Engineering Fracture Mechanics

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Lecture No. # 16

Westergaard Solution Stress Field for Mode-I

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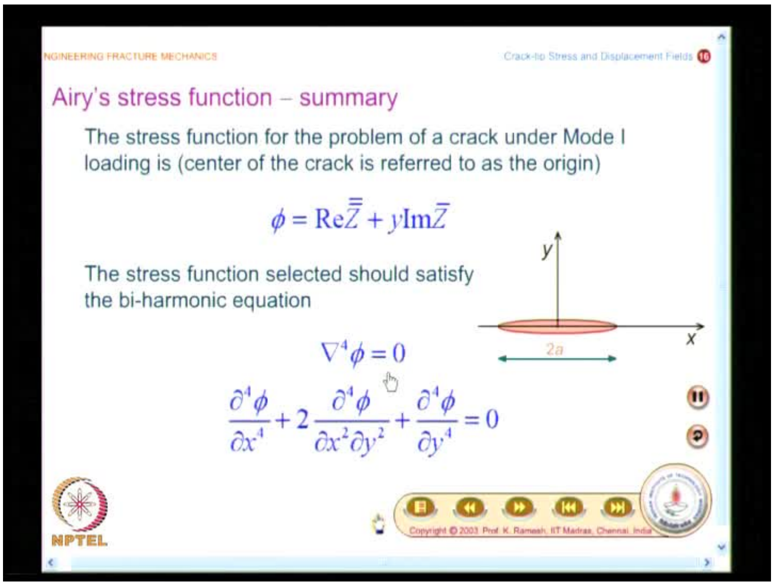

Let us continue our discussion on crack-tip stress and displacement fields. We have seen elaborately, certain theory of elasticity; once the stress function is specified, the problem is completely solved. And, in the case of a problem of a crack under Mode1 loading, the Airy's stress function is given as real part of Z double bar plus y imaginary part of Z bar; the Z is a Westergaard stress function. We are yet to see, for the case of a Mode1 problem, what is its form.

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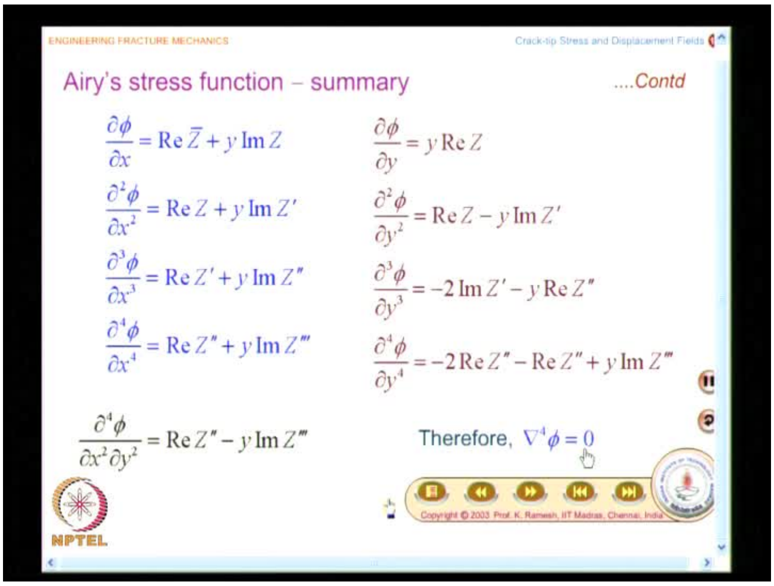

But, what we have done in the last class was, we had elaborately seen that, whatever the function phi that we have taken, it indeed satisfies the bi-harmonic equation del power 4 phi equal to 0; and, in the process, we have developed all the differentials, dou phi by dou x, dou squared phi by dou x squared, dou cube phi by dou x cube, dou power 4 phi by dou x power 4.

In fact, if you have done any clerical error in the last class, you could see these and edit them appropriately; and when you look at here, this dou squared phi by dou x squared is given as real part of Z plus y imaginary part of Z prime. This directly gives you the stress component sigma y and dou squared phi by dou y squared will give you stress component sigma x, and what we looked at was, when you substitute these appropriating in the bi-harmonic, the bi-harmonic equation is satisfied.

So, we have made sure that we are working with the stress function which is a valid candidate for solution; at this stage, let us look at it in this way; we will also look at the boundary condition, then we will compare the solution what we have obtained from the stress field we obtained, with that of the experiments. Then we will investigate what kind of corrections that we need to make. At that stage, we will come back and raise a very fundamental question, whether the stress function we have selected is indeed comprehensive or not.

