

Engineering Fracture Mechanics
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Module No. # 06

Lecture No. # 25

Evaluation of SIF for Various Geometries

In the last few classes, we had looked at exhaustively, the development of multi-parameter stress field and displacement field equations. Particularly, when you want to process the experimental data, the multi-parameter stress field, as well as displacement field comes in handy. And, I said, most of the fracture theories still focus on the importance of the singular term; only some of the recent theories, really look at the second term. So from that point of view, for the rest of the course, we would focus only on the singular solution; and the focus becomes important to find out the stress intensity factor for a variety of geometry.

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The slide, titled "Concentrated load", illustrates a crack of length $2a$ on the surface of a material. A concentrated load P is applied at the center of the crack. The coordinate system has the x -axis along the crack and the y -axis perpendicular to it. The stress intensity factor K_I is defined as the limit of the product of the square root of the crack length and the stress field function $Z_I(z_0)$ as the crack length approaches zero:

$$K_I = \lim_{z_0 \rightarrow 0} \sqrt{2\pi z_0} Z_I(z_0)$$

The slide also shows the intermediate equation for a crack of length $2a$ under a load P at a distance z_0 from the center:

$$K_I = \lim_{z_0 \rightarrow 0} \frac{\sqrt{2\pi z_0} Pa}{\pi(z_0 + a)\sqrt{z_0}\sqrt{2a}\left(1 + \frac{z_0}{2a}\right)^{1/2}}$$

Finally, the simplified equation for the stress intensity factor under a concentrated load is given as:

$$K_I = \frac{P}{\sqrt{\pi a}}$$

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And what we had seen in the last class? We had looked at the problem of a load at crack surface, you have a concentrated load, which is trying to open the crack faces.

And for this case, fortunately, there was a stress function available. And once you have the stress function, it is possible for you to find out the stress intensity factor by using this expression. And what is the advantage, if I have an analytical approach? I am able to get an expression for the value of stress intensity factor.

So, without doing much computation, by looking at that expression, we could comment what the expression tells you; and what is the striking feature we saw? We got the stress intensity factor, K_1 as P by root of πa ; and what you find immediately is the crack length is below the value of load P . So, what you find is, as the crack length increases, the stress intensity factor decreases; it is a very important observation, if I did not have an analytical expression, if I able to solve only by numerical analysis, then I will have to get the result for various crack lengths, then fit a sort of an empirical relation, then observe what is the nature.

So, in any problem if you have an analytical approach and you have analytical expression, it is a most convenient form of relation that you can handle. And what you will also notice is, you know, I have a solution for this case, you could find out and use this for many other problems which are similar.

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The slide is titled "Symmetric wedge load" and "Wedge load on cracked surfaces". It features a diagram of a crack of length $2a$ on the x-axis, with a symmetric wedge load applied. Two forces P are shown acting at distances s from the center of the crack. The stress function is given as $Z_1 = \frac{2Pz}{\pi(z^2 - s^2)} \left(\frac{a^2 - s^2}{z^2 - a^2} \right)^{1/2}$. Below the equation, the stress intensity factor is indicated as $K_1 =$.

And in the last class, we had also taken up another case, where I have a symmetrically loaded crack faces; you have two forces at distances plus s and minus s from the center

of the crack; and we got the stress function in this fashion; and as far as stress function goes, somebody gives you the stress function, and these stress functions are defined with crack center as the origin. And now, you have a methodology, once the stress function is known, shift the origin to the crack tip, then apply the limits. In fact I had asked you to do this as an exercise, I am not quite sure how many enough you have really solved it, and anyway we will have a look at the expressions.

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The stress function referred with respect to the centre of crack as the origin, subjected to two wedge loads of P N/m acting symmetrically is given as:

$$Z_1 = \frac{2Pz(a^2 - s^2)^{\frac{1}{2}}}{\pi(z^2 - s^2)(z^2 - a^2)^{\frac{1}{2}}}$$



So, this is the stress function that we had started with.

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Shifting the origin to the crack tip, by the expression

$$z \rightarrow z_0 + a$$

$$Z_1 = \frac{2P(z_0 + a)\sqrt{a^2 - s^2}}{\pi[(z_0 + a)^2 - s^2]\sqrt{(z_0 + a)^2 - a^2}}$$

$$= \frac{2P(z_0 + a)\sqrt{a^2 - s^2}}{\pi[z_0^2 + a^2 + 2az_0 - s^2]\sqrt{z_0^2 + 2az_0}}$$


And the next step is, you have to shift the origin to the crack tip by the expression z becomes z naught plus a , those who have done this in the rooms, please verify the expressions.

So, when I substitute for z equal to z naught plus a , I get Z_1 as $2P$ multiplied by z naught plus a root of a squared minus s squared divided by π multiplied by z naught plus a whole squared minus s squared multiplied by square root of z naught plus a whole squared minus a squared. This is fairly simple to simplify, and the next set of expressions you write this in an expanded fashion, and you also simplify this; when you simplify this this reduces to z naught squared plus $2a$ z naught. And now you can apply the limits; if you get zero by zero then you got to L-hospital rule and then proceed in finding out the limits; once you have express Z_1 in this fashion, it is possible to apply the limits.


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From the definition of SIF,

$$K_1 = \lim_{z_0 \rightarrow 0} \sqrt{2\pi z_0} Z_1(z_0)$$

$$= \lim_{z_0 \rightarrow 0} \sqrt{2\pi z_0} \frac{2P(z_0 + a)\sqrt{a^2 - s^2}}{\pi [z_0^2 + a^2 + 2az_0 - s^2] \sqrt{2az_0} \sqrt{1 + \frac{z_0}{2a}}}$$

$$= \frac{2Pa\sqrt{a^2 - s^2}}{\sqrt{\pi(a^2 - s^2)}\sqrt{a}}$$

$$K_1 = \frac{2P\sqrt{\pi a}}{\pi\sqrt{a^2 - s^2}}$$


So, the next step is, you have to apply the limits and we know from the definition of stress intensity factor, K_1 is given as limit z naught tends to 0 root of two pi z naught multiplied by the stress function expressed in terms of Z naught. So, when I rewrite this expression, this is also further simplified, so I have $2Z$ naught and root of $2Z$ naught, these two get canceled and when I put z naught equal to 0, you will be able to get a final expression for the stress intensity factor K_1 . And that expression turns out to be $2P$ root of a squared minus s squared divided by root of pi multiplied by a squared minus s

squared into root of a, which could be further simplified as K_1 equal to $2P$ root of πa divided by π root of a squared minus s squared.

