Engineering Fracture Mechanics
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Module No. # 04
Lecture No. # 23
Validation of Multi-parameter Field Equations

In the last class, we had looked at multi-parameter stress field equations and we also saw one example problem, where the utility of multi-parameter stress field equations was demonstrated. And what are the equations we saw?

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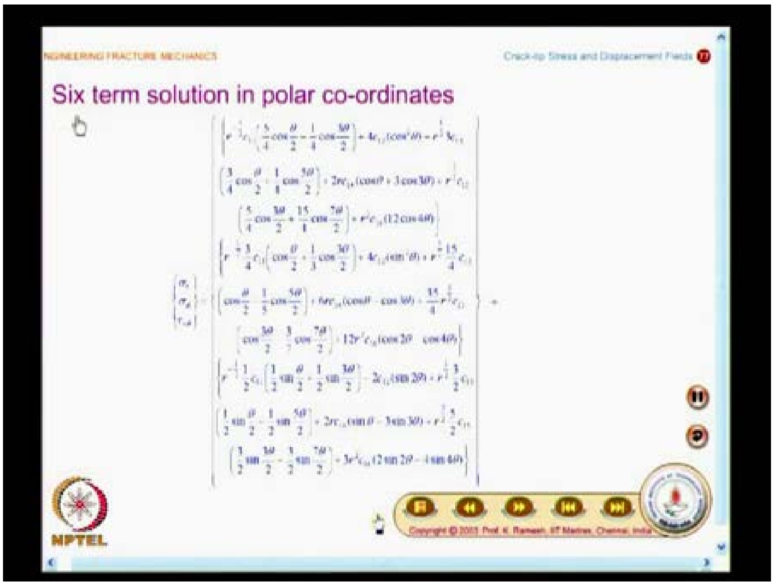

We saw the equations by Williams and you have the six-term solution in polar co-ordinates; you get the stress components sigma r, sigma theta and tau r theta; and this corresponds to the mode 1 loading.

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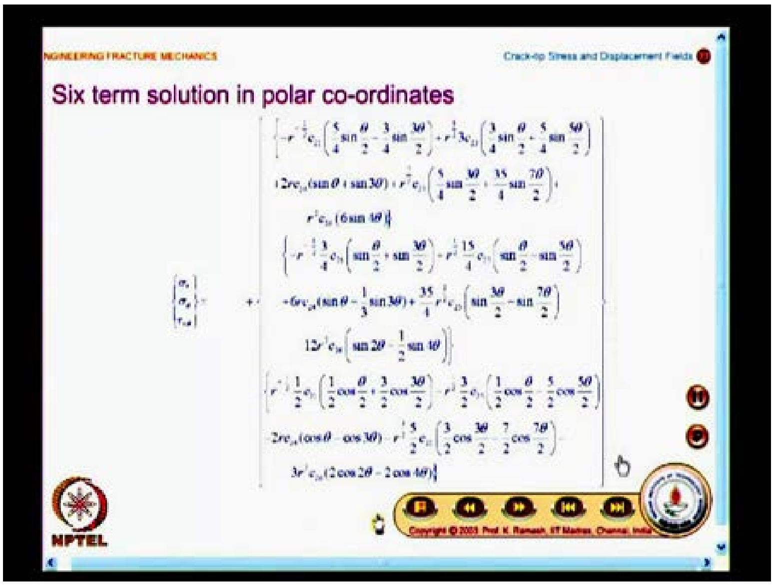

And you have another set of terms, which corresponds to mode 2 loading. This equation was, is quite clumsy and we are not in a position to write it in a generic form.

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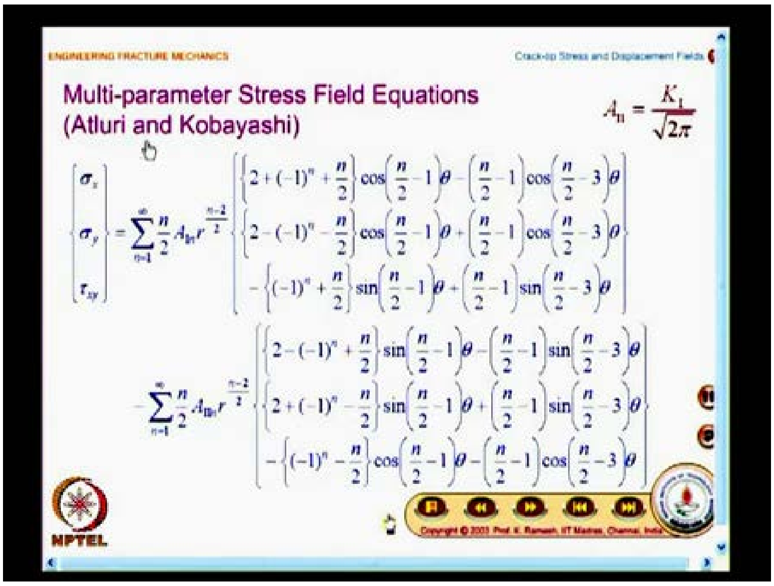

And the credit for bringing out multi-parameter stress field equations in an elegant fashion goes to Atluri and Kobayashi. And here, you have the terms expressed in terms of variable n, n varies from 1 to infinity; and you have terms corresponding to mode 1 and terms corresponding to mode 2. The advantage of such an expression is, it is easy to write a computer software, which would take as many terms as possible, for data processing.

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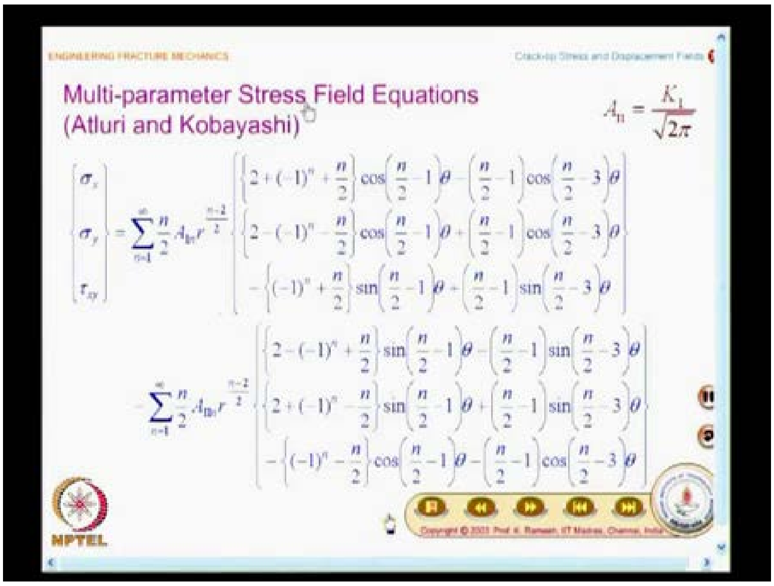

And in these solutions, if you take as many terms as possible, it closely models the stress field. That argument is valid. We had looked at modification of Westergaard equations by Tada, Paris and Irwin; they also got a series solution, but that series solution had only one stress function, capital Z, which was inadequate. This was pointed out by Sanford. He added capital Y; then you got the generalized Westergaard equations. And those equations are valid for the given problem on consideration; and you could take as many terms; and the more terms you take, the data processing would be more relevant from experimental analysis point of view. All this show, that we are converging into an understanding that multi-parameters stress field equations are a necessity. Though, the origin of these approaches are different, they converge to a unique solution. We will also see that.

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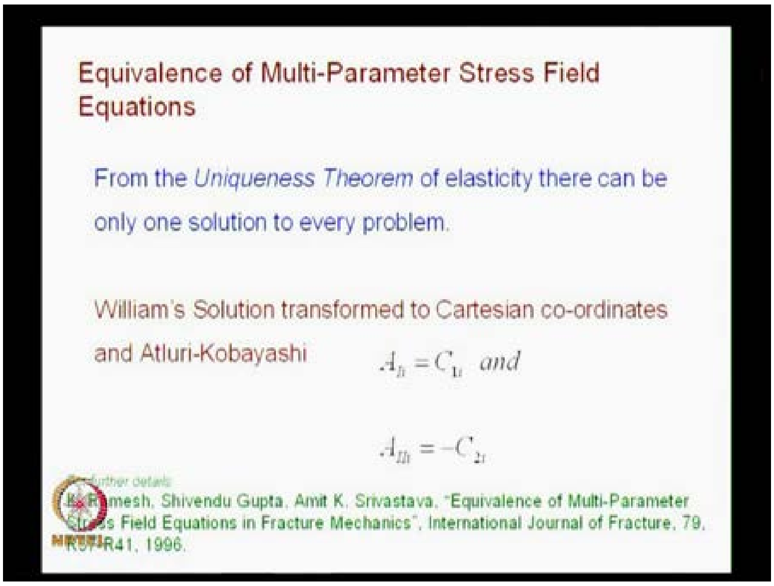

So, what you will have to keep in mind is, from the uniqueness theorem of elasticity, there can be only one solution to every problem. This you have to appreciate and this is very important. And whatever we have got from Williams' solution was in polar co-ordinates. If those stress components are transformed to Cartesian coordinates and compare it with Atluri-Kobayashi; you take the reference as A 1i and A 2i; A 1i is nothing, but C 1i of Williams' solution and A 2i is nothing, but minus C 2i.

