Engineering Fracture Mechanics
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Lecture No. # 19
Stress Field in Mode - 2

In the last class, we had looked at what is crack opening displacement, followed by two methodologies to find out the energy release rate. In the first methodology, we took up a specific problem and evaluated the crack surface displacements, using that energy release rate was calculated. In the second case, we have looked at the basic definition of energy release rate, as the energy released per unit extension of a crack front per unit thickness per crack-tip.

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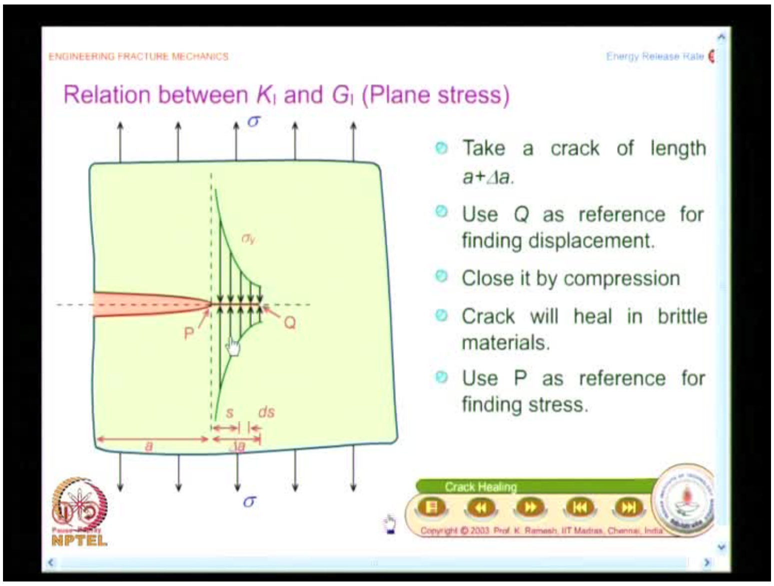

So, we have looked at the problem in a much more fundamental fashion, then we evaluated the relation between the stress intensity factor K and the energy release rate G. And if you watch the animation that gives you what is the kind of approach that we have adopted to establish this interrelationship. What you have actually looked at is you have looked at a small extension of crack-tip delta a instead of… it is extending from a to delta a. We have looked at the case of taking a crack of length a plus delta a and closing a portion of the crack-tip, we justified in the case of brittle materials healing has been observed. So, we are really talking about reversible processors. So, whatever the energy required to close that small extension of the crack would be the energy released when the crack extends by a small amount.

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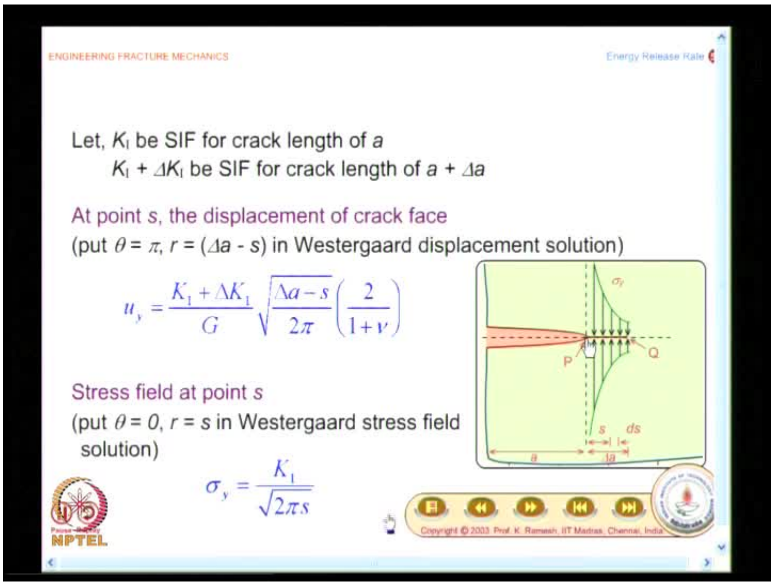

Then we looked at the mathematics. So, in the processes, you have been able to utilize the mode one displacement field as well as the stress field, appropriately used in writing the energy expression. So, you have to be careful in specifying, what is the value of theta when you are trying to find out the displacement. For the displacement, we have taken Q as the origin. So, theta turns out to be pi, whereas for the evaluation of stress theta is just 0 degrees, because we are taking P as the origin for evaluating the stresses.

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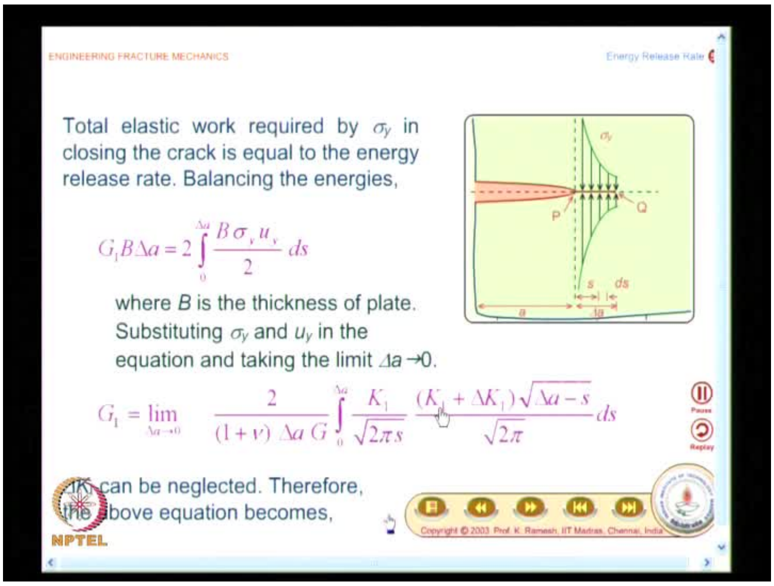

So, using this you could write the energy expression based on fundamentals, then we also simplified for small extension of crack lengths, though there would be a change in the stress intensity factor, we would neglect delta K 1 in comparison to K.

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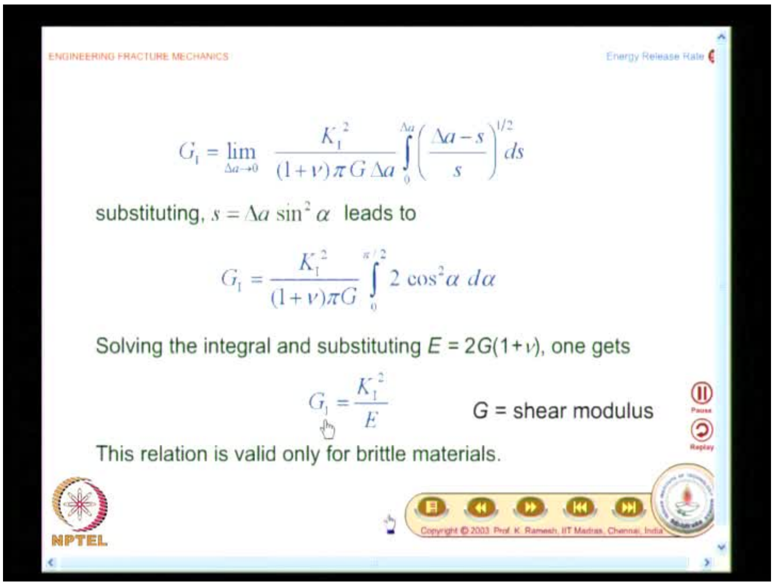

So, upon simplification, you get a very important result which interrelate energy release rate G as well as the stress intensity factor and you know there is also expression written here; G is shear modulus, you also use G with a suffix for representing energy release rate.

In fact, if you look at some of the earlier publication, in order to distinguish between the two, they even use a stylish G to denote the energy release rate, I think with the context in mind and also these appropriate suffixes. It is possible for you to distinguish are we talking about energy release rate or whether the G is refereeing to shear modulus.

