Engineering Fracture Mechanics

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Lecture No. # 21

William's Eigen Function Approach

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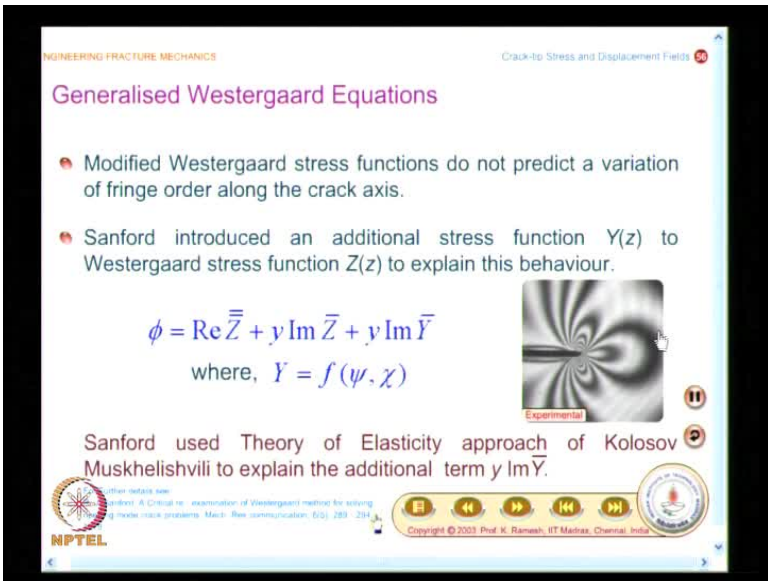

In the last class, we have looked at generalized Westergaard equations, and one of the key points in that was, we noted, that modified Westergaard stress functions do not predict a variation of fringe order along the crack axis. And this is the feature that you observe in an actual experiment. I have been showing these photo-elastic fringes for quite some time, I am sure you must be, now, very familiar in appreciating the fringe contours.

So, what you have here is, this is the crack, and you have the fringes which are forward tilted; as you go close, the fringes become almost straight, and along the crack axis, you find frontal loops. So, the idea is, though we want tau xy to be 0 along the crack axis, we find that modified Westergaard equations do not provide a variation of maximum shear stress. In an actual experimentation, you do find fringe loops, and they correspond to variation of maximum shear stress along the crack axis.

You know, this is very unusual, in the case of fracture problems, they question the basic stress function itself. In one of our problems in solid mechanics, the moment you get a stress function, you simply go to evaluating the stresses and then to displacements. You never go back and then question, whether the stress function was reasonably good enough. But, in the case of fracture problems, what we find a peculiarity is, the original solution was not bad; the only thing is, the original stress function was inadequate; it was not able to explain most generic feature of fringe pattern, nevertheless, it was able to capture some of the key issues related to what happens in the neighborhood of the crack.

See, you should not discount whatever Westergaard done was wrong; it is not like that. The problem is so complex, you have been able to unravel certain aspect of it, and if you want to go deeper into the problem, you need to have a relook and find out how well the solution can be improved. And one of the key observations was provided by photoelastic experimentation, and what we will do is, we had some discussion on whether the stress function we obtained or the stress field that we have got from the stress function, is valid for a uniaxial field or a biaxial field; this kind of discussion we had. Now, what we will do is, we understand very well, that the near vicinity is reasonably taken care of by the singular solution.

Now, we would look at, by comparing the geometry of the fringe patterns, when we go from Westergaard, Irwin and generalized Westergaard equations, what way it aids in improving, processing experimental data? We will have a different look at that; and, what Sanford did? He introduced a stress function Y z, and the Airy's stress function is recast as real part of Z double bar plus y imaginary part of Z bar plus y imaginary part of Y bar.

And Y is given as a function of psi and chi, Sanford justified that the introduction of an additional stress function is necessary, because, if you look at the Kolosov Muskhelishvili formulation, in general, any Airy's stress function is given as two analytical functions in suitable combinations. So, from that logic, he was able to justify why we need to have additional stress function y; and we would see, what was the implication of it. And, you have to keep in mind, the kind of fringe patterns that you come across in the most general situation.

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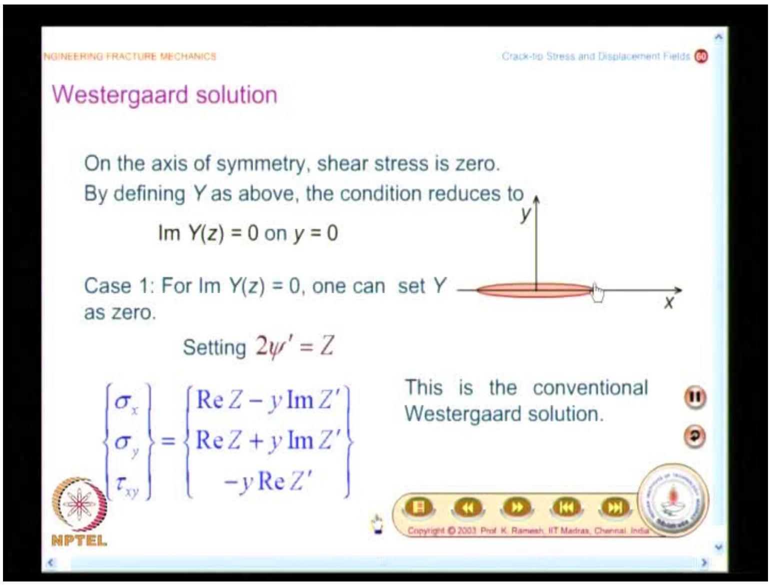

And, what do you see as a Westergaard solution? In the generic formulation, if you take the imaginary part of Y to 0, on y equal to 0, then you get the conventional Westergaard solution. And, we had already looked at, when we plotted the fringe pattern, the fingers were symmetrical about the x-axis, as well as the y-axis; that means, fringes were straight. See, very close to the crack-tip, you have plastic deformation. No mathematical equations are available to model that, so, you cannot collect data in that zone. The only way you can collect data in an experiment is, away from this zone; away from this zone, I should have sufficient data points for me to collect.

So, one way of looking at Westergaard solution is, it provides you very little data for experimental processing. Because, we have already seen, fringes are prominently forward tilted, and you find the straight fringe is very close to the fringe pattern, and you have to ensure, whether this is very close to the crack-tip, so that, plastic deformation can effect. So, you have to exclude that unelastic deformation zone, and you have to live in that singularity dominated zone, and I had already mentioned the size and shape of the singularity dominated zone is problem dependent.

