Engineering Fracture Mechanics

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Module No. # 04

Lecture No. # 17

Displacement Field for Mode - 1

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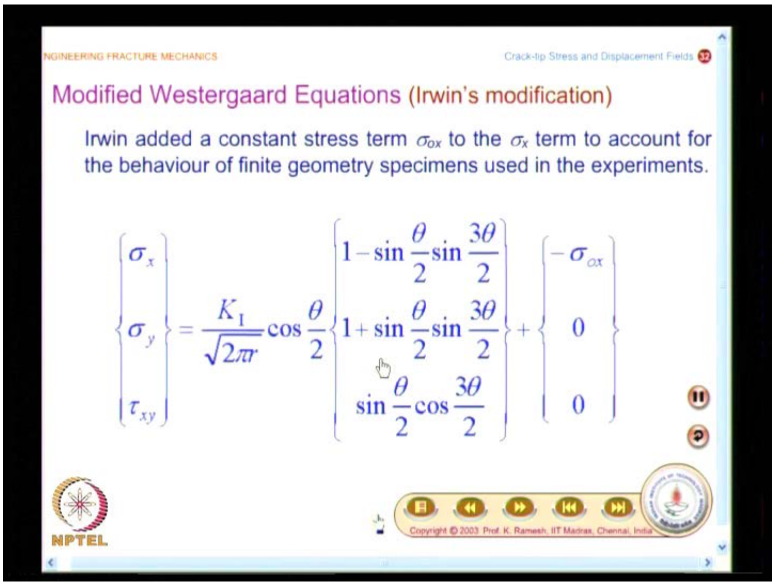

In the last class, we had developed the stress field equations and particularly we saw the stress field in the very near vicinity of the crack-tip; we have seen the solution by Westergaard; we also saw certain modifications on the Westergaard solution and one of the important modifications you have on Westergaard's equations is the modification by Irwin.

In fact, while interpreting the results of Wells and Post, he added an extra term and this is to essentially the sigma x stress term quantity minus sigma naught x. Although, the final objective may be to find out K 1 from the experiment, you will have to use the experimental information to find out sigma naught x as well as K 1, and then, report the value of K, only that value of K is representative of the actual stress fields in an experiment; this is the way you have to look at it.

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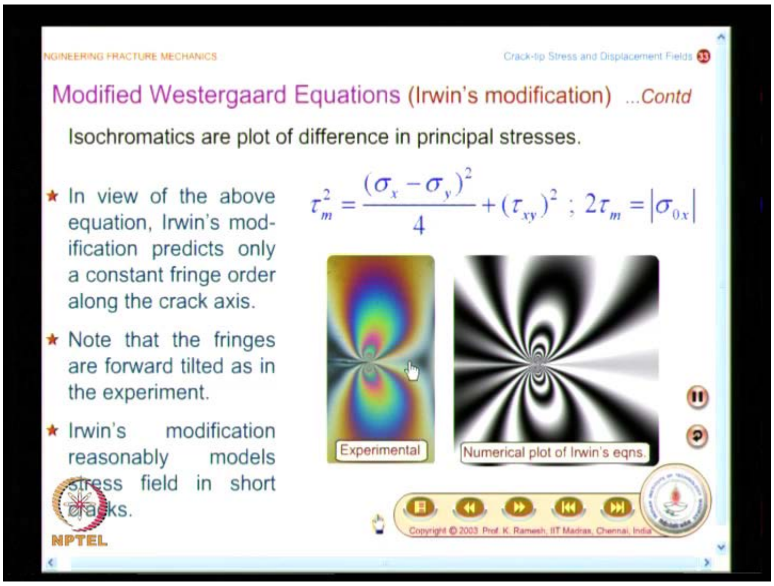

And we have also seen the kind of fringe patterns that you get by using this stress field equation; we will have a look at them again. We have calculated the maximum shear stress along the crack axes, turns out to be a constant value; this is the generic expression. When you plot these, you had fringe patterns like this, and when we compare it with the experimental fringe patterns, it matched quite well.

And this is what I said, a simple modification by Irwin has helped you to capture even the tilt of the fringes, this is becoming straight, which was inclined like this tilt of the fringes was inclined like this and it is becoming straight as you go close to the crack-tip and that is what you see in the fringe patterns. But along the crack axes, it also gave only a constant value of the maximum shear stress, whereas in the experiment even in this case, there is a subtle variation. You could see prominent variation in problems, where the crack is long or where the crack is situated in a stress concentration field.

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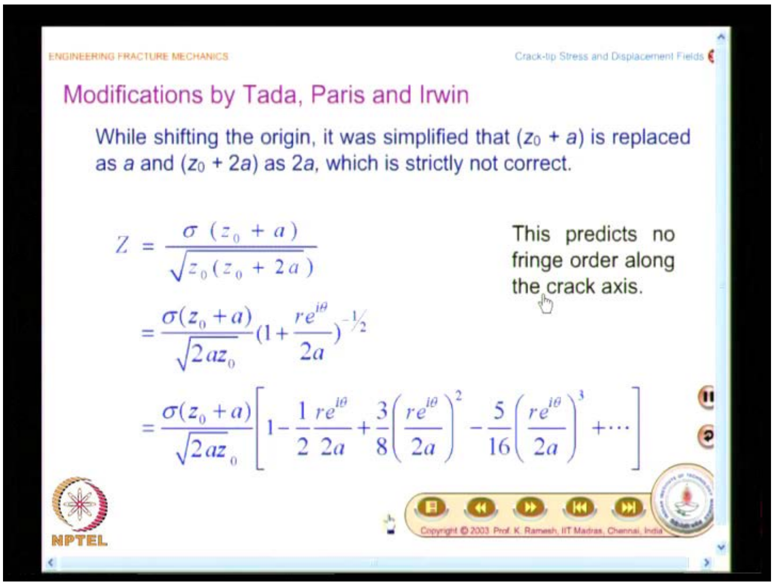

In addition to this, we also saw what are the modifications done by Tada, Paris and Irwin. What they looked at was, they looked at while shifting the origin, we had made a simplification, and instead of simplifying the way that we had done, we could express the denominator as the binomial series. So, you essentially get a series solution. And if you look at whether it has helped in satisfying what is seen in the experiment, if you really look at, it has not done so. Though, it is not shown here along the crack axes, it predicts no fringe order very similar to the original Westergaard equation; though, this modification was straight, this modification was not useful.

When you have serious terms, you normally think, if I take the first term, the solution is to certain level of accuracy; if I take the several other terms, the accuracy keeps on increases, that kind of a generalization you cannot make it here; the first term is still what you have in your conventional Westergaard solution. Now, what we will do is, we will also have looked at some aspects of the basic solution by Westergaard.