So, what you find from this exercise is, once you have a stress function from the definition it is possible for you to get an analytical expression for stress intensity factor; it is a fairly straight forward exercise. Is there a way to verify that this expression is correct? Because we have one problem, we have seen already, you have the concentrated load at the center and for that case you make s equal to zero. And now you have two times the applied load, so divided by two; so you will finally get K_1 equal to P by root of πa . So, this is an indirect check that we are proceeding in the right direction.

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ENGINEERING FRACTURE MECHANICS

SIF for Various Geometries and Loading

Asymmetric wedge load

A pair of wedge loads at distance s on the surface of crack

Play

$Z_1 = \frac{P}{\pi(z-s)} \left(\frac{a^2 - s^2}{z^2 - a^2} \right)^{\frac{1}{2}}$

$K_1^A = \frac{P}{\sqrt{\pi a}} \left[\frac{a+s}{a-s} \right]^{\frac{1}{2}}$

$K_1^B = \frac{P}{\sqrt{\pi a}} \left[\frac{a-s}{a+s} \right]^{\frac{1}{2}}$

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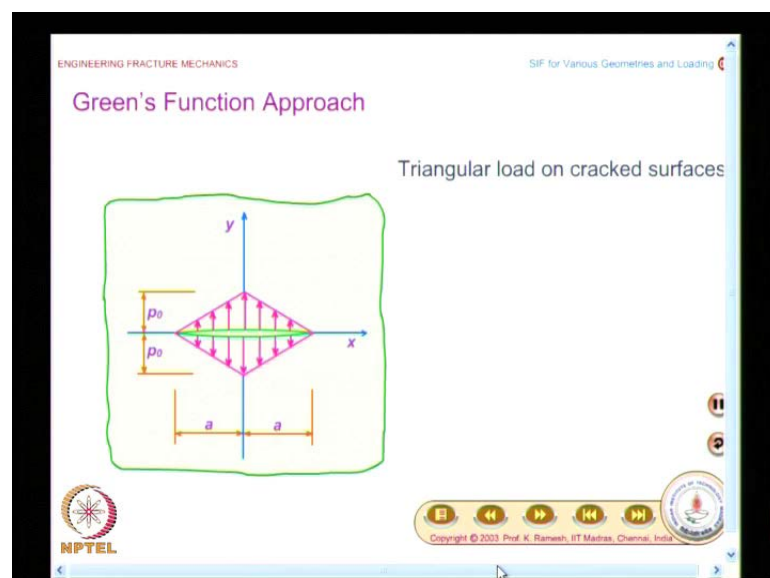
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And **you** now you have the third case, where I do not have a symmetric load, I have an asymmetric wedge load; and you can see the animation, the crack faces are open; and the load is applied at a distance s from the origin; make a neat sketch of this. And I have label the crack tip as A and B; there is a purpose behind it, see when I have an unsymmetric load, what do you anticipate? See in all the problems we have seen so far, at the crack tip we have evaluated the stress intensity factor; if I have a center crack, we have always seen a **through** thickness crack, the stress intensity factor in both the tips were same. We had seen a case where you had uniformly applied load at remote loading; then we had looked at the bi-axial loading situation; then we had symmetric loading on the crack faces in all those cases what we saw? When you have a center crack, both the

crack tips had the same stress intensity factor; which will not be the case when I have a unsymmetric load like this. I have loading at a distance s from the center; so obviously, the crack tip A will experience a higher stress intensity factor, and crack tip B will experience a lower stress intensity factor. In fact, even this stress function was provided by Westergaard in his original paper. And you have the stress function given as P divided by π into z minus s multiplied by a squared minus s squared divided by z squared minus a squared whole power half. So, once the stress function is given, it is now **child's** play for you; shift the origin to the crack tip, and then apply the definition of a SIF; If necessary, go for a L-hospital rule and get the limits, finally get the expression for stress intensity factor; in fact, I am not going to do that, I am going to provide you only the final expressions; and I have already mentioned, that you will have different expressions for the stress intensity factor at tip A and stress intensity factor at tip B. For tip A, it is P divided by root of πa multiplied by a plus s divided by a minus s whole power half; and for tip B, which is given as K_{1B} , you get this as P divided by root of πa minus s divided by a plus s whole power half.

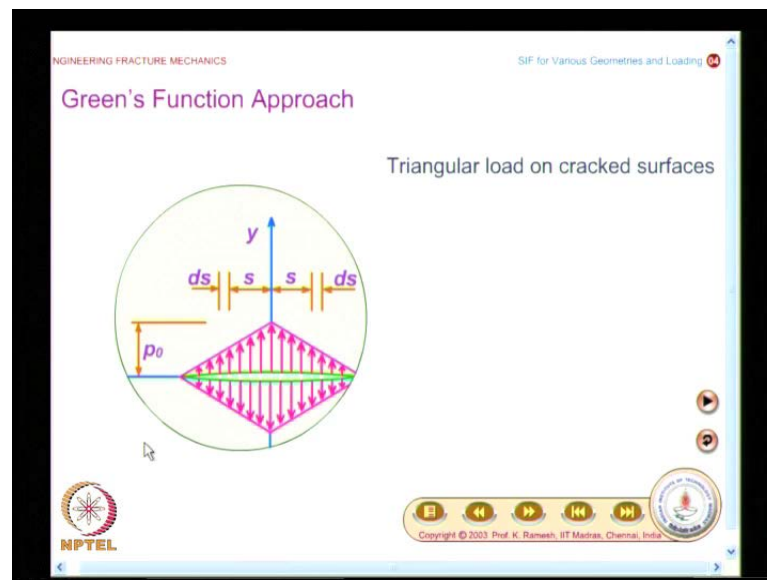
Is there a way to check this result? You have a provision, suppose you make s equal to zero both the expressions reduce to what we saw for the concentrated load at the center. So, this shows that the crack problem is reasonably understood, and we are able to play with different types of loading and find out the appropriate stress intensity factor. And you know if you look at the development of any of these approaches.

