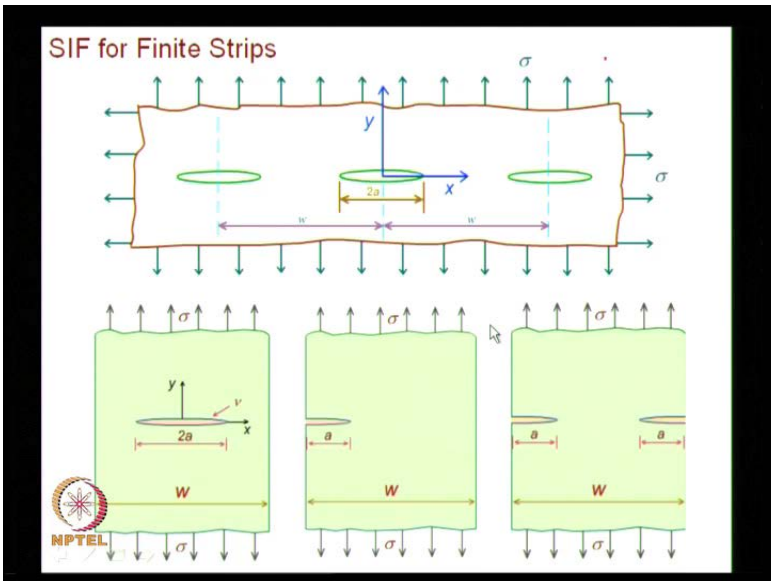
Engineering Fracture Mechanics
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module No. #06
Lecture No. #26
SIF for Embedded Cracks

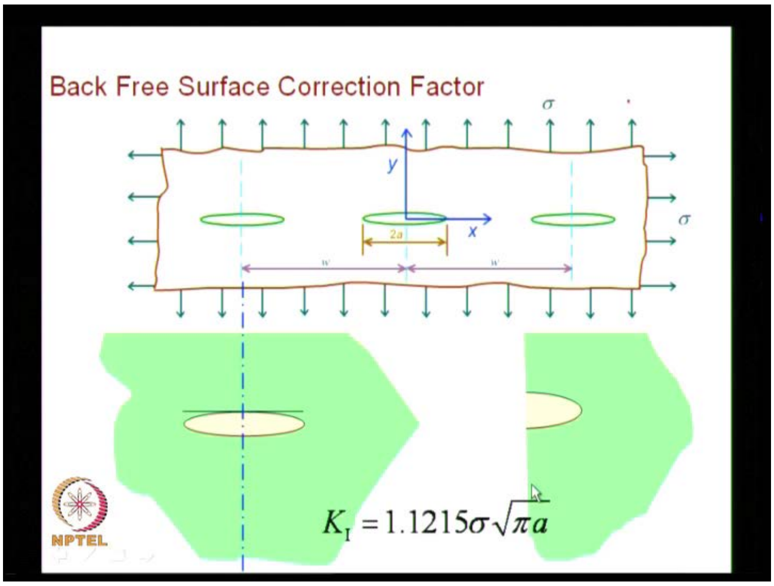
(Refer Slide Time: 00:19)



We have been looking at stress intensity factor for variety of geometries, and in the last class, we had looked at the problem of collinear cracks evenly spaced in an infinite strip. As I said, from an academic point of view, it would look like an extension of writing a stress function to a fictitious problem.

However, what we saw in the last class was using the solution of this problem, we could construct solutions for a CCT specimen - center cracked tension specimen. If you cut it along the y-axis and in between this, you will be able to get a single edge notched specimen. And if you cut it along the y-axis here, and also along the y-axis here, you will get the specimen of double-edged crack.

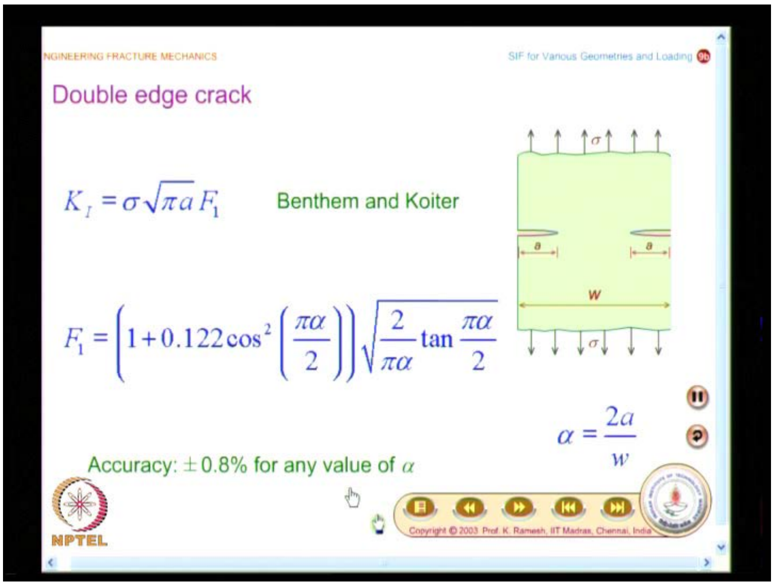
(Refer Slide Time: 01:21)



One of the issues which I pointed out to you in the last class was a comparison between a center crack specimen and a single edge notch specimen. In the case of a center crack specimen because of symmetry, you need to have the slope to remain horizontal here. On the other hand, when I cut this, there is no restrain on this and the crack will open up more. The slope will not be at 0 degrees. This implies that you will have a higher value of stress intensity factor in those classes of problems and it is labeled as back free surface with respect to the crack tip essentially having a factor of 1.1215.

Many times, people just use it is as 1.12 and is known as back free surface correction factor. Our focus is to graduate through the thickness cracks to surface cracks as surface cracks are more realistic.

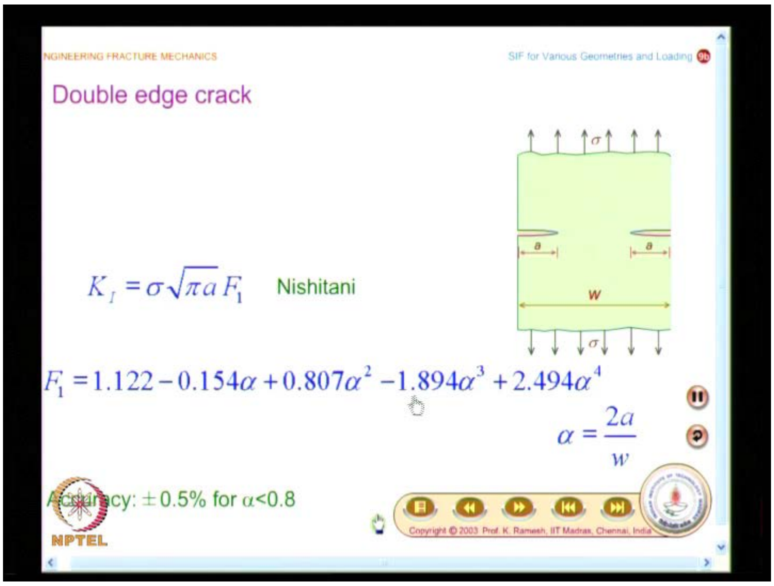
(Refer Slide Time: 02:52)



In the last class, we had looked at stress intensity factor for the center crack specimen as well as a single edge notch specimen. Now you have the solution for double edge crack with the definition of K 1 as sigma root pi a multiplied by the function F1. According to Benthem and Koiter, the function F1 is given as 1 plus 0.122 cos squared pi alpha divided by 2 whole multiplied by square root of 2 by pi alpha tan pi alpha divided by 2. Alpha is defined as 2 a divided by w.