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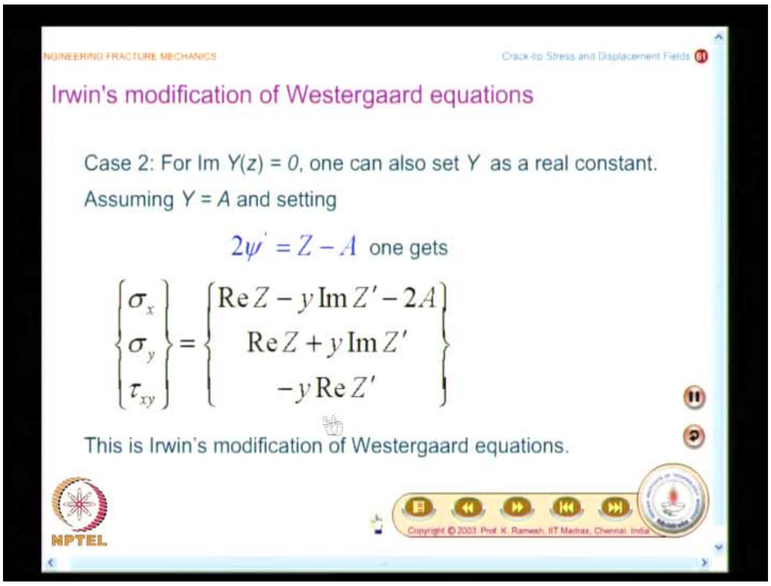

But, you will have to have a concept, that what way, this equation could be valid from an experimental point of view is, you are able to collect data, only a very small zone close to the crack-tip. On the other hand, in the Westergaard generalized formulation, if you set Y as a real constant, assuming Y equal to A and 2 psi prime equal to Z minus A, you get the Irwin's modification of Westergaard equation.

And, in this, what we saw? If prominently shown, forward tilt are the fringes or backward title are the fringes, whichever way you take, the sign of sigma naught s, and this models short cracks; in the case of SEN specimen, it is forward tilted; in the case of RDCD specimen, it is backward tilted. So, what you will have to understand is, from an experimental point of view, it provides you a little more zone for you to collect data, that are also studies what should be the range of theta m and what should be the range of the distance r in relation to the crack length. When you collect the data, the evaluation of k and sigma naught x is valid from experimental point of view.

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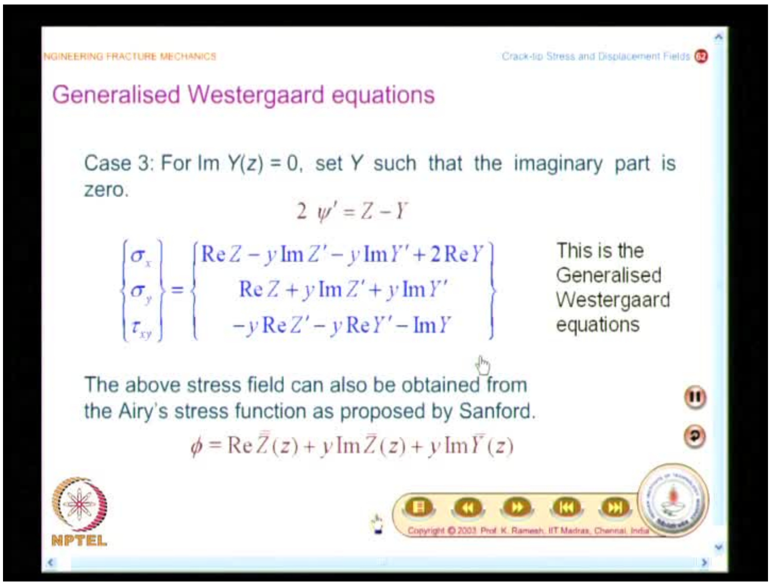

So, what you will have to look at is, by going to Irwin's formulation, we are able to enlarge a zone of data collection in the experiment. And finally, what Sanford pointed out was, you have to ensure only imaginary part of Y to 0; if you set 2 psi prime equal to Z minus Y, and you find the most general form of Westergaard equations, and this also models variation of fringe orders along the crack axis.

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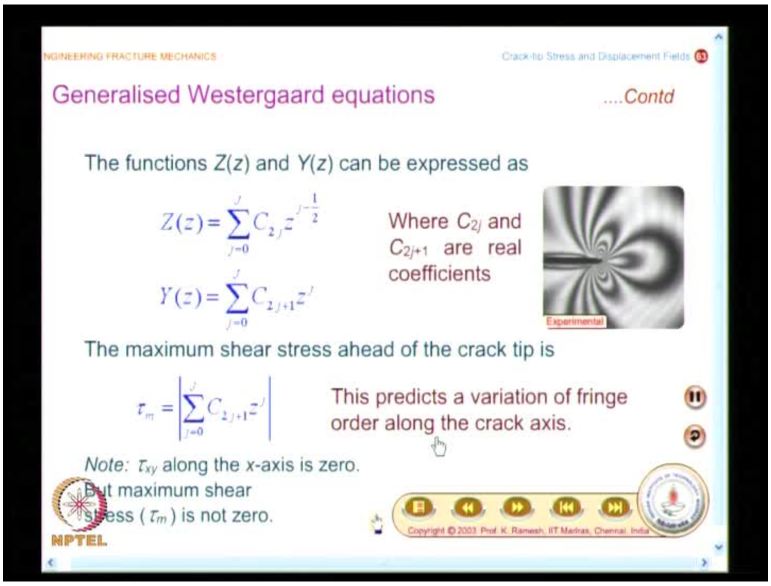

Rather than coming from Kolosov Muskhelishvili root, I could also get these set of equations by directly differentiating them appropriately. I have the Airy's stress function, by suitability differentiating it, I would be able to get this; until this, you have all written it down in the last class. Now, we will proceed, what is the form of Y as well as Z. The Z and Y are taken as, see this polynomial series, I have Z as sigma j equal to 0 to J C 2 j z power j minus half; and this is very similar to, if you look at the first term, it is nothing but your singular solution, you will have z power minus half, that is, rooter singularity embedded in this, and you have higher order terms.

If you look at the modification by Tada, Paris and Irwin, this will be very similar to that; not exactly the same, but very similar to that; but only taking Z as a series form will not yield either the forward tilt of the fringe loops or the fringe loop ahead of the crack; neither the forward tilted loops nor the fringe loop ahead of the crack will be able to get it.

Sanford pointed out, that you need to necessarily bring in a function Y z, this is also given in a series form; this is given as sigma j equal to 0 to J C 2 j plus 1 z power j. So, if you look at the first term, it will be like your sigma naught s, and you have higher order terms; in the case of z function, what you get? First term is a singular term, then you have a higher order terms. So, if you have the stress functions defined in this fashion, and also simultaneously taking Z as well as Y, Sanford was able to show the expression for maximum shear stress, turns out to be like this.

You have sigma j equal to 0 to J C 2 j plus 1 z power j, and when you have an expression like this, this predicts a variation of fringe order along the crack axis. In fact, in a class, later, I would show how taking a multi-parameter solution can capture all the fringe features of the experimentally obtained patterns. So, what you find here is, when you take Z as well as Y and ensure that imaginary part of Y is 0 on Y equal to 0, you satisfy shear stress tau xy 0 along the crack axis, at the same time, maximum shear stress varies along the crack axis.