So, the solution given by Williams and Atluri-Kobayashi are not different; they are one and the same, as an equivalence. And this was brought about, that the, first time by my students. This appeared in International Journal of Fracture and the topic of the paper was 'Equivalence of multi-parameter stress field equations in Fracture Mechanics'. You know, this is the logical step forward. Because when you have multiple solutions for a same problem, they have to be identical; if they are not identical, then you have to go back and check your mathematics and see whether there have been any mistakes in the development of the solution. And this again gives us a confidence that, we are proceeding in the right direction as far as crack problems are concerned.

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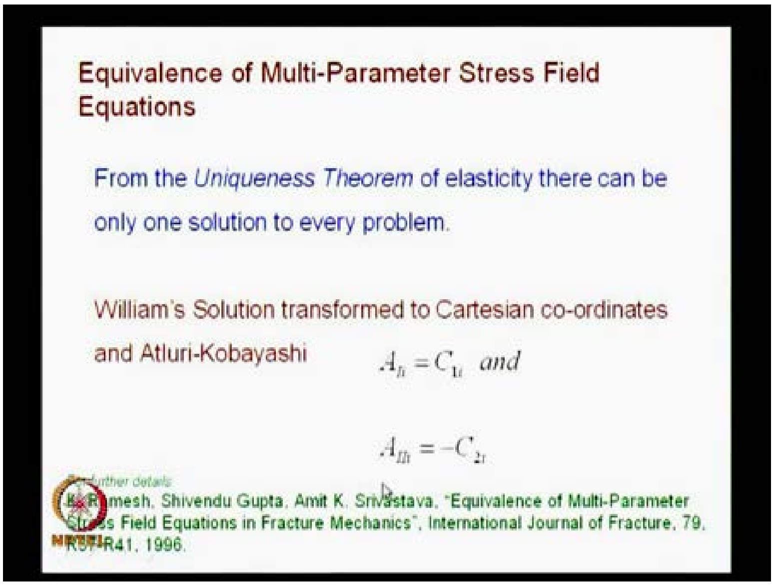

Now, you have seen, what is the kind of inter-relationship for the Atluri-Kobayashi and Williams solution.

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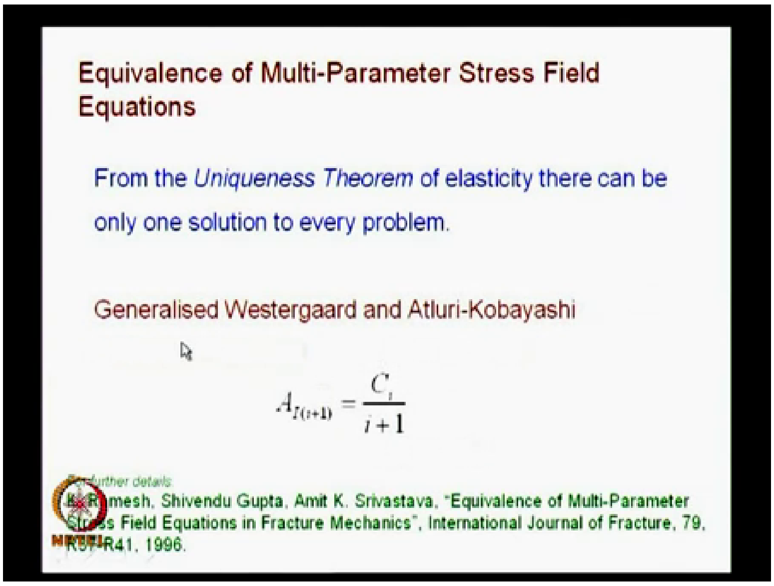

You can do a similar exercise for Westergaard solution also. And in the class, we have developed for the mode 1 situation and the inter-relationship is the coefficient A I of i plus 1 equal to C i divided by i plus 1. For the case of mode 2 loading, I have given you the airy stress function. I would appreciate that you take that as a home exercise.

So, this shows that the multi-parameters stress field equations are not different; one methodology you had complex variable approach and another you had a Eigen function approach; they converge to similar solutions.

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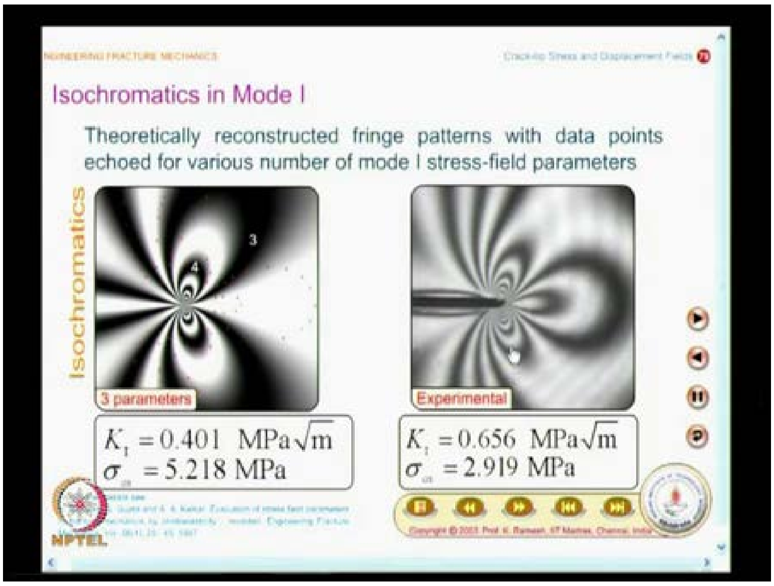

Now, what we will see is, we would see the utility of this. And we had taken up in the last class, the problem of a mode 1 situation and you have the experimental fringe pattern, which is shown in this, right side of the screen. And what you have here is, you have forward tilted loops as well as a frontal loop. On our left side, you have the fringe patterns theoretically simulated. I would like you to make a neat sketch. Please take your time, make a neat sketch. Whenever you go for a two parameter solution, the two parameters solution can capture only the forward tilting of loops; it cannot capture the frontal loops that you see in an experiment. And that is what is shown in this. And when you use a 2 parameter solution, the value of K 1 is 0.445 MPa root meter, whereas, the actual K 1 for this problem is higher, that is 0.656 MPa root meter. And what you see in this picture, is the red dots, which are actually data points taken out from the experimental fringe pattern. And what I want you to do is, you would draw the sketch for two parameter, draw the sketch for a 6 parameter solution. The intermediate ones, you just have an observation and you could refer this paper, I will magnify it.

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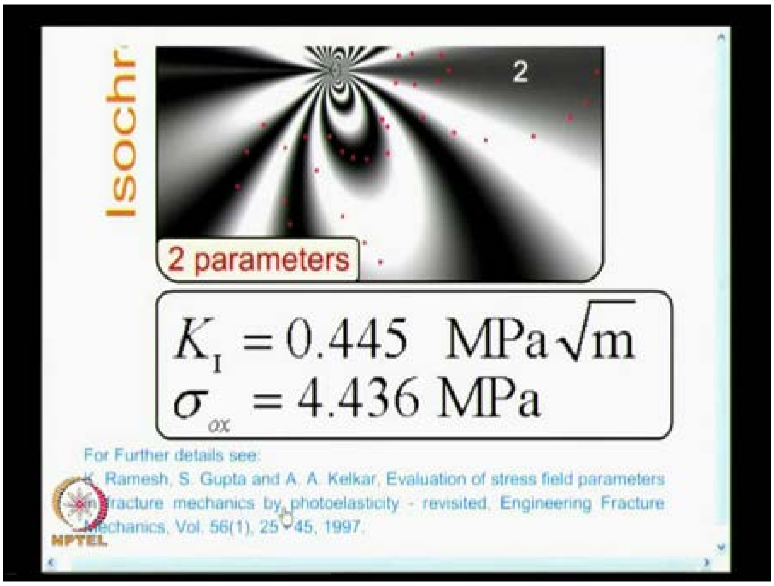

This paper has all the details. This is on Evaluation of stress field parameters in fracture mechanics by photoelasticity-revisited. Because this was the first paper, which utilized the equations of Atluri and Kobayashi and demonstrated it, its utility for a variety of problems. This appeared in Engineering Fracture Mechanics and there, you have the solution given as a function of number of parameters. All these fringe patterns are available.