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Once they obtain a stress function, they keep tweaking it, and then bring in certain modifications; and likewise, you know, you could use this for solving a variety of problems. Next, we will take up a problem of a crack which is loaded with a special type of loading.

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You can observe that I have a crack which is loaded with the triangular loading, and the loading is symmetrical, you must observe all that, and the maximum value of this is p_0 , that is the pressure that is acting; and you have the axis label, please make a sketch of this, because this is needed for our further discussion.

So, I have distance as s label and you have to find out the force that is acting; so, you will have to look at what happens over incremental distance ds ; and once you have this value of p_0 at the center from the triangular loading, it is possible for you to write what would be the value of the pressure at any distance. And now, you know, you will have to identify which would be the starting point for me to get a solution for this problem. Because we have looked at a concentrated load acting at the center of the crack; then we saw two symmetric loads acting on the faces of the crack; then we saw an asymmetric load, and if you look at the loading here, the loading is symmetric. So, it would be prudent for me to use the solution that we have got, where you have symmetric loading on the crack faces; there it was a concentrated load, now it is a distributed load, it is a very simple exercise, you have to recast the type of expressions. So, based on this

diagram, it is possible for you to write that expression. The overall problem is given, in this I have a distance of crack as a and it is possible for you to write dP equal to a minus s divided by a multiplied by p_0 .

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ENGINEERING FRACTURE MECHANICS

SIF for Various Geometries and Loading

Green's Function Approach

Triangular load on cracked surfaces

$$dP = \frac{(a-s)}{a} p_0 ds$$

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ENGINEERING FRACTURE MECHANICS

SIF for Various Geometries and Loading

Green's Function Approach

Triangular load on cracked surfaces

$$dP = \frac{(a-s)}{a} p_0 ds$$

$$dK_1 = 2 \frac{(a-s)}{a\pi} p_0 \sqrt{\frac{\pi a}{a^2 - s^2}} ds$$

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This would be the pressure at any point, you are having at a distance a minus s ; and by looking at the expression which we had for the symmetric load, I could write this; we are writing dK_1 . I have this load as 2 into a minus s divided by a into π p_0 square root of π a divided by a squared minus s squared ds .

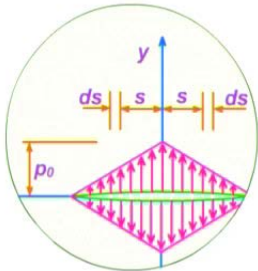
Just flip your pages back and then see what we had written as the expression for K_1 for symmetric loading, the same expression I am using it, and replacing it in terms of dP ; and we say, what all the value we get, we get that as dK_1 . See once you come to a course in fracture mechanics, you know, you will also have to do all this mathematics; for all of this, you know, you need to recall the table of integrals. I have the expression for dK_1 and if I have to find out what is K_1 , I have to integrate it over the crack length;

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ENGINEERING FRACTURE MECHANICS SIF for Various Geometries and Loading

Green's Function Approach

Triangular load on cracked surfaces



$$K_1 = \int_0^a 2 \frac{(a-s)}{a\pi} p_0 \sqrt{\frac{\pi a}{(a^2 - s^2)}} ds$$

$$K_1 =$$

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I have to do it from 0 to a 2 into a minus s divided by a pi p naught root of pi a divided by a squared minus s squared into ds.

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The slide is titled "Green's Function Approach" and "SIF for Various Geometries and Loading". It features a diagram of a crack of length $2a$ on the x-axis, with a triangular load p applied to the crack faces. The load is zero at the crack center and reaches a maximum value p_0 at the crack ends. The diagram shows the crack opening along the x-axis from $-a$ to a , with the y-axis perpendicular to it. The load is represented by a series of arrows of varying lengths forming a triangular shape. To the right of the diagram, the integral equation for the stress intensity factor K_1 is given as:

$$K_1 = \int_0^a 2 \frac{(a-s)}{a\pi} p_0 \sqrt{\frac{\pi a}{(a^2-s^2)}} ds$$

Below the integral equation, the text $K_1 =$ is followed by a blank space and a small icon. The slide also includes the NPTEL logo and a copyright notice: "Copyright © 2003 Prof. K. Ramesh, IIT Madras, Chennai, India".

Can you just make an attempt to simplify, at least get what is the integral that we will have to evaluate; you may not possibly remember the expressions from the table of integrals, that I would help you. And this is a very interesting problem, the problem I have taken is quite interesting, I have taken with a purpose.

Here you are having a pressure varying as a triangular loading; I could also have another interesting situation, where the pressure p naught is constant over the entire length of the crack; I could also get the solution for K_1 , even for that case, from this situation.

Right now, what I want you to do is, when I have this expression for K_1 simplify it and then see what is the integral that you will have to evaluate; only if I know the value of that integral, I would be able to find out what is the value of K_1 .

So, I would like you to look at that as one exercise; another exercise is, I am not having a triangular variation of the pressure, but I have constant pressure; that is also an important problem. See, if you had looked at, when we wanted to find out energy release rate based on displacement of crack faces, we had a discussion that the crack is opened by a pressure which is equal to σ ; and we said some expression for the value of K .

And we would see, what is the kind of expression for K we would get it; so, both the expressions are important. And what you will find is, in each of these cases the integral will be different. See in books, what you will find is, they will just give this, and finally

give the expression for k; many times, you know, if you read like that, assimilation of ideas will not be there. Because the only thing what you know, what you have been trained is, doing some little bit of mathematics, which you had forgotten that is all at this stage; stress function you are not writing, if somebody gives you a problem, if I ask a question develop a stress function for it, it is a herculean task.