For such expression the accuracy is put as plus or minus 0.8 percent for any value of alpha. One thing you have to keep in mind is when going from infinite geometry to finite geometry as the stress concentration factor increases stress intensity factor also increases. That is the reason why you have to find out SIF for finite geometries if it is less than what it is in the infinite geometry.

(Refer Slide Time: 04:52)

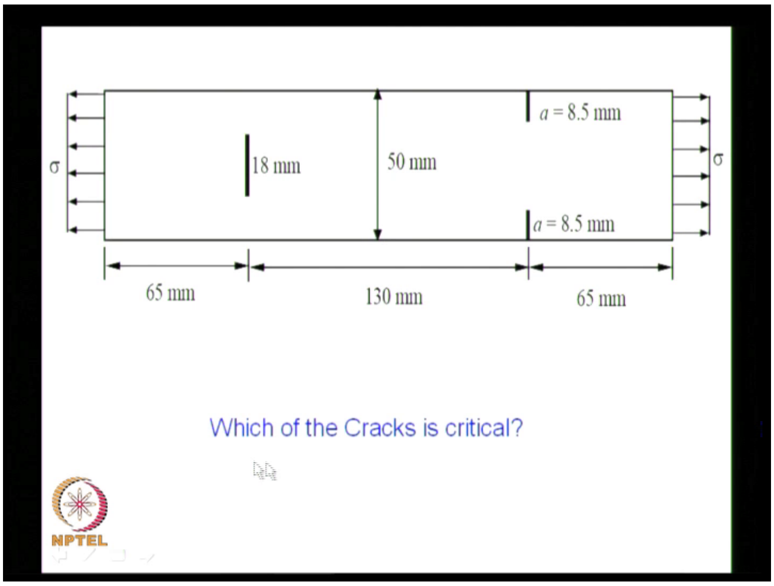


As the problem is complex you will have multiple solutions. You have another set of expressions given by Nishitani. The function F 1 is little more elaborate and you find the accuracy improved from point eight percent to plus or minus point five percent and the function F 1 is given as 1.122 minus 0.154 alpha plus 0.807 alpha square minus 1.894 alpha cube plus 2.494 alpha power 4.

In the beginning of the course somewhere around 10 classes we have taken up a problem of a center crack and a double edge crack. There we were not sure which crack is more critical as you were not armed with calculation of SIF. So we had only seen an experimental result where the fracture started at the double edge crack.

.

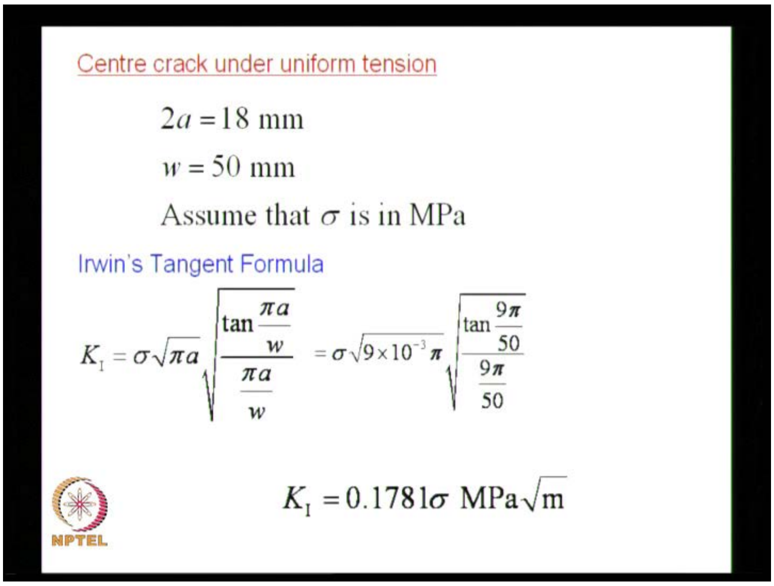
(Refer Slide Time: 06:28)



Now as you are having the equations just go back to that problem and then evaluate the value of SIF for a center crack as well as the double edge crack when the center crack is 18 millimeter and the double edge crack is eight point five millimeter. Double edge crack summation is shorter than the center crack.

Let us see whether our experimental observation matches with our analytical computation.

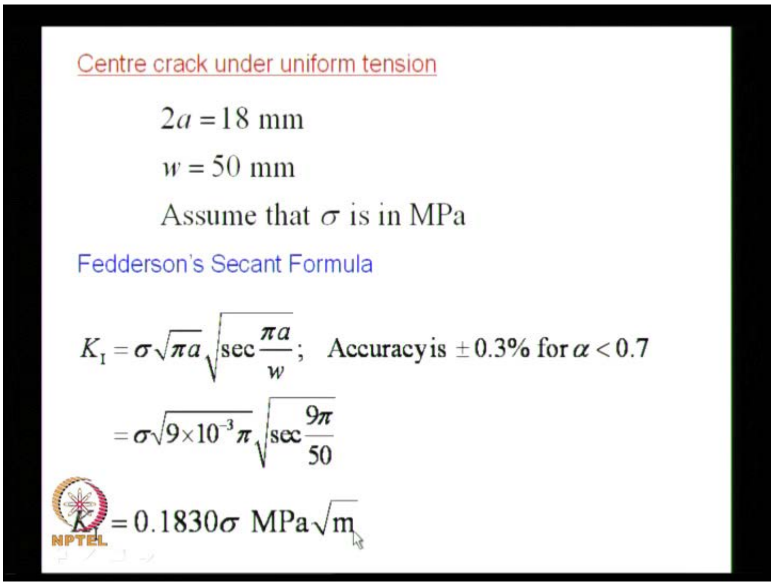
(Refer Slide Time: 07:28)



What we will do is we will try to evaluate the parameters for all the formula that we have learned. We know Irwin's tangent formula and the parameters needed for calculation. Here a and w are available.

