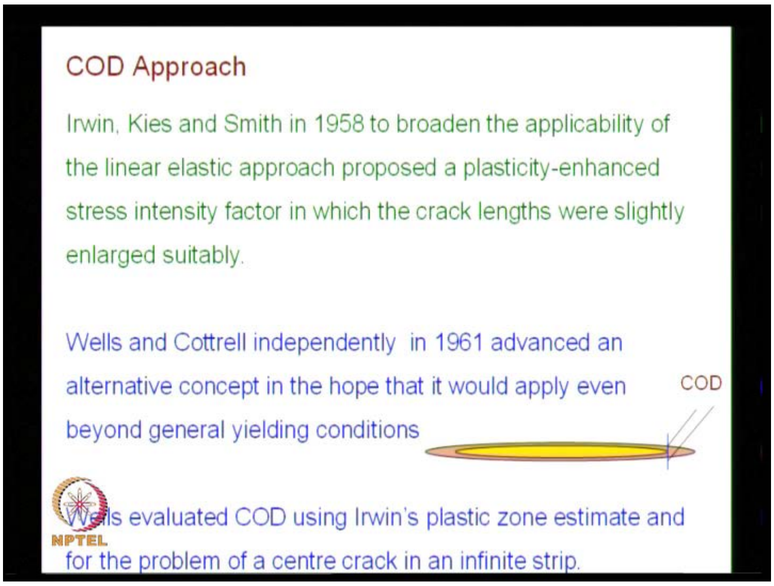


So, this is the goal, if you have this kind of a situation, how will you go and do it? And what was initially done was, that was the work of Irwin Kies and Smith in 1958, to broaden the applicability of the linear elastic approach, and proposed a plasticity-enhanced stress intensity factor in which the crack lengths were slightly enlarged suitably.

See, you have to look at this was the kind of work in 1958, when fracture mechanics was in the initial stages, where significant contributions and new ideas have been proposed by the group lead by Irwin. And they had come out with a very simple modification, what they have said is; you have to get up plasticity-enhanced stress intensity factor, in this what they had done is; we have already look at in the contest of a surface crack instead of taking the crack length as a, they have taken a small extension of crack length which is dictated by the plastic deformation near the crack-tip.

This was one kind of an approach, another kind of approach was promoted by Wells and Cottrell, they have done it independently in 1961. They advanced an alternative concept in the hope that, it would apply even beyond general yielding conditions. We would take this, when we discuss j integral and another measure for elastoplastic fracture mechanics is the COD approach. We will just see that, these two approaches were there to understand the plastic deformation near the crack-tip. And from the result of Irwin wells evaluated the crack opening displacement and here it is used in a different context.

We would have a pictorial representation now, we will also spend some time a little while later, from the point of view of Irwin you imagine the crack is to be longer than the actual crack. The actual crack is only this much so, at the tip the would be opening, which is labeled as COD.

So, people had coin new parameters, particularly from the point of view of particularly from the point of view of taking the laboratory results, useful for analyzing actual structural components.

So, in this chapter you will confine our attention to, what was the discussion done by Irwin? What is the approximate shape of plastic zone? And also what is the model given by ductile in finding out extent of plastic deformation ahead of the crack-tip? And before we proceed into that, it is worthwhile to recall, how plane stress and plane strain terminologies are used in fracture mechanics?

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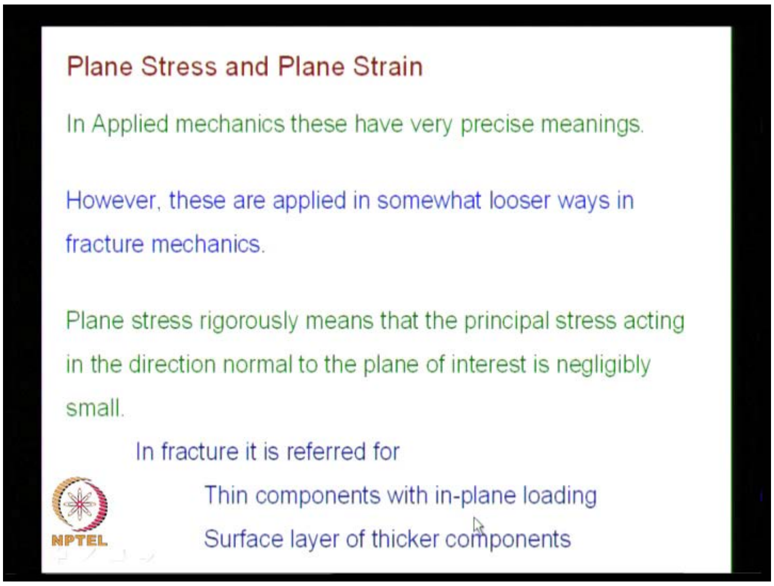


If you look at in applied mechanics these have very precise meanings, we know what is the plane stress, we have looked at the stress tensor as well as strain tensor. The moment you come to fracture mechanics these are applied in somewhat looser ways, you have to recognize that and keep that in your mind.