If you look at the history and the development of fracture mechanics, people struggle to arrive at stress function, and we had seen even for the very famous problem of the central crack, **Sanford** has to come and rescue that you have to have one more stress function capital y, only then the problem is complete; and it had taken quite a number of years. So, the only exercise that you can possibly do is, do the mathematics without skipping steps; that would also give you some kind of attachment to the result that you get, rather than looking at the final expression, it would help you to do that. I hope you have got the expressions.

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Using Table of Integrals,

$$\int_0^a \frac{\sqrt{a-s}}{\sqrt{a+s}} ds = \sqrt{(a+s)(a-s)} \Big|_0^a + 2a \sin^{-1} \sqrt{\frac{a-s}{2a}} \Big|_0^a$$

$$= a \left(\frac{\pi}{2} - 1 \right)$$

When pressure p_0 is a constant

$$\int_0^a \frac{1}{\sqrt{a^2-s^2}} ds = \sin^{-1} \frac{s}{a} \Big|_0^a$$

$$= \frac{\pi}{2}$$


So, in the first case, when I have the loading is triangular, you would have to look at the integral of root of a minus s divided by a plus s into ds; and when you do that, it turns out to be from the table of integral an expression like this. The first term is square root of a plus s into a minus s within the limit 0 to a; and second term is plus two a sin inverse root of a minus s divided by 2a from 0 to a; and when you substitute the limits and simplify, you get an expression that this is equal to a into pi by 2 minus 1. You know, this you will

have to substitute it back and get the expression for K. I am not giving you the expression for K, we will have a look at it.

And what would be the type of integral that you will have to evaluate, when the pressure is constant? The expression is different, and the expression turns out to be integral 0 to a 1 by square root of a squared minus s squared into ds, from the table of integrals, you have this as sin inverse s by a with the limits 0 to a and once you substitute you get the expression as pi by 2. Here you are only looking at, what is the kind of expressions that you get from the integral value; these are not the final expressions for K, we will have a look at the final expression for K.

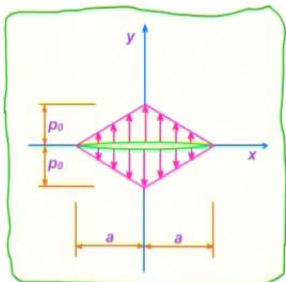
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ENGINEERING FRACTURE MECHANICS

SIF for Various Geometries and Loading

Green's Function Approach

Triangular load on cracked surfaces


$$K_I = \int_0^a 2 \frac{(a-s)}{a\pi} p_0 \sqrt{\frac{\pi a}{(a^2-s^2)}} ds$$
$$K_I = \left(1 - \frac{2}{\pi}\right) p_0 \sqrt{\pi a}$$

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The slide is titled "Green's Function Approach" and "SIF for Various Geometries and Loading". It features a diagram of a crack of length $2a$ along the x-axis, with a triangular load p_0 applied perpendicular to the crack faces. The load is zero at the crack center and maximum at the crack ends. A differential element ds is shown at a distance s from the center. The stress intensity factor K_I is given by the integral equation:

$$K_I = \int_0^a 2 \frac{(a-s)}{a\pi} p_0 \sqrt{\frac{\pi a}{(a^2-s^2)}} ds$$

The simplified equation is:

$$K_I = \left(1 - \frac{2}{\pi}\right) p_0 \sqrt{\pi a}$$

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So when you have the pressure loading varying as a triangular fashion, the expression for K reduces to $1 - \frac{2}{\pi} p_0 \sqrt{\pi a}$.

On the other hand, when I have pressure is constant, you can go and simplify, the result reduces to $p_0 \sqrt{\pi a}$; a very important expression, because you know, pressure loading is very common in many of the engineering applications.

And what do you find here, whatever the expression for stress intensity factor, for a center crack subjected to uniaxial loading or bi-axial loading, because we have always said the stresses on the horizontal direction is not contributing to the quantity K ; the K is still not effected, there you got that as $\sigma \sqrt{\pi a}$, you have an expression very similar to that when the crack faces are opened by pressure. See, here we have looked at a mathematical approach, and then got the expression for a distributed loading, as well as, for a constant pressure.

Later, I will solve the problem by a method of superposition, there you will again get the same answer. The reason is, for you to appreciate method of superposition, this answer will give you some kind of a confidence; because you have already seen the result is, $p_0 \sqrt{\pi a}$, I am going to get the same result by method of super position. So, you **learned the no answers of method of** superposition from this result, then apply or extend method of superposition for other class of problems.

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The slide, titled "Evenly Spaced Collinear Cracks in an Infinite Strip", illustrates a mechanical problem. It shows a horizontal strip of width w and height $2a$ (with a central crack of length $2a$) subjected to a uniform tensile stress σ . The coordinate system (x, y) is centered on the strip. Below the diagram, the stress function is given as $Z_1 = \frac{\sigma \sin\left(\frac{\pi z}{w}\right)}{\sqrt{\sin^2\left(\frac{\pi z}{w}\right) - \sin^2\left(\frac{\pi a}{w}\right)}}$. The stress intensity factor is expressed as $K_I = \sigma \sqrt{\pi a} \left(\frac{w \tan\left(\frac{\pi a}{w}\right)}{\pi a}\right)^{1/2}$. The slide is attributed to Irwin, Westergaard and Koiter and includes NPTEL and IIT Madras logos.

And as I mentioned earlier, you know, when stress functions are developed, the developers of those stress functions also looked at, by what suitable modifications, they could extend the utility and solve more variety of problems. I would like you to make a sketch of this.

And Westergaard in 1939 itself, in his original paper, he had obtained and reported this stress function; this is meant for evenly spaced collinear cracks in an infinite strain. See, for the first look of it, what you get? In which class of problem, we are going to get evenly spaced cracks? That is not the way developers look at it; they have a stress function by tweaking it slightly, they are able to solve this kind of a problem.