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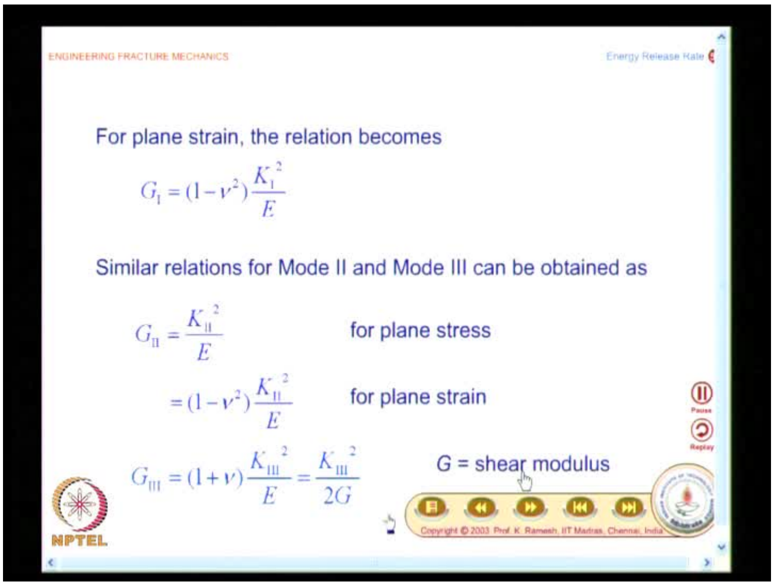

Now, we have developed this for the case of a plane stress situation. I have already mentioned earlier, once you develop expressions for plane stress with a simple modification, it is possible to get the relevant quantities in plane strain. If I have the young’s modules you replace young's modules E by 1 minus nu squared. So, if I do that I get for plane strain situation the relation is G 1 equal to 1 minus nu squared K 1 squared by E. So, you know for plane stress as well as plane strain for the case of mode 1 loading. What is the interrelationship between energy release rate and stress intensity factor? Once you have developed for mode 1 on a similar fashion, you could also get the energy expression for mode 2 as well as mode 3. In this class itself, we will try to develop the stress field in mode 2 as well as stress field in mode 3. So, in those cases we will have K 2 and K 3 replacing the role of K 1.

So, we have to find out the interrelationship between G 2 and K 2 G 3 and K 3. So, for the case of mode 2 situation in plane stress, the expression turns out to be K 2 squared by E; very similar to what you have for mode 1 for the case of plane strain. This turns out to be 1 minus nu squared K 2 squared by E again same as mode 1 and for mode 3, there is a slight difference you have G 3 related to K 3 s 1 plus nu times K 3 squared by E that is equal to K 3 squared by 2G and you have to keep in mind G is a shear modulus.

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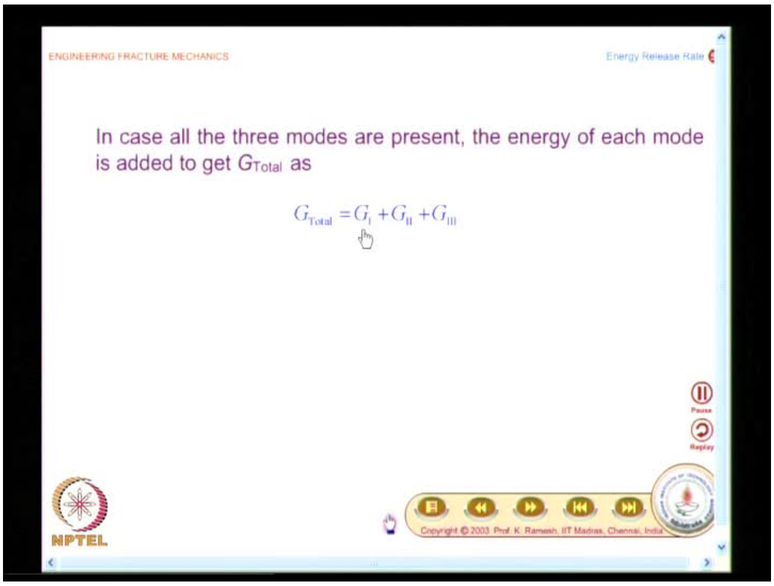

So, now we have expressions relating energy release rate and stress intensity factor for all the three modes of loading. We have looked at them separately in a generic situation, you would have all the three modes appearing with different proportions, so in such a case, you will have G total as G 1 plus G 2 plus G 3 - see mind you - it is an energy quantity. So, I can add all three of them I cannot add K 1 plus K 2 plus K 3 I cannot do that when I use principle of superposition if different loads provide mode 1 loading then I can add for those sub problems K 1 a plus K 1 b plus K 1 c so on and so forth. In fact, we would have a separate chapter on evaluating stress intensity factor for finite bodies, where we would use principle of superposition and we would handle that carefully.

Being an energy quantity, when I have individual energies, I could comfortably add. In fact, some of the finite element course provide, you the total energy rather than energy pertaining to G 1 that is mode 1 or mode 2 or mode 3. Then they have to do special efforts to segregate them and then evaluate the stress intensity factors separately.

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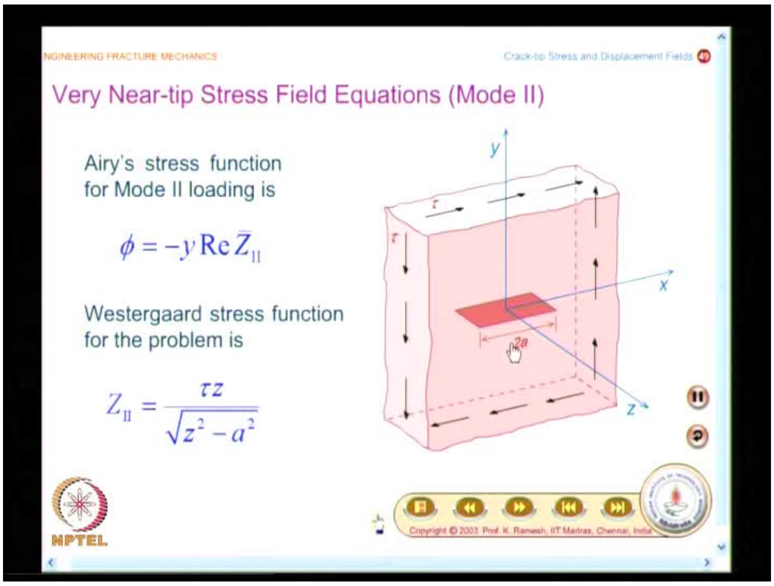

So, now, we move on to find out the stress field in the case of a mode 2 situation and you know our focus is very clear we are going to evaluate very near-tip stress equations and you have a picture shown here, where you have a shear stresses on the boundary, it is an in plane shear stress and what is depicted here is we are talking about a through the thickness crack. The crack is present for the entire thickness of the body they are very simple to analyze that is a reason we are doing it and you have a shearing mode in plane shear mode. The crack surfaces shear like this and you have to note down, how the axes are marked, you have the x-axis and y-axis form in the plane of the specimen and there z-axis is perpendicular to that. You know this problem is very similar to what you could do for mode 1, we have already learnt exhaustively for mode 1, here I am gone to only mention the salient results I am not going to derive them you could take that as an exercise in your rooms and verify whether the final results are ok.

In all this problems, what is the first step? The first step is to know, what is the Airy's stress function for the problem? For mode 2 loading it is given as phi equal to minus y times real part of Z 2 bar, because now we are confining our attention to only mode 1, mode 2 and mode 3.