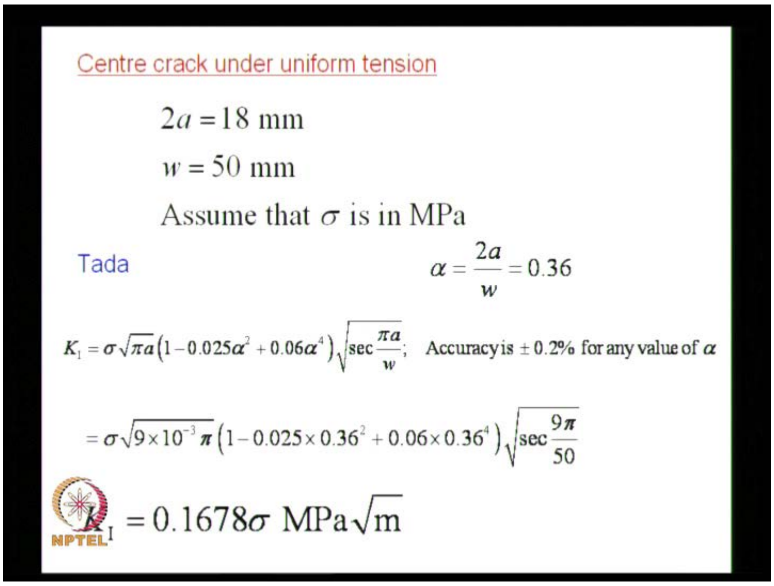
After substituting the relevant quantities, the stress intensity factor K 1 is equal to 0.1781 sigma MPa root meter. Now what we have to do is to get the results for other formula and compare it with the double edge crack.

(Refer Slide Time: 08:35)



Now let us go to the formula by Fedderson's which is very famous secant formula and it is sigma root pi a multiplied by secant pi a divided by w. It is an empirical relationship based on heuristic arguments which gives you the value of K1 as 0.1830 sigma MPa root meter which is slightly higher than the tangent formula.

(Refer Slide Time: 09:19)

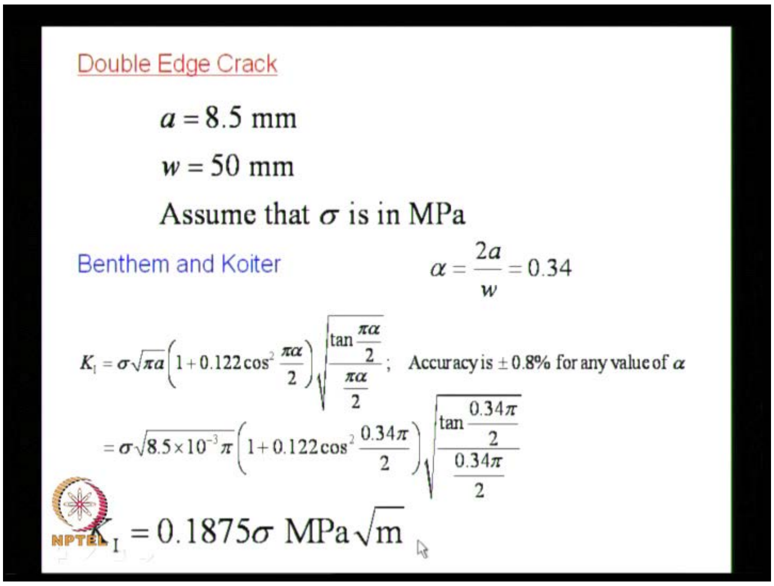


Here we have one more formula that is Tada's formula. In Tada's formula alpha is taken as 2 a divided by w and after substituting the values you will get alpha is equal to 0.36.

The formula is sigma root pi a multiplied by 1 minus 0.025 alpha square plus 0.06 alpha power four whole multiplied by secant pi a divided by w. After substituting the relevant quantities the result is 0.1678 sigma MPa root meter.

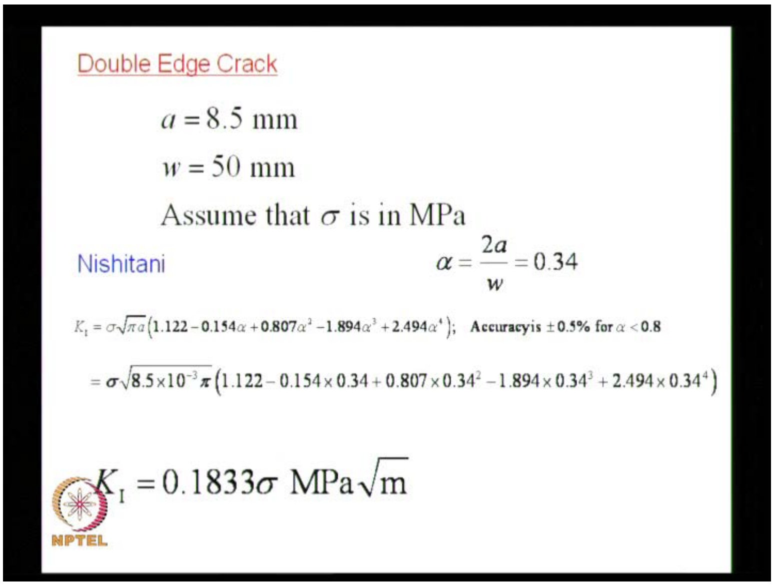
So in the case of center crack we had three formulae each one giving different values and the highest value we got was 0.183 from Fedderson's formula. Now we will look at the calculation for a double edge crack.

(Refer Slide Time: 10:25)



So in the double edge crack you have a equal to 8.5 millimeter, w equal to 50 millimeter and the expression of Benthem and Koiter. The value of alpha is point three four and after substituting it and simplifying the equation the value of K 1 is 0.1875 sigma MPa root meter which is definitely higher than the highest value we got for the center crack.

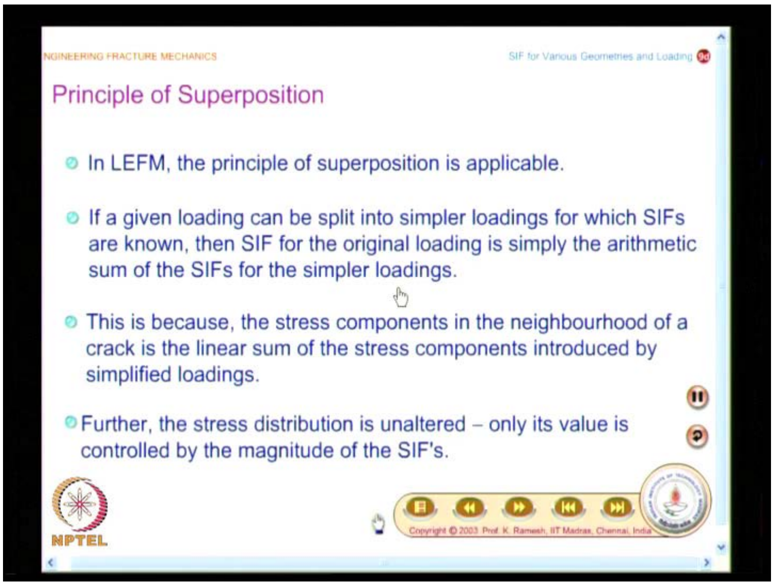
(Refer Slide Time: 11:35)



In this we saw one more formula by Nishitani and what I get here is K 1 equal to 0.1833 sigma MPa root meter.