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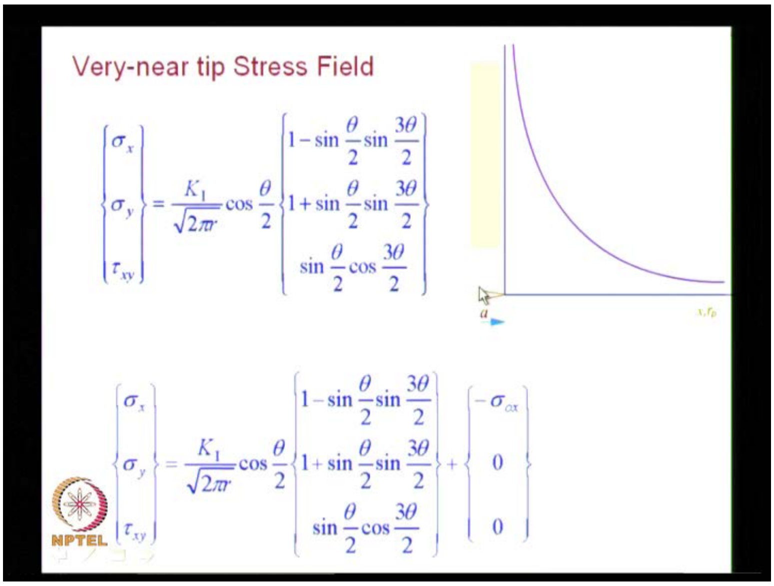

Let us see what all we understand from the basic solution here. I have what is known as stress the intensity factor and there is the function of r and theta that is given. As the loading is changed, you would find the value of K 1 changes; the value of K 1 really determines the strength of the stress field. Suppose I have multiple loads which are giving you a mode one situation, then from principle of super position, you can add the K 1s for different type of loading and what happens when r is closed to 0? The value was asymptotically increased to infinity. In a real material it cannot go to infinity, it can only reach a value dictated by the material behavior; it may be if it is the perfectly plastic material, it may reach the yield strength.

If it is the work hardening material because of work hardening, it will be some other value of stress level; this is what would happen exactly at the crack-tip, how small or how big this zone is problem dependent. Another aspect what we looked at was, this solution is actually valid very close to the crack-tip, because we have made a simplification z naught is much less than a and that is how you have got.

Suppose you look at this, when r tends to infinity what happens to the stress values, they essentially go to 0. So, you are only capturing the stress field very close to the crack-tip and what was the modification done by Irwin? He simply added one extra term and I had mentioned earlier, that you need this to interpret the experimental fringe pattern, though your interest may be to find out only K, you process the data from experiments and calculate sigma naught x as well as K 1 and take that value of K 1 as representative for the problem on hand. But people also have looked at this interpretation in many different ways. If you look at the derivation from Westergaard's approach for the mode 1 problem, whatever the solution that we have got was for a biaxial stress field solution; it is not for a uniaxial stress field.

If you look at Irwin modification, this could be considered as a modification for a uniaxial stress field, because this minus sigma will go to at infinity cancel with the sigma and the biaxial load. So, people thought Irwin's modification could be taken as a uniaxial case representative solution, but there are also other arguments.

See if you look at really at the crack-tip, the value of sigma x is very high. So, near the crack-tip sigma x minus, whatever the sigma would still be very small. So, there has been discussion in the literature, how to take the solution for uniaxial stress field. If you really look at the development when you looked at what is K 1, we set for a biaxial loading situations sigma root by a. So, we have an analytical expression on what is the value of K 1.

Now the question is this is generated for a biaxial situation, can I use it for uniaxial situation? This kind of question comes. So, what people have done is, for finite body problem, people develop boundary collocation solution and obtained expression, it had a factor a by w a function in terms of a by w, which is represented as beta. So, beta sigma root of pi a, that way you can handle finite body problem.

So, what you will have to keep in mind is, in the near vicinity of crack-tip, the basic solution of Westergaard is still very useful. And what is the other thing that you get when r tends to the crack-tip, you will look at what happens, along the line theta equal to 0 that is a crack axis. So, what I get here, these terms go to 0; I get sigma x and sigma y both behave in this manner; you have sigma x equal to sigma y in the near vicinity of the crack-tip and later on when we are going to look at what is the size of the plastic zone, I had already given a problem in your review of solid mechanics, find out the level of stresses when you have sigma x equal to sigma y in plane stress as well as plane strain that comes from what happens near the crack-tip. The Westergaard solution gives you near the crack-tip at the crack-tip both sigma x and sigma y are like this; they asymptotically increase to infinity and goes to 0 when you go along the x direction, but what would really happen to the crack- tip?

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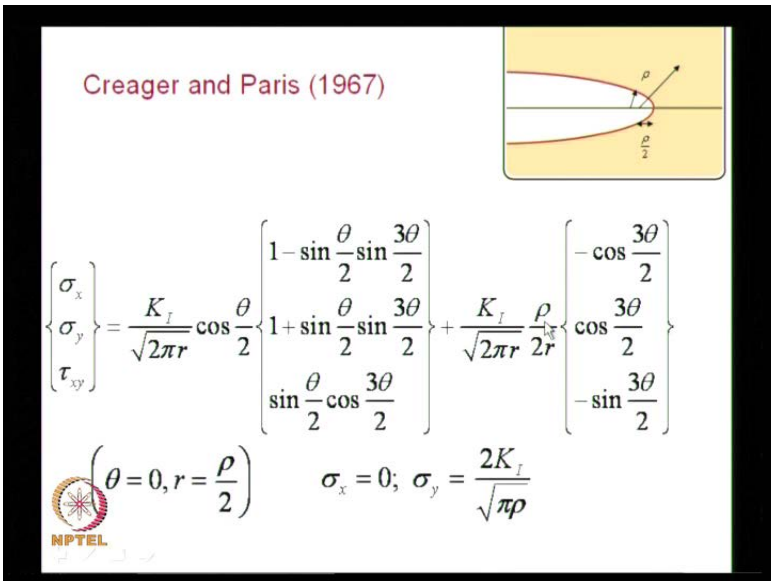

The crack-tip is not going to be sharp; in a real material, crack-tip would be blunt. So, people also looked at solution from a different perspective, this was done by Creager and Paris 1967, who tried to find out the stress field when you have a blunted crack-tip. How does the solution change? You had this Westergaard part of the solution plus you have K 1 by root of 2 pi r rho by 2 r multiplied by minus cos 3 theta by 2 cos 3 theta by 2 minus sin 3 theta by 2.

What is the difference between the two? In the earlier case, crack-tip was very sharp; in the second case, the crack-tip is blunt. So, when the crack-tip is blunt, what would happen to the sigma x stress component? It has to go to 0, because it becomes a free surface; when it is the sharp point, the story is different. And when you look at the Creager solution, what they have done is they have considered this as a root radius rho and the crack-tip is taken as a distance rho by 2 from the edge of the crack. This is how the measurements are done.

So, if I have to find out what happens at the crack-tip, I have to substitute the value of rho by 2. So, when I substitute a value at theta is equal to 0 and r equal to rho by 2, I get a solution which is understandable from your common sense. You get a solution, where sigma x equal to 0 and you also get sigma y becoming a finite value; it is not infinite any more. Ever since Inglis pointed out cracks are very dangerous, several scientists have contributed in understanding what happens at the crack-tip.