In the contest of applied mechanics, plane stress rigorously means that the principal stress acting in the direction normal to the plane of interest is negligibly small. In fracture what do you call as plane stress, it is referred for thin components with in-plane loading, and we also do one more thing, we say the surface layer of thicker components behave in a plane stress fashion.

You know this is something unusual, this is so convenient for us to develop certain concepts in fracture mechanics.

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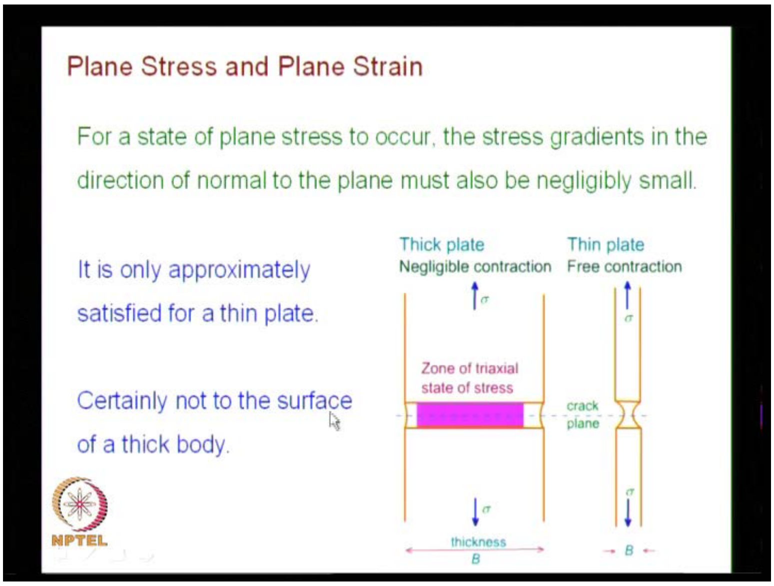


So, we have adapted a slight variation in the definition of, what is plane stress and plane strain in the context of fracture mechanics. So, you have to recognize that, there is a difference. And you will also have to note, for a state of plane stress to occur the stress gradients in the direction of normal to the plane must also be negligibly small.

This condition is not completely satisfied even in thin plates, it is only approximately satisfied for a thin plate. See, because of plastic deformation you have the neighboring elastic region will refuse to deform to that extent. So, you will have sort of a tension compression type of situation in the plastic zone as well as the elastic zone.

So, you will have gradient of stresses developed in the thickness direction so, in the contest of fracture mechanics though we say thin plates subjected to in-plane loading can be equated to a plane stress situation, from the definition of applied mechanics the definition is strictly not corrected, it is only approximately satisfied in a thin plate.

Certainly it does not satisfy the conditions in the surface of a thicker body. You have gradients existing into thickness direction that is quite alright, because we are handling a very complex problem situation, we have to carry forward.

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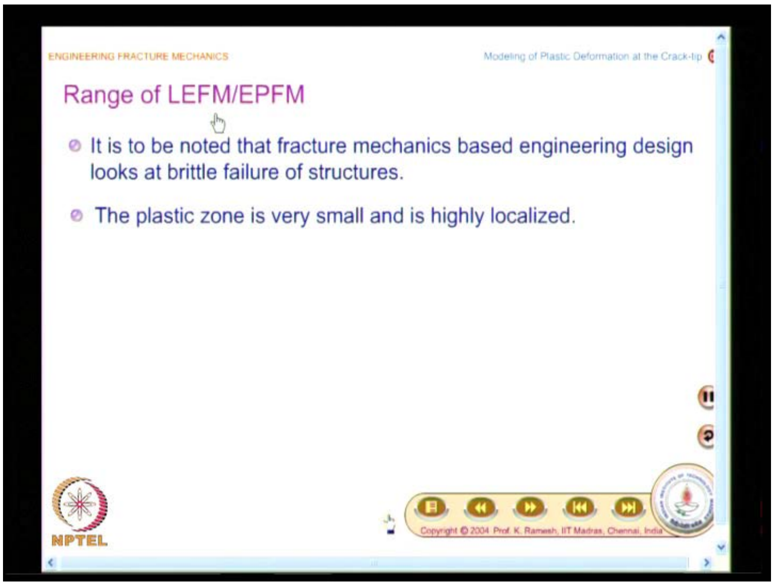


So, we need to make certain approximations that make our life simpler. And before we get into the discussion of plastic zone, we have already seen the range of linear elastic fracture mechanics and elastoplastic fracture mechanics, based on the plastic zone.