So, this is the Westergaard's stress function, we have to look at how this stress function is coined and one of the advantages. When you are using an analytic function is that all analytic functions are candidates for Airy's stress function, they will satisfy the bi-harmonic equation and the beauty here is the Westergaard's stress function for the in plane shear stress problem turns out to be as simple as this what is the change here between these Westergaard's stress function for mode 1 and mode 2; only the stress is change from sigma to tau. So, that is the advantage so, he struck gold mind. You know he was able to go in a stress function for mode 1 with slight modification, he could solve mode 2 situations with slight modification further one can also solve mode 3 problems. Though the formulation would be based on displacement, the form of the stress function remains simple and same only. The stress quantities change in each of them and another important issue you have to keep in mind is you are analyzing for all the situations through the thickness crack and your crack front is straight. If the crack front is straight then the stress intensity factor remains constant on that front. Suppose, I have a penny shaped crack which was solved by Sneddon around the same time, when people where focusing more on fracture mechanics. The 3-dimensional problem of a penny shaped crack in a solid was solved by Sneddon. There, you had a curved crack front, so the crack front here is straight, you have to keep that in mind these are all very simple situations for us to analyze easily.

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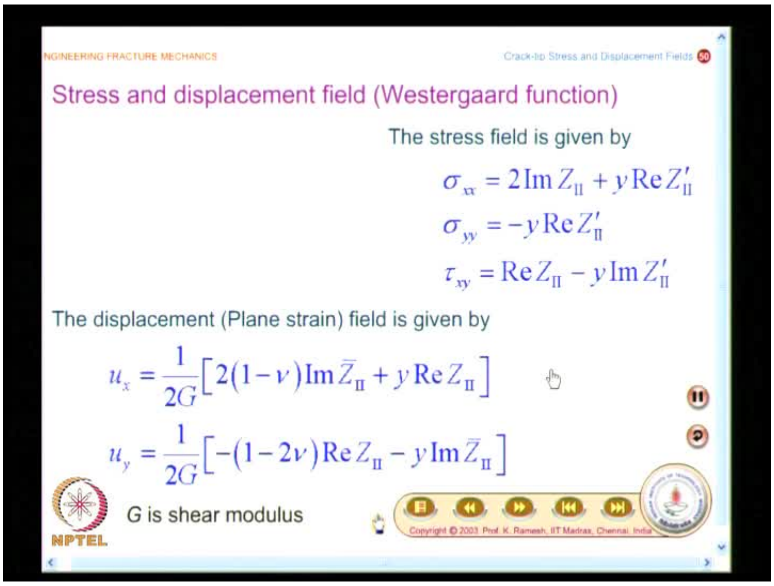

As mentioned, I am not going to verify whether the Airy's stress function satisfies bi-harmonic, if you look at the form of the function, it would easily satisfy. I leave that as an exercise to you. So, based on that as we have done for the case of mode 1, we get the expression for sigma xx sigma yy tau xy. In terms of the stress function, now we have a choice, whether we want to look at the stress field or displacement field with crack center as the origin or crack-tip as the origin, depending on the way I express the stress function. And when you express it in this fashion this is with respect to the center of the crack as the origin. So, I have sigma xx equal to 2 imagine part of Z 2 plus y real part of Z 2 prime sigma yy equal to minus y times real part of Z 2 prime tau xy equal to real part of Z 2 minus y imaginary part of Z 2 prime. So, we have the stress field and next one is you will have the displacement field and this is for the plane strain situation is given. The displacement field u x equal to 1 by 2G multiplied by 2 into 1 minus nu into imaginary part of Z 2 bar plus y real part of Z 2 and the displacement component u y is given as 1 by 2G multiplied by minus of 1 minus 2 nu into real part of Z 2 minus y imaginary part of Z 2 bar. See, though it appears tiring to write such long expressions after you complete the course, you will find that this information is quite useful, because you will find you have a compendium of all the relevant information for you to carry forward. So, it is worth writing this expression.

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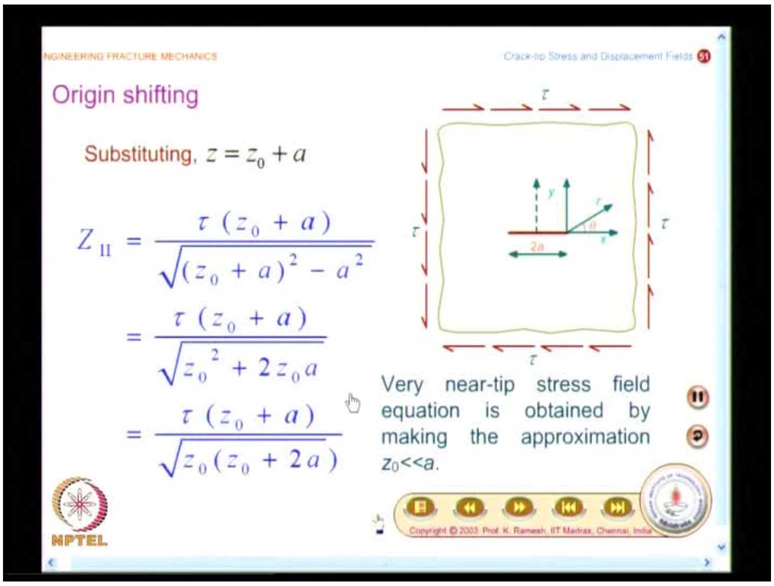

So, in the case of mode 1 what we did? We got the stress field then the next step was shift the origin that means from center of the crack to the tip of the crack. We would do this for mode 2 as well as mode 3, for all the cases, we would shift the origin to the crack-tip that brings in the approximation that brings in whatever the stress field that we get is valid only in a zone very close to the crack-tip, how close? It is problem depended.

Now, this is the settle point there are no exact values to say the solution is valid till what distance, the question of how close is left to the problem under consideration.

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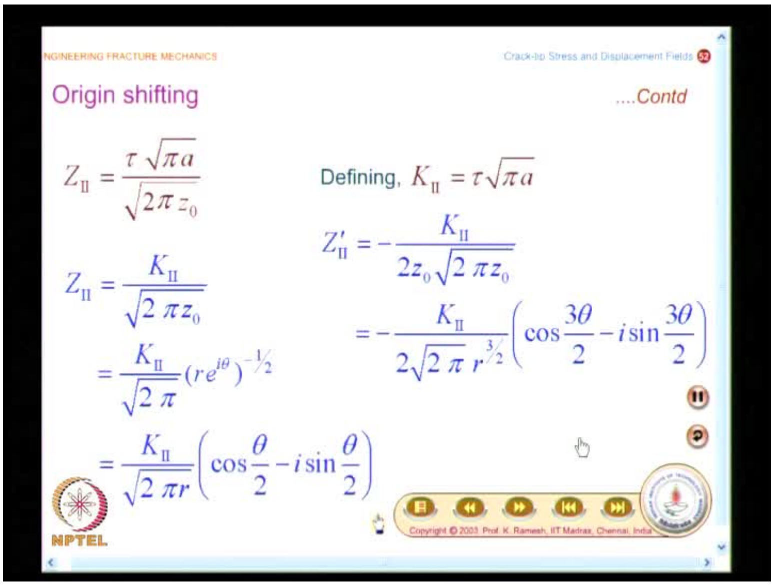

So, you shift the origin from the center of the crack to tip of the crack and the approximations are very similar, like what you have done for a mode 1. So, instead of z naught plus a you would say z naught is very small and the stress function turns out to be like this tau into root of pi a divided by root of 2 pi z naught. This is what you had seen in the case of mode 1 and here again, we would coin the definition of stress intensity factor K 2 and K 2 is defined as tau of root of pi a. So, I can recast this expression as Z 2 equal to K 2 by root of 2 pi z naught which could be modified as K 2 by root of 2 pi multiplied by r e power i theta whole power minus half.

From your knowledge of such functions, you can recast them like cos theta by 2 minus i sin theta by 2. Now, finally, we would like to get the expressions in terms of r n theta; this I had already pointed out we would be essentially evaluating Cartesian stress components, but these are calculated based on the position r n theta because complex numbers are easily representable as r e power i theta. So, that is the reason why we are writing it in terms of r n theta.