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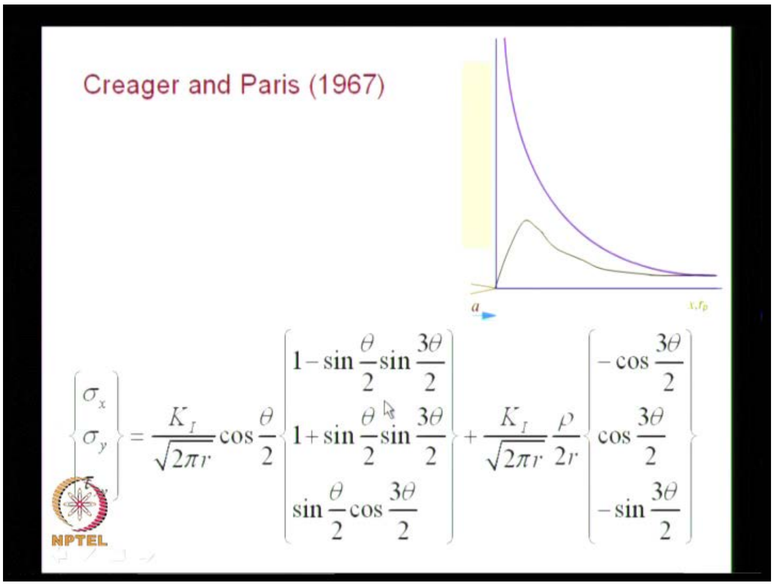

So, the solution by Creager is quite useful. The important aspect here is you have this has blunt and this blunting is accounted for, by taking a root radius rho. And if you look at the stress field, it would be something like this. You have a blunted crack-tip here; shown a sharp, but imagine that it is the blunted crack-tip and I have some variation of sigma x like this, it is 0 at the crack-tip, and the sigma y and sigma x are different and sigma x reaches the maximum value at a short distance away and this is nothing but a Creager solution that I have shown earlier. So, the focus here is you are getting a sigma x value 0 at the blunted crack-tip and sigma x reaches the maximum at short distances away and your sigma y value is not infinity, but it is very large, but it is the finite value.

Now, what we will look at is, we will go and find out the displacement field. We would still use the Westergaard solution itself; from the Westergaard methodology, you may not be able to get it for a uniaxial loading situation. But we have seen, whatever the final solution you get is reasonably good enough to represent the crack-tip behavior quite well.

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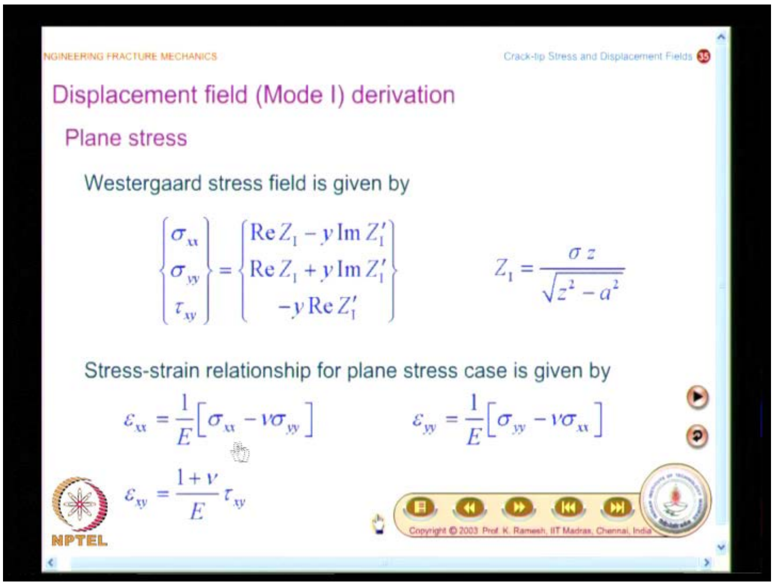

So, we go to the displacement field and here again what we will do is, we will develop it based on the stress function themselves and what is done here is since we are focusing on mode 1, the stress function given by Westergaard is represented as capital Z 1 and Z 1 prime and so on. In the earlier discussion, we had kept it as capital Z, because Westergaard used that basic solution to get the stress field for a variety of problems.

Now, we are going to focus on mode 1, mode 2 and mode 3; in order to distinguish the relevant Westergaard stress function, we would represent them as Z 1, Z 2, Z 3 as the case may be. So, here we are having this capital Z 1 and what is the advantage by taking Z 1 in this fashion? Origin of the reference axis would be the center of the crack-tip.

If I modify the Z, you could also see what happens at the crack-tip. In fact, when I want to get the displacement field, I would like to get the displacement field with crack center as the origin as well as crack-tip as the origin; I would use them in appropriate manner when I want to understand the concepts in fracture mechanics. One of the interest is, we want to see under given loading how the crack opens up.

For that finding out, the displacement field with crack center as the origin is useful and we would develop it for plane stress as well as plane strain. So, in the case of a stress formulation, we get the stresses first; from stresses, we will get the strain components;

the stress strain relationship for plane stress is given as follows and you have to be very careful in putting the appropriate stress quantities.

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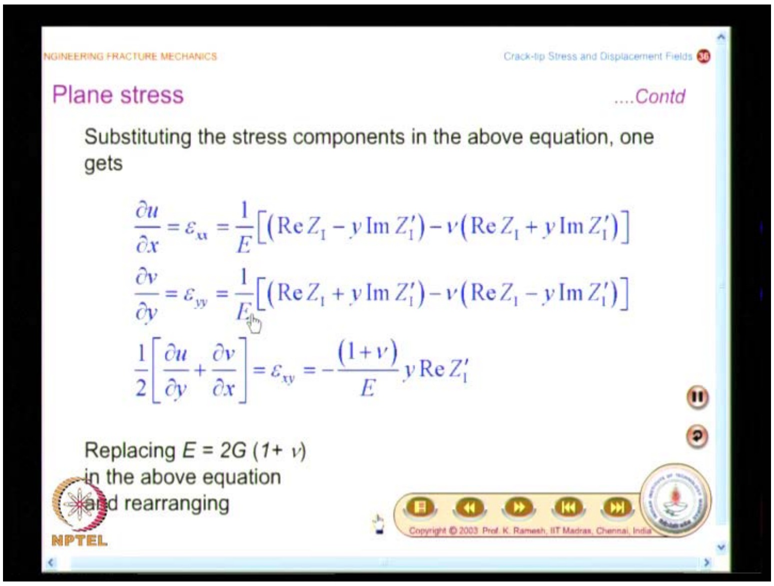

So, epsilon x x equal to 1 by E into sigma x x minus nu time sigma y y; similarly, you can write for epsilon y y as well as epsilon x y. So, in the case of stress formulation, we get the stresses first; using stress strain relationship, evaluate the strains and the strains are related to displacement quantities. So, from strains we will have to find out the displacement. And we know from your solid mechanics, that epsilon x x is nothing but dou u by dou x and epsilon y y is nothing but dou v dou y and your epsilon x y is one half of dou u by dou y plus dou v by dou x and what we have done now is from the stress strain relationship, we have written the strain quantities. So, epsilon x x turns out to be 1 by Young's modulus multiplied by real part of Z 1minus y imaginary part of Z 1 prime minus of nu times real part of Z 1 plus y imaginary part of Z 1 prime, and epsilon y y is very similar; there is a change in the sign here and there is a change in the here; otherwise the basic expressions look very similar.

I have this shear strain in which we have replaced the value of G in this manner. We know E equal to 2 G into 1 plus nu so that is substituted and we have written it in terms of E here, minus 1 of 1 plus nu divided by Young's modulus y real part of Z 1 prime.

See finding out the displacement from strain is not a simple task; you have to be very careful. If I have only dou u by dou x and dou v by dou y, there is no other interrelationship, then the methodology is straight forward; I can directly integrate expression one and expression two and say this is the displacement field.