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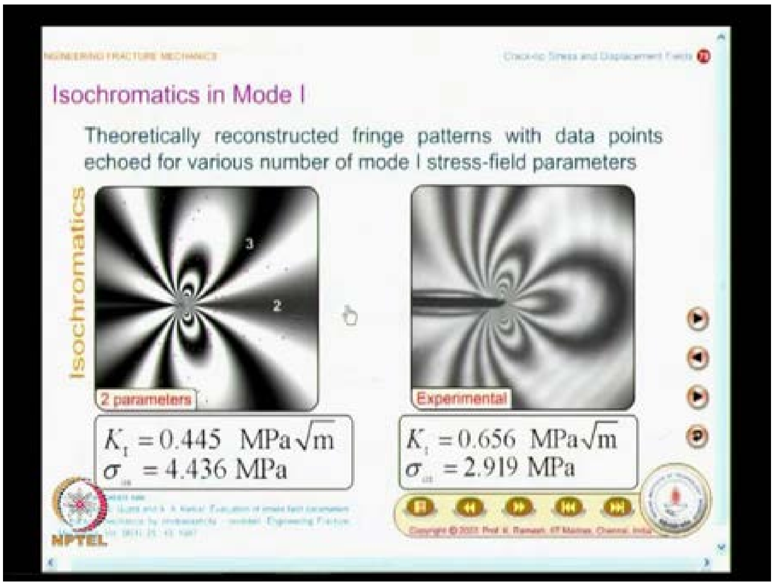

But nevertheless, you have a look at, for various loading situation. And in this case, you are able to see very clearly the red dots I had of the crack-tip; this for a 3 parameter. Even a 3 parameter solution is not able to capture the frontal loops. From 4 parameters onwards, it captures the frontal loop. 5th parameter and 6th parameter, you have reasonably capturing all features of the experimental fringe pattern. And the result is also quite close. K 1 is 0.653 and the final solution is 0.656. And your sigma naught x is 2.917 and the final value is 2.919. This is obtained for a eight term solution. So, what you get here is, by taking more number of terms in the series, you are in a position to capture the experimental features, which also gives a confirmation, that the stress field solution what we have obtained, indeed models the stress field in the near vicinity of the crack-tip.

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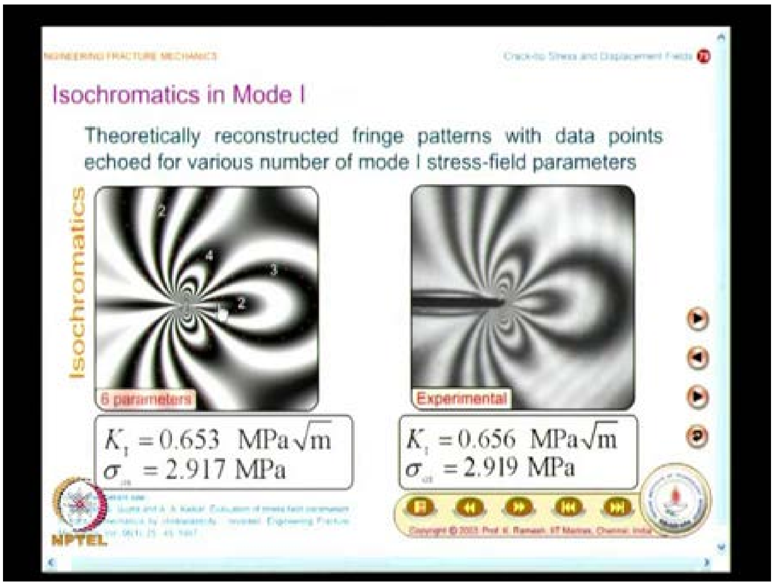

I have always been mentioning, photo elasticity provides a clue from the geometric appearance. I have the crack-tip, you have fringes crossing. If people have not noted this, they would not have raised the question while using the Westergaard solution. And you should also look at the history. See, the experimental is started from a Westergaard solution; Irwin added a constant term. So, they were only looking at the Westergaard solution. Unless Westergaard solution is modified, they would not have arrived at the final solution. Whereas, the Williams' solution was complete. But I also mentioned, in a summary of his discussions, he unfortunately quoted, the second term has to be 0, when the straight edge has no stresses; that is, like uniaxial stress field; which is an unfortunate conclusion. If he had not made that, then people would have used Williams right from 1957 and model the stress field in the near facility of the crack-tip. So, if we look at the history, a modification was necessary; this was introduced by Sanford.

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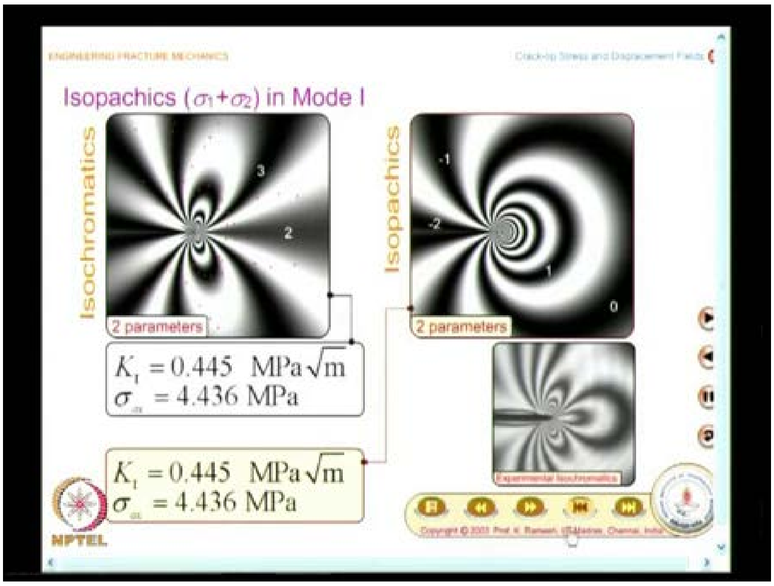

And we will also have a look at, how the fringes appear in the case of holography. You observe the fringes; this screen has the experimental isochromatics. And here you have, experiment and theoretically reconstructed fringe patterns for sigma 1 minus sigma 2, corresponding to photo elasticity. With the same solution, here theoretically sigma 1 plus sigma 2 is plotted, so that the comparison could be made. And what I want you to appreciate here is, as the number of terms changes, you see more and more density change of the fringe. You are not observing any striking difference in the geometry of the fringe pattern; the geometry of the fringe pattern remains more or less same; only the density changes, which is equivalent to the load is higher or lower. That is the way you can interpret. It would not have promoted a researcher to think, that the theoretical solution, what we have developed is not comprehensive.

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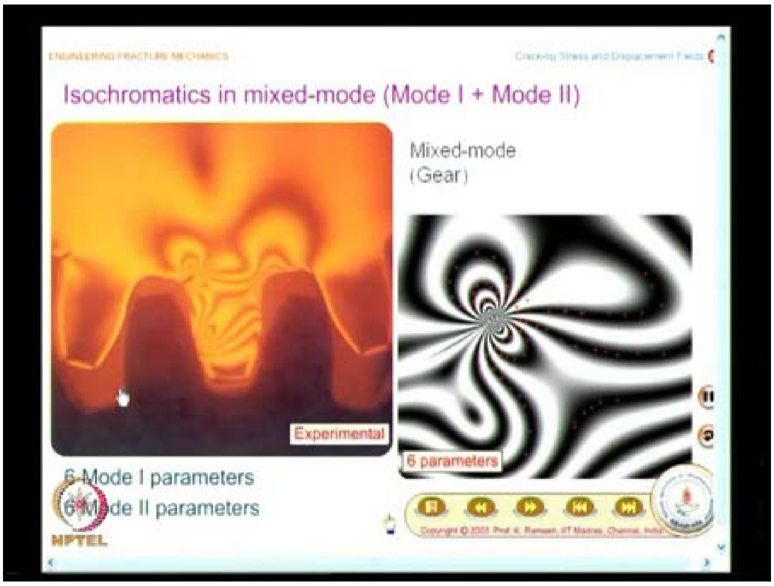

Now, we will take up the case of a mixed-mode situation. You know, it is a very complex problem. In fact, this was done at IIT Kanpur and the student who fabricated this rig was one Mr. Pankavala. Very complicated exercise, which models even how the gear is mounted on a shaft. Here you have a gear made of aluminium; that is why you do not see fringes on this. This is the gear made of epoxy and you have a crack in the tensile root fillet. You know, first of all you have to recognize that this is a finite body problem. Second one is crack is situated in a stress concentration zone. Third aspect is, I have one stress concentration related to contact stress here.