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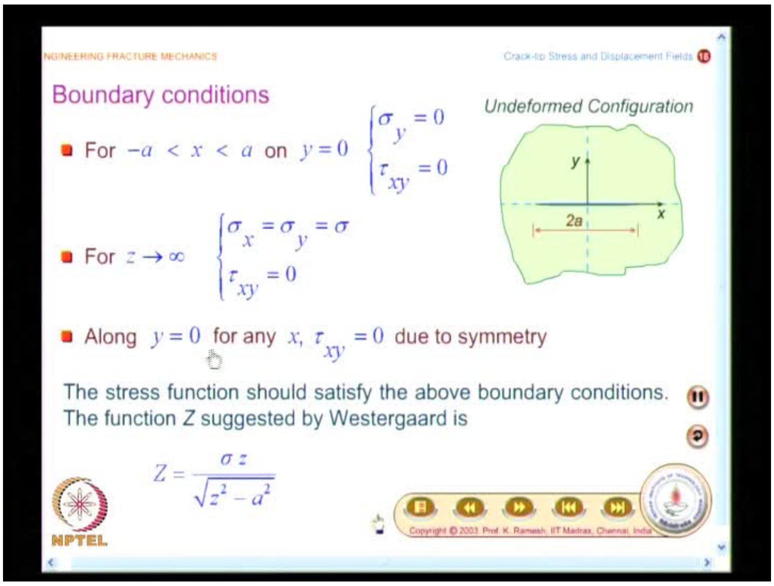

So, we will keep that question in abeyance for the time being then we move on to defining the problem. We have to look at the boundary conditions, and I had mentioned for visualization purpose, you see the crack, phases opening as an ellipse, and I also said there is close link between ellipse and crack problem. Once we developed the displacement field equations, we would go and find out how the crack phases open up, mathematically; and we will also see, what is the kind of curve that we are going to get?

These boundary conditions are very simple and straight forward. So, on the crack phases, you need to have sigma y is equal to 0 and tau xy equal to 0; at infinity, we have already noted that we are applying biaxial stress field, so, this should be satisfied. And, when Westergaard reported his solution in 1939, he actually considered a family of problems in which along y equal to 0; for any value of x, the in-plane shear stress should be equal to 0;in fact, he solved the set of problems about 8 or 9 in one single paper.

He solved conventional problems as well as problems involving crack, problems involving single crack as well as multiple cracks; and what is the kind of stress function that he had proposed for the case of central crack in an infinite strip? The stress function takes the form Z equal to sigma z divided by root of z squared minus a squared.

You know, this stress function looks very, very simple; and if you look at, for Mode 2 as well as Mode 3, the form is still maintaining; that is the greatest advantage of

Westergaard’s approach. Not only this, this stress function is meant for a single crack, with simple modification of this, he was able to get the stress function for the case of evenly spaced cracks, several cracks’ and, if you look at, when you want to find out a solution for a finite body or a body with single edge crack or double edge crack, for all of them, series of cracks would be the starting point to arrive at a solution; so, the beauty of Westergaard’s approach is, he had coined the stress function Z; with simple modifications, you could get solution for a variety of problems. In the subsequent discussion also, we will keep this as Z and get the expression for sigma x sigma y dou x, later on we will substitute the specific stress function; this is how we will proceed.

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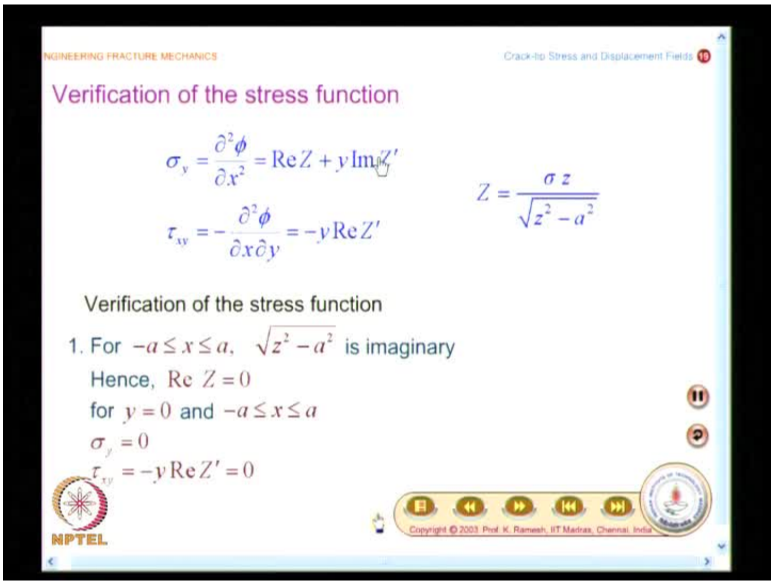

And what you will have to look at is, when you say, along y equal to 0, your shear stress should be equal to 0, is easily satisfied; if you look at the basic expression for shear stress, it is nothing but minus y times real part of Z prime; so, when y is equal to 0, shear stress component is automatically satisfied. We will have to verify whether sigma y is 0 on the crack phases; and what happens to sigma y and sigma x at infinity? And, I had drawn your attention that the expression for sigma y and sigma x, there is only a sign change of the second term; otherwise, you will have to worry about what is Z and what is Z prime.

What you will have to do is you will have to verify whether the boundary conditions are satisfied by the Westergaard’s stress function; in fact, you have already got the solution, we are only re-verifying whether the solution is correct, because by satisfying the boundary condition, we are not evaluating any coefficients. In the case of a beam problem, we saw several coefficients, and I said you evaluate the coefficients by satisfying the boundary conditions. Now, our focus is only to re-confirm whether the solution obtained by Westergaard is correct or not; and what you will have to look at? I have to find out what is sigma y on the crack phases, and this has two terms. The first term is real part of Z and the second term is y imaginary part of Z prime. On the crack phase, y is equal to 0, so, the second term goes to 0.