What I have here is, I have an expression for dou u by dou x; I have an expression for dou v by dou y; I also have an expression interrelationship between them dou u by dou y plus dou v by dou x is given as epsilon x y. So, whatever the solution for u and v, I get this equation also to be satisfied that is to be taken care of in your mathematics.

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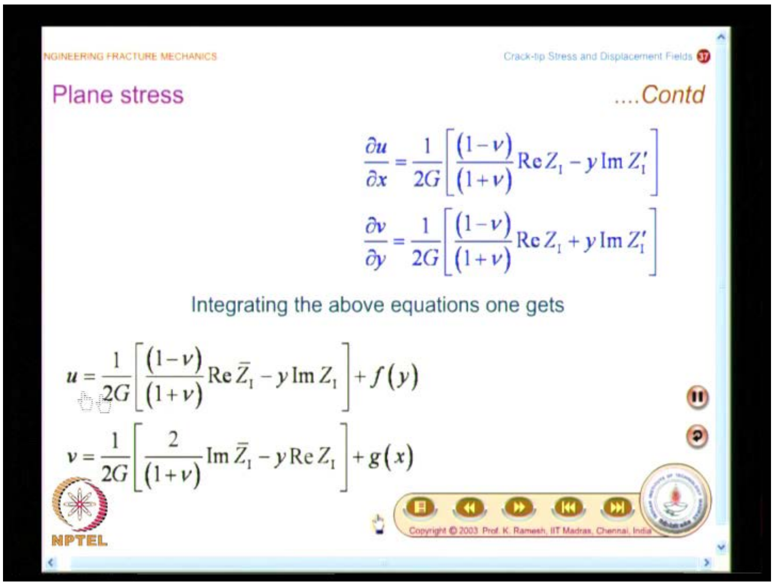

Now, what we will do is, instead of keeping this as Young's modulus, we will replace it terms of g that is what we are going to do it the next slide and group the terms. So, what you have here is dou u by dou x equal to 1 by 2 times G of 1 minus nu divide by 1 plus nu multiplied by real part of Z 1 minus y imaginary part of Z 1 prime.

Then you have expression for dou v by dou y, so these are carefully grouped so that you have a the Westergaard stress function terms are group together. I have this as 1 by 2 G into 1 minus nu divided by 1 plus nu of real part of Z 1 plus y imaginary part of Z 1 prime; we will still retain this as Z 1 and carry on with the calculation.

Finally, we would look at one set of solution with crack-tip as origin; another set of solution with crack center as the origin; we will get the expressions in terms of r and theta until then, we will just keep them as capital z itself. So, on integration, the expression for u and v turns out like this; these are long expressions, please take some time to write them.

So, I have u equal to 1 by 2 G multiplied by 1 minus nu divided by 1 plus nu real part of Z 1 bar minus y imaginary part of Z 1 and I have added a function here; this is what we have to keep in mind, we have expression dou u by dou x. So, when I get an integration of that, I will have an integration constant, that integration constant is actually a function of y when I have the original expression in terms of dou u by dou x. As far as integration with respect to x is concerned, this behaves like a constant.

So, when I go to dou v by dou y when I integrate, I get a integration constant as function of x. So, I have v equal to 1 by 2 G of 2 by 1 plus nu multiplied by imaginary part of Z 1 bar minus y real part of Z 1.

So, keep track of when I do it with respect of y, there is a flip over in the functions as well as the sign change; this you have to look at the Cauchy-Riemann condition and make your understanding clear. So, the solution of u and v would be complete only if I also evaluate what are f y and g x; if I know these two, then I can get the expression for u and v.

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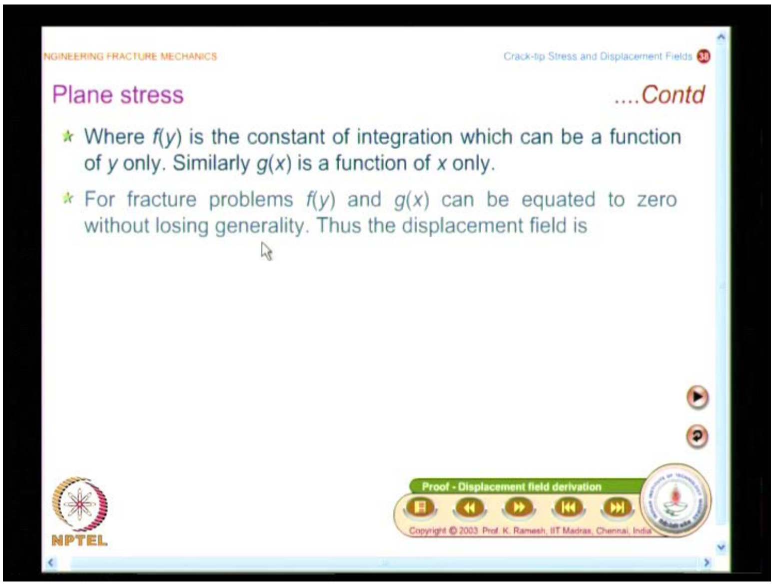

And that is what is summarized here; f function of y is the constant of integration which can be a function of y only. Similarly, g x is a function of x only. And I made a statement, that for fracture problems function of y and g of x can be equated to 0 without losing generality that makes our life simple. But what we will do is, we would really go and find out what is the reason behind it.

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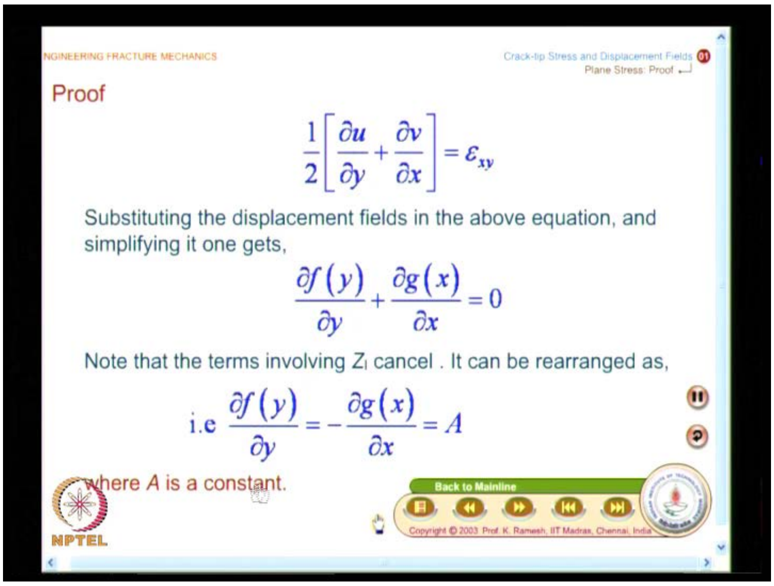

So, for that I need to look at what will happen to this expression. Because we had seen, we also have interrelationship between displacements in the form of your epsilon x y. So, I will have to evaluate one half of dou u by dou y plus dou v by dou x. You have already got the expression for the u and v in terms of the integration constants, which are actually functions of y and g of x; I am not doing that detailed; so go back into your room and then check it.

So, when I do that, I get this equal to 0. So, this you will have to substitute it and then I would like you to verify it in your rooms. So, dou of f of y divided by dou y plus dou of g of x divided by dou x equal to 0, this is what you get when you substitute it and this can be rearranged as dou of f of y divide by dou y equal to minus of dou of g of x divided by dou x equal to A, and A is a constant and we will investigate what is the influence of this. Suppose I go back and substitute the displacement field what happens to it?