What we will have to look at is, how the solution was looked at by later investigators and what is the substance they extracted out of it. It is a very important problem from that perspective, if you look at the problem as such, it is an academic interest. And for this, you have the stress function given as Z_1 equal to $\sigma \sin \pi z$ divided by w whole divided by square root of $\sin^2 \pi z$ by w minus $\sin^2 \pi a$ divided by w . And you are sufficiently trained now, the moment you are the stress function, it is possible for you to find out the expression for stress intensity factor.

In fact, Westergaard did not know it; he provided the stress function but he did not evaluate the stress intensity factor in 1939; because at that time, stress intensity factor

concept was not available, it was only introduced by Irwin. So, the expression for stress intensity factor is credited to Irwin; there was a paper in 1957, where he had formalized all this. And for this problem of evenly spaced collinear cracks, the expression for stress intensity turns out to be $\sigma \sqrt{\pi a}$, this is magical, this till remains, which is multiplied by another factor, which is like w by πa multiplied by $\tan \pi a$ divided by w whole power half.

And what is w ? w is the spacing between the cracks, I have, this as given as w and length of the crack is $2a$, and I have the origin fixed at the crack center, and you have cracks extending in this direction, as well as, extending in this direction. And the expression for K is credited to Irwin Westergaard and Koiter.

Because for any problem, the stress function takes a predominance, it was reported by Westergaard; from the stress function, Irwin extracted the value of K , an expression for K ; and this is known as a tangent formula, because this is in terms of $\tan \pi a$ divided by w , it is known as a tangent formula. And what way this expression was used? That is what, we have to look at it.

And from now on, you will always see this kind of expression, involving the crack length and a geometric parameter of the problem.

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The slide is titled "Evenly Spaced Collinear Cracks in an Infinite Strip....Contd". It features two diagrams. The left diagram shows a vertical strip of width w with a central crack of length $2a$. The strip is subjected to a uniform stress σ applied vertically. The corners are labeled A, B, C, and D. The right diagram shows a similar strip with two cracks, illustrating the spacing w and crack length $2a$. Below the diagrams is a list of conditions:

- In view of symmetry, on edges AB and CD no shear stress can exist
- Further they have to remain straight
- This leads to the development of σ_x stresses on these boundaries

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Is this you have to read it as a by w ratio; for all our future discussion, we will look at what is the value of a by w or two a by w, things of this nature. So, what is so important about this problem? Now, take up a line A B and C D, which are in between the cracks; and what would happen to lines A B and C D?

Because of symmetry on edges A B and C D, no shear stress can exist. See, for all these cases, I would like you to make sketches; that is the reason why I am talking slow. So, you should be able to draw the sketch and also, assimilate the concept and discussion. You have evenly spaced collinear cracks; and I take a line A B and C D, which are in between the two cracks, from arguments of symmetry they have to remain straight; if you have shear stress, they cannot remain stretched. So, shear has to be zero. And for the lines to remain straight, you need to have stresses developed in the horizontal direction.

So, what I get out of it? I am able to cull out a finite strip from an evenly spaced collinear cracks. So, in fact the base solution of evenly spaced collinear cracks is the mother of all solutions. Now, what is the problem I have? I have a plate with a central hole and we have already discussed sufficiently, that the stresses in the x direction does not contribute to changing the value of K; and you have to take it that, this is a schematic; in fact, accurate plots have been obtained by Irwin and his co-workers for various values of a by w ratio, they have obtained this plot. When the a by w ratio is small, the sigma x is smooth; only when a by w is very large, you find the sigma x values are very high on the crack axis.

So the argument here is, as long as the a by w ratio is small, the advantage of solution for evenly spaced collinear cracks can be taken as the value of K for a finite strip; because all practical situations, you have only finite body problems. And this is the kind of argument they had put forth and we would see for that.

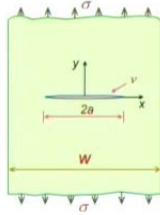
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ENGINEERING FRACTURE MECHANICS

SIF for Various Geometries and Loading

Crack in a Plate of Finite Dimensions

- In a plate of finite dimensions, an edge of the plate may be closer to the crack tip.
- The edge may have considerable influence on the stress field in the vicinity of the crack tip and on the stress intensity factor.
- If the influence of stress components parallel to the crack face on K_I is ignored, then K_I for collinear cracks in an infinite strip could be approximated as solution of K_I for a finite strip.


$$K_I = \sigma \sqrt{\pi a} \left(\frac{w}{\pi a} \tan \frac{\pi a}{w} \right)^{1/2}$$

Irwin, Westergaard and Koiter

NPTEL

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So, from looking at a problem of infinite plate with a collinear evenly spaced cracks, we are able to discuss, what is the kind of SIF for a crack, in a plate of finite dimensions. Now, this is shown with a finite... You have the edge straight. I have a center crack, I have a load applied as sigma; and mind you, a problem like this, is labeled as CCT specimen - Center Cracked Tension specimen - you know they had abbreviated like we had seen DCB specimen - Double Cantilever Beam specimen - RDCB specimen, then you have the CCT specimen, then you have Single Edge Notch specimen – SCN - specimen, if you look at literature in fracture mechanics, it reads like this.

And in those days, when Irwin developed, this was the specimen they had used for finding out the fracture toughness also, you have to keep in mind. So, this was used for material testing; so, they needed expressions, which are analytically, reasonably good to find out the value of K. And what you have to keep in mind is, when I have longer and longer cracks, the edges will have a considerable influence on the stress field; So, the solution need to be improved further.

But this is how the initially the solution was looked at. So, when a by w ratio is small, we result for evenly spaced collinear cracks can be taken as the value of K1 for a finite strip also. We would do modification; we would not stop here; we would do the necessary modification. And I would like to emphasize again, the influence of stress components

parallel to the crack phase on K_I is ignored. This is the kind of approximation that we make.

And you know, this all happened at a time when people were having only slide rule for calculation, they were not having a computer to do the calculation; and in 1966 Esida solved this problem numerically, and he arrived at a solution with 36 terms, it is too difficult to handle.

So, Brown and Strawley, later on, he simplified that expression and arrived at an approximate solution. Then there were many other developments.