So, for us to find out the stress field, I need to have Z 2 as well as Z 2 prime and Z 2 prime turns out to be like this minus of K 2 divided by 2 root of 2 pi r power 3 by 2 multiplied by cos 3 theta by 2 minus i sin 3 theta by 2.

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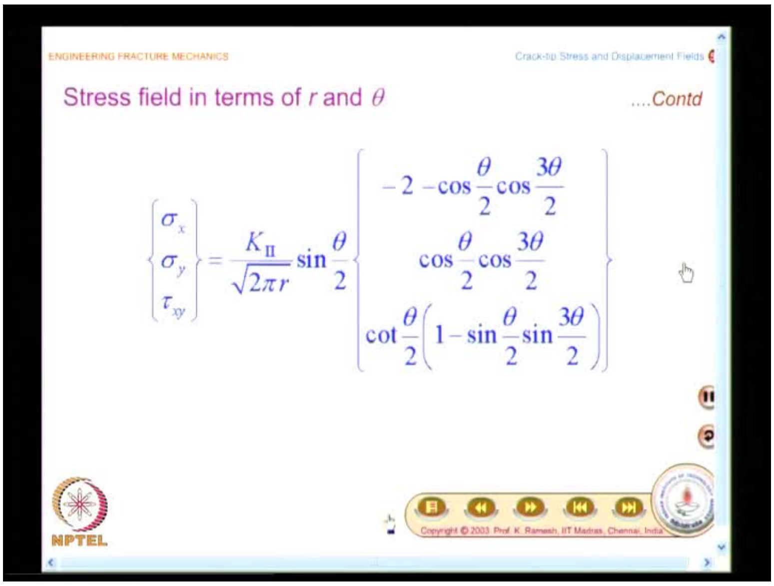

You know half way in the course at least the singular stress field equations you would get it by heart, you would be seeing them again and again and since the form is similar, it is easier for you to remember them. The singular solution at least higher order solution be difficult to remember, you need to look at the hand books or your notes but singular solution is easy for us to remember.

And the very near-tip stress field see, the whole section was started as very near-tip stress field here. It is just mentioned as stress field in terms of r and theta, but take it as very near-tip stress field. So, I have sigma x sigma y tau xy that is related to that is given as K 2 divided by root of 2 pi r sin theta by 2 and you have for sigma x minus 2 minus cos theta by 2 cos 3 theta by 2 for sigma y cos theta by 2 cos 3 theta by 2 for tau xy cot theta by 2 multiplied by 1 minus sin theta by 2 sin 3 theta by 2. And if you want to be mathematically precise, you will have addition of higher order terms which are not possible from your Westergaard's solution; we will have to look at other methodologies and fill in the gap, because a simple binomial expansion like what Tada parries and Irwin did what not fruitful. So, we will have to come back to that discussion again.

Where in the case of mode 2, we will quickly see after looking at the displacement field we will see the isochromatics. Let us see, how the analytical expressions provide the isochromatics and what way they compare with experiments. We will have a different type of story here in the case of mode 1, we found that Westergaard singular solution was not sufficient to capture even for simple short cracks. So, you will have to look at this stress field more closely and one of the aspects you can look at is that distribution is dictated by this and this strength of each of this magnitudes is dictated by only one single parameter; see this is something very unique to fracture problems, this was identified by Irwin in 1757 and 1957 again is a very crucial year in the development of fracture mechanics many good fundamental papers were published during that year. And something very unique see if you have a different types of loading with finally result in plane shear mode or opening mode or tearing mode in all these cases the distribution does not change.

The distribution of mode 1, mode 2 and mode 3 will be different but if you look at mode1. Mode 1 could be caused by different types of loading - for all of those loadings, the distribution would remain same strength is dictated by the relevant K, which is not seen if you have other type of this continuities, if we have a cutout the story is different. So, this is something unique and that is why the near field or very near-tip stress field equations are very popular, because if you are really looking at crack propagation, which is very, very important from fracture mechanics point of view it is dictated by what happens in the very near vicinity.

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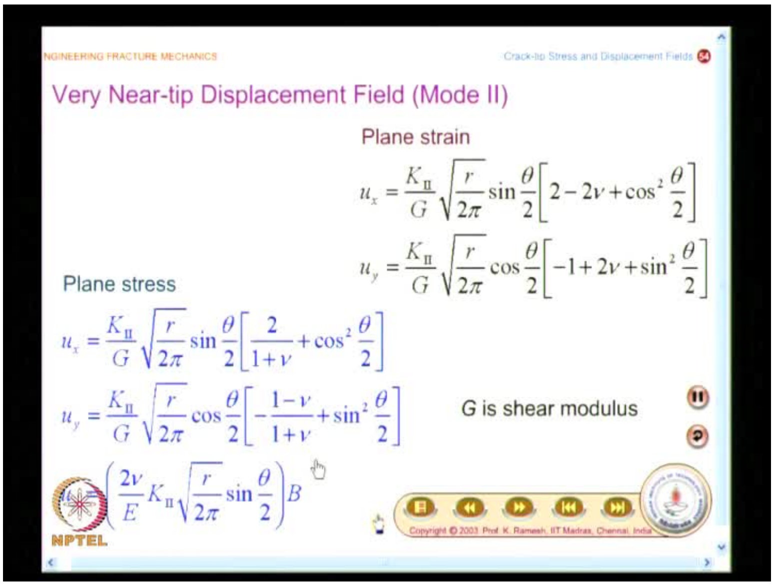

See that is a reason, why we are looking very close to the crack-tip after seeing the stress field, we will now move on to displacement field. These are again long expressions please be patient to write them, I have u x equal to K 2 by G root of r by 2 pi sin theta by 2 multiplied by 2 minus 2 nu plus cos square theta by 2 and what you find here like what we had seen in the case of mode 1 here again and r tends to 0. The solution bounded that is how displacement should be and you looking at displacement solution it cannot have infinite values like what we had got for stresses.

And stresses also we said for real materials it would reach the yield stress or it might for card something like that will happen so, ultimately it will have some finite value. And you have the expression for u y that is given as K 2 by G root of r by 2 pi cos theta by 2 minus 1 plus 2 nu plus sin squared theta by 2 and I will also write the expression for plane stress and for plane stress, you will also have the u z displacement component. In fact, in some classes earlier you had seen for the case of a mode 1, how near the crack-tip a dimple gets form - that indicates two things, it indicates the stresses are very high and near the crack-tip the Poisson effect is very significant, because the stresses are very high whatever the expression you get because of Poisson effect they appear very strong and you have u x equal to K 2 by G root of r by 2 pi sin theta by 2 multiplied by 2 by 1 plus nu plus cos squared theta by 2.

Then u y equal to K 2 by G root of r by 2 pi cos theta by 2 multiplied by minus 1 minus nu divided by 1 plus nu plus science squared theta by 2 and u z equal to 2 nu by E multiplied by K 2 root of r by 2 pi sin theta by 2, if you are considering a plate of thickness B you have to multiplied by B.