So, only from this prospective, I said the conventional Westergaard solution was in adequate; it is not wrong, it was inadequate to represent all the features, what you observed in an experiment. And, one more thing is, you have to keep in mind, we had some discussion whether Westergaard solution is valid for biaxial stress filed or uniaxial stress field, then we argued in fashion. Now, what we will have to keep in mind is, for any problem, you will have to go in for multi-parameter solution. The discussion of uniaxial and biaxial is not going to take as any further; because, what is dictated at the crack-tip is how the crack phases are displaced relative to each other. That is what is determining whether you have mode 1, mode 2, mode 3 or combination of mode 1, mode 2, mode 3. So, the success of Sanford's approach is by bringing the stress function Y, and also taking both the Z and Y as series functions, he was able to get analytically, an expression for tau max, which varies along the crack axis.

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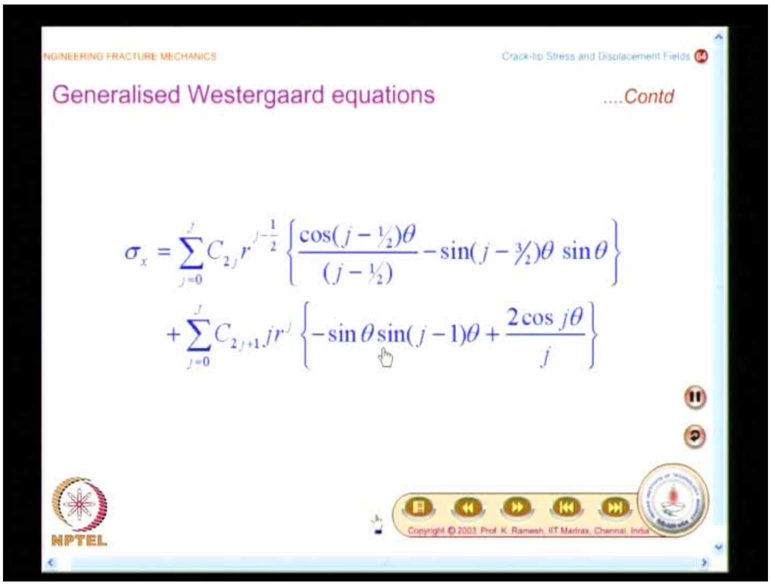

Now, you can also get the expression for stresses. Please write this down; in the next two classes, you have to write long expressions; these are culled out from such papers, and some of these are specially worked out by my students. You may not be finding them in published literature, and your notes will be comprehensive when you have expressions like this.

From generalized Westergaard equations, you get sigma x equal to sigma of j equal to 0 to j C 2 j r power j minus half multiplied by cos j minus half theta divided by j minus half minus sin j minus 3 by 2 theta sin theta. You also have a second term, this is again a summation, j equal to 0 to j C 2 j plus 1 j into r power j multiplied by minus sin theta sin j minus 1 theta plus 2 cos j theta divided by j; you have an expression for sigma y, this also I will read it out for you.

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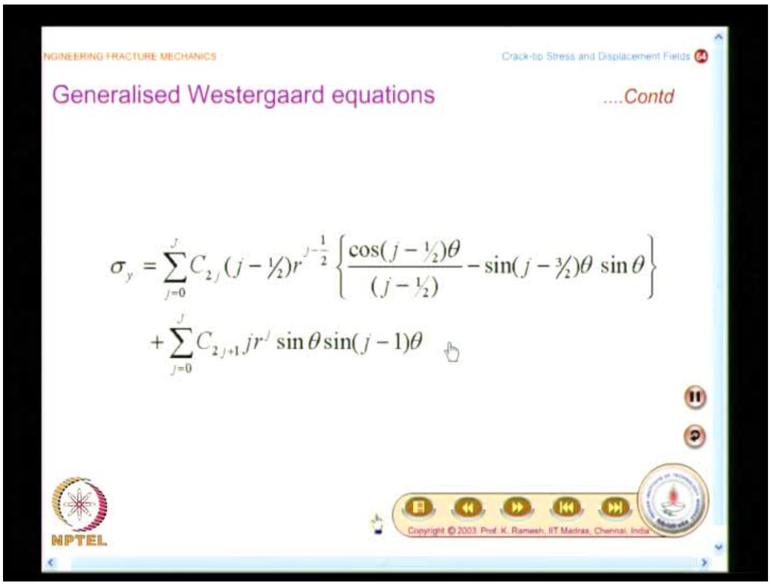

I have sigma y equal to summation over j equal to 0 to J C 2 j multiplied by j minus 1 half multiplied by r power j minus half multiplied by cos j minus half theta divided by j minus half minus sin j minus 3 by 2 theta sin theta; the second term is summation over j equal to 0 to J C 2 j plus 1 j r power j sin theta sin j minus 1 theta.

I am sure you will find that these expressions are very clumsy; it is indeed, so, when you have such a complex set of stress functions, the final expressions would be clumsy. And possibly by next class, we would have an elegant expression for multi-parameter stresses filed. You will also go to that; but before going to that, we will look at what was the stress field obtained by Sanford through his generalized Westergaard equations.

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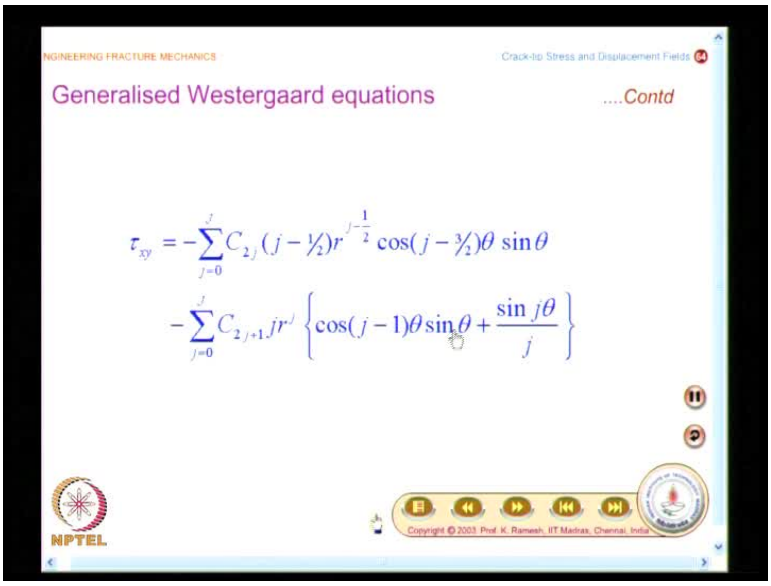

Now, we will go for what is shear stress tau xy, that is equal to minus j equal to 0 to j C 2 j multiplied by j minus half r power j minus half cos of j minus 3 by 2 theta sin theta minus summation over j equal to 0 to j C 2 j plus 1 j r power j multiplied by cos j minus 1 theta sin theta plus sin j theta divided by j. And, you know, you will also have the full expressions shown; so, if we have made any typographical error, you can relook at them and make the suitable changes.