So, I will have to go and find out whether, on the crack phase, real part of Z exists or not. If I am able to show that you will have only imaginary part of Z, then my job is done, and that is indeed so; because, you are taking on the crack phase, x is between minus a to plus a; and, when you look at this expression, root of z squared minus a squared, this is imaginary.

See, while discussing, I interchangeably use z or Z. So, depending on the context, you make an interpretation; but nevertheless, while you write, ensure you make a distinction between z and Z; otherwise, you will really get confused. So, what we are able to now show is, on the crack phase, sigma y is 0 as well as shear stress is 0, so, the boundary condition 1 is fully satisfied.

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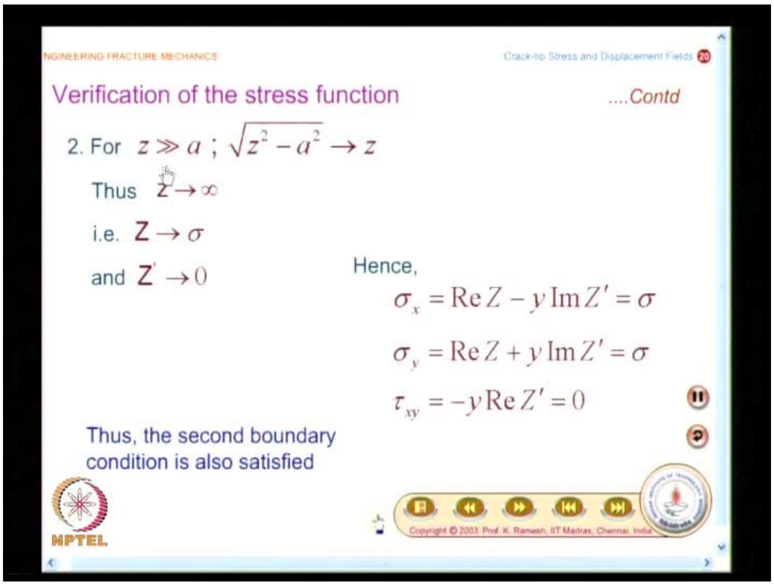

And let us look at the boundary condition 2; we are going to look at what happens at the far field; that means, z approaches infinity or in other words z is much larger than a. So, I could simplify this expression, approaching z; so, that means, what happens to the stress function? Stress function becomes just a number sigma; it is a constant; it is not a function of Z.

So, when it is not a function of Z, Z prime is 0; because if you look at the expression for sigma x or sigma y, you need to know what is Z and what is Z prime. Now, we know all of them at infinity. So, when you look at this, the expression for sigma x is given as real part of Z minus y imaginary part of Z prime, that goes to sigma; and, there is only small sign change in the second term, sigma y also becomes sigma.

So, this you will have to keep in mind, you should never forget this. This step clearly brings out that Westergaard solution is indeed for a central crack in a biaxial tension specimen; it is not subjected to any axial tension. Your boundary condition clearly shows that you are looking at biaxial problem; biaxial tension is the load that you have applied, and shear stress is obviously 0, because Z prime is 0 at infinity; and, we have already looked at, on the y equal to 0, tau xy is 0;so, all the three boundary conditions are satisfied. See, as of now, how does the solution look like? You know, I am having Z, I am satisfying what happens at the crack phase 0, and I am satisfying what happens at the infinity; so, that means, it gives you a resemblance, that the solution what you get at this stage, is valid for the entire field; it is also like a close form solution; and where does a difference comes? You would see sooner.

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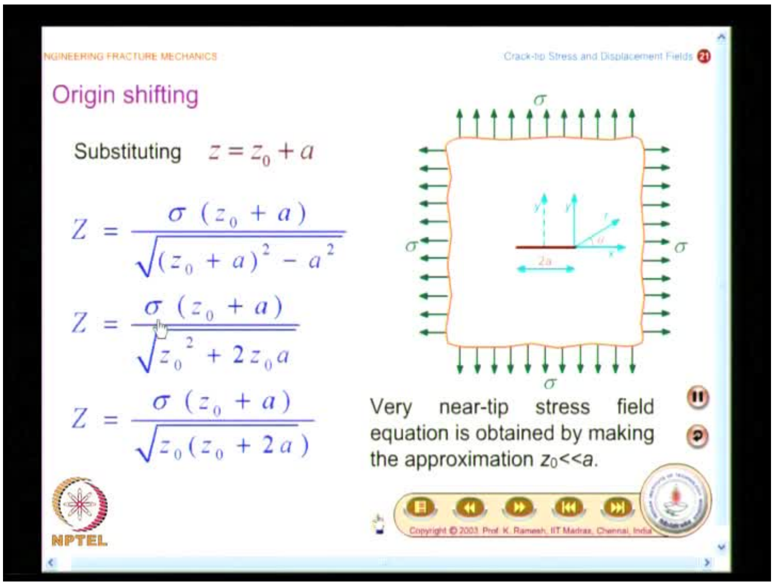

Why do we say that we are talking about near-tips stress field or very near-tip stress field? So, that comes, when I do the origin shifting; whatever the stress function that we have looked at, is given for the axis (x, y), where the origin is at the center of the crack-tip.