So, there could be interaction of this, with the crack-tip stress field. Even for such a complex situation, you find a 6 parameter, involving 6 mode 1 parameters, and 6 mode 2 parameters, has reasonably captured the fringe field. In fact, this is a success. It is a demonstration, that we have reasonably understood the near field stress state of the crack. That is the way you have to look at. And I will also show, as a function of number of parameters, how the solution changes. And I would like you to make one or two sketches; that is essential.

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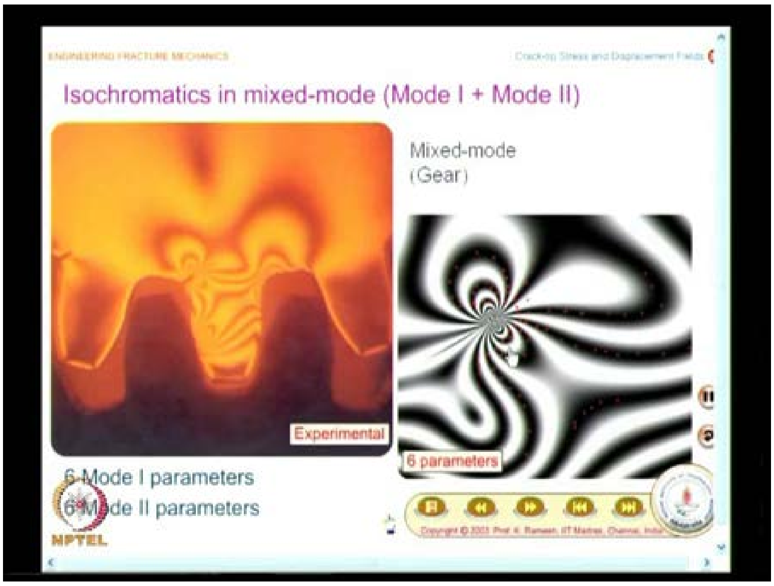

So, this shows the overall gear and the, theoretically reconstructed. Here you have 2 parameter solution.

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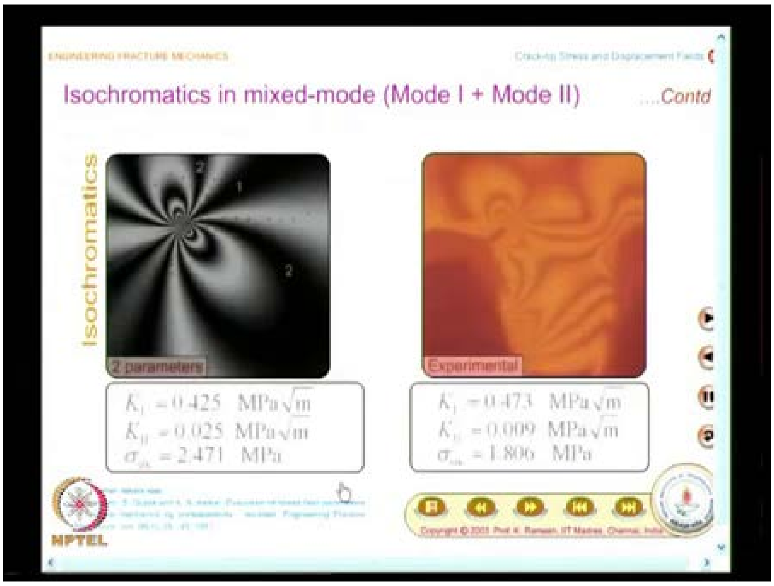

Here again, you notice, the 2 parameter solution is able to provide only the tilt; the size of the fringes are different; that is also captured. The tilt is different with respect to the crack axis. This is also captured by your 2 parameter solution. The 2 parameter solution is unable to capture the other features.

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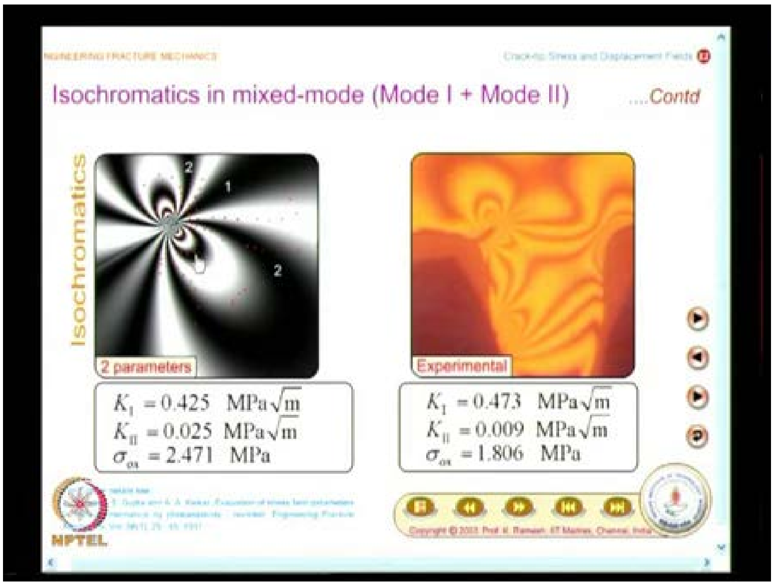

And you have the data collected from your experiment. And this data is utilized to find out the coefficients in a non-linear (( )). Please make a sketch of this. And you should also make a reasonable sketch of the experimentally observed fringe pattern. Actually, this feature is because, you have a stress raise (( )) here; that is why the fringes here are distorted. Even this distortion, your multi-parameter solution is able to capture. And even the software is very elegant to write. Because you have the parameters given as function of n, by changing the n, you are in a position to take 2 terms, 3 terms, 4 terms and gradually increase the number of terms. And you could also see the values of K 1, K 2 and sigma naught x for each of the parameter solution; for 2 parameter solution, the values are like this; for the converge solution, the values are like this. It is primarily a mode 1 situation; the final mode 2 component is almost close to 0. And sigma naught x also changes.

You know, this, you will have to keep in mind. So, what you will have to keep in mind is, as I have several terms, depending on the appropriate conditions, these coefficients differ. But, I would still say, this is useful as a… Instead of a very near-tip stress field, you could call it as a near-tip stress field equations. And this is reasonably good enough. And it also gives you confidence that we are in the right direction. You have an experimental proof, that these equations indeed give you useful result. Our focus is finally on what is the value of K 1 and K 2, from the experimental fringe pattern. But in order to do this, you may have to model a 6 parameter mode 1 and 6 parameter mode 2 solution, and finally, arrive at what is the value of K 1 and K 2. Now, you have the 3 parameter solution; you have the 4 parameter solution now; and you have the 5 parameter solution and 6 parameter solution. You could see, as we increase the number of terms, the fringes smoothly fit into the experimentally observed data points. You also make a reasonable sketch of this. You do not have to draw the shading like this; you just draw this as a contour.

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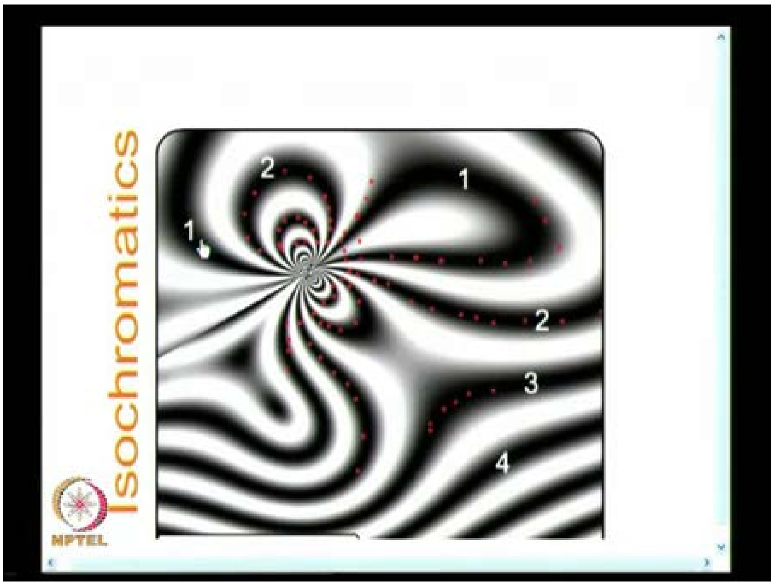

If necessary, I could also enlarge it for you. And that gives you, very clearly, the data points. Even here, you will find 1 or 2 data points are slightly off. Now, this happens. In an experiment, there would be some sort of a scatter. So, this is a demonstration that the multi-parameters stress field equations are indeed correct for a mode 1 situation, as far as a combination of mode 1 and mode 2. Here mode 1 is predominant, that is small value of mode 2 is present. I would like you to draw the sketch in the near vicinity. And this is what I would like you to do. Even if you draw a line and even the fringe ordering is put; I have fringe order 1, 2 and I had of the crack it is fringe order 1 and you have the fringe orders like this. I have this as fringe order 3, 4, 5 and here also fringe order increases, mainly because, you have this is as the contact stress field. So, here again the fringe orders increases. By any standard, it is a very complex fringe field. And you will have to appreciate that this is very complex fringe field. And it is a demonstration, that the multi-parameters stress field equations are able to capture even such a complex situation.