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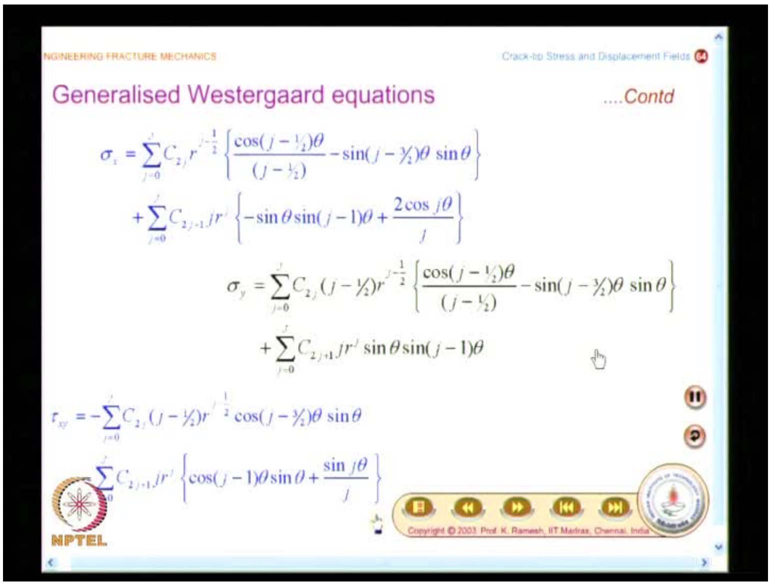

So, now, I have these as series functions, and if you segregate the terms carefully, this has the singular term like what you have in Westergaard, plus we have a higher order terms; and the question remains, how many higher order terms are needed for modeling a problem? That is again problem dependent.

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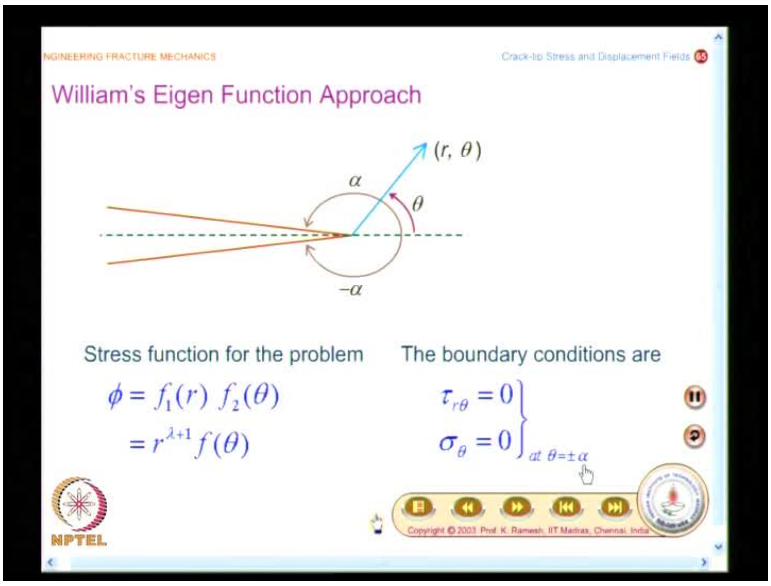

The success here is, you are able to get a series solution, which explains variation of maximum shear stress along the crack axis; and this is done in the complex domain, you know, this Westergaard solution was given in 1939 and Sanford's modification came in 1979; you also had another approach by William's, this was there in 1957, and he looked at the problem from a different perspective. In fact, he considered a wedge, he considered a wedge like this, whose surfaces are free and it had a remote loading. Suppose, I look at the wedge in such a manner, I make the angle in this fashion, make it as large as possible, when I make alpha equal to 180 degrees, and minus alpha equal to minus 180 degrees, it becomes a crack.

So, what he analyzed was, he wanted to analyze a wedge problem, this is what he published in 1952, which was later modified for crack problems in a paper in 1957; and what he took? He took crack surfaces that are free, unloaded crack surfaces, and he approached the problem from polar co-ordinates; and, for this, what he had taken the stress function? He is taken as function of r multiplied by function of theta, so, phi is given as r power lambda plus 1 function of theta; so, we will have to find out what is this function of theta and what is the value of lambda, and this is also known as William's Eigen Function Approach.

So, what you have is, you have lambda is known as an Eigen value, and for each one of these lambda, you will have a corresponding function, that is called the Eigen function. So, in all of these problems, you know, what you can do in the class is, to write the boundary condition; and, what we are going to do is, we are going to solve this problem in polar coordinates, so, I would be essentially evaluating sigma r, sigma theta and tau r theta in polar co-ordinates. So, I will have expression for sigma r, which would be expressed in terms of r and theta; that is a way we have always been looking at. Only in the Westergaard solution, we had Cartesian stress components, they are expressed in terms of r and theta, but here, you would have the stresses sigma r, sigma theta and tau r theta, expressed in r theta. Now, you will have to find out how to define the boundary conditions.

You know, I have already mentioned what happens on a free surface. On a free surface, the stress factor would be 0; but stress tensor cancel exists, so, when I have a radial line like this, I have given the clue by calling that as a radial line, which component of stress is permissible on this line. Whether it is sigma, theta or sigma r, you will have to be very careful about, that is, its sigma r or sigma theta; sigma r can remain, sigma theta cannot remain; this is the radial line, you have to note that. So, the boundary conditions for this problem are tau r theta is equal to 0 and sigma theta equal to 0, at theta equal to plus r minus alpha.

See, what we will do is we will write a very generic expression. After writing the generic expression, we will substitute alpha equal to pi, that would make the problem for crack; and, mind you, here, they are not discussing anything about what happens at infinity. The books say we are only talking about a uniform loading at that place. You are only worried about how the crack phases are. The crack phases are free, that is all you specify. Not only this, when you look at the solution, you will find this is a planar problem; the solution will automatically take you for combination of mode 1 and mode 2.