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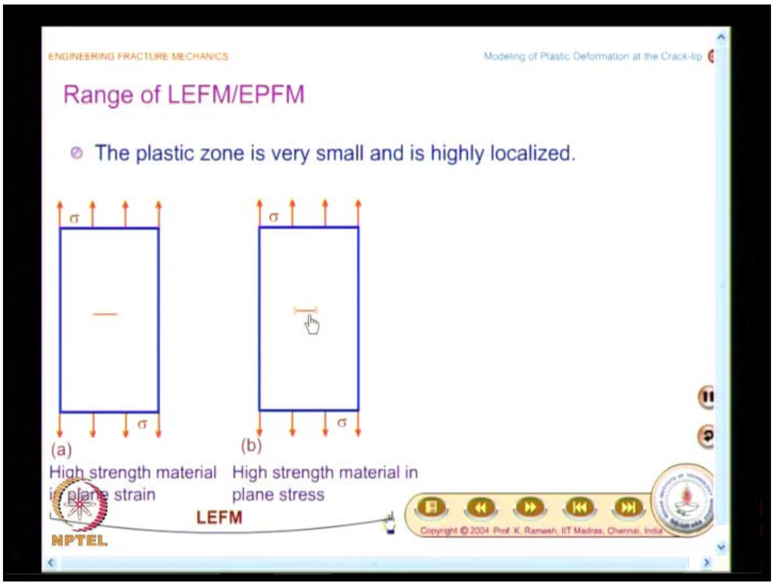


And you have to recognize the plastic zone is very highly localized only when you magnify you are able to see, and this is the situation that exist in high strength materials in plane strain. And in the case of high strength material in plane stress, you have little more plastic zone but, compare to the plate dimensions and also the crack length the size of the plastic zone is very small. So, only for these class of problems we will make suitable approximations and find out certain modifications to our way of calculating stress intensity factor.

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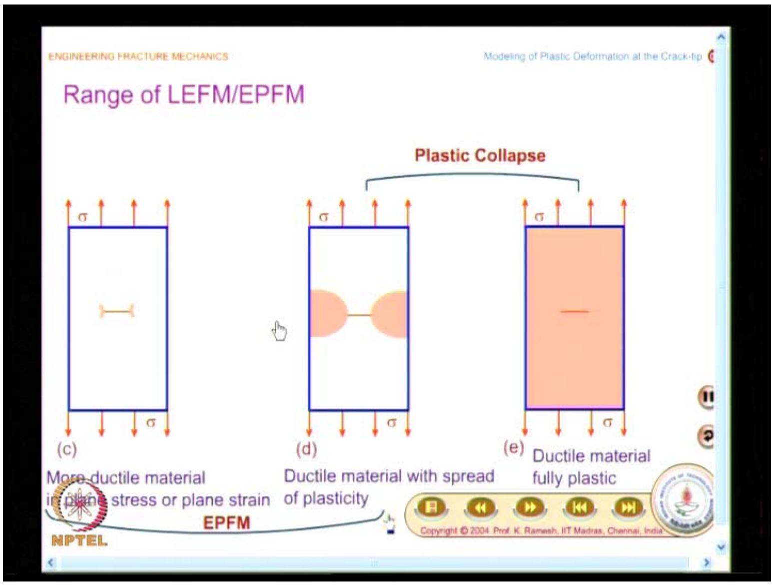


So that, LEFM could be applied to a broader class of high strength ductile alloys, the moment you have a very high plastic zone you have to going for elastoplastic analysis, and if the plastic zone is much larger compare to the crack and other specimen dimensions you will have to going for analysis based on plastic collapse.

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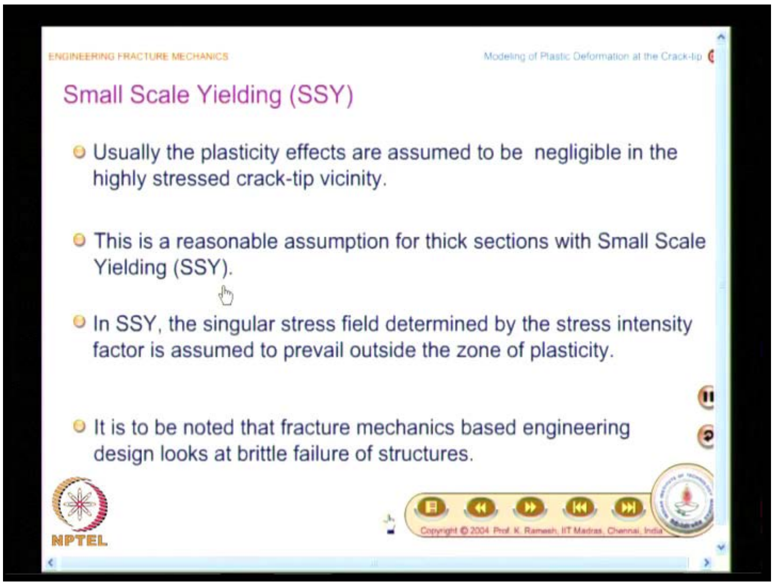

This we have already seen, this is just reviewed for the sake of in which domain we are going to leave, we are not even going to look at such large plastic zone, you are going to look at very small plastic zone, that is why we have characterize this as small scale yielding, whatever the discussion that we do it is applicable only when this aspect is satisfied.