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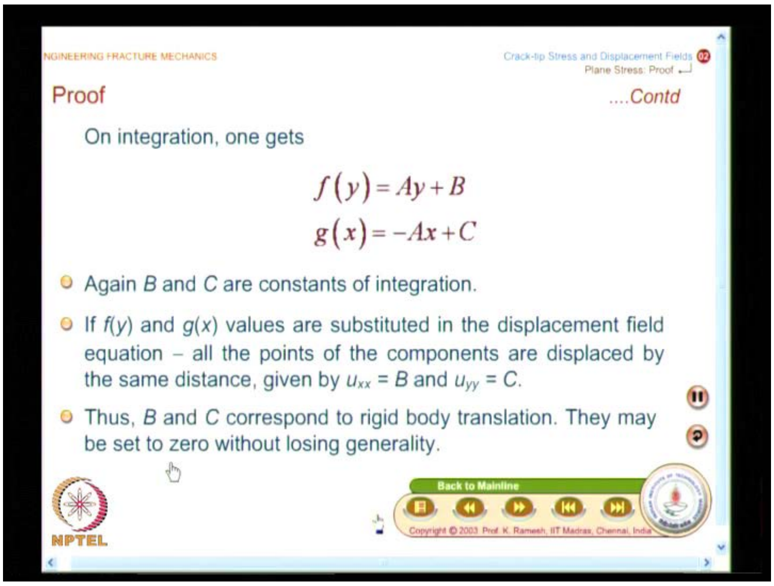

So, when I integrate this, I get f of y equal to A y plus B and g of x equal to minus A x plus C and these are constants; they are constants of integration. So, now, I know what is f of y and g of x; so when I substitute in the expression for u and v or u x and u y which ever way you look at it, I would like you to do that as an exercise in your rooms. I get u x as a constant B and u y as another constant C.

And our solution will not be affected if I take without losing generality, these be set to 0. You know in your normal theory of elasticity, though you have solved the problems based on stress formulation, only for a very few cases probably you would have gone ahead and evaluated the strain as well as the displacement. If you had done that exercise, here also the exercise is similar.

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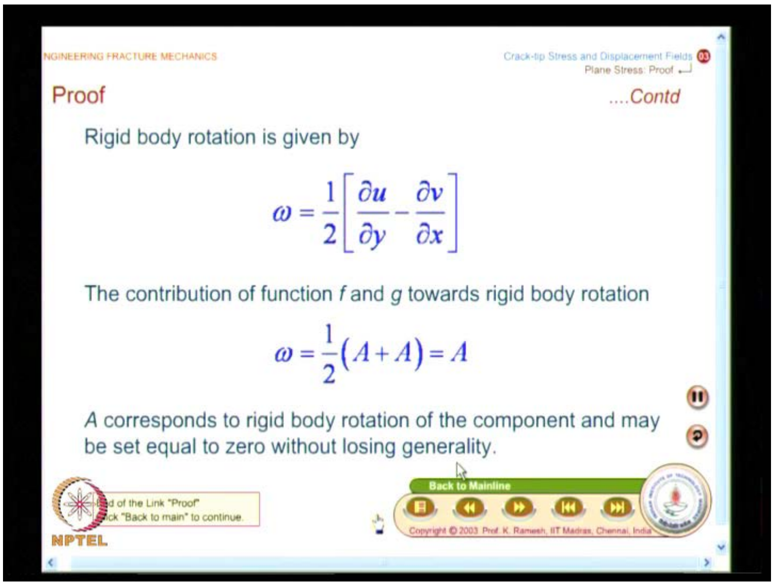

And now we look at what is rotation. The rotation is given a symbol omega that is nothing but one half of dou u by dou y minus dou v by dou x. So, the contribution of function f and g towards rigid body rotation is what you are getting it here, omega equal to one half of A plus A which reduces to a constant A. A corresponds to rigid body rotation of the component and may be set equal to 0 without losing generality.

So, what we have actually looked at it is, we have said when you integrate dou u by dou x, I would have an integration constant appearing as the function of y; when I integrate dou v by dou y, I would have an integration constant appearing as function of x, which is g x. Without losing generality, by a detailed analysis, it shows they represent only rigid body translation and rotation, we could make them.

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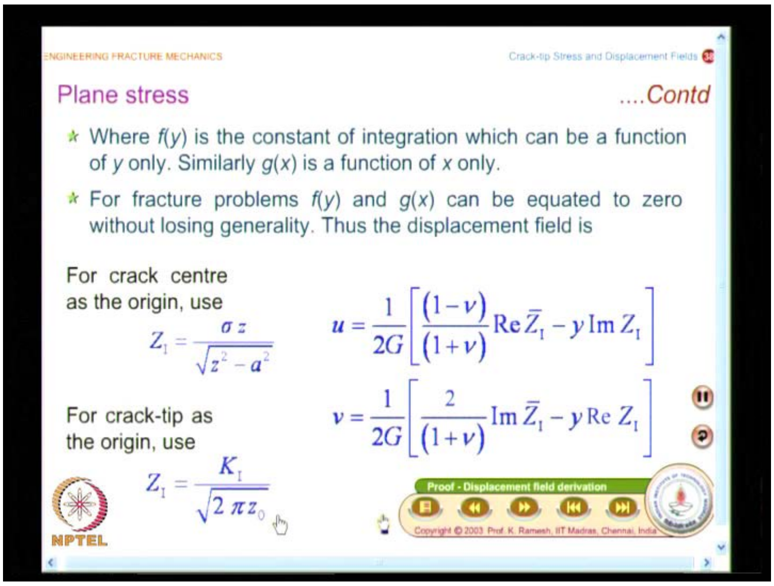

So, the displacement field is finally obtained in this fashion, u is given as 1 by 2 G of 1 minus nu divided by 1 plus nu multiplied by real part of Z 1 bar minus y imaginary part of Z 1, and v is given as 1 by 2 G of 2 by 1 plus nu imaginary part of Z 1 bar minus y real part of Z 1. You know our interest is to get the displacement field with crack center as the origin as well as crack-tip as the origin.

So, for crack center as the origin, use the stress function as Z 1 equal to sigma z divided by root of z squared minus a squared; for crack-tip as the origin, use Z 1 equal to K 1 by root of 2 pi z naught. This I would be emphasizing it, because when you take up a book, they give an expression, you may not know unless it is very clearly said, whether those expressions are related to crack center or crack-tip. This we have to keep in mind and also whether it is related to plane stress or plane strain.