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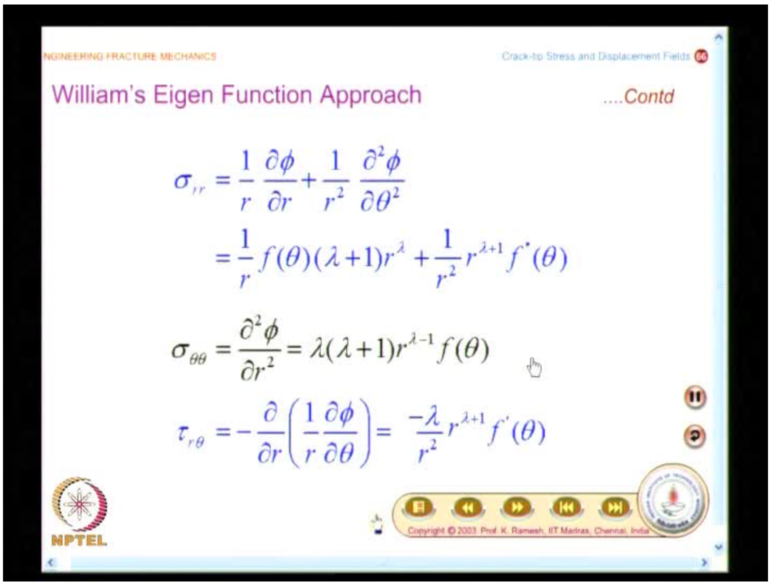

Now, let us brush-up our fundamentals in handling stress function in polar coordinates. If stress function phi is given, you can write sigma rr, and sigma rr is nothing but 1 by r dow phi by dow r plus 1 by r squared dow squared phi by dow theta squared, and we have already taken phi as r power lambda plus 1 function of theta, so, when I substitute this, the expression for sigma rr turns out to be 1 by r f of theta lambda plus 1 r power lambda plus 1 by r squared r power lambda plus 1 f double prime theta.

I think the double prime is not very clear. You have to write this carefully, f double prime theta; and we also know how to evaluate sigma theta, and sigma theta theta is given as dow squared phi by dow r squared that is equal to lambda into lambda plus 1 r power lambda minus 1 f of theta. Because, we have already assumed what is the nature of the function phi; once you know that nature and substitute it here, you get an expression of this form. We can also get the shear stress tow r theta, that is given as minus dow by dow r of 1 by r dow phi by dow theta, that is equal to minus lambda by r squared r power lambda plus 1 f prime theta.

See, we are writing boundary condition on what? We are writing a boundary condition on sigma theta theta and tau r theta. And, we already known sigma theta theta has to be 0, when alpha is specified, when theta is specified as some value of alpha. So, when you say, on the crack phase, sigma theta theta is 0, which implies what? From this expression, these quantities cannot go to 0, so, when I say sigma theta theta is 0, it implies function theta is 0; on the other hand, when I say tau r theta is 0, we will have f prime theta is 0.

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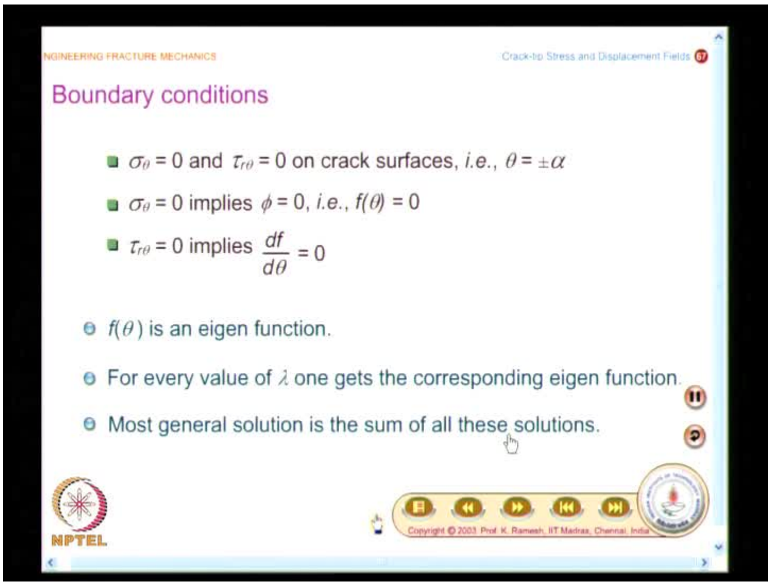

This is how he would use in writing the boundary conditions, and whatever I have mentioned is summarized here, on the crack surfaces, sigma theta equal to 0 and tau r theta is equal to 0, that is, when theta is equal to plus or minus alpha, when I say alpha, it is still a wedge only; if I say alpha equal to pi, it becomes a crack. So, you get, in one case, function theta equal to 0; in another case, the first differential of the function is 0. And as I mentioned earlier, you have to note down that f of theta is an eigen function, and on all these class of problems, this is how we write the most general solution; for every value of lambda, one gets the corresponding eigen function.

And the most general solution is the sum of all these solutions. So, what does this method guarantee? You are going to get a series solution, and you will also look at, that time, when William's reported, what way he reflected up on the series solution; that also, we have to look at it. Though it was developed in 1957 and people use this for writing certain boundary collocation answers, people have not really appreciated the t-stress; which we will have to reflect up on it.

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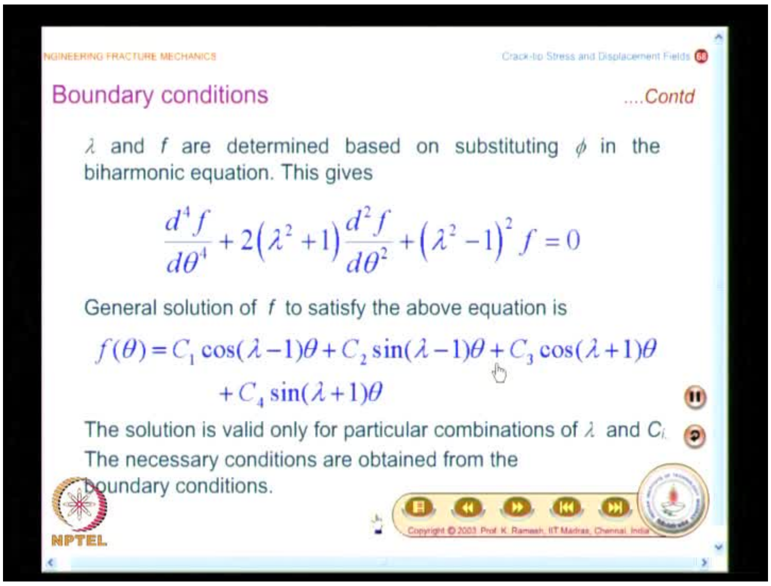

So, what you are going to have is the most general solution, is the sum of all these individual solutions; and we will write down the equations, you know, when I want to see that phi is a valid candidate, for stress function, it has to satisfy the bi-harmonic equation. And, this biharmonic equation, in terms of the stress function that we have taken, turns out to be d power 4 f divided by t theta power 4 plus 2 times lambda squared plus 1 d squared phi d squared f by d theta squared plus lambda squared minus 1 whole squared f equal to 0.

What we have done is, we know how we have taken the Airy's stress function, phi; you substitute it in the bi-harmonic equation and you get the final resulting equation in this fashion. So, by solving this, we will be able to find out the eigen values as well as the eigen functions; and the most general solution for this is to be given as f of theta equal to C 1 cos lambda minus 1 theta plus C 2 sin lambda minus 1 theta C 3 cos lambda plus 1 theta plus C 4 sin lambda plus 1 theta.