Usually the plasticity effects are assumed to be negligible in the highly stressed crack-tip vicinity. This is the reasonable assumption for thick sections with small scale yielding, and we will also take up in detail. What is small scale yielding? After looking at Irwin's correction, for the time being you can consider, in small scale yielding the singular stress field determine by the stress intensity factor is assumed to prevail outside the zone of plasticity.

This is the reasonably a good assumption, we are making an assumption to make our life simpler, as long as this assumption is valid certain improvements in our stress intensity factor calculation can accommodate this class of materials, that is the way you have to look at it. And what is reminded here is the fracture mechanics based engineering design looks at brittle failure of structures, structure has a whole fails in a brittle fashion but, what happens at the crack-tip? Certain amount of plasticity effects also dictate what happens, and how this is handle? That is the way we have to look at it.

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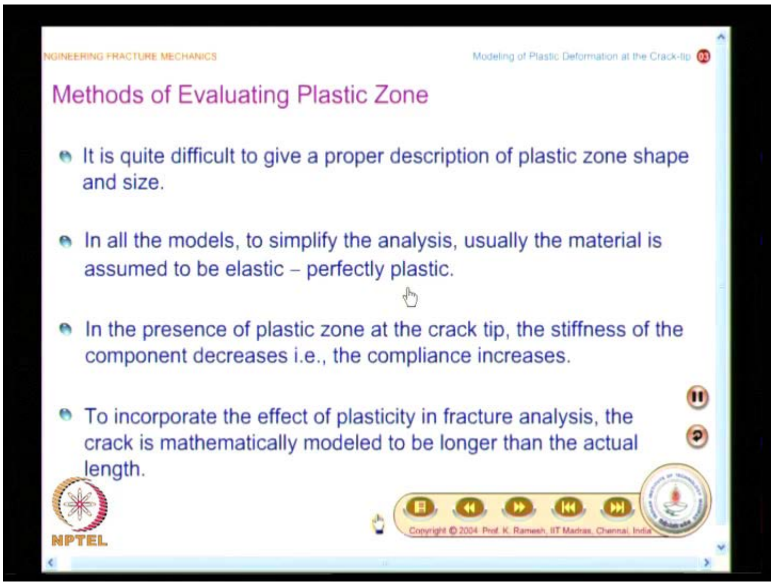

And how to evaluate the plastic zone, it is a very difficult aspect, it is difficult to give a proper description of plastic zone shape and size. We will look at simplistic models and the determination of shape has a connotation, because you would be able to quickly compare, what happens in aplane strain situation and a plane stress situation. And in all the models to simplify the analysis usually the material is assumed to be elastic-perfectly plastic. In reality many materials exhibit strain hardly, we are not considering that for a simplistic analysis, we simply say the material is elastic-perfectly plastic. And what happens, because of the plastic zone at the crack-tip, the stiffness of the component decreases that is the compliance increases.

So, only to accommodate this kind of an observation Irwin said, that the crack is to be mathematically modeled to be longer than the actual length. So, the question is how to find out the extension of appropriate crack length? This could be done in many days, we would look at a very simple model and then improve our calculations.

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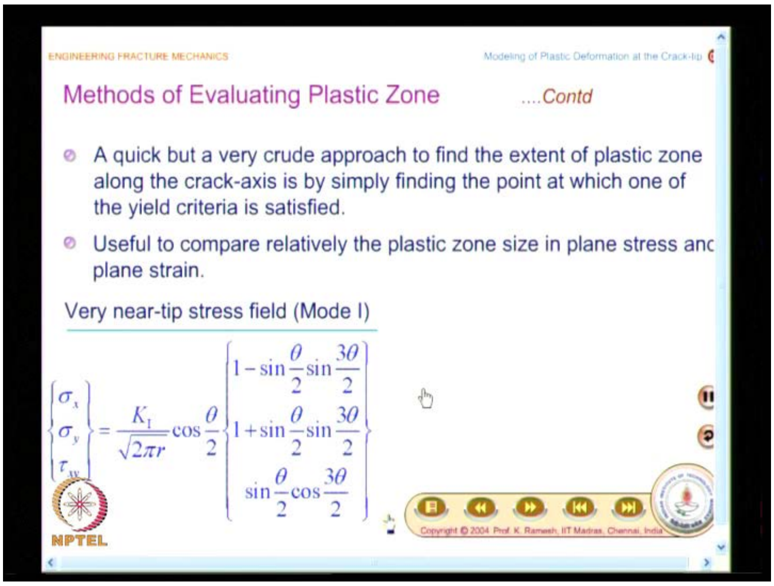


So, one of the quickest methods but, definitely a very crude approach is to find the extent of plastic zone along the crack axis by simply finding the point at which one of the yield criteria is satisfied.