So what you find is that the least value that you have got from the double edge crack is higher than the highest value you got from the center crack, but they were close. From the above discussion it is clear that the configuration of the crack is also important in deciding the value of SIF.

(Refer Slide Time: 12:45)



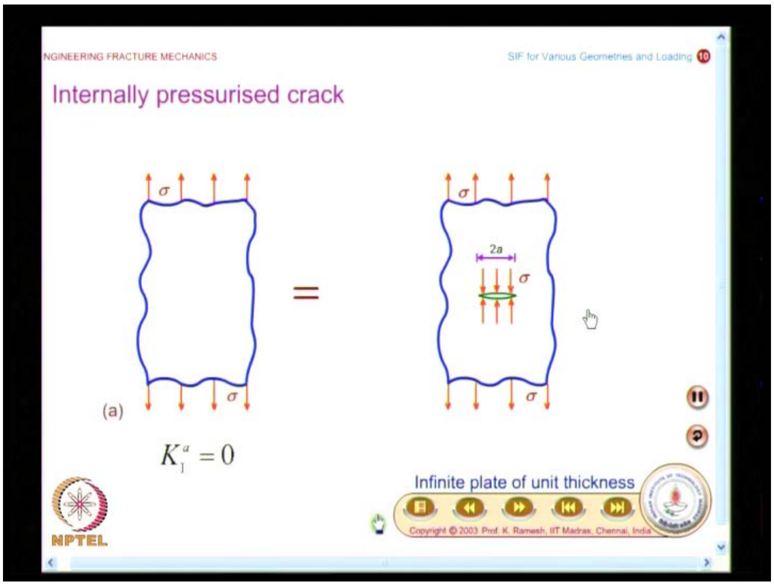
Another issue we have is about the critical crack which we will do by taking up another approach to SIF evaluation using the principle of superposition. Here principle of superposition is applicable because of linear elastic fracture mechanics. If a given loading can be split into simpler loadings for which SIF's are known, then SIF for the original loading is simply the arithmetic sum of stress intensity factors for simpler loadings.

As I had mentioned earlier, the stress field is dictated by the function of R and theta which is identical. The strength is controlled by the stress intensity factor. Suppose I have multiple loadings, each one of them gives the same mode one type of loading on the crack faces.

When the crack faces open up, I could add the stress intensity factors which we are going to do in principle of superposition. Here the challenge is how to reduce the given problem as sum or sum and subtraction of simpler problems. We are able to do this because the stress component in the neighborhood of a crack is the linear sum of the stress components introduced by simplified loadings. Further the stress distribution is unaltered, only the value is control by the magnitude of the stress intensity factors.

Because of this we are in position to add or subtract as the case maybe of the stress intensity factors. Here we are looking only at a particular type of loading. If we do mode one loading we can do for mode one loading and if we do for mode two loading those things we can add. We cannot add mode one and mode two, it can be done only in energy approaches.

(Refer Slide Time: 15:14)

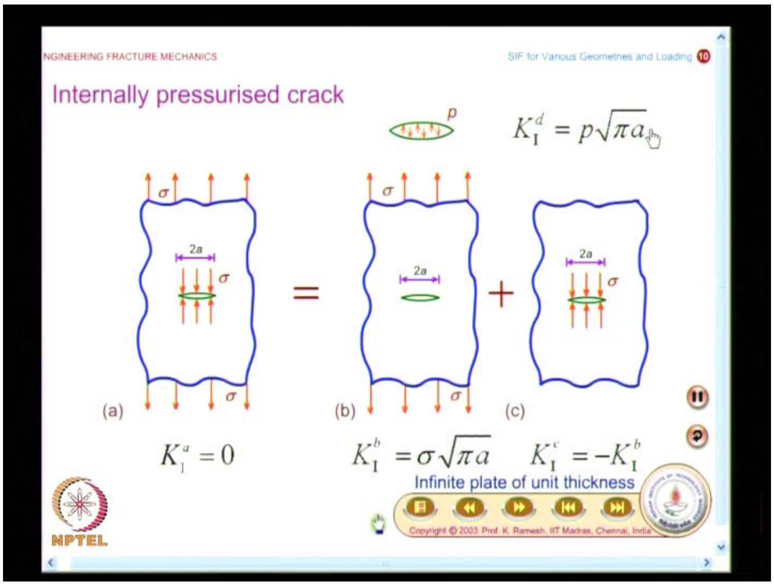


Now let us take a problem of internally pressurized crack. I have taken a simple strip subjected to uniform tension sigma and there is no crack in this. So when I do not have a crack I am justified in saying K 1a is equal to zero.

We had a discussion on green's function approach where we have seen when there will be internal pressure and what would be the value of stress intensity factor. The reason why I have taken the same problem is to get the idea of how people construct sub problems in principle of superposition.

I can also represent the same problem with a different type of loading. I have a crack but the crack is closed by another set of loading which is same as no crack at present. Now the question arises should I apply only sigma or two sigma or three sigma. In whatever way you put it, it is not going to change as this will cancel each other and the crack remains close.

(Refer Slide Time: 17:20)



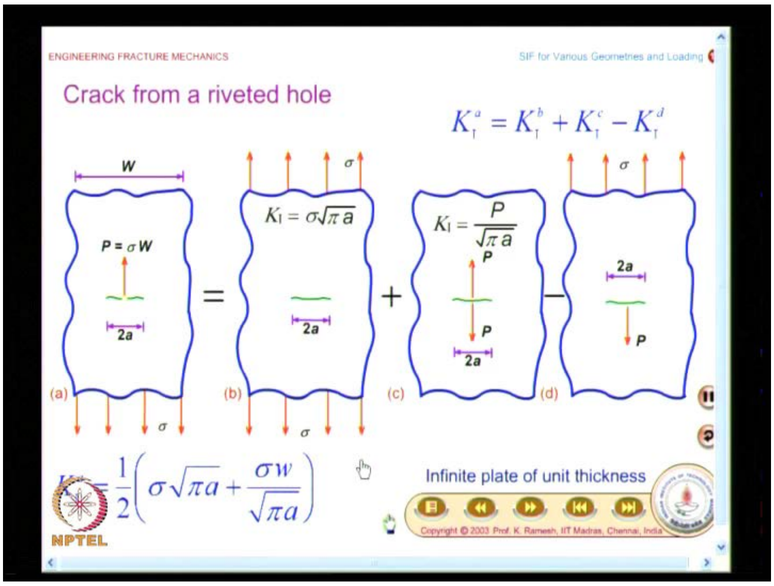
So now what we will do is rather than splitting this into some problems, we will make this as problem a and try to split it into sub problems for which we know the stress intensity factor.