Now, we shift this origin to the tip of the crack, and to aid your visualization, we have also put; you have tension on the y-axis as well as tension in the x-axis. So, it is subjected to bi-axial tension at infinity. Just because people solved the problem of a plate with a circular hole, plate with an elliptical hole, with a uniaxial tension, then Griffith, when he developed the energy release rate, he took a central crack in a tension strip; people extrapolated blindly that Westergaard solution is also meant for a uniaxial tension strip with a central crack; it is not so. We have looked at the boundary condition; we have re-convinced ourselves, that it is actually for a biaxial tension. Now, if we want to get the stress field, I need to take this step and how does this modification affect us?

So, when you substitute z equal to z naught plus a, where z naught is this distance measurement from the crack tip, your Westergaard's stress function would change and it would be something like this. As engineers, what we would do is, we want to find out the stress field in the very near vicinity of the crack-tip; for that, what we are going to do is, we are going to take? z naught is very, very small, in comparison to the crack length.

So, whatever the solution that I am attempting is, for the case, when my domain is very small, close to the crack tip; so, when I do this, this expression could be simplified, and you can simply write this Z as, z naught plus a would become simply a, because, z naught is small, and when you say z naught is small, z naught square will be very, very small, so, we will eliminate this; so, I will have only sigma a divided by root of 2 z naught a, which could be recast in this fashion.

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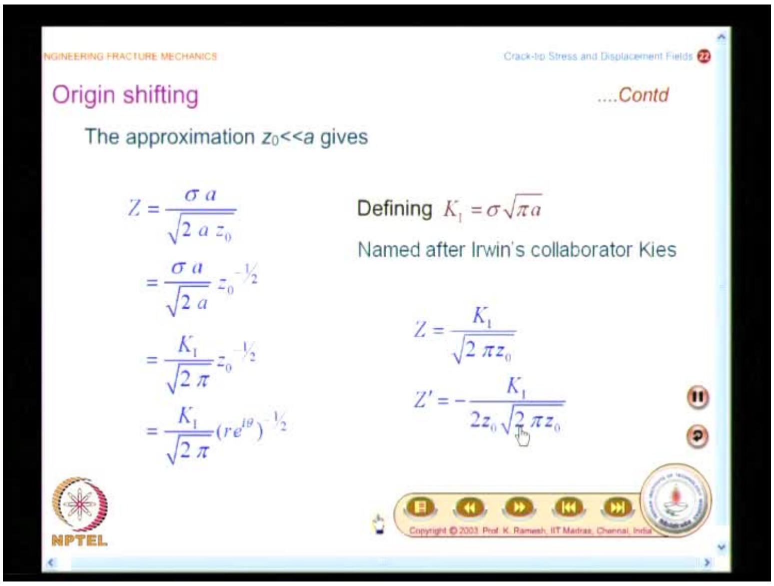

See, you have to be very careful in looking at these steps. We have looked at the boundary conditions; the boundary condition has shown that you are really looking for a biaxial tension problem; that is number 1. But when we actually want to find out the stress field, we are bringing in an approximation, that domain in which we want to find the solution is much smaller compared to the crack line, and we simplify; so, at this stage, whatever the solution we are going to get with this stress function, would be valid only very close to the crack tip. So, it is not a close from the solution in the sense, which you come across a theory of velocity, it is only a near field solution.

And now, you will have to look at further simplifications. We would also bring in the introduction of a new parameter K, because we are discussing Mode1 situation. We will take this as K1 equal to sigma root of pi a; I have already mentioned that this was introduced by Irwin in honor of his collaborator Kies. You have a terminology sigma root pi a. You know, if you look at fraction mechanism, literature people have debated whether pi should be included in this or not; people thought even sigma root a would be sufficient. There is a school of thought, because, you should also know that; without pi also, this could have been discussed, and a whole of fracture mechanics could have developed; but pi has come to stay, because Irwin has defined it only like this. So, with this modification, the stress function also can be recast.

So, when you substitute this definition, this will reduce to K1 by root of 2 pi z naught power minus half, and z naught can be written as r e power I theta; this could be further simplified in terms of cos theta plus i sin theta; that is how we will proceed. Finally, we will get the stress field in terms of r and theta; that is what we are proceeding at. And we would also look at a basic definition of what is a stress intensity factor.

Because, we have what is called the Westergaard's stress function. The moment I write the stress function in this way, you should recognize, because, we have said z naught is very small compared to the crack length. This will provide you only a near-tip stress field; once you know Z, Z prime is straight forward to right, there is no difficulty at all. And I have also mentioned, while discussing the problem of a central crack, Westergaard also provided a series of stress functions which could be used for multiple cracks, crack with wedge load; all of these we would see in the next chapter.