We will not do any elaborate elastoplastic calculation, at the beginning of the course I had given you a review of solid mechanics, there you had look that, when the stresses are equal, when the two principal stresses are equal at the crack-tip for a plane stress situation as well as the plane strain situation for which you have Poisson ratio as 1 by 3, I have asked to calculate, what are the levels of stress given by the yield theories?

So, if you perform a simple tension test you are doing a uniaxial test and you have the stress develop is sigma y s, But if I have multi-axial situation the individual stress magnitudes will be much higher than the yield strength, this you had actually look that as part of a review of solid mechanics, and I am going to use that result right away. And if I use this kind of an approach of simply finding out the point at which on set of yield criteria satisfied, which is very crude. Nevertheless, this helps us to compare the relative plastic zone size in plane stress and plane strain. And I had mentioned earlier for rest of the course we will worry only about the singular stress field, and you know this singular stress field for mode one crack, and if I have to go and find out the plastic zone the first step is you have to determine the principal stresses.

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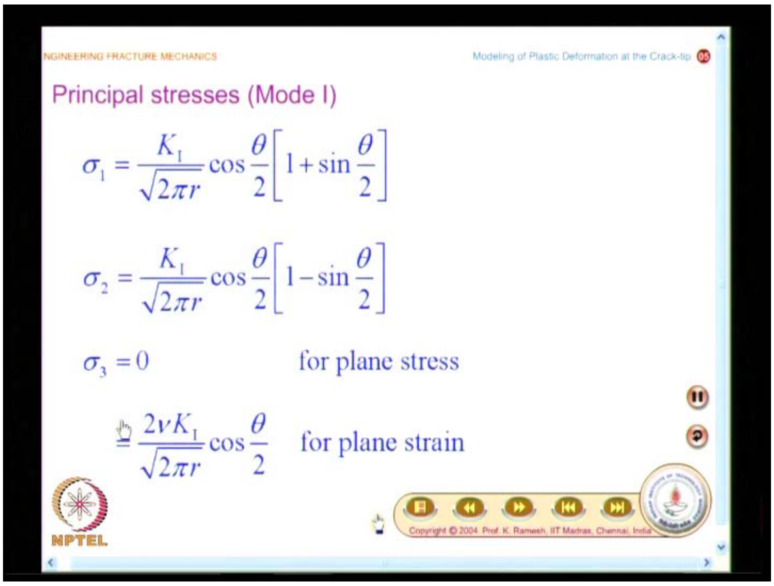


Now, you have the stress field, you know how to find out the principal stress from this, I would like you to work it out and then calculate the expression for principal stresses near the crack-tip for the mode one situation. You have the expressions for the stress field and it easy to calculate the principal stresses.

So, we will now look at the expression for the principal of stresses, sigma 1 is given as K 1 by root of 2 pi r, cos theta by 2 multiplied by 1 plus sin theta by 2. And sigma 2 is K 1 by root of 2 pi r, cos theta by 2 multiplied by 1 minus sin theta by 2, and in the case of a plane stress situation sigma 3 is 0, in the case of plane strain situation, which is seen in thick plates, I have sigma 3 given as 2 nu K 1 divided by root of 2 pi r cos theta by 2.

When theta equal to 0 you can find out when nu equal to 0.5, it will be same as your sigma 1 sigma 2, what you have here. And this explains the triaxial situation near the crack-tip, we had seen it pictorially earlier now we are looking at a mathematically so, the moment you have a crack you have to recognize that triaxial situation is possible.

This is something very significant, we have a plate with a hole you essentially say you have a biaxial stress field, a uniaxial stress field changes to biaxial stress field. The moment you have a crack it can become a triaxial situation.

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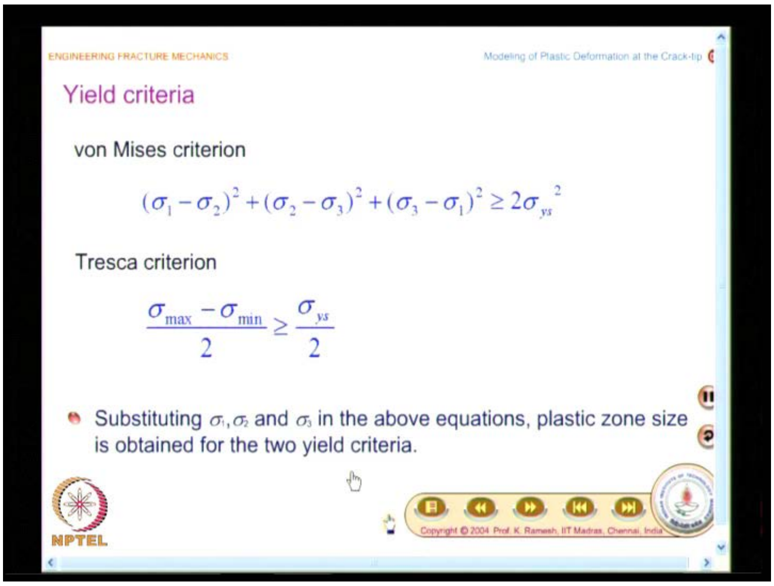


Suppose, the crack-tip is blunt instead of happening at the crack-tip the triaxial situation will take place slightly ahead of the crack-tip - that is the way you have to look at it. And what is the material model that we have used? Elastic-perfectly plastic, and the yield criteria are two you know this, just for review we are just looking at a the von Mises criterion utilizes all the three principal stresses.