We will split this as one problem where we have a crack which is subjected to uniaxial tension. When we take up another problem where the crack is closed by the stress sigma, we can find the stress intensity factor for problem c as we know the value of stress intensity factor of problem b denoted as K 1 b which is equal to sigma root pi a and of the problem a which is equal to zero. We can simply say it is zero because K 1 c is nothing but minus K 1 b as K 1 b plus K 1 c is equal to zero. Here we get a negative sign which can be compressed so as to apply forces on the crack surfaces.

So to get that kind of a scenario, I reverse the sign of the load and equate this to the pressure. So for the pressurized loading of crack faces K 1 is P root pi a.

So once you know how to write sub problem, the problem is solved. Here I have taken up an infinite geometry because I do not want to worry about the correction factors which are considered in finite geometry.

(Refer Slide Time: 20:19)



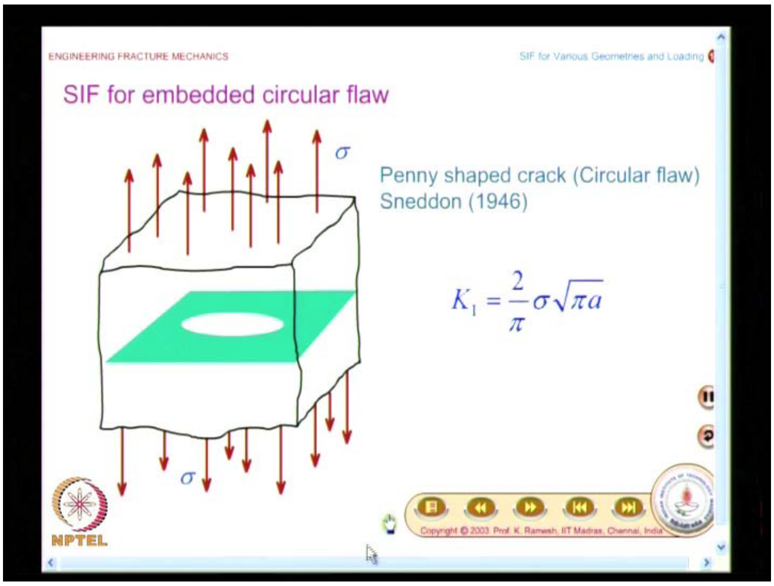
Now let us take up a problem of a crack from riveted hole. Suppose, I have a crack emanating from this hole and I have a rivet which is like a pin which would exact a force like this on one direction and you have some pulling of the whole plate by a stress sigma. The load P is given as sigma w which is load per unit length and here we are considering a unit thickness. Now write the sub problems and ensure that you know at least part of the solution for K. If you write a sub problem where you do not know solution for K, then the purpose is difficult.

So the whole challenge lies in writing out the sub problems. Let us see how the sub problem can be written. So I take a problem of a crack in a tension strip as one problem where these stresses have to be canceled and I do not have any loading on the crack face. So the problem for which I know the solution is the force acting at the center of the crack on either side. This can be added as another problem and when I look at these two problems, if I have to match this to the problem depicted in figure a I need to subtract another problem which is very similar to the question itself one concentrated force and its distributed load. So it is put in the right hand side to subtract.

So I have K 1 for the situation a is equal to K 1 b plus K 1 c minus K 1 d where I do not know what is K 1 d.

So when I do that my expression turns out to be one and half of sigma root pi a plus sigma w divided by root of pi a.

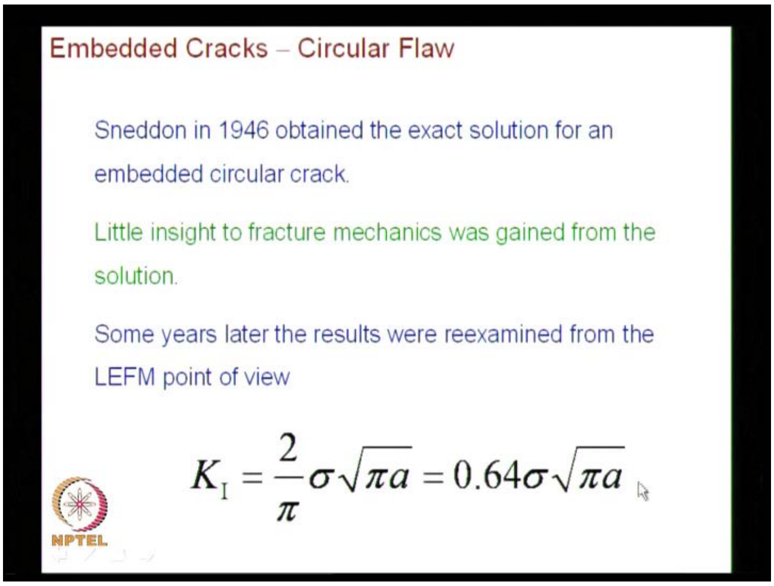
(Refer Slide Time: 24:37)



And now what we will move on to embedded flaws from through the thickness cracks. And if you look at the literature, they have first taken up the problem of embedded circular flaw in the case of stress concentration problem and the first problem people solved was plate with a circular hole. Once after they have understood how to solve a problem like this they graduated to plate within elliptical hole.

In a similar way in this case we find problem of a circular flaw in an infinite body. Here as shown the edges vary indicating that it is a flaw in an infinite object and this was developed in 1946. This is where initial stages of fracture mechanics developed. In fact Sneddon in 1946 provided only the stress filed and was able to solve it mathematically he could provide the stress filed at that time the expression of K 1 was not available which is equal to 2 divided by pi sigma root pi a.

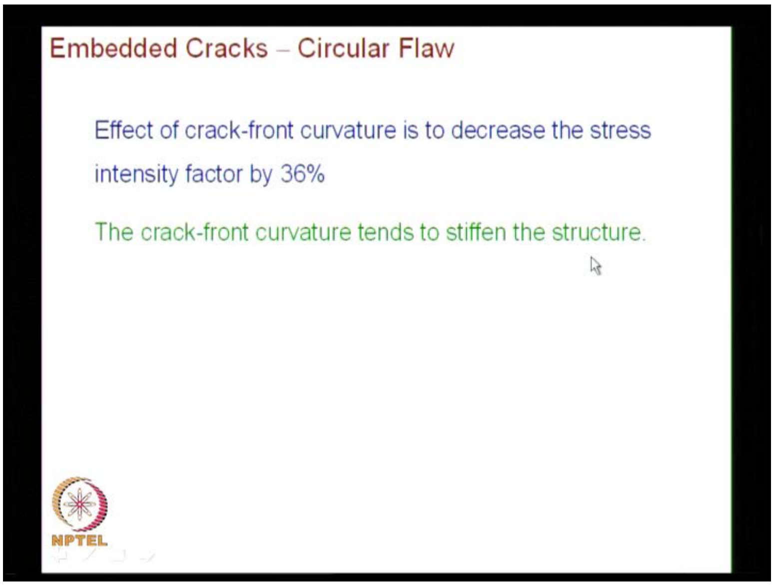
(Refer Slide Time: 26:21)



To summarize here, Sneddon in 1946 obtained the exact solution for an embedded circular crack. Little insight to fracture mechanics was gained from the solution. However, some years later the results were reexamined from the LEM point of view and you have got the result for K 1 equal to 2 divided by pi sigma root pi a written in another form as 0.64 sigma root pi a which is smaller than SIF for center crack specimen.