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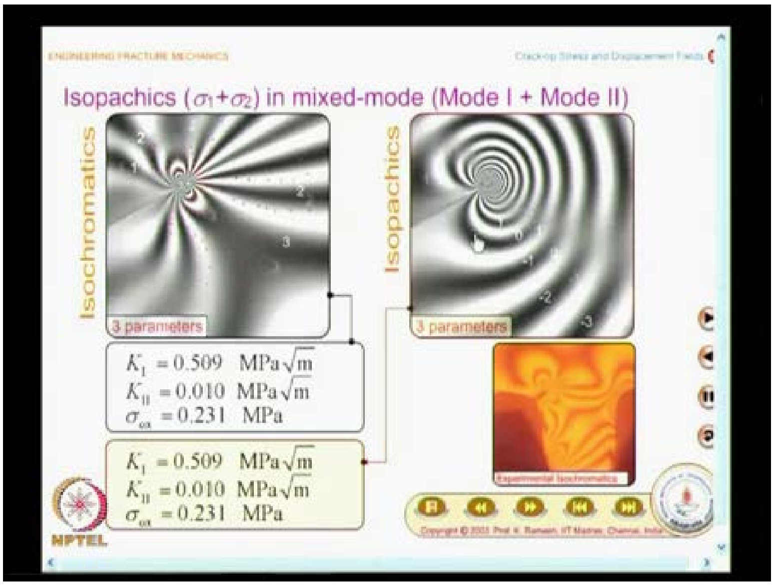

Now, we would also look at, what is the kind of isopachics, that is sigma 1 plus sigma 2 contours. Here again, you will see only the density of the fringes change. There is no major change in the geometry of the fringe pattern. On the other hand, the geometrical changes of the photo elastic fringe patterns are very significant. You cannot miss that aspect, whereas, here you may try to find out some very subtle aspects. And by and large, it is only density change.

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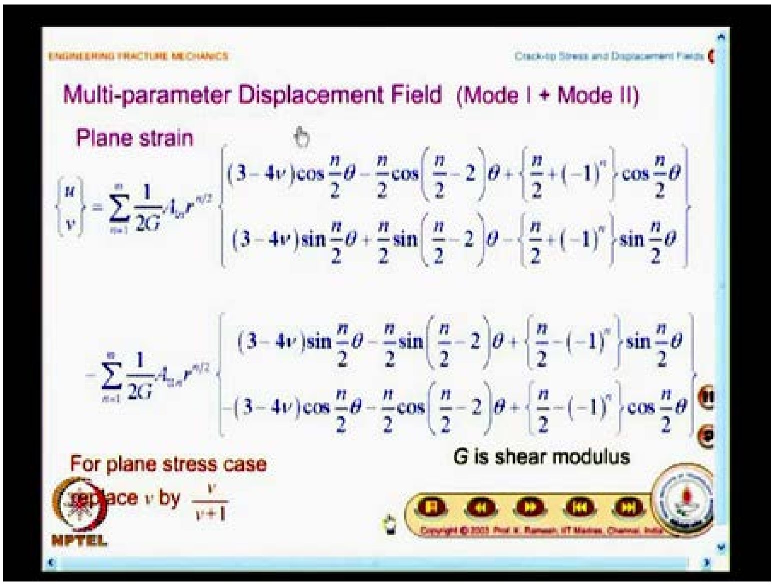

Having looked at the multi-parameter stress field, we would also look at multi-parameter displacement field equations. This is for mode 1 plus mode 2 and this is obtained for plane strain situation.

I have the displacements u and v. This is again given as summation of two series, one corresponding to mode 1; the other one corresponding to mode 2. n varies from 1 to infinity. 1 by 2 G of A 1 n r power n by 2 multiplied by 3 minus four nu cos n by 2 theta minus n by 2 cos n by 2 minus 2 theta plus n by 2 plus minus 1 whole power n multiplied by cos n by 2 theta; the v component you have this as, 3 minus four nu sin n by 2 theta plus n by 2 sin n by 2 minus 2 theta minus of n by 2 plus minus 1 whole power n multiplied by sin n by 2 theta. And you have this for mode 2, given as minus n equal to 1 to infinity 1 by 2 G A 2 n r power n by 2, for the u component it is the 3 minus 4 nu sin n by 2 theta; mind you, I read this as nu, do not confuse this as v; the font appears as if both are similar; there is only a subtle difference; it is 3 minus 4 nu; nu is the Poisson's ratio; sin n by 2 theta minus n by 2 sin n by 2 minus 2 theta plus n by2 minus of minus 1 whole power n sin n by 2 theta; minus 3 minus 4 nu cos n by 2 theta minus n by 2 cos n by 2 minus 2 theta plus n by 2 minus of minus 1 whole power n cos n by 2 theta. And you have to note that, G is the shear modulus. Once I have a plane strain, using the same set of expressions, you could construct multi-parameter displacement field equations by replacing nu by nu by nu plus 1.

So, now you have, at this level of the course, you have multi-parameter stress and displacement field equations. These are very comprehensive. These have been demonstrated that, they model the experimentally obtained fringe patterns. We have seen it for stress field. We would also see it for a displacement field. If you go to displacements, Moiré is one of the techniques, which is widely used, and we would see how the displacement field looks like for u displacement and v displacement, for the problems that we have looked at. We will see for the mode 1 as well as combination of mode 1 and mode 2. And what is done here is, you have the coefficients for this displacement field again, as a 11, a 12 and so on. These are already determined from the stress field. The experiment, what is done, is only from photo elasticity. You have these coefficients and these coefficients could be used to plot even the displacement field.

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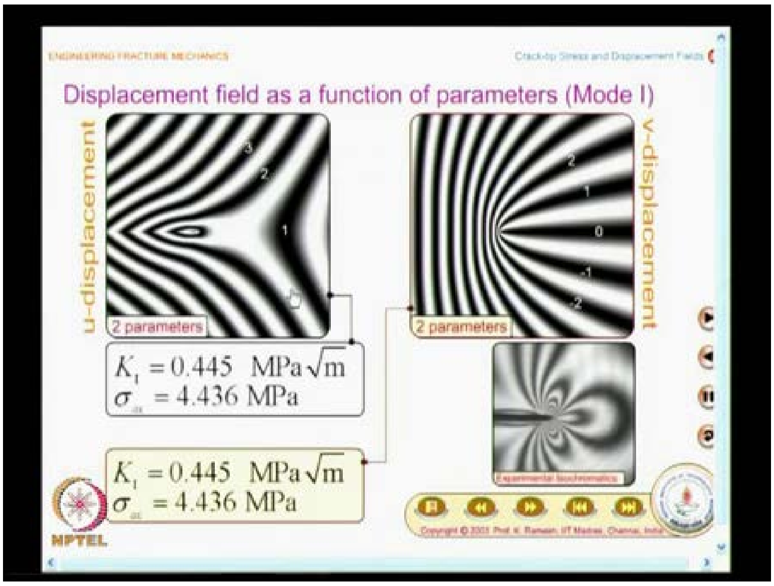

And that is what you see for the mode 1 situation. And here again, I want you to note down, this is the v displacement, and you also have labeling of fringes. Once we come to displacements, these are called isothetics and the fringes will have both positive and negative numbers.