We would see some of those solutions. What you will have to recognize is, for a b/w which is small, this is a reasonably a good approximation; because you know, you will have to come out of infinite body problem to finite body problem, how do you go that? People have cleverly made this argument and then arrived at this. That is what I said, evenly spaced collinear cracks may not be important from a practical point of view, but from that, you could arrive at solution for a center crack, we would see, you could get it for an edge crack, as well as, double edged crack; that is the starting point, from that point of view, it becomes important.

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The slide is titled "ENGINEERING FRACTURE MECHANICS" and "SIF for Various Geometries and Loading". The main heading is "Finite plate with a central crack".

- General solution is of the form,
$$K_I = \sigma \sqrt{\pi a} F_I(\alpha)$$
- where $\alpha = \frac{2a}{w}$
- For a finite plate, Feddersen has obtained the expression of SIF as
$$K_I = \sigma \sqrt{\pi a} \left[\sec \left(\frac{\pi a}{w} \right) \right]^{1/2}$$

Accuracy: $\pm 0.3\%$ for $\alpha \leq 0.7$; 1% for $\alpha = 0.8$

The diagram shows a rectangular plate of width w and height h with a central crack of length $2a$. The crack is along the x-axis, centered at the origin. The plate is subjected to a uniform tensile stress σ applied vertically. A coordinate system with x and y axes is shown, with the crack extending from $x = -a$ to $x = a$.

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So, the general solution you could write it in the form, $\sigma \sqrt{\pi a}$, a function of α and α equal to $2a$ by w ; for this case, I have the crack length $2a$, width is w .

And what Feddersen did was, I mentioned you have the solution of Esida, he **is** did a fitting for the results, and he obtained an expression for K, which is given as sigma root by a, that is magical, this is available, multiplied by secant of pi a divided by w whole power half; and this is known as a secant formula. This, he obtained purely from heuristic arguments; and what the literature says is, later on people tried to attach a theoretical development, which did not really succeed; but nevertheless, this is fairly accurate, you find the accuracy is plus or minus 0.3 percent, for alpha less than 0.7 and it is within 1 percent, for alpha equal to 0.8.

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ENGINEERING FRACTURE MECHANICS

SIF for Various Geometries and Loading

Finite plate with a central crack

Tada has reported the function F_1 as,

$$F_1 = (1 - 0.025\alpha^2 + 0.06\alpha^4) \sqrt{\sec \frac{\pi a}{w}}$$

Accuracy: less than 0.2% for all α 's

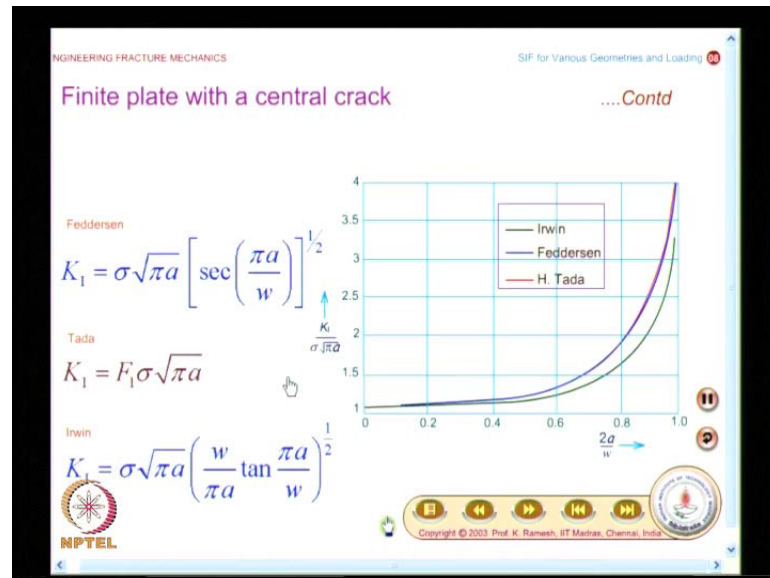
The diagram shows a rectangular plate of width w and height h with a central crack of length $2a$. The crack is oriented along the x-axis. The plate is subjected to a uniform tensile stress σ applied vertically. A coordinate system with x and y axes is centered on the crack. The NPTEL logo is visible in the bottom left corner, and a copyright notice for Prof. K. Ramesh, IIT Madras, Chennai, India is at the bottom.

You know, **the**, whatever the final expressions I give and accuracy, these are from the hand book written by Murikami, stress intensity factor hand book has this information. And you also have another approximation that is given by Tada, which has an improved accuracy. The function is given as 1 minus 0.025 alpha squared plus 0.06 alpha power four multiplied by root of secant pi a divided by w. And this claims that the accuracy is within 0.2 percent for all values of alpha.

See, what I would like to caution is, these are not from analytical solution, you must understand that; people have done a numerical analysis either by boundary collocation or finite element, as the case may be, and whatever the results that they have got, they have fit an appropriate empirical relation; and these are time tested; people have found them

useful, So, you could take that for finding out the stress intensity factor for finite geometry.

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And these expressions are also compared in a graphical form, and also make a note of this graph.

So, I have x axis is given as $2a$ by w , it is given as 0, 0.2, 0.4, 0.6, 0.8 and I go up to 1.0; on the y axis, what is plotted is, K_I divided by $\sigma \sqrt{\pi a}$, I have 1, 1.5, 2, 2.5, etcetera. And this is how the variation of SIF is predicted by Irwin's expression, and I have already said that Irwin's expression is the simplest and crudest of all of it. But it was the very good starting point; and in fact CCT panels, they had use this expression for finding out fracture toughness also.