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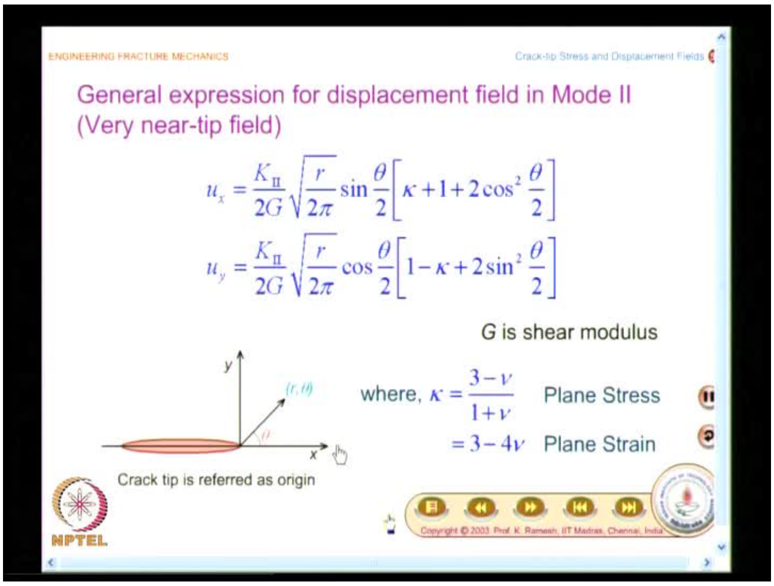

So, this gives you the displacement field and this is referred with respect to the crack-tip as the origins - make a note of it - with crack-tip as origin. These expressions are given and you could also a bundle these two and get a common expression, books give you these information in all these ways. So, you need to have them in your notes. So that when you take up a research paper you would be able to grasp, what is mentioned in the paper, if you know the fundamentals, depending on the way it is written you would have an advantage I have u x as K 2 by 2G root of r by 2 pi sin theta by 2 multiplied by kappa plus 1 plus 2 cos squared theta by 2 then u y equal to K 2 by 2G root of r by 2 pi cos theta by 2 multiplied by 1 minus kappa plus 2 sin squared theta by 2 and kappa is given as 3 minus nu divided by 1 plus nu for plane stress 3 minus 4 nu for plane strain.

And you have a pictorial representation that the displacement field is given in such a manner that crack-tip is taken as origin. And this is very important, I am emphasizing it again and again it may look very trivial to you when you take up book only then you will realize people give it either as a tip or as a center you will also have to distinguish whether the solution is for plane stress or plane strain these are all very important from practicing point of view, if you want to practice fracture mechanics, you want to get the solution as quickly as possible.

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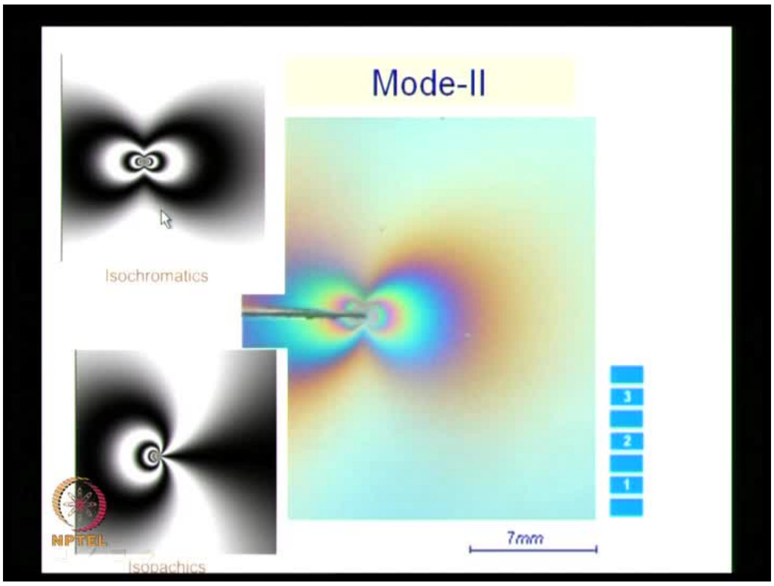

Now, we move on to investigating what is the kind of stress field that I get in an experiment for a mode 2 situation, and how does the Westergaard solution compare with it; we have already made a sketch of this in one of your earlier classes, these are the photo elastic fringe patterns seen for a mode 2 situation, and these are isochromatics they are recorded in color and I will show the Westergaard solution which is shown in black and white see in photo elasticity, if we use a monochromatic light source, I would only see black and white fringes, if I use a white light source which is a multi wave length source, then I will see fringes of different color. So, do not confuse that experimental fringes are colorful Westergaard solution is black and white. So, Westergaard solution is useless do not make to such drastic conclusions, here we have only looking at geometry features.

And look at this what is the first observation, do you agree the singular solution is good enough to model the fringe feel seen in an experiment, definitely so. The geometrical features of the theoretically calculated fringes as well as experimental fringes match very well, absolutely no problem - see there is no issue of you have bi-axial loading or uni-axial loading none of those confusions exist here and even when you later look at the multi parameters stress field solution, you would find for mode 2, the second terms is 0.

So, only in that context I am going to go back to mode 1 and then see what is the way the second term as influenced a change in the geometrical appearance of photo elastic fringes, see when I say photo elastic fringes have provided that geometric change - you should not think the time bi-est because, I am a photo elastician, I always harp on photo elasticity for anything, if you look at holography which gives the sigma 1 plus sigma 2 I get the fringe pattern like this. You will appreciate, how photo elasticity is good, only when I take a comparison for a mode 2 situation it gives you a fringe pattern - something like this. And this is as interesting, like what you get for isochromatics.

So, there is no problem; and in the case of mode 2 there is also no confusion of second term being present, if I take up a case of a mode 1 then I will have to investigate how isochromatics are and how isopachics are kindly make a neat sketch of these geometrical features, you do not have to shade them or anything just get the fringe pattern, the way it appears. And that would be very useful, see when you come across some photo elastic fringes related to fracture, you can associate what type of mode of loading it represents. And this also gives you a comfort that what we have been discussing as Westergaard solution are indeed good, because from fracture mechanics point of view what happens in the near vicinity of the crack-tip is more important then what happens a distance as away, from that point of view the Westergaard solution is still quite useful. The singular solution itself is good enough, when you are looking at the mode 2 situations.

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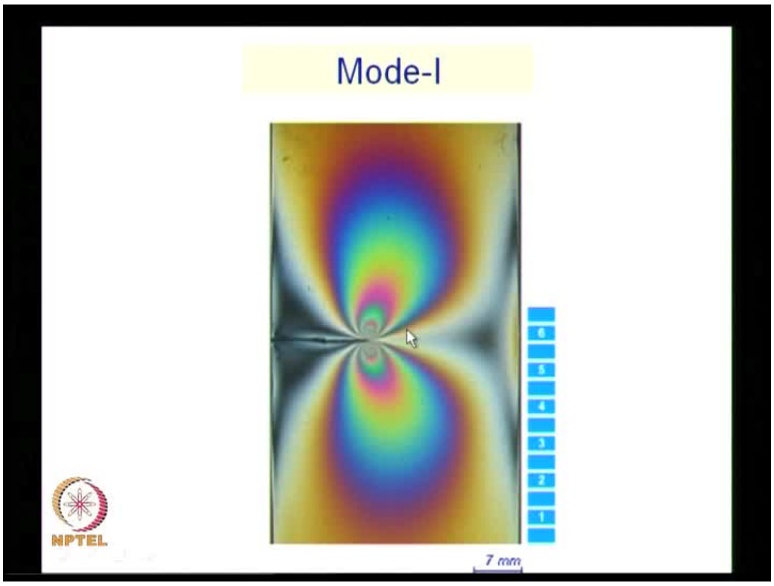

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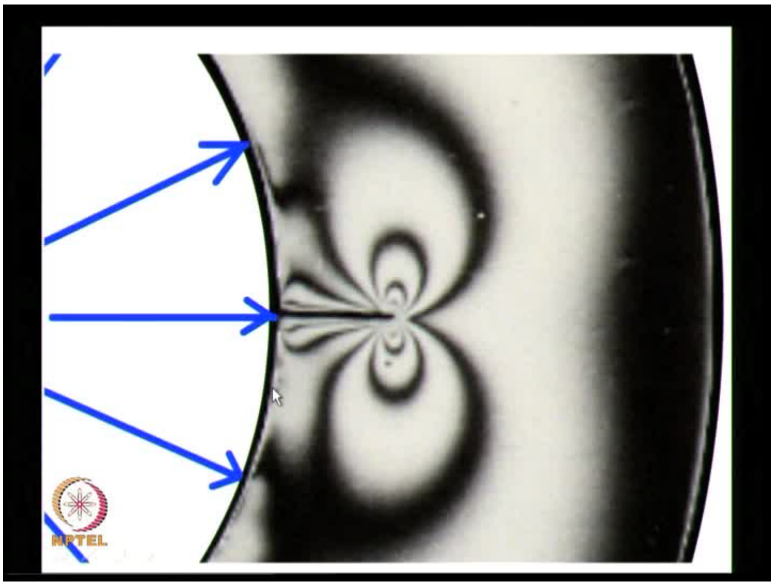