And this is where you have the crack. Make a sketch of this. This is a very typical sketch of v displacement field. And as I increase the number of parameters, you will find there would not be any perceptible change in the geometry of fringe patterns. This is the kind of experiments that you have done based on Moiré. Even here, there are no visible changes in the fringe patterns; only the density changes. If the density increases, what happens is what you are having here. For example, for the 6 parameter and 3 parameter, the numbers of fringes have increased. And here again, the number of fringes have increased. So, from that point of view, photo elasticity has indeed propelled the research of fracture mechanics in the right direction. And what you see in this screen is, you have the photo elastic fringe pattern, because the coefficients are determined based on the photo elastic analysis. And these coefficients are used to plot the displacement field by taking as many numbers of terms as possible. And this is a typical v displacement field and this is the typical u displacement field. For the case of mode 1, we have gone up to 8 parameters; I will also go up to 8 parameters. And you find there are no major geometrical changes on the fringe patterns. So, even if you draw one such figure, it is good enough. And, in fact, this is a very popular, the v displacement is quite popular.

The moment you come across v displacement field like this, you should be able to recognize, that you have a crack and you have the displacement field represented in this fashion.

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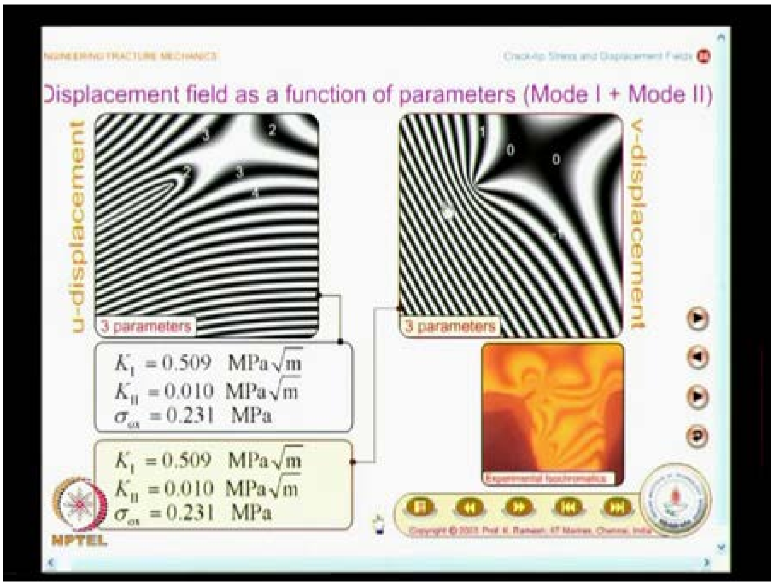

On similar lines, we would also see the fringe pattern for combination of mode 1 and mode 2. Here again, the process is same; from the photo elastic fringe pattern, the coefficients forming the series were obtained. Using that, the displacement fields are reconstructed. See, for 3 parameters, 4 parameters, 5 parameters and 6 parameters. So, you have this as a typical fringe field for a mixed mode situation, mode 1 plus mode 2. These are called isothetics. You have u displacement isothetics and v displacement isothetics.

See, at this stage, what we have done is, we have looked at threadbare, the development of stress field and displacement field equations for crack problems. We have started with singular solution for the stress field; then we graduated to multi-parameter stress field equations. We also saw multi-parameter displacement field equations. And this would be the last class, where we would be talking about multi-parameter solution. Because in fracture problem, it is a singular term that is dominant and all the future fracture theories, which are existing right now, used only this first term for the analysis. Now, some theories have been developed, where they take the second term, which they called it as t, which they call it as q. So, fracture theories based on second term also are appearing.

May be in future, you may find the necessity for using higher order terms. Then the higher order solution will become important. For all our discussion from today onwards, we would use only the singular stress field; we would use it to find out the plastic zone; we would use it to find out fracture theories; and we would also be interested in finding out, what is the value of stress intensity factor for a variety of problems.

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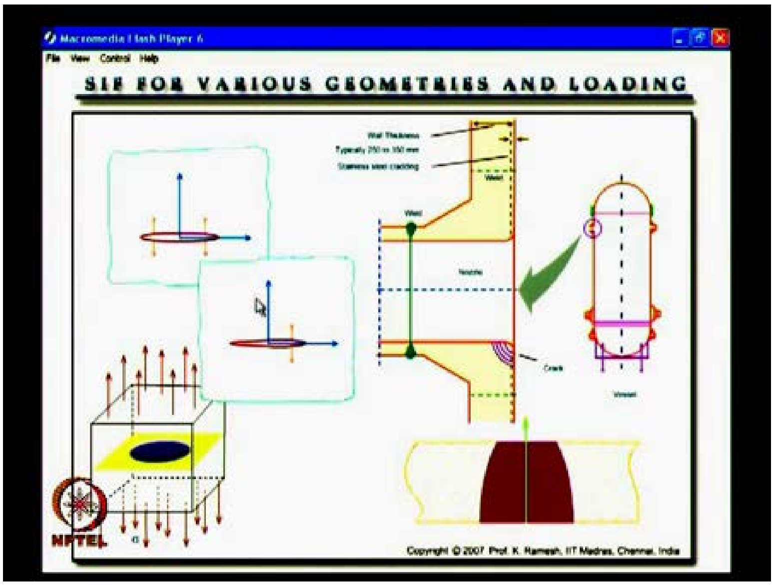

So, we will take up stress intensity factor for various geometries and loading. See, so far, we have looked at only those class of problems, where the crack surfaces are free. Whereas, here, what you find is, the crack is opened by two symmetrical loads. And you have another situation, where the crack is opened by a point load here. Then, you have an embedded crack in a solid. Then, you have a real life situation, where you have a pressure vessel, which has a nozzle and you have a crack coming out from the corner; it is a corner crack.

In fact, in one of the classes, the students have asked, you are analyzing only through the thickness crack, whereas, in all practical geometry, you have cracks on the surface, appearing from corners; how these theories could be utilized in those situations. In fact, we would do how to find out stress intensity factor for this important class of problems. There is also another interesting aspect. When you have a surface crack like this, we would understand, as part of the discussion in this class, that crack will primarily tend to move in the thickness direction. Why? This we will have to understand. You will get an answer, when you look at, how the stress intensity factor varies on the crack front. See, I had already mentioned, why we take through the thickness crack to start with is, they are simple to analyze. And we have got, for a center crack in an infinite plate, the value of stress intensity factor as sigma root pi A; that is a very standard expression for mode 1. For mode 2 it is tau root pi A and so on. So, that is a base solution; for any finite geometry you will have a function multiplied, which would be a function of A by w. So, what we will look at is, when you have a through the thickness crack, when you have an embedded crack, when you have a surface crack, how the stress intensity factors changes? And what is the kind of mental picture that you should have, whether the stress intensity factor would decrease, when I go to surface crack, or would it increase in comparison to through the thickness crack? This kind of a knowledge base, that is what we are going to gain, with the discussion that we are going to do.

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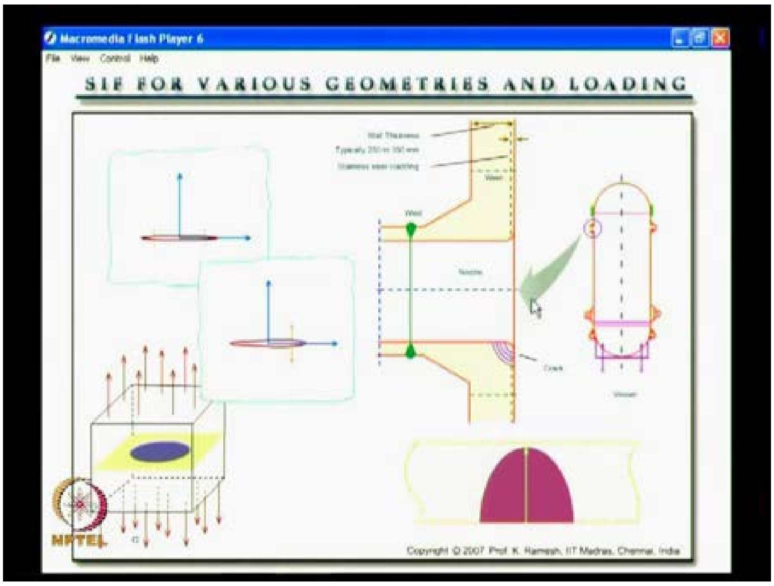

A clever user, what he will do is, he will not go into this chapter at all. He will go to a hand book, where you have summary of stress intensity factor for variety of loading situations; directly take the result and use it for your design. You may also do that, at a later stage, but in order to appreciate what is fracture mechanics, you need to go through this exercise.