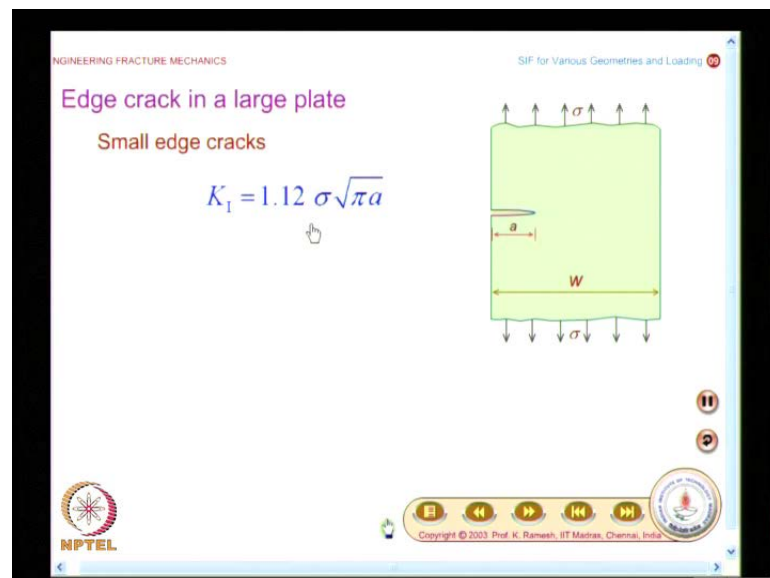
Now, we are not using CCT specimen for fracture test, we are using modified compact tension specimen, we would look at that, when we take up the chapter on fracture toughness testing; and how does the expression for Feddersen goes?

Though this expression is obtained based on heuristic arguments, this is reasonably accurate and in fact, it is preferred than the tangent formula; though the secant formula is preferred, but it is purely an empirical argument; and **by enlarge** the most accurate fit is by Tada; and you find only hardly a small difference between the graphs by Tada and Feddersen.

So, make a neat sketch of this. And what you find here? If you compare, whatever the solution by Irwin, if you take the reference as Tada, up to 0.4 the Irwin solution and Tada solution are very close; there not much of deviation. And if you look at the Feddersen solution, it is up to 0.85; they say that the solution of Feddersen is reasonably accurate, and it is also simple to calculate.

If I have to go to Tada, I have to evaluate a serious functions, whereas this is simpler to evaluate. Now, we have looked at what is the kind of stress intensity factor for a center cracked specimen.

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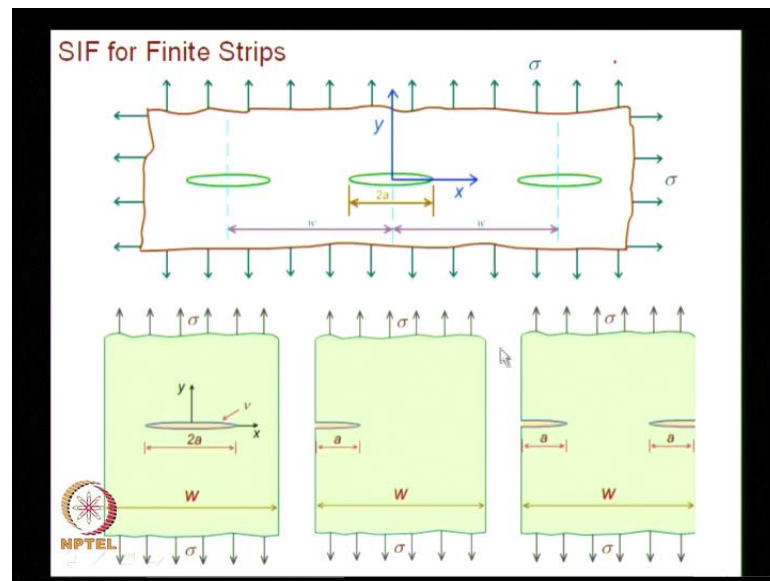
Next, we will move on to single edge notched specimen; and what is it that you see? For small edge cracks, what is it that you see? You have a factor, 1.12 attached to this, what does it signify? See, if you have to appreciate, how fracture mechanics got developed, structures design based on conventional design were breaking, and they found that cracks are more dangerous, and they have to arrive at a solution. At that time, numerical methods are not very well developed, and they could go and solve the problem left and right, that was not the case.

So, they have to use engineering judgment to appreciate which of the cracks is more dangerous. And you see the expression here; what is striking here? In the case of a center crack, the crack length is $2a$, you got the value of $\sigma \pi a$ root of πa , whereas you

have an edge crack which is only half its length, the stress intensity factor is higher than that; it is again through the thickness crack.

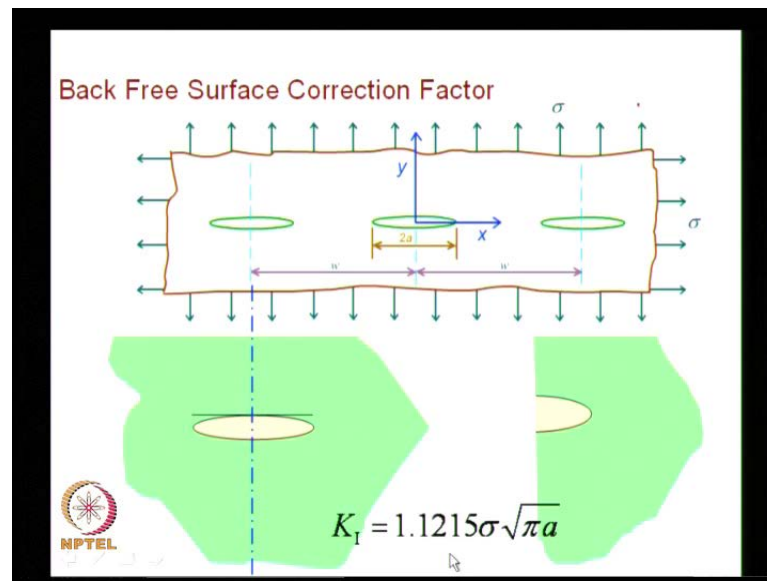
What is the argument behind it? It is the very interesting argument they had put forth, we will see that also.

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So, what you will have to look at is, collinear cracks evenly spaced is the mother of all solutions; from this, you arrive at, what is the SIF for a center cracked specimen. What is the SIF for a single edge notched specimen, and what is the SIF for a double edge notched specimen; you could see all of them, if I cut like this, I get this specimen; if I cut like this, I get this specimen; if I have a cut here and cut here, I have double edge notched specimen.