So, for every value of lambda, you will have a corresponding coefficients C I, and these coefficients have to be determined from the boundary conditions, and the most general solution will be the sum of all the individual solutions; and what we will have to do is, we have already looked at what is the meaning of sigma theta going to 0; what is the meaning of tau r theta going to 0? Now, we will adopt that we have the function f of theta, so, we look at f of theta as well as f prime theta, then you get the basic equations;

from that, you write the characteristic equation, find out the eigen values; that is how we are going to proceed; it is a very standard way of solving system of equations. We are not doing anything new, but if you follow the procedure, if you have that in your notes, you will feel comfortable when you review it later, that whatever you have done is mathematically rigorous.

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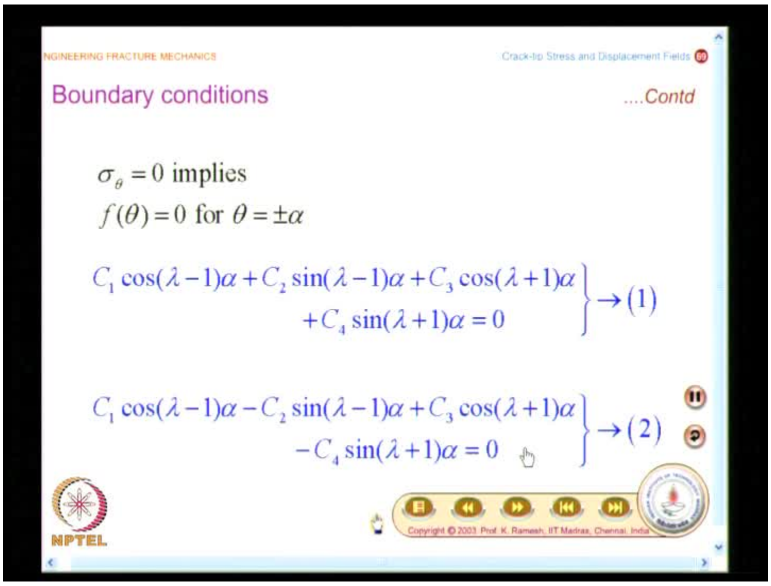

So, you will just apply the boundary conditions; I am going to say f theta equal to 0 for theta equal to plus or minus alpha, and these expressions are long. Please take your time to write down; I will read them for you C 1 cos lambda minus 1 alpha plus C 2 sin lambda minus 1 alpha plus C 3 cos lambda plus 1 alpha plus C 4 sin lambda plus 1 alpha 0. And, what I am going to do is, instead of plus alpha, I will make it as minus alpha; the sin terms will change; sin cos term will remains as such. So, I get the second expression as C 1 cos lambda minus 1 alpha minus C 2 sin lambda minus 1 alpha plus C 3 cos lambda plus 1 alpha minus C 4 sin lambda plus 1 alpha that is equal to 0.

So, what we are trying to do now is, we are trying to group the solution, find out how to estimate lambda, and then try to write the most general form of expression; still, we have not got what is the form of phi. We are going towards writing out the function phi in the most general fashion with coefficients; those coefficients have to be determined from your experimental model or from your numerical model.

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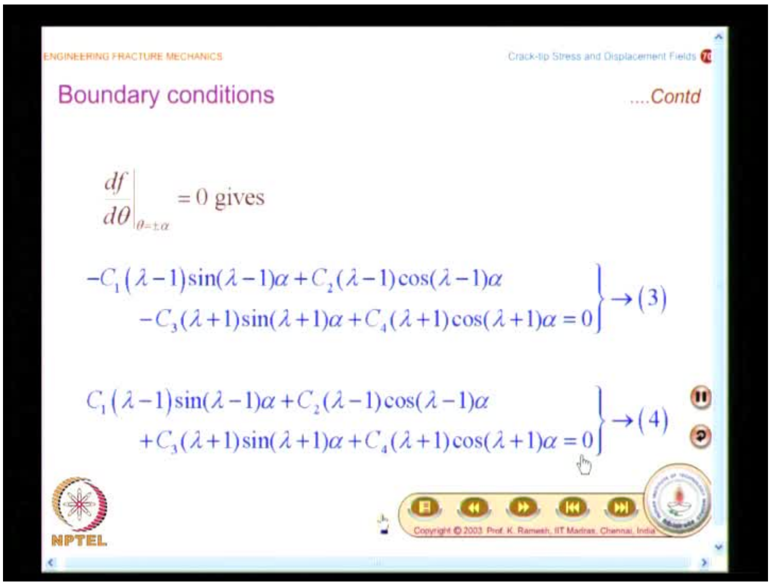

And the next expression what you have, is the first differential of the function f is 0; f is 0, when theta equal to plus or minus alpha. We have considered those phases as free, so, I have that as minus C 1 lambda minus 1 sin lambda minus 1 alpha plus C 2 lambda minus 1 cos lambda minus 1 alpha minus C 3 lambda plus 1 sin lambda plus 1 alpha plus C 4 lambda plus 1 cos lambda plus 1 alpha equal to 0. The next equation is C 1 lambda minus 1 sin lambda minus 1 alpha C 2 multiplied by lambda minus 1 cos lambda minus 1 alpha plus C 3 lambda plus 1 sin lambda plus 1 alpha plus C 4 lambda plus 1 into cos lambda plus 1 alpha equal to 0.

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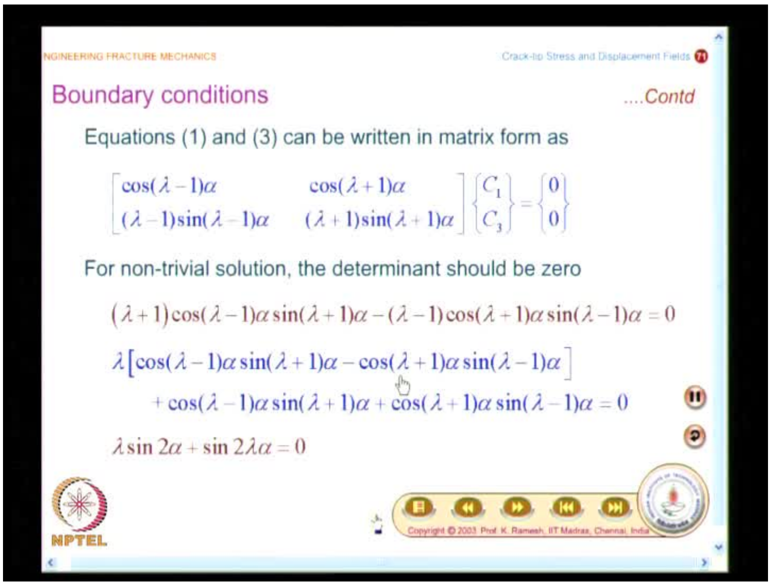

Now, you have to do some algebraic manipulations and you could group them into two categories, set of equations involving C 1 and C 3, and C 2 and C 4; if you do some algebraic manipulation, you could do that; and, after doing that, I get the expressions in this fashion. So, what I get after algebraic simplification is, you take this as algebraic simplification; it will not directly come from this after algebraic simplification, only it can be written in this fashion. So, I have this as matrix cos lambda minus 1 alpha cos lambda plus 1 alpha lambda minus 1 multiplied by sin lambda minus 1 alpha lambda plus 1 sin lambda plus 1 alpha C 1 C 3, and the right hand side is 0.