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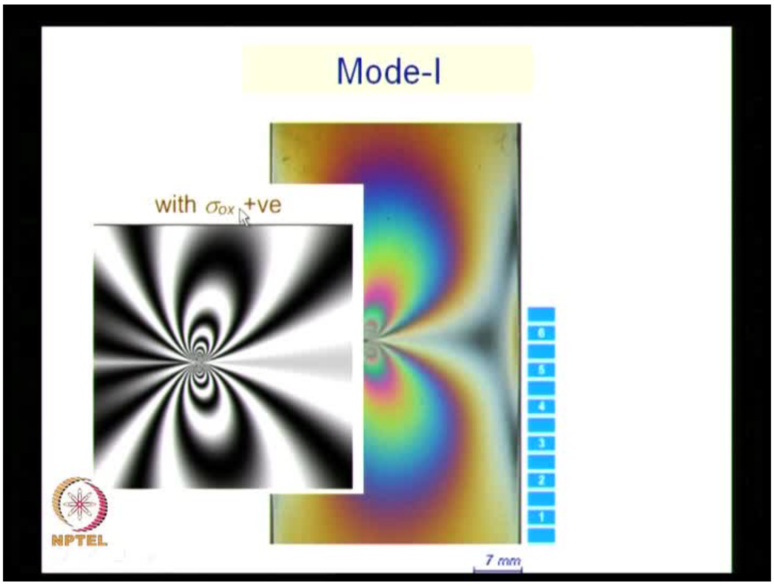

Now, let us go back and see, what happens in mode 1? The experimental fringes are forward tilted this is one case we had seen. Another case, We had seen that it was backward tilted, in this fringe backward tilted the second fringe and the third fringe is forward tilted and this is the crack that was appearing in a pressurized cylinder and we would see what way the sign are the second term which we are called it as correction to sigma x stress term as sigma naught x.

And I mention in analytical literature and numerical literature they call it as t stress. Originally it was introduce by Irwin, while discussing the results obtained by wells and posed way back in 1957. He introduce the sigma naught x and it appears as minus sigma naught x, when I say sigma naught x is positive, we are only talking about minus outside then sigma naught x that sigma naught x is positive. So, the net result would be minus of sigma naught x and what you find here is the fringes are forward tilted and the tilt angle becoming 90 degree as we go close to crack-tip, which is comparing very well with the experimental fringe pattern.

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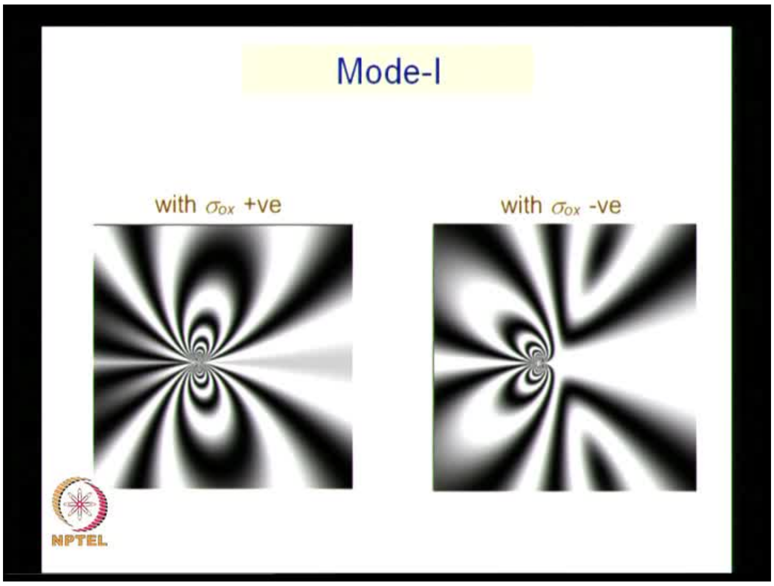

Now, let me change the sign of sigma naught x, what do you find - look at the fringes, you have to look at the fringes very closely; see no other experimental technique was so sensitive by just changing one sign of the term the entire picture changes. In this case it is forward tilted when I just change this sign it is backward tilted. You know automatically when people look at this kind of different fringe patterns even if they do not want to think the fringe patterns will make them ask a question, why it is behaving like this?

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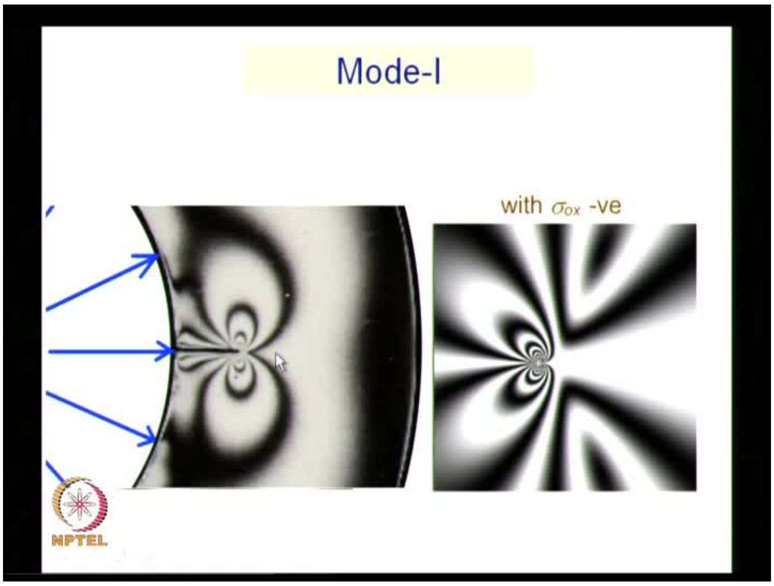

So, this is desirable I have fringes forward tilted, fringes backward tilted. Now, if I take up a problem which is particle I had a combination of both forward and backward tilted fringes. So, that means I need to go ahead, I cannot stop with even second term in the series, I have to take many terms in the series then I can handle this kind of situations but at least this provides a via media to look for higher order terms. Now, you may say you have shown only photo elastic fringe patterns, where we see such a drastic change because of the sign change, you show us one more fringe patterns simulated whether it shows any significant change or not.

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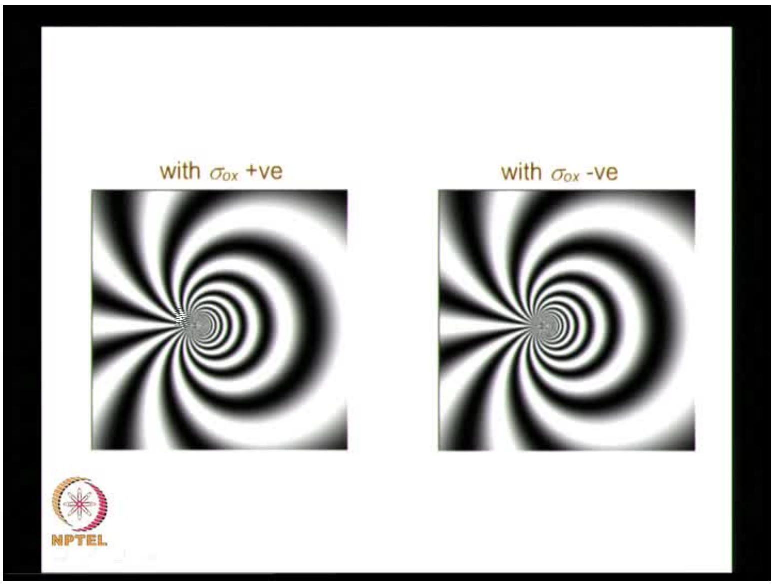

So, what we will do is we will take up isopachics for the same problem isopachics are developed for a mode 1 situation. So, I have this kind of a fringe patterns please make a sketch of it, this is for sigma naught x positive, when I make sigma naught x negative do you find any change, do not complain that I have just copied and pasted. This figure it is not done that way, it is done very realistically, very systematically it was simulated and then put.