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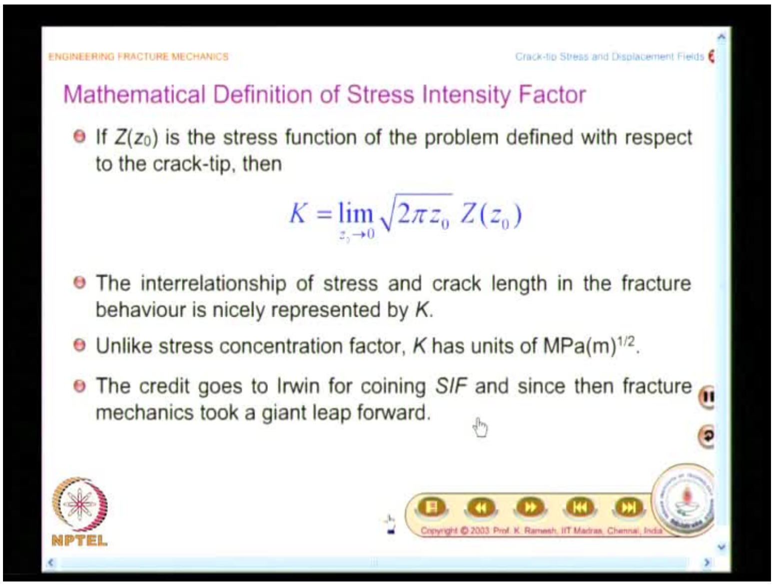

For all of them, you would have a form of stress function. So, once a stress function is given, from which, how to find out the expression for stress intensity factor, and what is the basic definition of stress intensity factor; with these two purposes in mind, we will look at how stress intensity factor is defined mathematically. So, if I have a stress function Z and, mind you, it should be referred with respect to the crack-tip as the origin; in order to distinguish, that origin is shifted, I am using the symbol z naught; if stress function is known, multiply that by root of 2 pi z naught and put the limit z naught tends to 0; then, whatever the expression that you get, will be the stress intensity factor for that particular problem. So, this is again an advantage from Westergaard's approach, he was able to provide analytical expression for a variety of problems.

So, people felt that they are able to apply fracture mechanics to certain situations in practice. And, I have already mentioned that K has a units of MPa root meter, which is quite different from our knowledge of stress concentration factor, which was just a number; in the case of stress intensity factor, you have a very funny unit where you get hurt. When you do the unit conversion from FPS system to SI system; you have to be very, very careful; if you are not careful, you can get erroneous results. And again, we will look at the advantage of the contribution by Irwin by coining SIF, fracture mechanics took a giant leap forward. So, what people felt was, energy release rate, as a concept, which was quite good, but it was very clumsy to evaluate and handle. On the other hand, by focusing on the crack-tip, Irwin coined Stress Intensity Factor, which provided a different way of looking at the crack problem; that provided greater insights and people were able to advance further in fracture mechanics.

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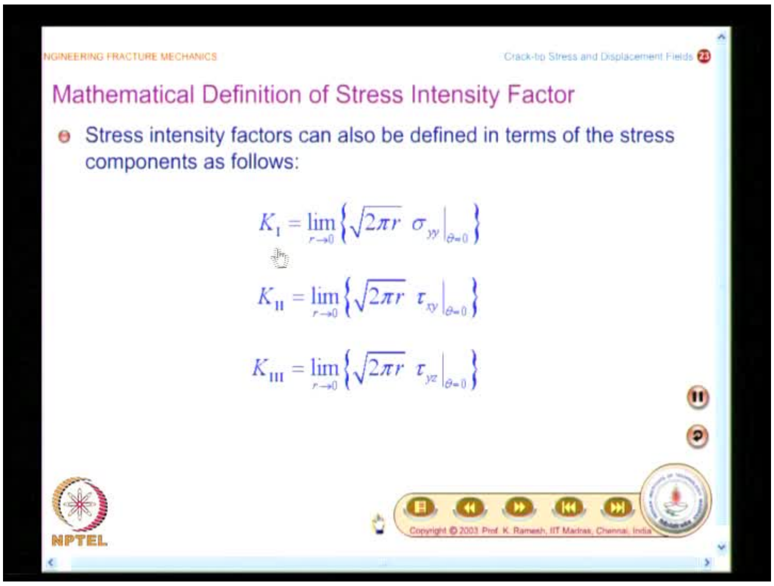

Soon, we are going to look at the stress field; based on that, we could also have definitions for the stress intensity factors. For the Mode1 stress intensity factor, you take the expression for sigma yy at theta equal to 0, pre-multiplied by root of 2 pi r; and in the limit r tends to 0,whatever you get is the Mode1 stress intensity factor. You can write for Mode2 as well as Mode3, and the difference is the stress component that you are going to look at. In the case of Mode1, you are looking at sigma yy at theta equal to 0; in the case of Mode2, you are looking at tau xy at theta equal to 0, which is in-plane shear stress; in the case of Mode3, you are going to look at tau yz at theta equal to 0, which is the out of plane shear stress. And these definitions would become useful when you are actually developing numerical methods and you want to find out what is the value of K; they could be used to find out some numerical solution, how to get K.