And, from your system of solving equations, when you have homogeneous equation for non-trivial solution, what is it that you have to do? That determinant should be 0; that determines what would be the value of lambda, and when you write the determinant, it turns out to be like this; it looks very long, but, if you group them properly, you would be able to write a simple expression. And, you multiply these 2 minus of these two, so, that is how these expression is written; and this could be rewritten in this fashion, I have lambda into cos lambda minus 1 alpha sin lambda plus 1 alpha minus cos lambda plus 1 alpha sin lambda minus 1 alpha.

So, I have something like cos a sin b minus cos a sin, so, when I have something like this, it is possible for me to simplify further. I have this as simplified to lambda sin 2 alpha plus sin 2 lambda alpha equal to 0; you have to read this as cos a sin b minus cos b sin a, because, lambda plus 1 is taken as, suppose I take this as b, I should say that this as cos b sin a, so, if you use that rule from trigonometry I, can simplify this as lambda sin 2 alpha plus sin 2 lambda alpha equal to 0.

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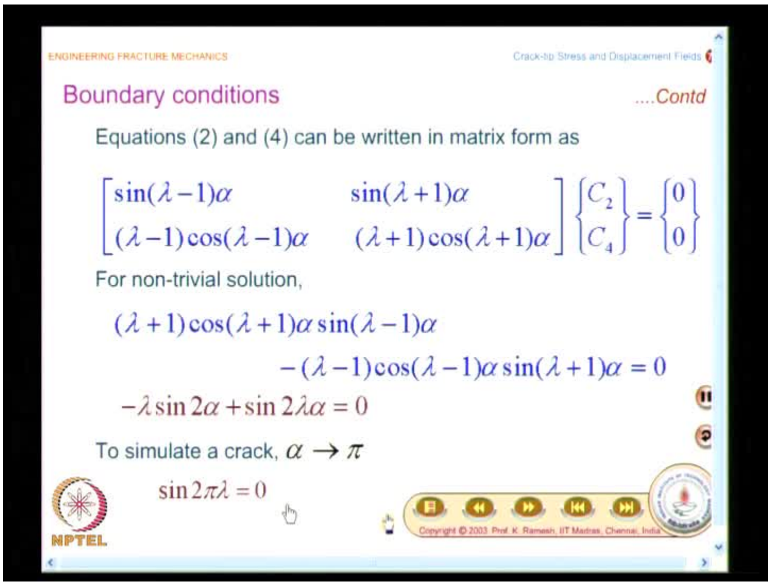

I would have another set of expressions, you know, this is one set of conditions I have got relating C 1 and C 3, because, I need to solve for all the four coefficients; and, when I group the coefficients C 2 and C 4, I could catch this basic equation sin lambda minus 1 alpha sin lambda plus 1 alpha and you have lambda minus 1 cos lambda minus 1 alpha lambda plus 1 cos lambda plus 1 alpha multiplied by C 2 C 4; that is 0,0 here. For non-trivial solution, you have to have the determinant, should go to 0; and which could be written in a fashion convenient for simplification. You group all the terms involving lambda and just take out this one group; all those terms from that, it is possible for you to simplify this as minus lambda sin 2 alpha plus sin 2 lambda alpha equal to 0; see, till then, the development is for a wedge also.

Now, what we will do is, we will substitute what happens to this equation. When alpha goes to pi, when alpha goes to pi, you get from both this expressions; this is the expression we have seen. The early expression was lambda sin 2 alpha, so, from both these expressions, you will get the final expression as sin 2 pi lambda equal to 0, so, by solving this equation, you would be able to get the Eigen values.

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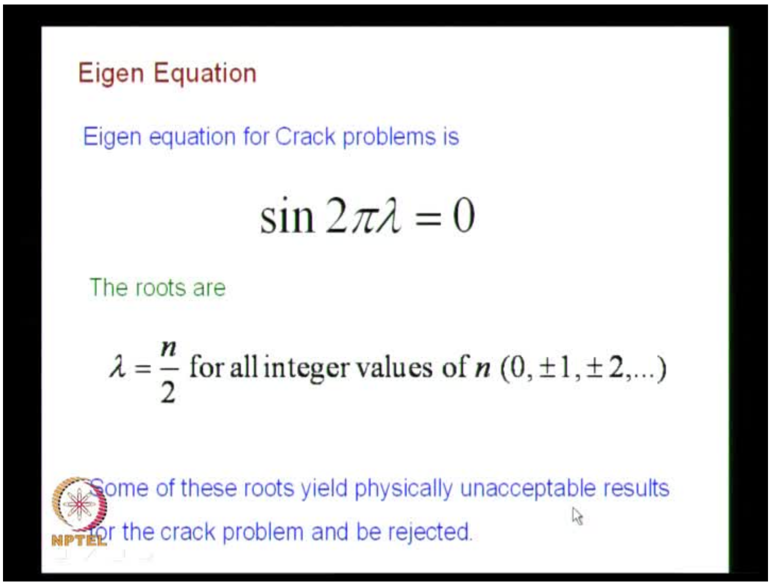

So, let us look at what are the roots of this equation; the roots are lambda equal to n by 2 for all integer values of n, 0 plus or minus 1 plus or minus 2 and so on. Now, we will have to find out of this, which are the roots are admissible. See, we are doing a mathematically rigorous procedure in the solution, so, at the end of it, whatever the solution you get, is going to sacrosanct and you can take comfort that mathematics has been rigorous. We are not just jumping into the solution by saying lambda equal to n by 2, and I have n 0 plus or minus 1 plus or minus 2 all roots. Let us look at what happens for n equal to 0 and negative values of the roots, and we will have to investigate whether any of them yield physically unacceptable results. If they yield physically unacceptable results, you have to discard them.