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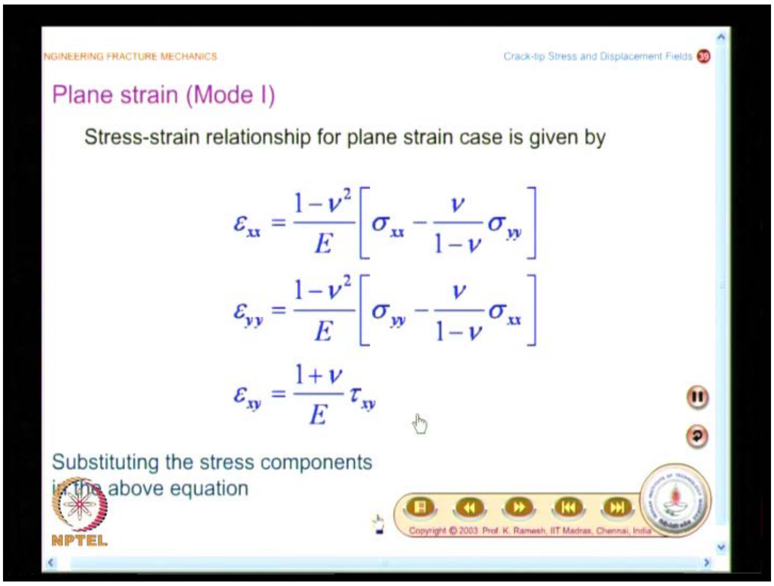

Now, let us move on to finding out the displacement field in the case of plane strain. And you all know the stress-strain relations, these are written here for convenience, epsilon x x equal to 1 minus nu squared divided by Young's modulus E multiplied by sigma x x minus nu by 1 minus nu sigma y y; epsilon y y equal to 1 minus nu square divided by Young's modulus multiplied by sigma y by minus nu times 1 minus nu sigma x x.

If you really look at these two expressions, can you get them from your plain stress case by making some modification to Young's modulus and Poisson ratio? In fact, you can do that; if you look at that expression, instead of Young's modulus, you take it as E by 1 minus nu squared, instead of Poisson ratio nu, you put it as nu by 1 minus nu. You will get from plane stress to plain strain.

And you know to save space in books, they might give one expression and say substitute it in this fashion, get it for strain plane; this is very common in elasticity books. And your epsilon x y given as 1 plus nu divided by Young's modulus into tau x y. So, I can substitute these stress components in the basic equation.

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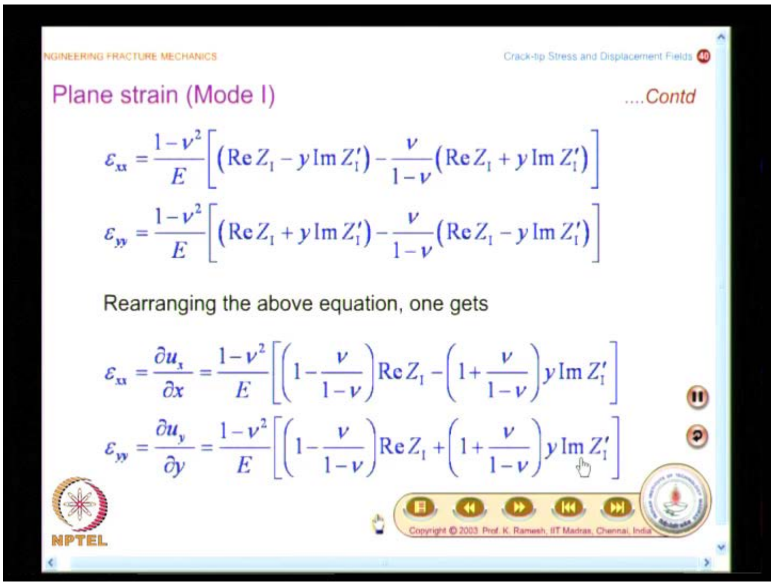

So, I get strain quantities related to the Westergaard's stress function as follows; it is quite long, please take some time to write this up. So, I get epsilon x x equal to 1 minus nu squared divided by Young's modulus multiplied by a real part of Z 1 minus y imaginary part of Z 1 prime minus nu by 1 minus nu real part of Z 1 plus y imaginary part of Z 1 prime, then I also look at what is epsilon y y, it is 1 minus nu square divided by Young's modulus into real part of Z 1 plus y imaginary part of Z 1 prime minus nu by 1 minus nu real part of Z 1 minus y imaginary part of Z 1 prime. So, what we will do is, we will group the terms involving Z 1 and Z 1 prime and get the expressions, because that will make our lives simple later.

So, I get dou u x by dou x equal to 1 minus nu squared by Young's modulus 1 minus nu by 1 minus nu multiplied by a real part of Z 1 minus 1 plus nu by 1 minus nu y imaginary part of Z 1 prime, and epsilon y y equal to dou u y divided by dou y equal to 1 minus nu squared by E multiplied by 1 minus nu by 1 minus nu real part of Z 1 plus 1 plus nu by 1 minus nu y imaginary part of Z 1 prime.

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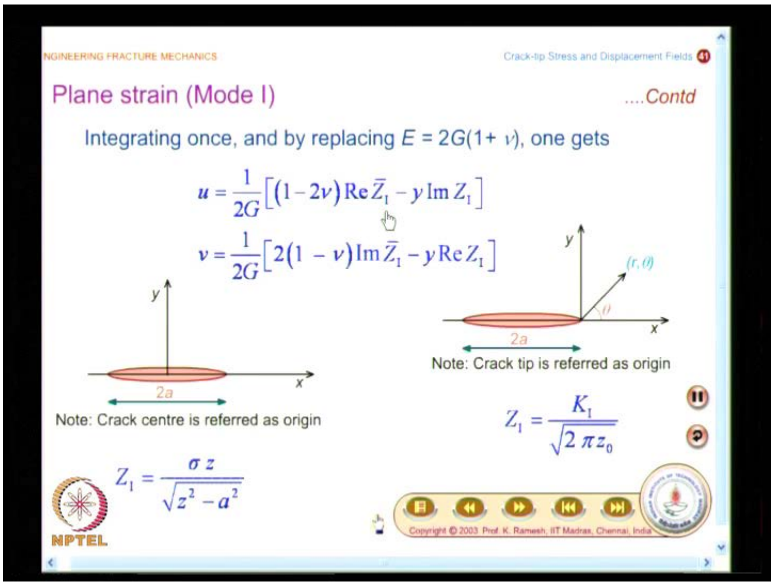

See the expressions are long; I do not expect you to remember, but nevertheless your notes should have these expressions. In fact, for your examination, I would allow you two sides of an A 4 size sheets, where you could have the formulae you think that are relevant can be written and come. Because in a course like this, you will have to encounter long expressions, there is no other go; it is not a test for your memory, but you need to have those equations. And I have been emphasizing, whether these equations are referred with respect to the crack-tip or crack center make those notes also appropriate and you will have a good set of equations summarized in your notes.

And while developing the plane stress situation, we have looked at the integration constants can go to zero so we take advantage of that understanding. I integrate the expressions for strains and get the displacement u as 1 by 2 G into 1 minus 2 nu multiplied by real part of Z 1 bar minus y imaginary part of Z 1.

Can you now go to your plane stress expression for nu can you get from plane stress expression for you to plane strain expression for nu? I have to make some small changes in the Poisson ratio; if you substitute nu by 1 minus nu in the plane stress equation, you will get the equation that you have got in plane strain and we have the displacement component v as 1 by 2 G 2 into 1 minus nu the imaginary part of Z 1 bar minus y real part of Z 1 and here again it is emphasized.

Since the expressions are still in terms of the stress function also make a sketch. But this is very illustrative; you are taking crack-tip as the origin and the relevant stress function is given as Z 1 equal to K 1 by root of 2 pi z naught and here, you have crack center is referred as origin and Z 1 is given as sigma z divided by root of z squared minus a square.