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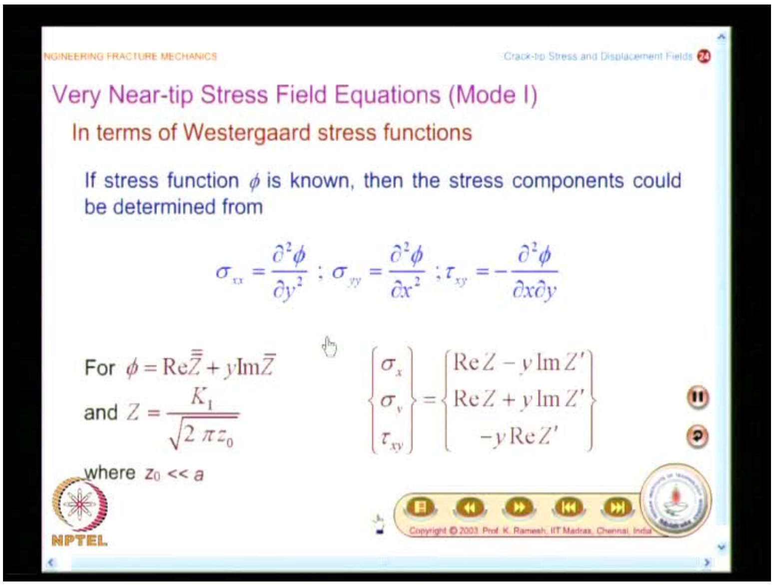

And, from now onwards, you know, you will have the title very focusly given as a very near-tip stress field equations. Because, we have shifted the origin, in the process of shifting the origin, we had brought in a simplification, that z naught is much smaller compared to the crack-tip;and in theory of elasticity, once phi is known, stress components could be determined. This is just to re-emphasize your understanding, and the stress field is now summarized in terms of the Westergaard stress function.

You have sigma x sigma y tau xy, which is equal to real part of Z minus y imaginary part of Z prime, then real part of Z plus y imaginary part of Z prime minus y real part of Z prime; and what is the corresponding Airy's stress function for this problem? It is this phi equal to real part of Z double bar plus y imaginary part of Z bar.

If you look at this expression, this is varied for the entire field, whether we choose Z as what was given by Westergaard, that is, sigma z divided by root of z squared minus a squared, or it becomes a near field solution when Z equal to K1 by root of 2 pi z naught. And, in fact, in the original paper, Westergaard, when he proposed, he showed for a class of problems, the stress field is like this; this is very generic. This becomes a closed form solution or a near-tip stress field solution depending on how do you express Z; if you express it like this, this is valid only for z naught is very small compared to crack-tip. So, that, that is what is emphasized here; so, you are really talking about very near-tip stress field equations.

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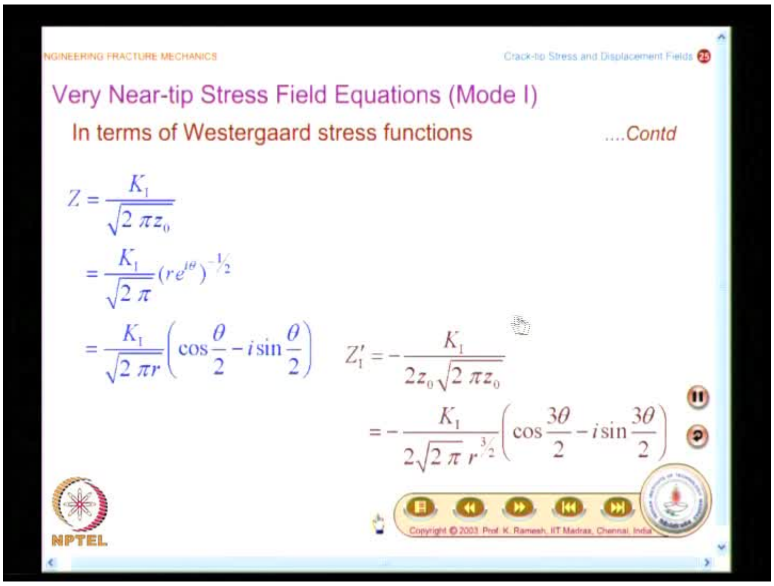

Now, we will get the expression in in terms of r and theta; this we had seen already, so, I we can write Z as K 1 by root of 2 pi r into cos theta by 2 minus i sin theta by 2.You know, after few classes, we will get accustomed to this kind of an expression, and you might eventually remember the first term; because, for me to get the stress field, I need to know what is Z and Z prime.