See isopachics provide sigma 1 plus sigma 2, isochromatics provide sigma 1 minus sigma 2 but you find surprisingly the isochromatic fringes are so sensitive to sign change of the second term in the series not only for the second term, the fringe geometry changes as more and more terms are added in the series. In fact, we would take that as a detailed discussion in one of the later classes.

So, what you have here is when you look at a technique like holography which provides you isopachics which is also very difficult to conduct you find there are no perceptible change for you to associate its behavior to the existence or nonexistence that is also not shown here is sigma naught x is 0, I do not have the fringe pattern, if I have the fringe pattern then I compare that as well.

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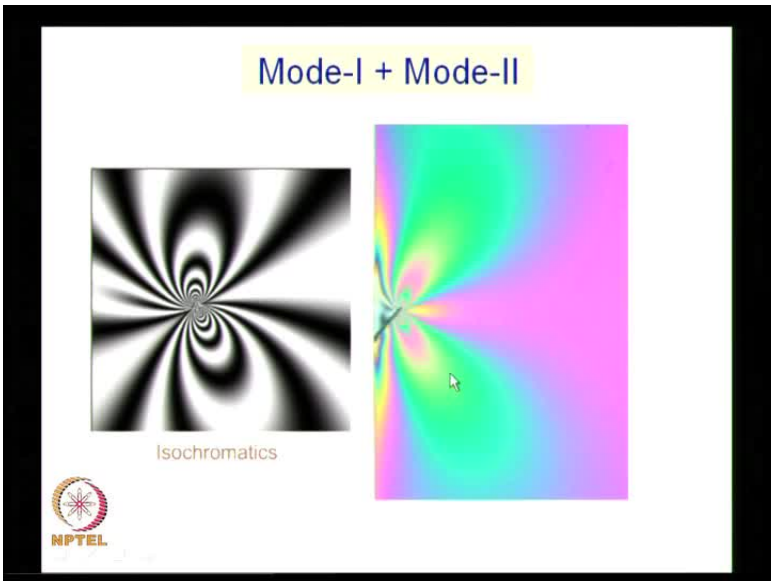

Now, having looked at mode 1 and mode 2, let us look at how the fringes appear in combination of mode 1 and mode 2 and this is the theoretically simulated. So, when it is theoretically simulated also make a sketch of it, you have a track oriented at this angle and this diagram is very illustrative you have the top fringes much bigger in size and bottom fringes are smaller in size. The difference is prominent and if you look at crack axis as a reference that tilt of the fringes in the top half and bottom half are different and how the solution was obtain this is simulated you say two terms solution of mode 1 plus 1 term solution of mode 2, please make a neat sketch of this and we will also look at the experimentally obtain fringe patterns they are not exactly reproduce for this case only for a qualitative comparison I am doing it.

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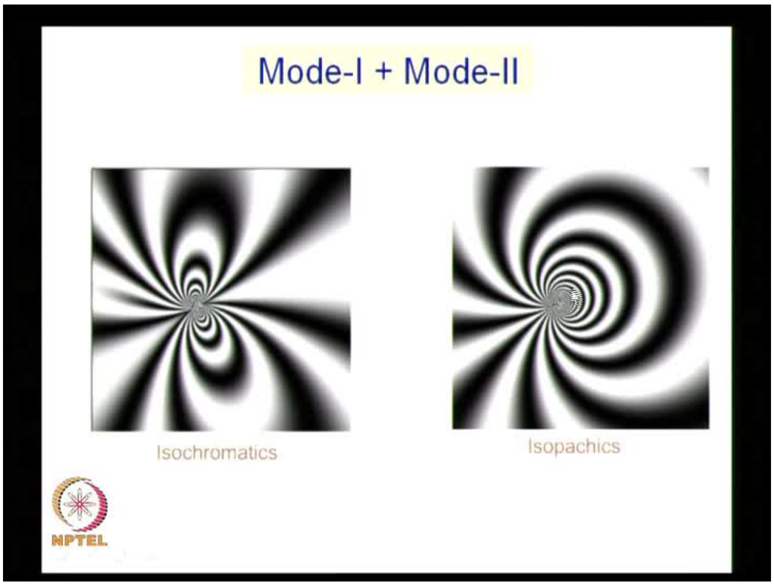

So, here we find the fringes are differently oriented, the size is also different and we could also look at the isopachics. So, when I look at the isopachics it is still like this no, it is very difficult to associate the influence of higher order terms, when you go for isopachics. So, you need to make a neat sketch of these fringe patterns. So, that provides you a perspective, how photo elasticity has been quite useful in the early development of fracture mechanics. In fact, photo elastic experiments are much simpler to perform than other interferometric techniques.

And they provide very useful information, key information first to verify and also to give you the necessary confidence that we are proceeding in the right direction, I hope you had sufficient time to take down the geometry of the fringe patterns.

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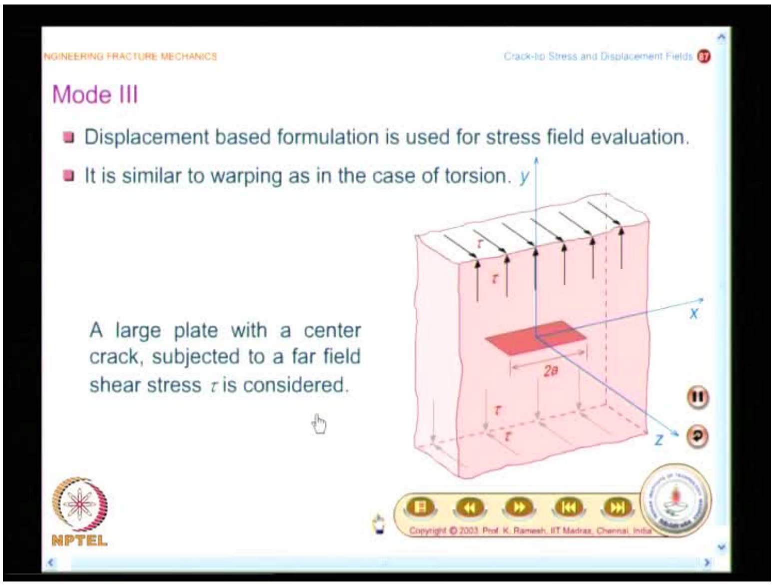

And now, we move on to discussing the mode 3 loading situation. This is again very simple straight forward and take down the sketch I have throw the thickness crack and what you have is you have out of plane shear and I would like you to note down, how the access or not x-axis is like this, y-axis is like this and z-axis is perpendicular to the plate and in contrast to stress formulation to solve this problem displacement formulation is convenient.

And once we go for displacement formulation, we have to find out how to write the displacement, we have already seen. In the case of a torsion problem, we had warping. Warping was only a function of x, y which is the plane of the body along the axis it remains constant, but it varies in the plane. A similar situation is observed for a mode 3 case also, we will have warping type of situation and we would have that as a function of x, y only. And the far field shear stress is given as tau, we are not attaching any symbol here but you can easily attach a symbol by looking at the figure.