The condition is sigma 1 minus sigma 2 whole square, plus sigma 2 minus sigma 3 whole square, plus sigma 3 minus sigma 1 whole square, greater than or equal to 2 sigma y s square.

This is the yield strength, and you want this to be within sigma y s, if it is greater than that yielding would occur. And when you go to Tresca criterion you have to recognize it is not sigma 1 minus sigma 2, it is written as sigma max minus sigma minimum divided by 2, and this becomes important in fracture mechanics problem, because at the crack-tip if you consider the crack-tip is sharp, sigma x equal to sigma y you are getting it, not only this, both of them are the same sign, when both of them are the same sign the other minimum stress is zero so, the zero plays a very important role.

So, when you are applying Tresca yield criteria to plane stress situation fracture problem you have to calculate the maximum shear stress carefully.

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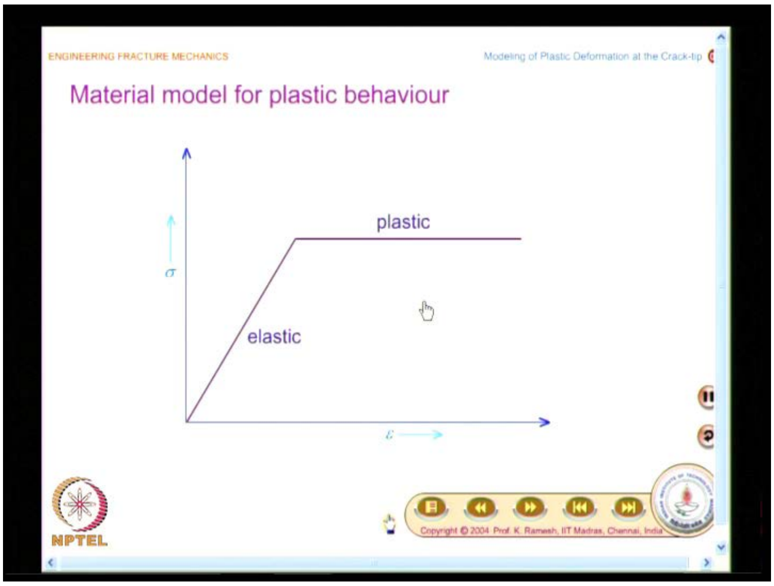


So, once you know the values of sigma 1, sigma 2, and sigma 3, either by using this criteria or this criteria you can always find out, what is the plastic zone size. And this is just to give a pictorial representation, what do we mean by elastic-perfectly plastic? You do not consider any strain hardening of the material, because this is simple for us to do the calculation.

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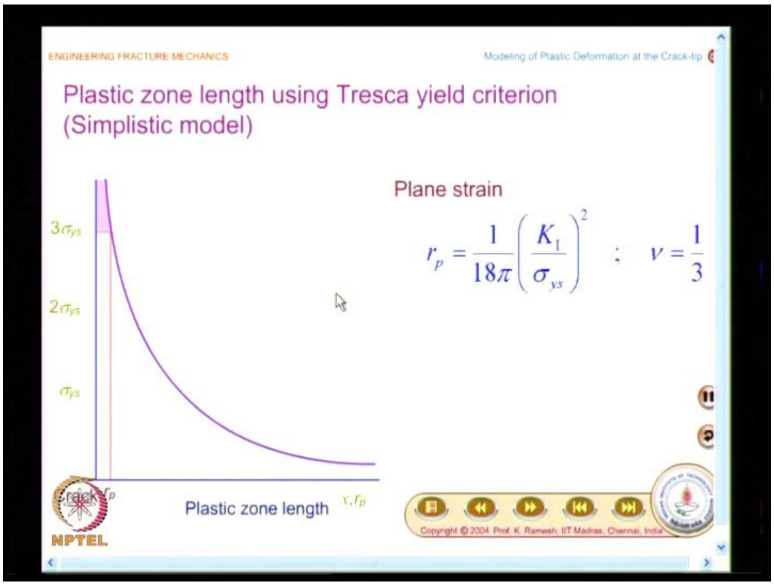


Now, what is plotted is you have a crack and I have the values of stresses put here sigma y, and if you go by your plane strain situation, when you use the yield criteria ,what you find is, the sigma y can take a very high value of 3 times sigma y s, when you have Poisson ratio is 1 by 3. See, in a simple tension test when the stresses reach sigma y s, yielding takes place. We have seen in the case of a crack problem you have a triaxial loading situation.