So in the initial development of fracture mechanics whenever any result is obtained people reflect many aspects of it. An embedded flaw is not as dangerous as through the thickness crack.

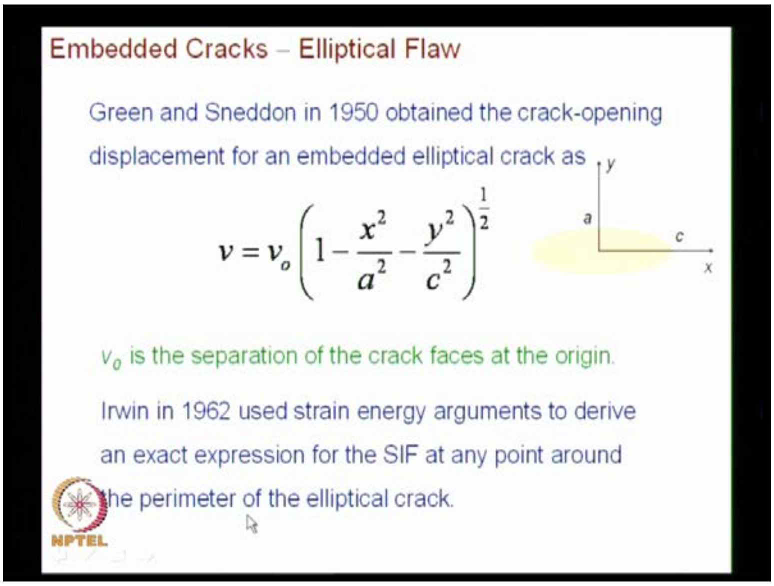
(Refer Slide Time: 27:36)



It is been analyzed that in the case of a circular flaw you have a curved crack front. The effect of crack front curvature is to decrease the stress intensity factor by 36 percent. In through the thickness crack, they could easily analyze the crack front and when they graduated to circular flaw fortunately the stress intensity factor was constant and was not changing. But after analysis they had brought in a conclusion that the stress intensity factor is reduced by 36 percent.

The crack-front curvature tends to stiffen the structure. As I said earlier, in the case of stress concentration problem people graduated from circular hole to elliptical flaw. Similarly, here also they graduate from circular flaw to elliptical flaw and this was done by green and Sneddon in 1950.

(Refer Slide Time: 28:55)

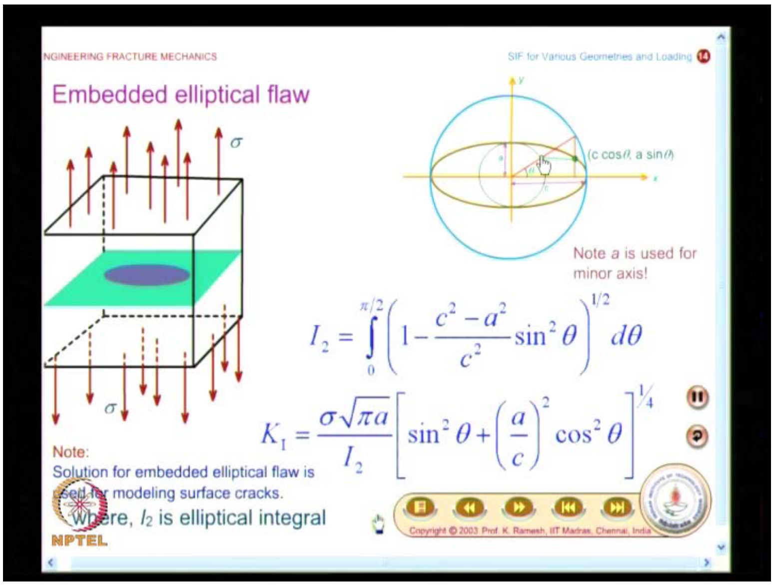


They initially obtained the crack-opening displacement for an embedded elliptical cracks as v equal to v naught multiplied by 1 minus x squared divided by a squared minus y squared divided by c squared whole power half, thickness crack the moment you come to embedded flaw and we would also graduate to surface flaw

In through the thickness crack, embedded flaw and surface flaws the major axis is given as 2 c and a minor axis as 2 a. This is a nomenclature primarily because it is the length of the crack. The length of the crack is taken as a and the length of the crack definition in through the thickness crack is in one way. In the case of embedded flaw or in the case of surface flaw a is more relevant. Here the semi major axis is c, semi minor axis is a and v naught is the separation of the crack faces at the origin.

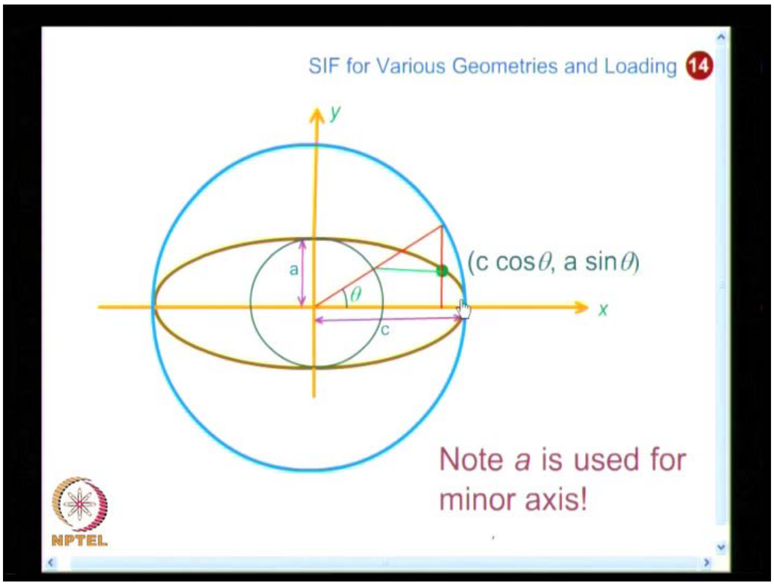
So initially Sneddon obtained only the crack opening displacement. Irwin in 1962 used strain energy arguments to derive an exact expression for the stress intensity factor at any point around the perimeter of elliptical crack which is very complex.

(Refer Slide Time: 31:39)



The stress intensity factor expression is obtained as shown here. The edges are given straight in the diagram but still consider this as an infinite body with in elliptical flaw. The value of K 1 is given as sigma root pi a divided by I 2 multiplied by sin square theta plus a divided by c whole square cos square theta whole power 1 by 4.Where I 2 is an elliptical integral which is in terms of theta . This primarily comes from how to locate a point on an ellipse.