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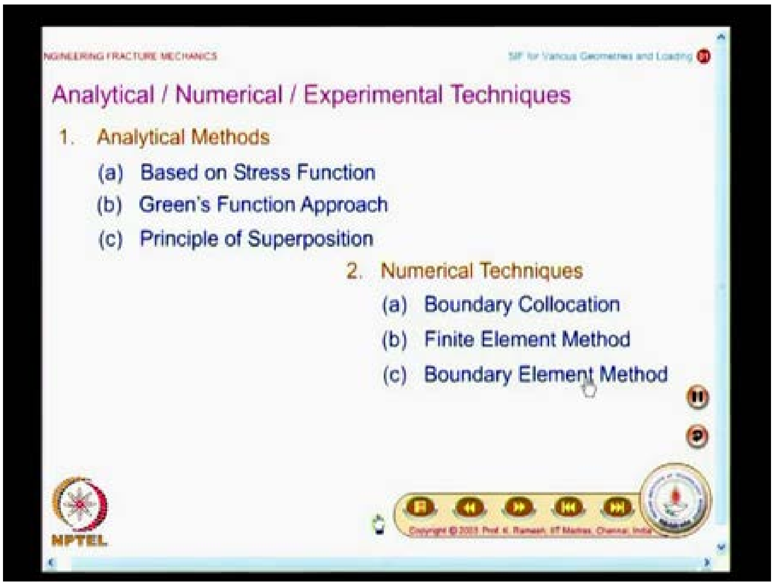

And in what way the stress intensity factors could be evaluated? It could be evaluated based on analytical methods. And one you, once you say analytical methods, it could be based on stress function.

In fact, only in this context, I said Westergaard stress function approach is quite useful. One of the advantages of Westergaard stress function is, you put coin stress functions, when the crack surfaces are loading. See, among the various solutions, if you are able to get solution by analytical method, the computational efforts are very, very less, when you want to use them. So, you have an expression, in terms of the geometry of the problem situation. What is the expression for stress intensity factor? Then we would also see Green's function approach.

Then, we would also see, because we are in linear elastic fracture mechanics, principle of superposition is valid. And we would see a wide range of problems that could be solved by invoking this principle. So, after analytical methods, you have numerical techniques. In numerical techniques, you have, what is known as boundary collocation. This is very widely used for reporting stress intensity factor for fracture problems. Then, you have finite element method and now boundary element method is also being used. And one of the aspects, you will find in this is, you know people would have developed the solution by boundary collocation or finite element method, but in order for the people to use their results, they have also tried to provide empirical relations, based on the result. This is one trend you will see in all fracture mechanics courses.

Though the source of the result may be from boundary collocation or finite element, they tried to represent the solution in the form of empirical relation, in the form of a series. You should not confuse that these series have been obtained from analytical bases. No, it might come from numerical basis. We would see that kind of expressions also.

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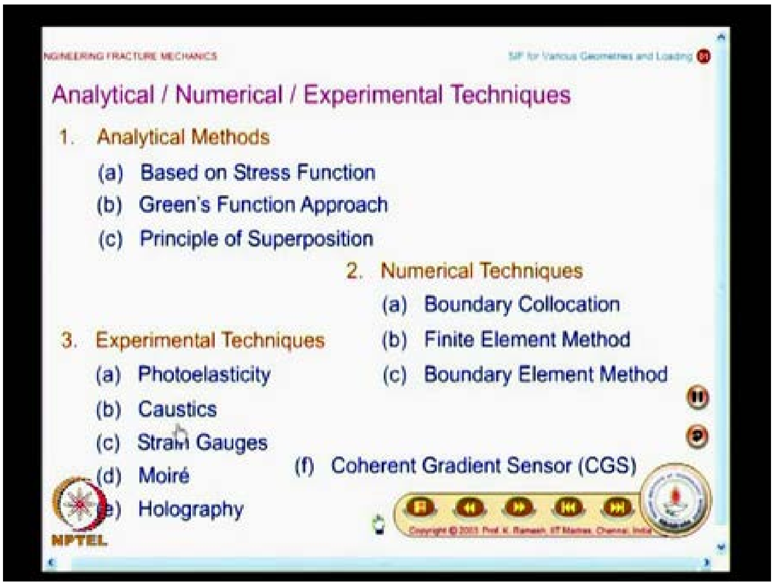

And as I mentioned earlier, you know, experimental techniques have contributed greatly to the development of fracture mechanics. And we have amply seen the role of photo elasticity. At times, I have also mentioned about the method of caustics. When we discuss the plane stress situation, in a normal plane stress situation, the lateral contractions would not be significant, but when you have a crack, in the near vicinity of the crack, you would have a dimple formation. This is exploited as a experimental technique and you have a method of caustics developed. And people have also attempted the method of strain gauges.

You know, this is very popular and easily available. If time permits, you would also see how to evaluate stress intensity factor using strain gauges. Then, you have use of Moiré; we have seen isothetics. Then, you can also see the experiments based on holography; that we have also seen isopachs, sigma 1 plus sigma 2 contours and a variation of caustics, known as coherent gradient sensor. This is again developed, particularly for fracture problems. And what we would do in this chapter is, we would primarily confine our attention, to analytical methods and some results from your numerical techniques, like boundary collocation and finite element method. If time permits, towards the end of the course, we would see how to evaluate the stress intensity factor based on experimental techniques.

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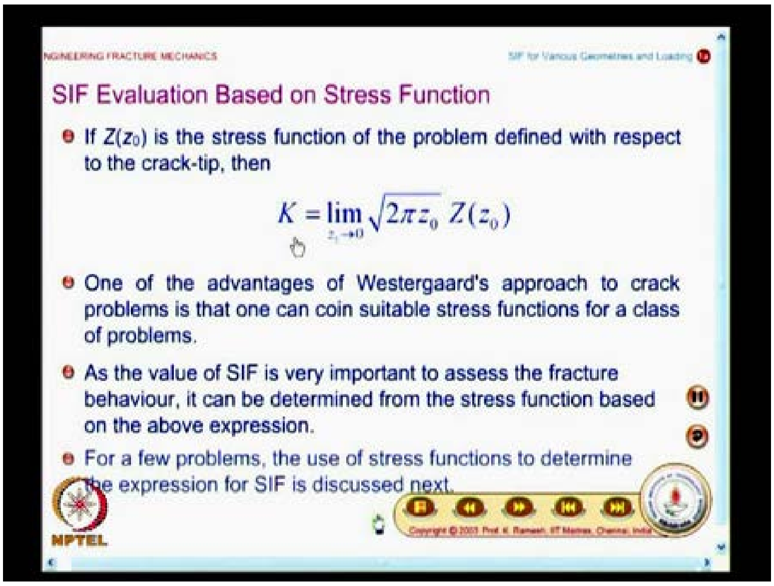

And, we will first take up evaluation of SIF based on stress function. And you have to recall, we have already defined the stress intensity factor in terms of the stress function capital Z. So, you have this as limit z naught tends to 0 root of 2 pi z naught multiplied by Z z naught, and as I had mentioned, if you are able to get the stress function Z, you could get the definition of K and the advantages of Westergaard approach is, you could get Z for a variety of problems. In fact, Westergaard himself, has given for the series of cracks, which was added by Irwin for a few problems. And as the value of SIF is very important to assess the fracture behavior, we focus on finding out the value of K.

And we would see how SIF can be determined. We would take up a problem like this. I have a crack, which is opened by a concentrated load. And what is a physical situation which you can think of? Suppose I have a riveted hole, and you have a crack form and it is getting opened, that kind of a situation could be modeled, based on a solution like this.

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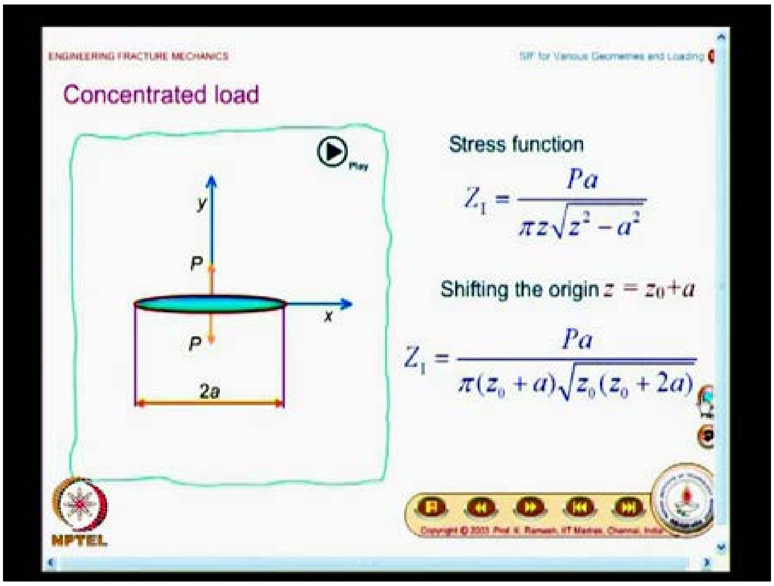

And we are fortunate that, there is a stress function available. The stress function for this class of problem is given as, Z 1 equal to P a divided by pi z multiplied by root of z squared minus a squared.