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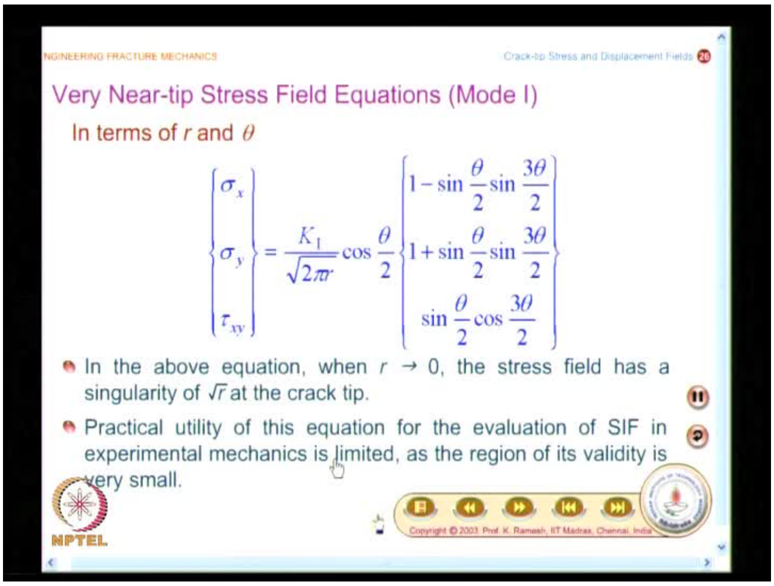

If I get them in terms of r and theta, I could substitute it in the expressions and get the stress field in terms of r and theta. Z1 prime is given as minus of K 1 divided by 2 z naught root of 2 pi z naught; and if you recognize, this is nothing but z naught power 3 by 2. So, I could write this as K 1 by 2 into root of 2 pi r power 3 by 2 cos 3 theta by 2 minus i sin 3 theta by 2.With such an expression, it is possible for me to get the stress field; and stress field is given in terms of r and theta and it is as follows. You have K 1 by root of 2 pi r cos theta by 2 multiplied by 1 minus sin theta by 2 sin 3 theta by 2. I would appreciate you go to your rooms, and then substitute the values of Z and z prime appropriately and verify whether you are able to get the final expressions like what I have shown here; the second term is 1 plus sign theta by 2 sin 3 theta by 2, and tau xy is sin theta by 2 cos 3 theta by 2; and this gives certain understanding, what happens near the crack-tip. See, if you look at the components, sigma x sigma y tau xy, the strength of all these are determined by the value of K.

So, the strength of the field is dictated by this; that is why you call this as a stress intensity factor and a field variation is defined by this (r, theta).The magnitude is dictated by a single parameter, and another one is, when r goes to 0, these values reach infinity and you have 1 by root r; and the singularity is known as root r singularity in fracture mechanics literature in plasticity situation. You will have an r singularity; the singularity strength would be different for different cases; people have noted that. And, root r singularity is a very famous terminology which you have come across very often in fracture mechanics. The elasticity solution gives the stress field approach as infinity, as 1 by root r, right? Now, what you have, we have been successful in getting the stress field is very close to the crack tip, because, we have made an approximation z naught is very small in comparison to a; is it useful for experimental? If you rise that kind of a question, the answer is No; because, the region of its validity is very small.

In fact, conventional books on fracture mechanics provide you only this solution. They do not even discuss the higher order terms; in fact, we would develop higher order terms in a few classes afterwards; because, right now, we will look at stress field then displacement filed. From displacement field, we will get back to our energy released rate chapter; we would look at how to calculate the energy based on the crack phase displacement, and we would also find and identity between stress intensity factor and energy released rate. We will do all of them, then come to Mode2; look at Mode2, then at length, discuss the higher order solution for fracture mechanics.

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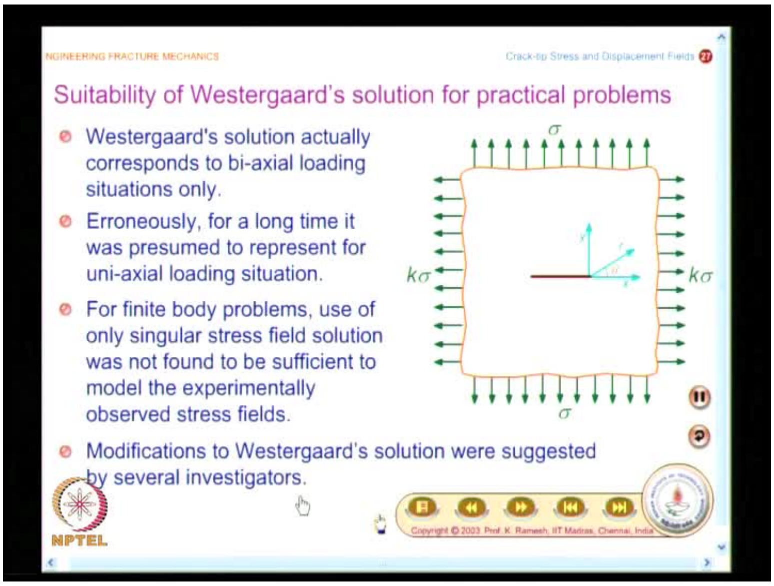

So, we will have to wait until you come to that stage; that is needed. If you want to evaluate the stress intensity factor based on experimental mechanics, so, we would do that. The important point I would like to emphasize is, the Westergaard's solution is actually meant for a bi-axial loading; that, you will have to keep in mind; since, you have got the stress field close to the crack tip. For finite body problems, use of only singular stress field solution was not found to be sufficient to Model the experimentally observed stress fields. See, if theory of elastic can provide solution only for infinite geometry, all practical geometries are finite. Ultimately, I have to get the solution for a finite geometry, because, the solution is very close to the crack tip; people have extrapolated this for finite geometry also, but you need to make appropriate corrections.