In a triaxial loading situation particularly, when you consider plane strain situation the individual stress magnitudes can be very high. It can be as high as three times the values of the yield strength, and you also define there is a length, you are looking at what happens along the crack axis, simply mark this point as a length.

This is erroneous, we would improve upon it later first we have said, we would simply find out the yield strength value, based on that find out what is the length, based on this find out what happens as a function of theta, that would give the plastic zone shape which is very approximate, because we are not considering redistribution of load. We are simply plugging in mathematically whichever the points which reaches the yield strength value - that is not going to happen physically.

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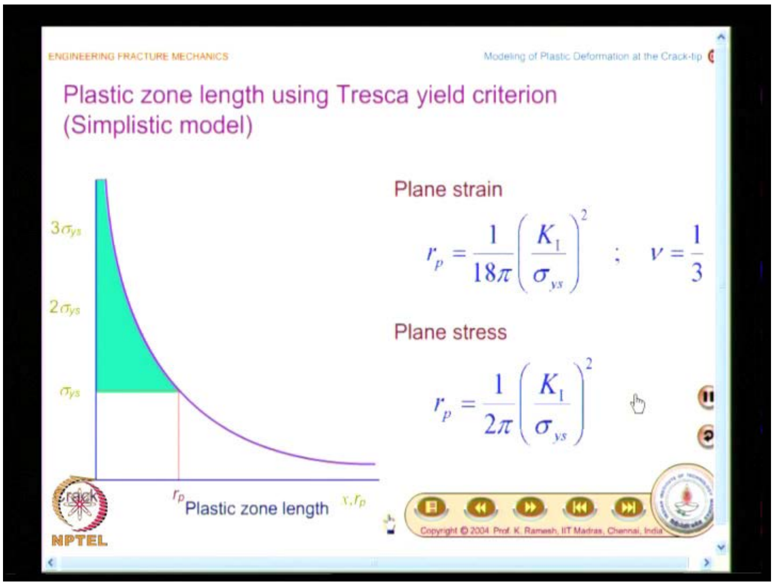


But definitely it gives an approximate shape of the plastic zone near the crack-tip, and the value of r p, turns out to be 1 by 18 pi multiplied by K 1 by sigma y s whole square, and this is particularly for the value of Poisson ratio equal to 1 by 3. And the moment you come to a plane stress situation, I had mention that sigma y equal to sigma x, and the other stress is what? Other stress sigma 3 is 0.

So, that tells you the maximum stress that you will have to look at is only sigma y s. And if you plot in this graph and locate the point where it reaches the sigma y s, it is here. So, in the case of a plane stress situation the plastic zone length ahead of the crack is very long, and this is given as 1 by 2 pi multiplied by K 1 by sigma y s whole square so, a very simplistic analysis has shown the relative importance of plastic zone in the case of a plane stress as well as plane strain.

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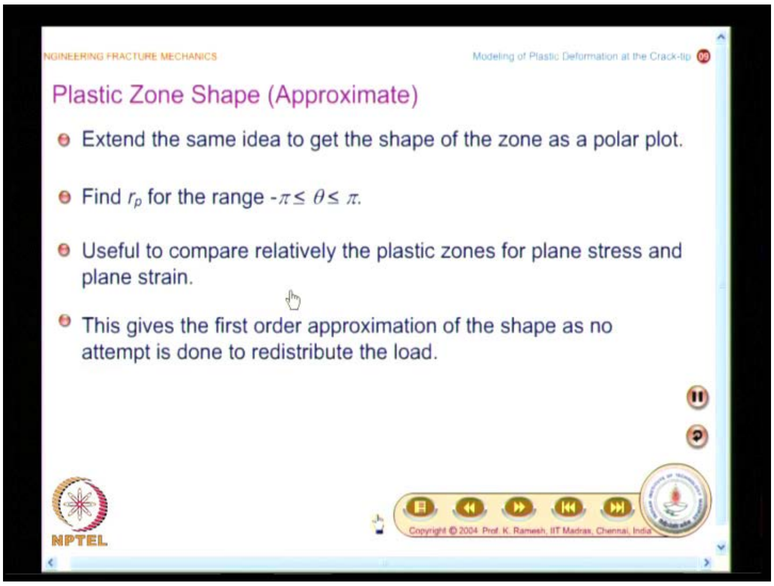


The plastic zone is very large in the case of plane stress; it is very small in the case of plane strain. See, these discussions are needed even to decide, what is the appropriate thickness to conduct fracture toughness testing. Definitely, this is the very simplistic model, but the advantage of a simplistic model is we can also go and estimate, what is the shape of the plastic zone. Let us look at that and even to start with the title is given as approximate.