(Refer Slide Time: 32:57)



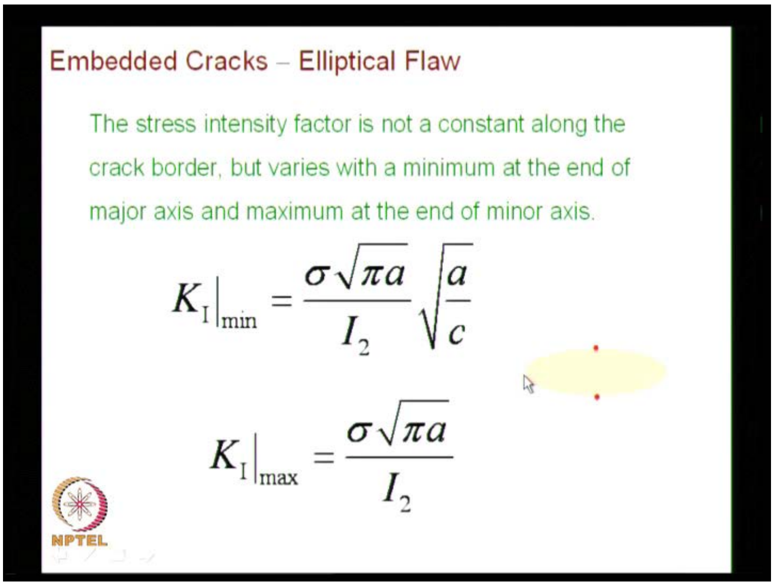
In the above diagram I have an ellipse and I have drawn a circle with the major axis and also drawn a circle with the minor axis.

Suppose I want to locate a point on the ellipse. I draw a normal through this which cuts the ellipse at this point as well as the perimeter of the circle and when I join this point to the center this is the angle theta. In other words when I take an angle theta I will hit the circle here when I drop a normal form to the x axis it will cut this point. The location is given as (c cos theta, a sin theta) which is a parametric representation and can be easily determined. The value of K 1 having an expression involving theta corresponds to the point shown as green on the ellipse. So as you change theta you will get various points.

The value of stress intensity factor from that expression and how to locate the point for a given theta is depicted in the figure. In fact if you go back and look at the various methods of drawing an ellipse like you can take a circle and compress it into forming an ellipse then this distance is the major axis and the short distance is the minor axis. So for any vertical line you maintain the same ratio in terms of major and minor axis and mark points.

You also have a an expression for the elliptical integral that is given as I 2 equal to integral 0 to pi divided by 2 evaluating the integral of the function 1 minus c squared minus a squared divided by c squared sin squared theta whole power half multiplied by d theta. When you have the location as theta you should know how to locate the point on the ellipse.

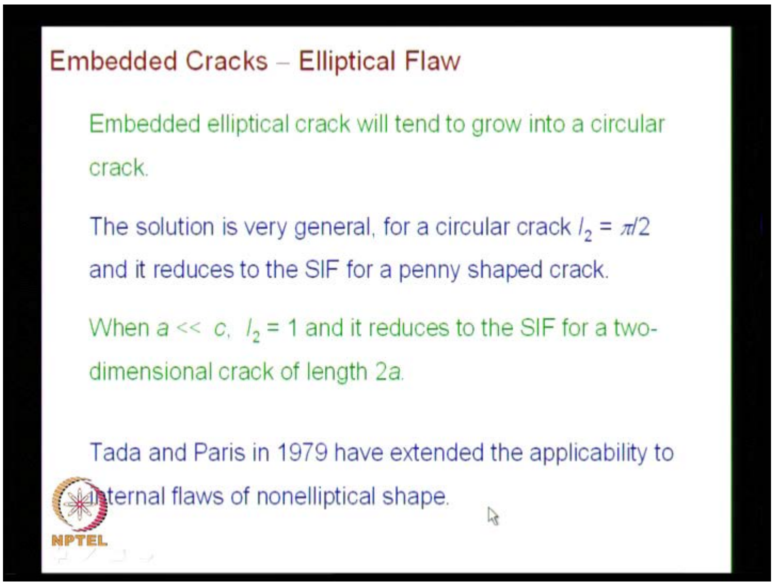
(Refer Slide Time: 35:51)



So first observation you get is the stress intensity factor on the elliptical front varies from point to point and you have a minimum as well as the maximum value. The maximum value of stress intensity factor which occurs on the minor axis decides which way the crack will grow. If you do the computation you find the stress intensity factor minimum along the major axis and it is given as sigma root pi a divided by I 2 multiplied by root of a divided by c.

So K 1 max is sigma root pi a divided by I 2. Now let us see what will happen to an elliptical flaw when the loads are increased. At some load it has to propagate, so the variation we have is that the stress intensity factor which is minimum on the major axis increases and reaches the maximum value at the minor axis and then returns back to minimum value.

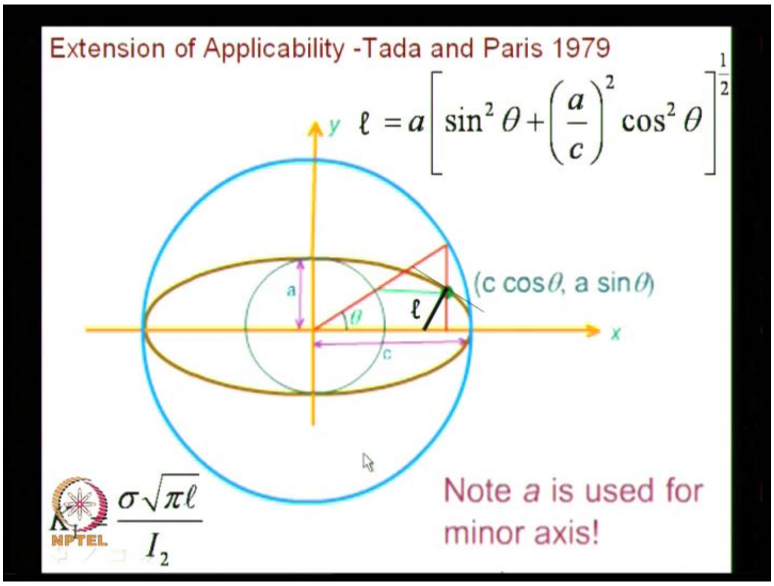
(Refer Slide Time: 37:47)



So the summary here is that an elliptical flaw would try to become a circular flaw. The solution that Irwin has obtained is termed as very general solution. For a circular crack the elliptical integral I 2 becomes pi divided by 2 and it reduces to the stress intensity factor for a penny shaped crack.

A circular crack can become a straight crack. When a is far less than c, I 2 becomes 1 and it reduces to the stress intensity factor for a two- dimensional crack of length 2 a. Many people have tried to extend the applicability of the results obtained by Irwin and this was done by Tada and Paris in 1979 who explained how to handle non-elliptical shape internal flaws.