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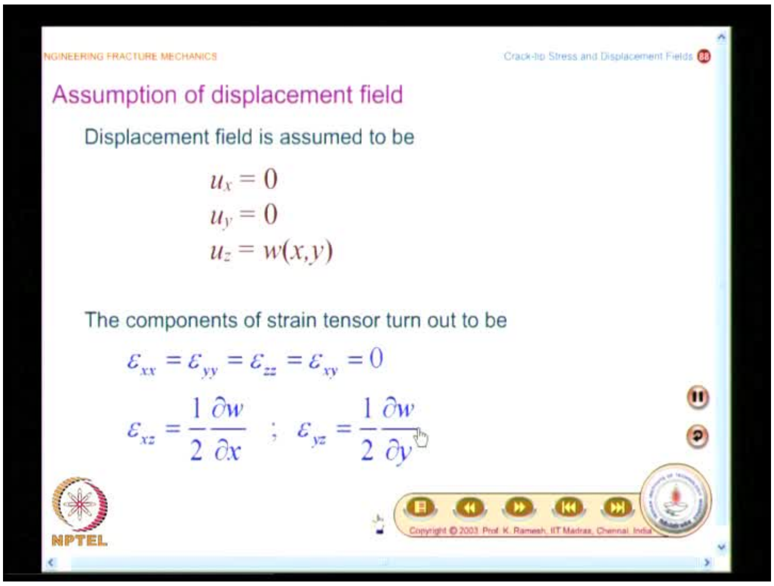

So, first thing is we have to assume the displacement field, the x component of displacement is 0, y component of displacement is 0 and the u z component is a function of x y only. And this has worked; people have taken this as a possible displacement field and in displacement formulation. What you do, you have the displacements from displacements, find out strains, from strains find out stresses and because I evaluate displacements satisfying the boundary conditions there is no need for compatibility conditions, compatibility of displacement is automatically satisfied.

Having known this displacement field, you can write components of strain tensor, you can do it yourself do it yourself and verify with my solution. Now, that is what I would appreciate I do not want you to blindly copy from what is shown on the slides, I want to do it on your own and then verify is a very simple that will keep your mind alert while in the class. So, I have them here, because I have u x equal to 0 u y equal to 0 I have epsilon xx is 0 epsilon yy is 0, because u z is a function of only x, y epsilon zz is also 0 and epsilon xy is 0. So, the non-zero strain components are only epsilon xz that is given as 1 by 2 dou w by dou x and epsilon yz is given as 1 by 2 dou w by dou y.

So, in this case, what we will have to do is - we will have to find out a stress function for w that is how the problem is solved and you will find the form of stress function used for w is very similar to what we have develop for mode 1 and mode 2 stress function, there it was used as a stress function. Here a similar form will be used for displacement information. Now, I have displacements as well as strains the next step is evaluate the stresses.

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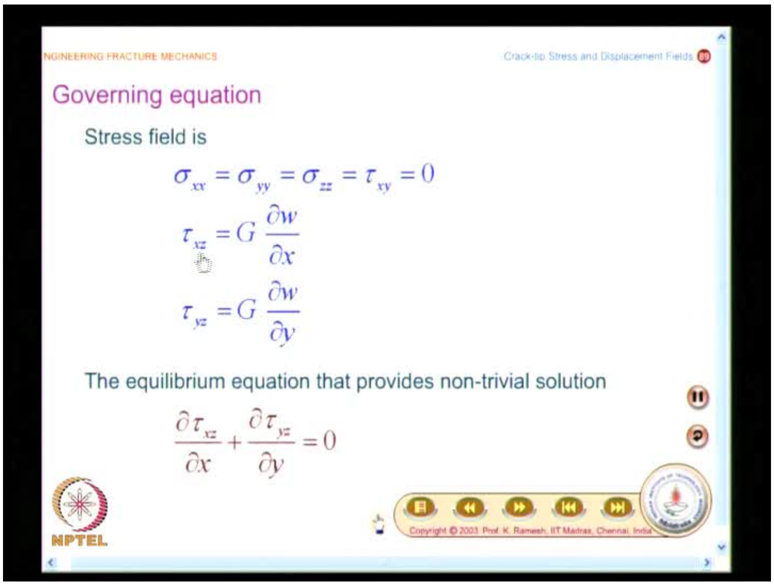

And you find sigma xx equal to sigma yy equal to sigma zz equal to tau xy equal to 0 all these stress components go to 0. The non-zero stress components are tau xz that is equal to G times dou w by dou x tau yz equal to z times dou w by dou y, you know I want to get a non-trivial solution, then I look at the equilibrium condition. The relevant equilibrium condition is in the z direction dou xz divided by dou x plus dou yz plus dou y equal to 0 because your dou sigma z divided dou z is already 0.

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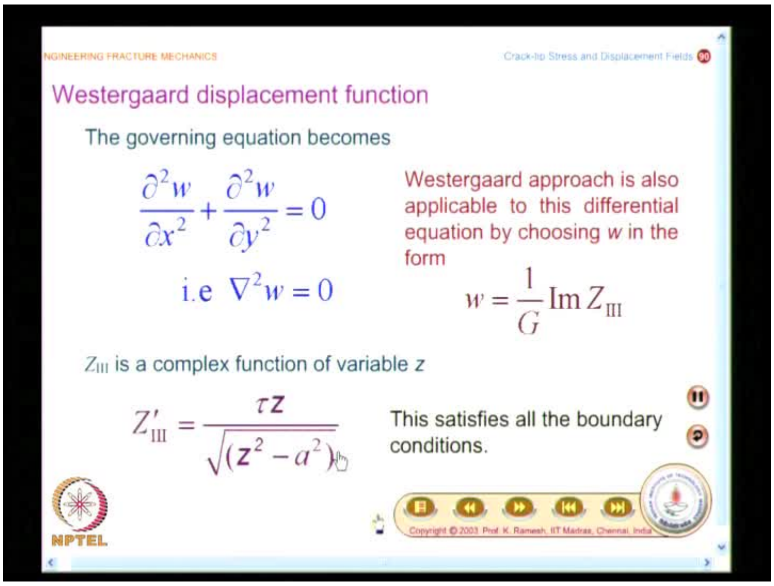

So, I have to satisfy this. So, I substitute the expression for tau xz and tau y z in this and the result is I get a Laplace equation del squared w equal to 0, it is a harmonic function if I if I have a harmonic function, then this equation will be satisfied and we have already seen all analytic functions are candidates for Airy's stress function. So, that is how the link is Westergaard approach is also applicable to this differential equation by choosing w in the form w equal to 1 by G imaginary part of Z 3.

And we have to know, what is Z 3? We will not worry about Z 3, but we would defined what is Z 3 prime because that is what is useful to us Z 3 prime, which is that differential first differential of Z 3 equal to tau z divided by root of z squared minus a squared very similar to what you had seen for mode 1, what you had seen for mode 2 same thing is appearing for mode 3, some aspect of it is very similar, there it was used as a stress function here it is used for displacement information and here it is not defined as Z 3, you are actually defending Z 3 prime in this fashion and this satisfies all the boundary conditions, which I would like you to verify. You should know, what are the boundary conditions, you should specify and systematically verify that this stress function satisfies all the boundary condition. I think you should take this as an exercise, try to arrive it. This completes solution for mode 3, because now you have all the information. So, in all the cases what we have done, we would move the origin from center of the crack to the tip of the crack do that then express the stress function in terms of r n theta and get the final expression for the stress field as far as the displacement field.

So, in this class we had looked at some aspects of energy release rate followed by development of stress field and displacement field close to the crack-tip for the case of mode 2, then we moved on to looking at the fringe field obtain by photo elasticity for mode 2, mode 1 as well as combination of mode 1 and mode 2 particularly we focused on, what is the influence on the geometric features, when the second term in the series changes in the case of mode 1 situation, then we moved on to discussing the formulation in the case of a mode 3 situation.

Thank you.