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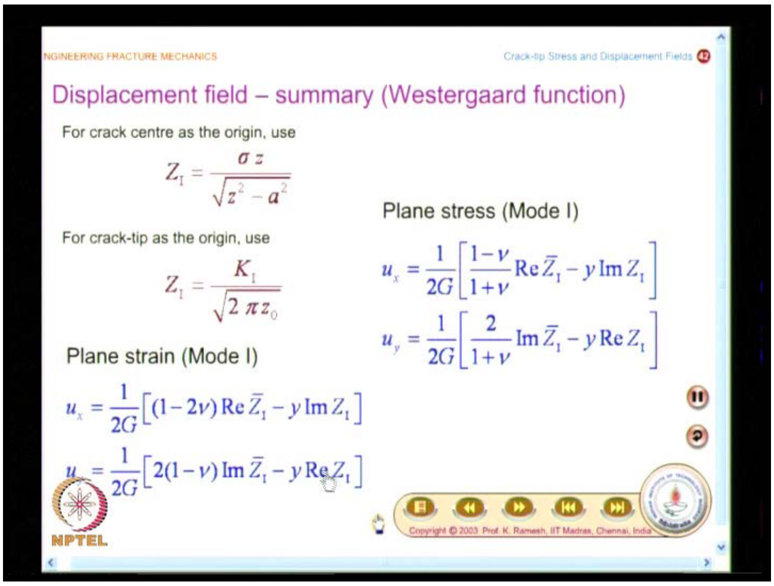

So, I can get referred with respect to the center using this stress function referred with respect to the tip with this stress function. So, essentially I will have to know, if I have to find out in terms of r and theta, I will have to know what is what is Z 1 and what is Z 1 bar. This is what I will have to do and this is just to verify your notes, it is all summarized here.

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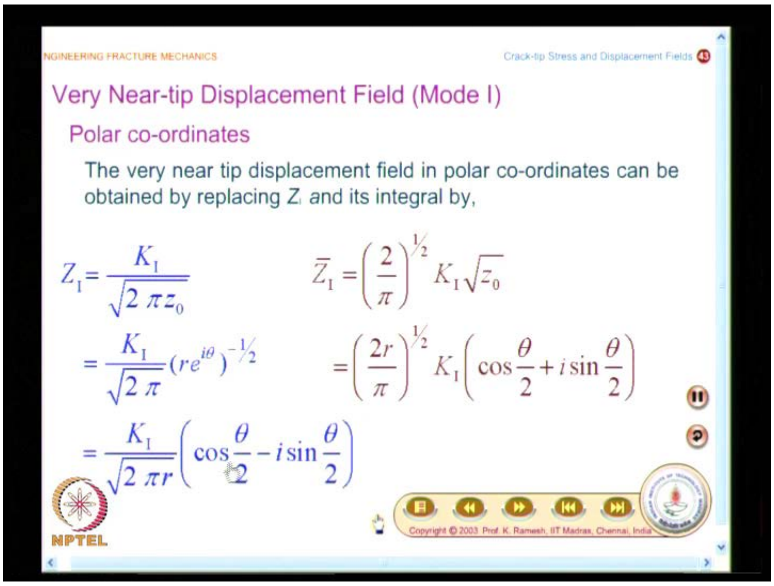

In this slide, you have the expression for the plane stress as well as for plane strain- the displacement field. If you have made any typographical errors, please verify and correct them while looking at this. See now, I am going into qualify the displacement field which is very close to the crack-tip, because I am going to use the stress function Z 1 as K 1 by root of 2 pi z naught, the moment I bring in this, you have to understand it is a very near tip field equation; in this case, it happens to be displacement field. And you have already seen how to write it in terms of cos and theta; so, Z 1 is nothing but K 1 by root of 2 pi r cos theta by 2 minus i sin theta by 2.

While developing the stress field equation, you have to come across the simplification. In the stress field equation, we were worrying about Z 1 and Z 1 prime and here we are going to worry about Z 1 and Z 1 bar; so I have this expression. So, when I have to find out what is Z 1 bar that is nothing but 2 by pi whole power half K 1 root of z naught and this could be written in terms of 2 r power half divided by pi into K 1 into cos theta by 2 plus i sin theta by 2.

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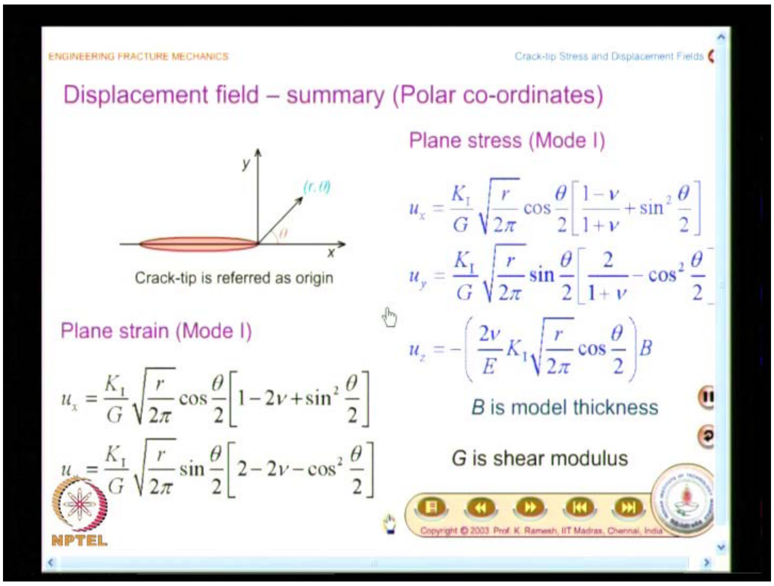

So, now, I have the expression of Z 1 as well as Z 1 bar in terms of r and theta. So, I can substitute it in the expression and get the displacement field in terms of r and theta as simple as that. And what is written here is I have the expressions for both plane stress as well as plane strain. In order to make your understanding very clear, the diagram re-emphasizes that whatever the solution you see for the displacement field is referred to crack-tip as the origin.

So, for the plane stress case, I have the expression for u displacement which is given as u x; v displacement, it is represented as u y, and w displacement, it is represented as u z. This is very important, you should not forget this; we will look at each one of them. So, u x is given as K 1 by G root of r by 2 pi cos theta by 2 into 1 minus nu divided by 1 plus nu plus sin squared theta by 2, and u y equal to K 1 by G root of r by 2 pi sin theta by 2 multiplied by 2 by 1 plus nu minus cos squared theta by 2 and when I come to u z, I have this as minus of 2 nu divided by E into K 1 root of r by 2 pi cos theta by 2 multiplied by B.

You know I had mentioned near the vicinity of the crack-tip because of very high trust values, you would find a lateral contraction; in fact, we saw that as dimple. And now what you have is, you have an expression for the displacement itself; in a normal problem, the displacement would be quite small; because you have a crack problem, I have the value of K and if look at the parameters close to the crack-tip, you find that these value is high significant; you cannot ignore it and if B is the model thickness, you will have to multiply this by B to get the u is the displacement. So, if I have to get this expression, I can simply substituted nu as nu by 1 minus nu in this and get this expression also. Take your time to write this down.

So, this gives you very near tip displacement field equations for the case of planes stress as well as plane strain. What is striking when you compare the displacement field with the stress field? You have the expressions before you. And we had seen in the case of stress field, the stresses go to very high values when you go reclose to the crack-tip. In practical materials, it cannot go very high; it has to reach the yield strength or whatever the work hardening phenomena that takes place. So, the stress values will still be finite, but very high. When you look at a displacement field, can you have very high displacement at the crack-tip? You cannot.