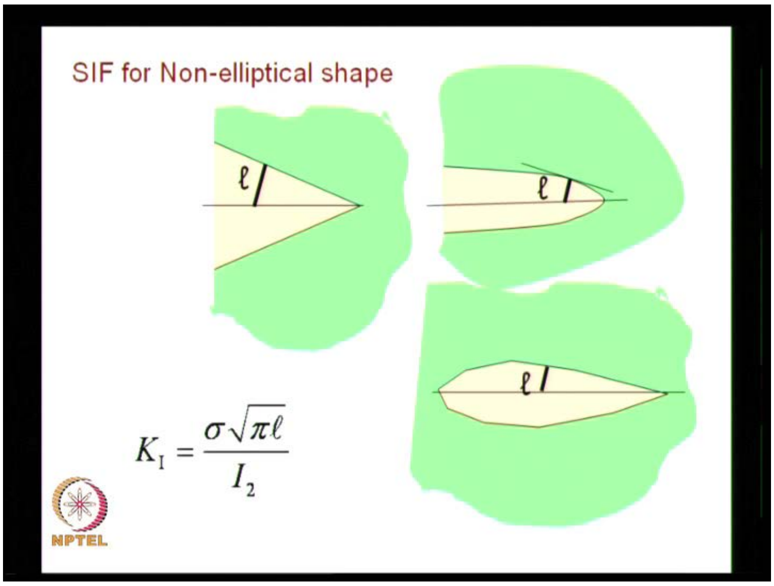
(Refer Slide Time: 39:42)



We have to go back to the diagram where we have identified the point on the ellipse for a given value of theta. According to Tada and Paris, draw a tangent at that point and drop a normal from there. Measure the length l of the line perpendicular to this tangent.

What they have noticed was that l is nothing but a multiplied by sign squared theta plus a divided by c whole squared cos squared theta whole power half. The expression for stress intensity factor could be simply written as K 1 equal to sigma root of pi l divided by I 2. So for any problem at any point on the internal flaw, if we determine the value of l we can estimate the value of K. They have extended this to other shapes also.

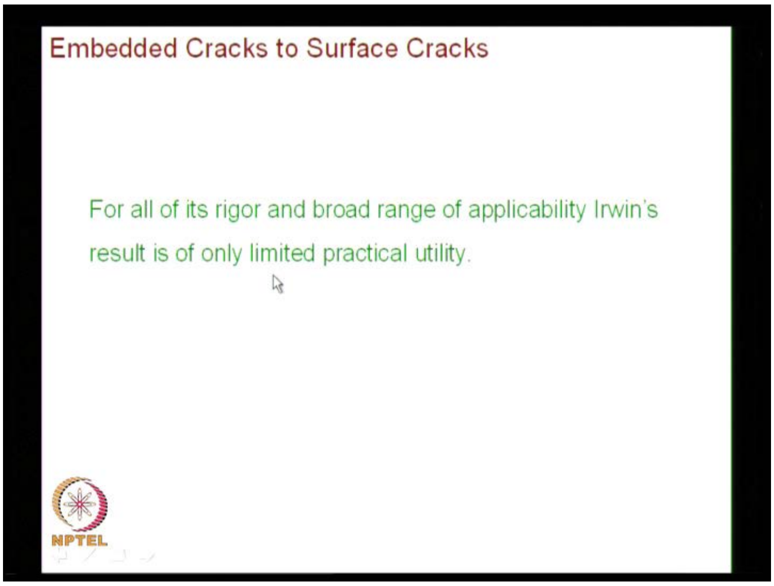
(Refer Slide Time: 42:05)



For different shapes measure the length l at the point you want to find out the stress intensity factor. Drop the normal and then find out the length l and if you have a smoothly varying type of situation put the tangent and drop the normal and find out the value of l then stress intensity factor is simply K 1 equal to sigma root pi l divided by I 2.

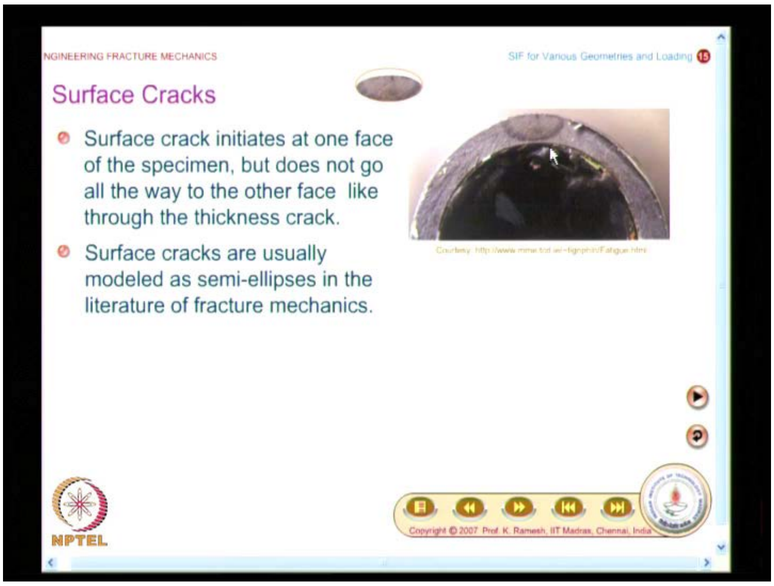
So from elliptical flaw you can go to non-elliptical shapes and come down to circle and then to a straight through the thickness crack.

(Refer Slide Time: 43:30)



For all of its rigor and broad range of applicability Irwin's result is of only limited practical utility. The result has to be extended to surface cracks as our ultimate objective is to find the stress intensity factor for surface cracks.

(Refer Slide Time: 44:15)



(Refer Slide Time: 44:34)

The very first question that has to be raised when you go to surface crack is how surface cracks can model it. Here we have a specimen with a surface crack and this surface crack shape is taken and put into an ellipse. The courtesy for the figure is given here.

So what you find here is the surface crack which is present in this object can be model as a semi-elliptical crack. Before I apply for surface crack I have to model what is the reasonable approximation for surface cracks and that is what is shown here.

Surface crack initiates at one face of the specimen but does not go all the way to the other face like through the thickness crack, surface cracks are usually modeled as semi-ellipses in the literature of fracture mechanics.

So in this class what we had done is we have looked at the stress intensity factor for a double edge crack and mathematically obtain the stress intensity factor for the experiment needed which was higher than the SIF of center crack. So the sum of the double edge crack length is only seventeen millimeter whereas the center crack length was 18 millimeter. Even though the double edge crack was shorter than center crack this precipitated the fracture. Then we moved on to embedded flaws, looked at stress intensity factor for a circular flaw then the elliptical flaw and then graduated to surface cracks. We have also shown surface cracks can be modeled as semi-elliptical.

Thank you.