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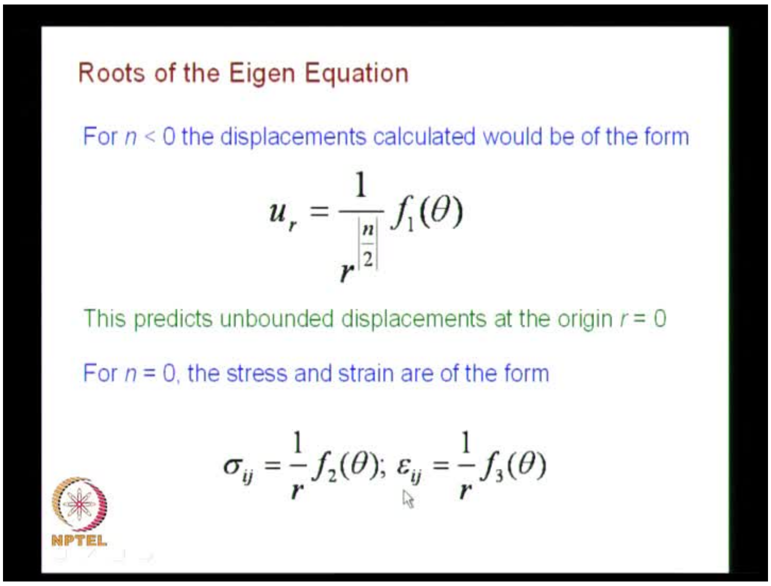

For n less than 0, the displacements calculated would be of the form u r equal to, mind you, we are dealing in polar co-ordinates, so, you will get u r, and u theta u r is of the form 1 by r power modulus of n by 2 multiplied by f 1 theta. So, what happens when r goes to 0, displacement is unbroken; in fact, while we were discussing Westergaard solution after stress field, we saw the displacement field; I drew your attention the stresses become singular at the crack-tip; whereas, the displacements are bounded, we had only root of r available in the numerator for displacement. So, when r goes to 0, displacement goes to 0. Whereas, if you admit negative roots, you find that displacement u r is unbounded, so, this has to be discarded. Suppose, you take the case for n equal to 0, what happens? The stress and strain are of the form sigma ij equal to 1 by r f of theta; this is f 2 theta, some other function of theta epsilon ij equal to 1 by r f 3 theta.

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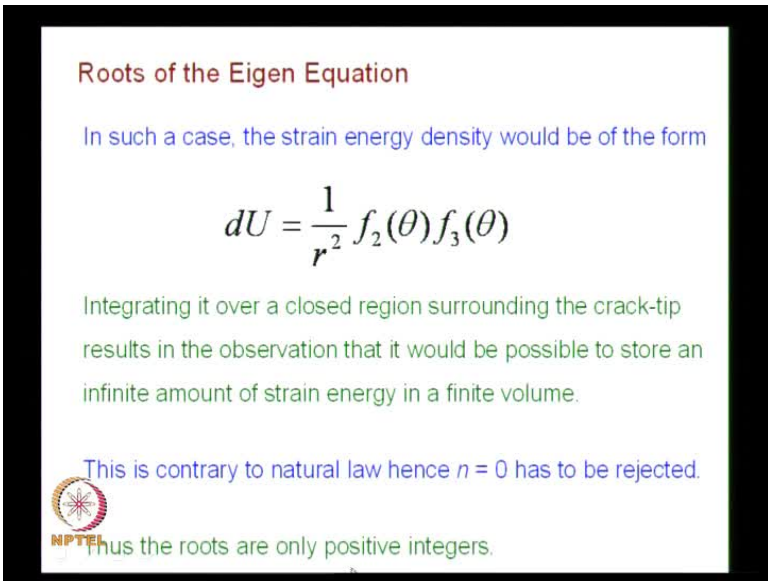

Once I know sigma as well as strain, again find out the strain energy density; and, if I look at the strain energy density, it would be of the form dU equal to 1 by r squared f 2 theta, f there theta. So, what is the implication? Suppose I take a closed contour, if I integrate it over a closed region surrounding the crack-tip, this results in the observation that it would be possible to store an infinite amount of energy in a finite volume, which is not possible. If it is not possible, what should we say? We should say n equal to 0 is not an admissible root; that is the conclusion. So, finally, what you get? The roots are only positive integers, so, we have looked at the basic Eigen equation, and from that, you have to find out the eigen values characteristic equation you have got from this; you have to get the eigen values, and based on these arguments, we find that the roots can only be positive integers.

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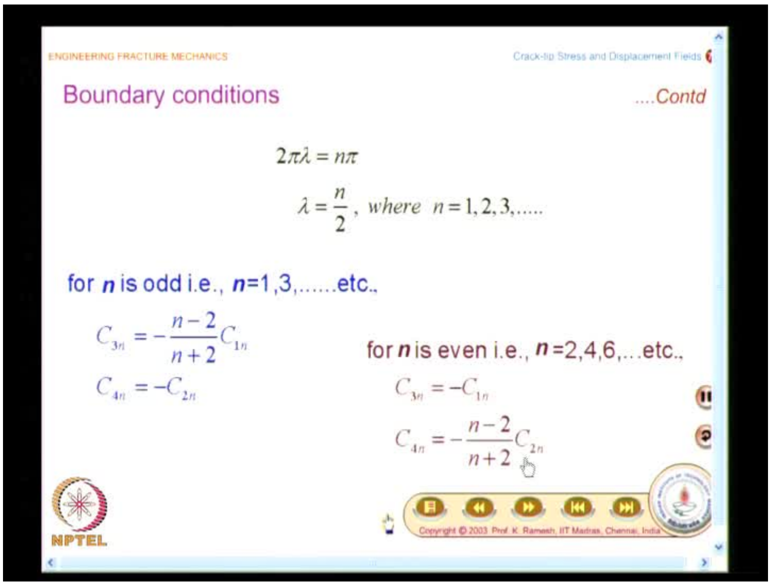

So, I have the boundary conditions, 2 pi lambda equal to n pi and lambda equal to n by 2, where n equal to 1, 2, 3; you cannot have negative roots, you cannot have 0 also; and, what you also find is, the coefficients or not totally independent, there is an inter relationship when n is odd; when n equal to (1,3), etcetera, you get C 3n equal to minus n minus 2 divided by n plus 2 C 1n, and C 4n equal to minus C 2n, when n is even, that is, n equal to 2 (4,6), I get C 3n equal to minus C 1n and C 4n equal to minus n minus 2 divided by n plus 2 into C 2n. So, now, we are ready to construct the Airy's stress function in the most general form for the problem that we have taken. I think we will do that in the next class, so, in this class, what we looked at was, we looked at generalized Westergaard equations and I pointed out by looking at Westergaard, modified Westergaard that is done by Irwin.

When generalized Westergaard, one way of observation is by going to the generalized formulation, you get a larger zone for you to collect data from the experiment which could be interpreted in the case of Westergaard. The zone of data collection is very small, the zone becomes slightly enlarged; in the case of Irwin, it becomes slightly more enlarged in the case of higher order solution. When you have generalized Westergaard equation, it can also be simplified to other two cases, so, that is why it is called generalized Westergaard equation. But, whatever the stresses that you have got, they were looking very clumsy, but we will see later.

In the next class, you would find out some identity between generalized Westergaard as well as William’s eigen function, and we will bundle them, put it in a nice fashion.

Thank you.