So, whatever the methodology that we have adopted to get the length of the plastic zone along the crack axis, extended to get the shape of the zone. And when you want to get the shape of the zone, make it as a polar plot. We have the expressions for sigma 1 and sigma 2 in terms of theta. So, we will express the value of r p, in the range minus pi to pi as a function of theta. And draw the polar plot that will help us to relatively compare the plastic zones for plane stress and plane strain.

This is the purpose, see if you have to really obtain the plastic zone, either you should go to an experimental approach, metal edges have done it, we will also see in the context of ductile model, they have arrived at shape of the plastic zone experimentally. Otherwise, you will have to go to a finite element calculation, exhaustive elastoplastic analysis you perform, and you do this for every kind of specimen, because all this things are depends on the kind of specimen as well as the loading, what we are now trying to look at is, a sort of an approximate understanding on relative shapes of these in plane stress and plane strain, you have to keep the focus in mind. So, it gives only a first order approximation of the shape, because the methodology does not attempt to redistribute the load.

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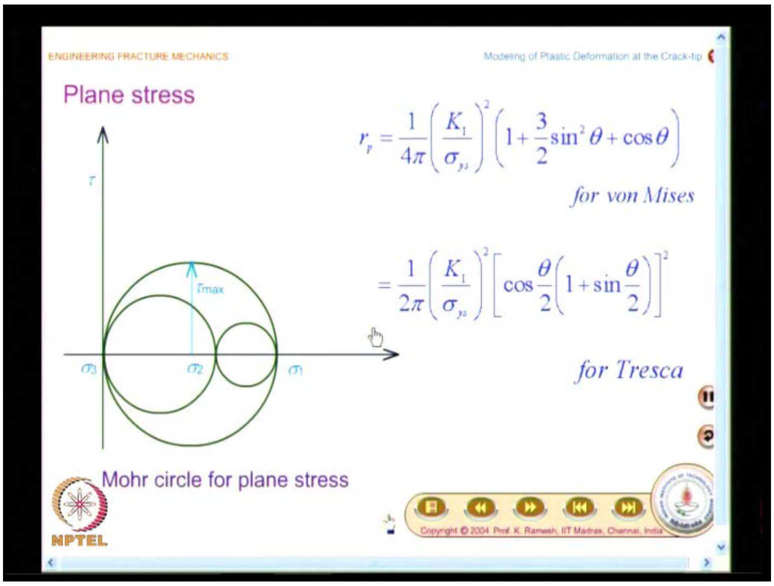


So, this is the key aspect. And for the case of plane stress, I had already drew your attention and that is drawn in the case of Mohr circle, you have the zero stress, this is to be taken into account. And you have the value of r p, as the function of theta is given like this and these expressions would change for the different yield criterion.

If I use von Mises, I get r p as 1 by 4 pi multiplied by K 1 by sigma y s whole square, multiplied by 1 plus 3 by 2 sin square theta plus cos theta. See, even for our well-developed mechanics of solids, different yield criteria provide different results.

So, in fracture mechanics we are dealing very complex situations so, you have to accept multiple solutions, you have to pick out which one is suitable for it, because in mechanism solids what we understand certain materials obey Tresca yield criteria. So, for those materials apply Tresca yield criteria, certain materials are obey von Mises yield criteria, this is an accepted practice and what we are now trying to look at is, how do the plastic zone shape differs with respect to plane stress as well as plane strain, and also with respect to the invocation of yield criteria by von Mises or yield criteria by Tresca? And for Tresca this turns out to be 1 by 2 pi K 1 by sigma y s whole square multiplied by cos theta by 2, 1 plus sin theta by 2 whole square, I am not reading the brackets you put the brackets as for the equation. Since you have these expressions, it is possible for you to do the polar plot. So, what I would appreciate is you make an attempt, you go to your rooms, you have the expression, you calculate for few values of theta, how does the shape look like? And the next class we will see, and what I would also appreciate is we have develop this for mode one situation, get similar expressions for mode two as well as mode 3 and try to come with polar plots of approximate shape of plastic zone.

So, in this class what we have discuss was, we have look at the final expressions given by Newman and Raju on the empirical relation of surface cracks, then we look at how triaxiality happens near the crack-tip. And what way we use that terminologies plane stress and plane strain in the context of fracture mechanics, what is the difference between the applied mechanics definition and a looser definition in the case of fracture mechanics.

Then we also looked at an inside into stress intensity factors for various situations, though the stress intensity factor for a surface crack is compare to through the thickness crack, I pointed out, because of triaxiality from fracture instability point of view you will have to use plane strain fracture toughness for surface cracks so, you have to keep it at the back of your mind, that surface cracks are always dangerous.

Then we moved on to the motivation for determining plastic zone a kind of the crack-tip, the essential idea is to extent fracture mechanics to larger class of material and people have looked at simpler modifications, and to that extent we have gone and looked at one of the very simple model to find out the extension of plastic zone along the crack axis, and we have also obtain the relationship as the function of theta and I have asked to plot this and come for the next class.

Thank you.