See, while developing the stress field also, what we did? We had the stress function; we shifted the origin to the crack-tip; then we obtained the near field solution. A similar exercise, you have to do here. So, you have to shift it to the crack-tip. For shifting the origin, substitute Z equal to z naught plus a; then you have the definition of stress intensity factor in terms of stress function, and put the limit z naught tends to 0. Then, you will get the expression for K. So, we will substitute z naught plus a, and how does the expression look like? The expression is of this form. I have this as P a divided by pi into z naught plus a multiplied by root of z naught multiplied by z naught plus 2 a. You know, this expression is simplified after substituting z naught plus a.

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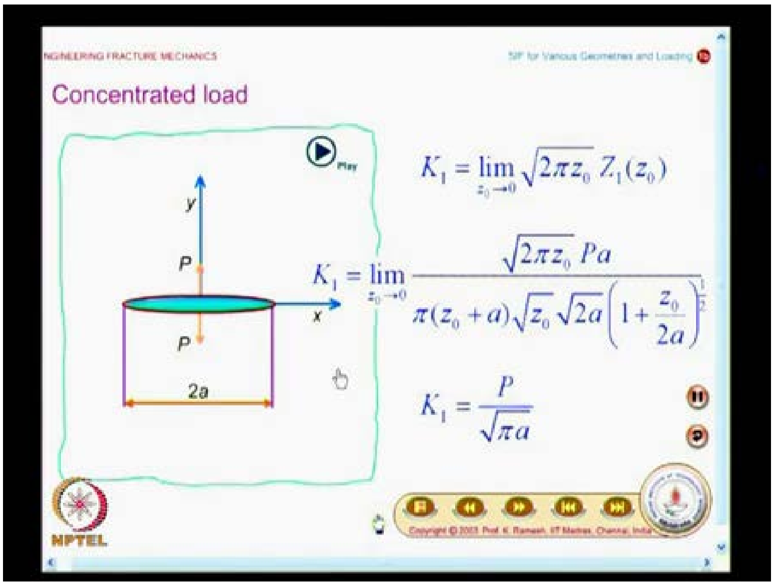

And what is the next step? I have to invoke the definition of stress intensity factor. The definition is like this, limit z naught tends to 0 root of 2 pi z naught multiplied by the stress function expressed in terms of z naught. This, you will have to keep in mind. And when I do this, I have the expression like this, limit z naught tends to 0 root of 2 pi z naught P a and this expression is recast; and you have like this pi z naught plus a multiplied by root of z naught multiplied by root of 2 a; it is multiplied by 1 plus z naught by 2 a whole power half.

Now, you know, I can simplify this and then get the value of K as like this, P by root of pi a, when I have concentrated load acting at the center of the crack-tip. Is there anything interesting from the result that you have got? Is there anything striking? There is a very important aspect. That is why I have taken up this problem. See, all along what we have been looking at? The stress and crack length are interrelated and as the crack length increases, stress intensity factor increases. When you have sigma root pi a for the center crack problem, K is defined like that. So, as a increases K also increases. What happens in this problem? I have K 1 equal to P by root of pi a. So, when a increases, K is going to decrease.

In fact, it is a very useful problem. Suppose, I want to study, my understanding whether the crack can close by itself. That is crack propagates and stops by itself. If you want to perform that kind of test and verify your fracture mechanics understanding, you could device an experiment based on this. So, as you pull the crack surfaces, as the crack increases, K decreases. So, after proceeding for some distance, the crack would stop. In fact, later we are going to study Paris law, which talks about the modeling of crack propagation, where we would see, that model is valid for both the cases of a center crack, where the stress intensity factor increases as the function of crack length; the counter example is stress intensity factor decreases as a function of crack length. We would see both the cases and convince ourself, that Paris law is useful.

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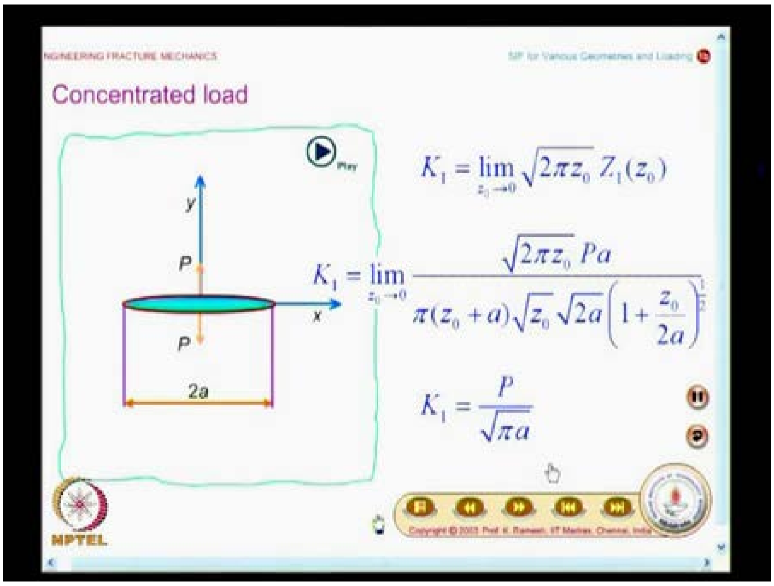

So, it is a very important problem, from that point of view. So, when I have it from concentrated load, this could also model for riveted hole, that kind of problems and you could also construct multiple solutions based on this. So, you have a problem like this.

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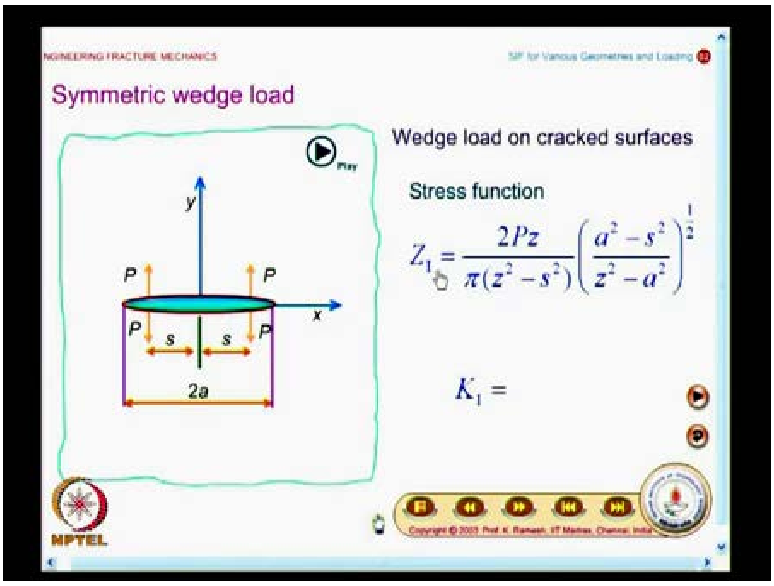

Now, we will take up another situation, where I have crack opened by symmetric loops, which are at distances s from the center. You make a sketch of this and what you find is, you have a Westergaard stress function, even for this problem. So, when you have Westergaard stress function for this problem, it is possible for you to solve. And evaluate what is a value of stress intensity factor in terms of parameters of the problem. What is the load applied, what is the geometry, so on and so forth. And the stress function takes the form like this, Z 1 equal to 2 P z divided by pi of z squared minus s squared multiplied by a squared minus s squared divided by z squared minus a squared whole power half.

You know, I would like you to take this as an exercise, because we have already seen what is the basic procedure; you shift the origin to the crack-tip; then bring in the definition of SIF, simplify and find out the value of K. I would leave this as the exercise. I hope that you do it and bring it in the next class. So, in this class, what we had looked at was, we had looked at a review of multi-parameter stress field equations. Then, we also said, those equations are not totally different, because in theory of elasticity, you have an uniqueness theorem, for one problem you will have one unique solution.

So, based on that, we have also looked at identity between the coefficients; between Williams as well as Atluri and Kobayashi and also generalized Westergaard equations. Then, we saw at length, what are the kind of fringe patterns that you come across and how photo elastic fringes are, fringes are different in comparison to isopachs which on contours of sigma 1 plus sigma 2.

Indeed the geometry of the fringe patterns had a significant change. Then, we moved on to finding out multi-parameter displacement field equations. We saw for plane strain as well as how to change it for plane stress. Then, we had a brief discussion on how to evaluate stress intensity factor for a variety of problems. We have just made a beginning and I said, from now onwards, all our attention and discussion would focus only with the singular term. It is very important and we will leave with it. Thank you.