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And when you cut like this, what happens? See, I have this as an ellipse, which is enlarged here; and when you look at, I have a slope, this is horizontal because of symmetry.

Now, if I have to get a single enlarged specimen, I have to cut it here. So, when I cut it here, what happens? See, the edge becomes free, there is no constrain that this slope has to remain horizontal. So, in essence, the crack will open up further; that is what is shown here, the slope need not remain horizontal and a crack will open up.

So, for the crack to open up, you need to apply more stress; this is the way, they have argued; and what they also call it is, I have that crack tip here, this is called a back free surface; and if the crack becomes large, you will have a front free surface. So, in the literature you will find back free surface correction, factor front free surface correction factor, because this is how engineers operate, the back free surface correction factor is given as 1.1215 which is simplified as 1.12; that is what you come across.

See, what you will have to keep in mind is, the problem is so complex, and they come out with beautiful engineering arguments. I would like you to make a sketch of it, you know this is very important, what you will have to note here is, for a, when you cut like this, embedding the central crack; because of symmetry, this slope will remain horizontal.

But if I cut along the center of the crack, which is what is the case, in the case of a single edge notched specimen or double edge notched specimens, you will have a higher stress intensity factor and the crack will open up more; and what would be the case? When I have a longer crack, this solution is not sufficient. So, what people have done is, people have gone to boundary collocation, which is the numerical technique; they exhaustively satisfied the boundary, as much as possible; so, you have a series solution available, that is how people have proceeded. See, people have not given up, when you have a complex problem, people were at it and arrived at very interesting solution.

You know, if you look at this, this would also be a training for you to apply, when you face very difficult situations; with the available knowledge, what kind of arguments I can put forth and extract as much as possible from what is available, which could be verified and corrected later with sophisticated solutions, when they become available. So, the correction factors approach is the way engineers operated; that is very much seen in fracture mechanics development.

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ENGINEERING FRACTURE MECHANICS

SIF for Various Geometries and Loading

Edge crack in a large plate

Small edge cracks

$$K_I = 1.12 \sigma \sqrt{\pi a}$$

Long edge cracks

$$\alpha = \frac{a}{w} \quad K_I = \sigma \sqrt{\pi a} F_1$$

$$F_1 = 1.12 - 0.231\alpha + 10.55\alpha^2 - 21.72\alpha^3 + 30.39\alpha^4$$

Brown and Strawley

Accuracy: $\pm 0.5\%$ for $\alpha \leq 0.6$

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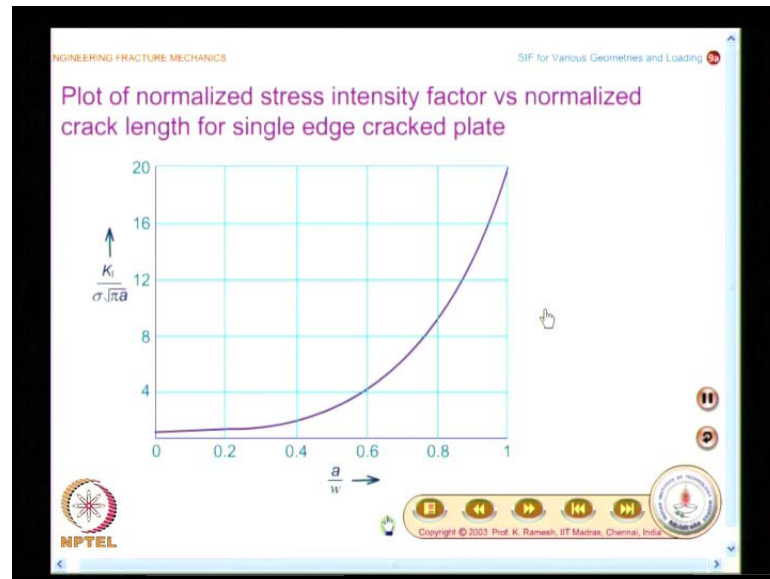
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So, when I have a long edge crack, I would have an expression like this, and this is credited to Brown and Strawley. And the function F1 is given as 1.12 minus 0.231 alpha plus 10.55 alpha squared minus 21.72 alpha cube plus 30.39 alpha power 4 and the accuracy is given as plus or minus 0.5 percent for alpha less than equal to 0.6.

And you will have to be careful, how the function $F1$ is defined, you have to look at, and you will also have to look at, how α is defined.

See, we have seen α as $2a$ by w in the case of center crack, in the case of a single edge notched crack, you have seen that as a by w .

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So, you have to use the correct value of α and then evaluate the expressions. And you also have a variation provided as a graph. So, that gives you, how SIF varies and the variation is much more sharper than what do you have in the case of a center crack.

So, what you find at this stage? A crack proceeding from surface is more dangerous, than a crack which is inside; that knowledge is very important, because it should alert you and in the worst case, I can always start with $\sigma \sqrt{\pi a}$ as a starting point; then put appropriate correction factors. This is how you will have to look at solutions of SIF and in this class what we had looked at was, we have evaluated the stress intensity factor by using the stress function definition and then we proceeded to find out the stress intensity factor for triangular loading on the crack faces. We said another important problem is, if I have a constant pressure on the crack faces, which is an equally important practical problem; we found the stress intensity factor as $p \sqrt{a}$; then we moved on to, evenly spaced collinear cracks in an infinite strip.

I said, to start with, it looks like an academic exercise and the beauty of **thus** is, how the later investigators utilize this, as a starting point to find out the stress intensity factor for a finite strip having a center crack, then a single edged crack; we will see in the next class, what is the solution for double edge crack.

Then what is the stage set for? The stage is set for, go back and verify the class experiment where we broke the specimen, paper specimen; because there our common sense failed, what you saw was different than what you would anticipate from common sense by just telling the length of the crack as the basis for fracture. It did not work.

Now, once you develop all this equations, we are armed with mathematical calculation to verify our observation in the experiment. So, please come with the calculators in the next class. Thank you.