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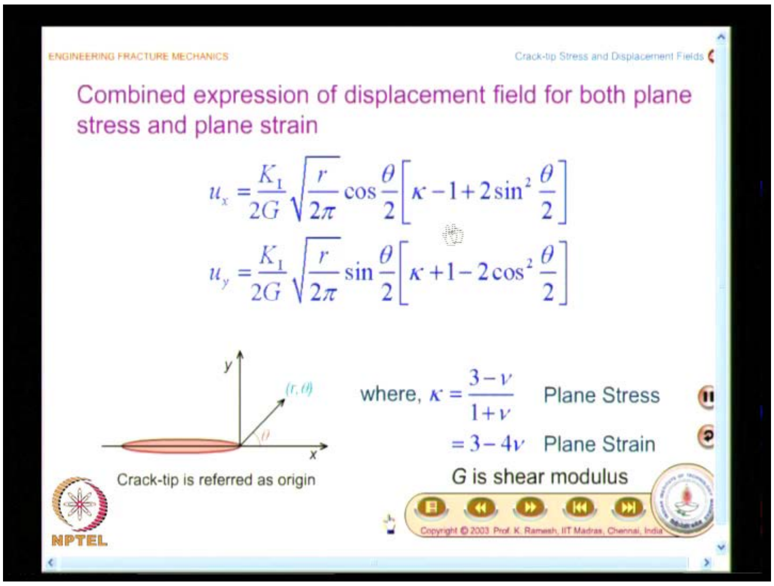

So, there is no singularity attached, when r goes to 0 what happens to these displacement components? The displacement components also go to 0. See this is another check for you to ensure that you earn the right direction. Your method of mathematics employed has been consistent, we got singular solution for the case of a stress field; we got a bounded solution in the case of displacement.

And you know books give these expressions in the various forms. You know it is desirable that you have the expressions available in all the possible ways. Books also give you expressions combined for plane stress and plane strain; you can write them down, u x is given as K 1 by 2 G root of r by 2 pi cos theta by 2 multiplied by, this is not K, it is to be understood as kappa minus 1 plus 2 sin squared theta by 2, then you have u y equal to K 1 by 2 G root of r by 2 pi sin theta by 2 multiplied by kappa plus 1 minus 2 cos squared theta by 2, where kappa is defined as 3 minus nu divided by 1 plus nu for plane stress; it is given as 3 minus 4 nu for plane strain and g is the shear modulus.

So, when you look at expressions like this, you should understand such expressions you have seen in the class; this is nothing but representing the displacement field in a convenient form essentially to safe pages that is what people have thought about it. But expressions are given differently in different books; so you should know how to look at them and mind you here for plane stress situation, u z is not given.

So, you have to know for the plane stress, you also have u z. And you know these expressions are verified several times to check for any typographical error, nevertheless I would request you to check for any clerical error in any one of these equations and you will have to filter out what expressions are important from your exam point of you, that is the different exercise .

Because I do not expect you to remember, but you should know what is important and what is not important, that judgment you will have to make, that itself is an understanding.

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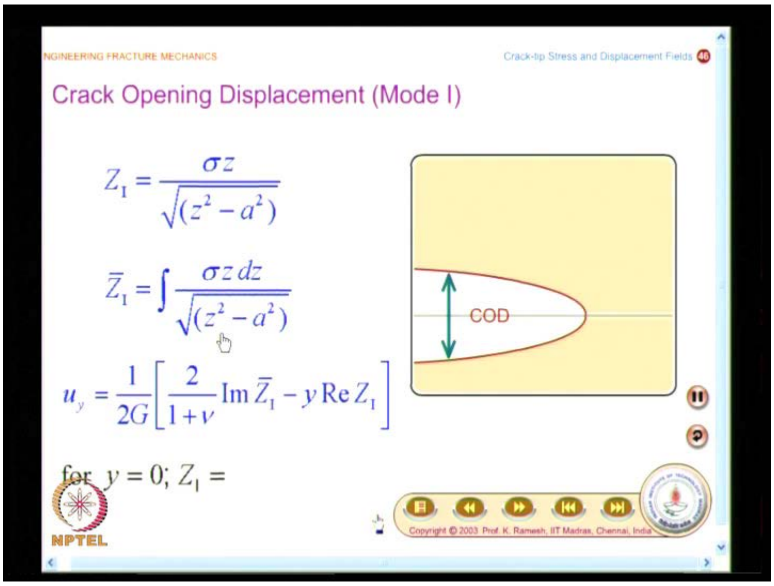

See one of the things that I have been mentioning in earlier classes was, when the crack opens, it opens like an ellipse; this is what I have been mentioning. You know when I make a statement like this, once you have got the displacement field, you will also have to evaluate it and qualify it and re-convince our-self yes, the crack opens indeed like an ellipse. So, we have to look that. And we already have the expression for Z 1 and when I want do this, I want to look at crack center as the origin.

So, I have taken Z 1 equal to sigma z divided by root of z squared minus a squared and what happens to Z 1 bar simple integration, I get this as sigma z multiplied by d z divided by root of z squared minus a squared and what is this displacement? This is nothing but the y displacement. So, I have to look at expression for u y and that is given as 1 by 2 G multiplied by 2 by 1 plus nu imaginary part of Z 1 bar minus y real part of Z 1. In fact, we are going to find out the displacement for the case when y equal to 0, because that is what defining the crack; so you really want to know for y equal to 0 what is the expression for the displacement.

And when you look at here, this simply reduces, because when you substitute y equal to 0, this term knocks off. So, essentially you will have to calculate, what is the imaginary part of Z 1 bar. And Z 1 bar is given in this fashion; now you have to go and integrate this equation and find out what is the final expression we get.

In fact for the next two three classes, you know I would appreciate that you go and refer the table of integrals. And also when you have functions with square root in the numerator or denominator, we will have substitution of variable and then do it, look at those steps also.

So, take your time to look at the table of integrals so that would help you to do the integration in the class and verify your solution with mine and finally arrive at the relevant equations that are of interest.

So, in this class what we had seen was initially, we looked at Irwin's modification of Westergaard's stress field; there we had seen his modification was very simple at the same time very significant to model the fringes seen in an actual experiment. So, from an experimental point of view, you will evaluate K 1 as well as sigma naught x and you will take that value of K 1 for all your fracture calculation.

And I also mention, there was a debate in the literature how to use Westergaard solution for uniaxial loaded situation. People have said, one could think of Irwin's modifications the candidate, but nevertheless in a later class since all bodies are finite, we will look at how to find out the stress intensity factor for finite body problems; for a infinite body with the biaxial loading, it turned out to be K 1 equal to sigma root pi a.

The basic structure would still remain same, but you would have additional factors and we have also looked at for the case of blunter crack-tip, how does the stress field changes; you cannot have sigma x at the crack-tip it has to be zero and you will also have a finite value of sigma y.

This was done by Creager and Paris, then later on we moved on to looking at the displacement field. We retained in terms of stress function so that we could get the solution with crack center as the origin or crack-tip as the origin. We have actually got the expressions with crack-tip as origin; we have looked at it for the plane stress case, plane strain case and also we looked at how to combine these kinds of expressions and see as one expression and we also emphasized the role of the displacement u z component in the case of plane stress.

Thank you.