So, later, boundary allocations techniques were developed, which really gave solution for finite body problems; here, the focus, what you will have to keep in mind is, singular solution is not sufficient; you have to look for much more than that. And, for these modifications to Westergaard's solution were suggested by several investigators, people looked at what kind of approximations we have made; have we made any mistake in satisfy the boundary condition, so on and so forth; we will look at one after another.

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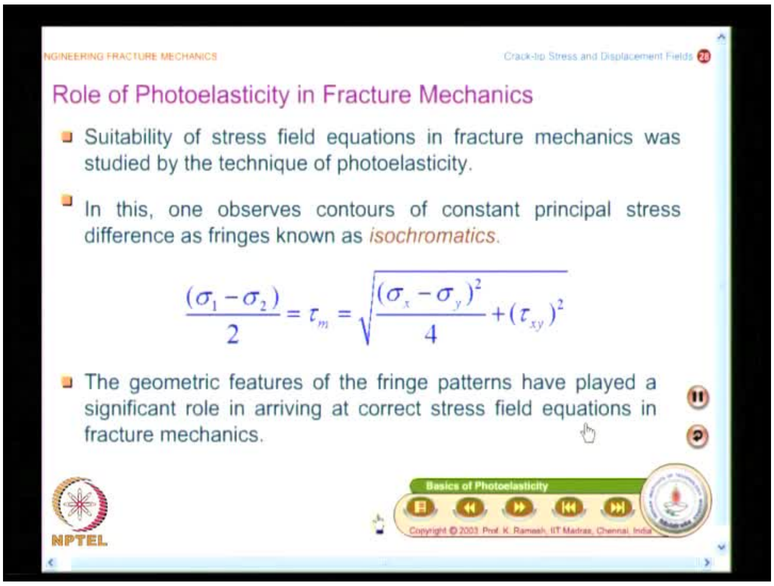

We will first see, what is the kind of result we get from Westergaard solution? We have seen the stress field, and I have been saying that we would look at use of photo elasticity in the advancement of fracture mechanics, and how we would use photo-elasticity for this purpose. We have to find out the suitability of the stress filed equations in fracture mechanics

How do we verify that? We have seen in the initial lecture, that photo-elasticity fringe contours are actually isochromatics which represent contours of constant principle stress difference, which could also be looked at as maximum shear stress. Actually, it is sigma 1 minus sigma 2 divided by 2 is what you get in the case of an iso-chromatics, and maximum shear stress is given as square root of sigma x minus sigma y whole squared divided by 4 plus tau xy whole squared. At this stage of the crack problem, we have the expressions for sigma x sigma y dou x y in the near vicinity of the crack tip.

So, we would be in a position to analytically calculate, what is the maximum shear stress? And, I also drew your attention, a difference between in-plane shear stress and maximum shear stress, very important. These are all certain concepts we wanted in-plane shear stress to be; 0 along the crack access; that we saw it as satisfied, but we did not want to have any such restriction for maximum shear stress.

Let us see what kind of solution we get. I have mentioned this earlier, it is a geometric feature of the fringe patterns, that have alerted the researchers in arriving at a suitable stress filed equations, to Model experimentally observed fringes better; and I make the statement, you have to see what kind of fringe patterns you get for Westergaard's solution; and what way you saw is, in the case of actual problems with the short cracks and long cracks, what kind of a comparison? Then we can make a judgment.

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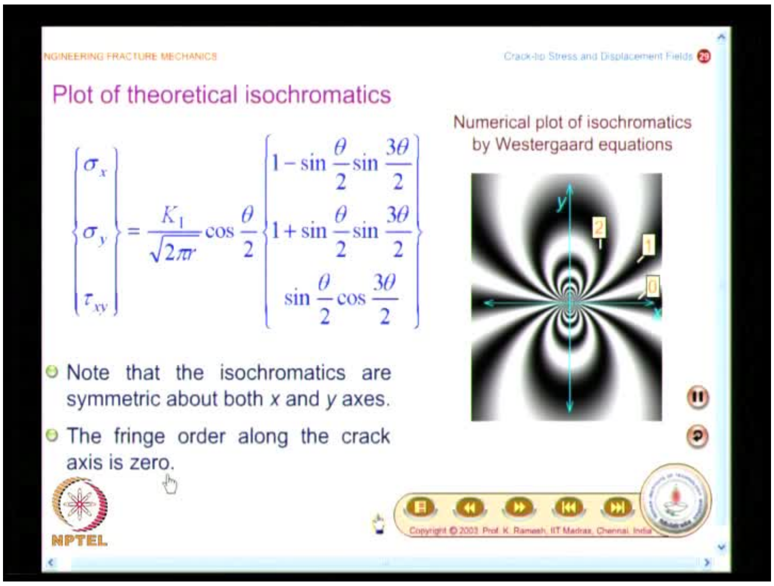

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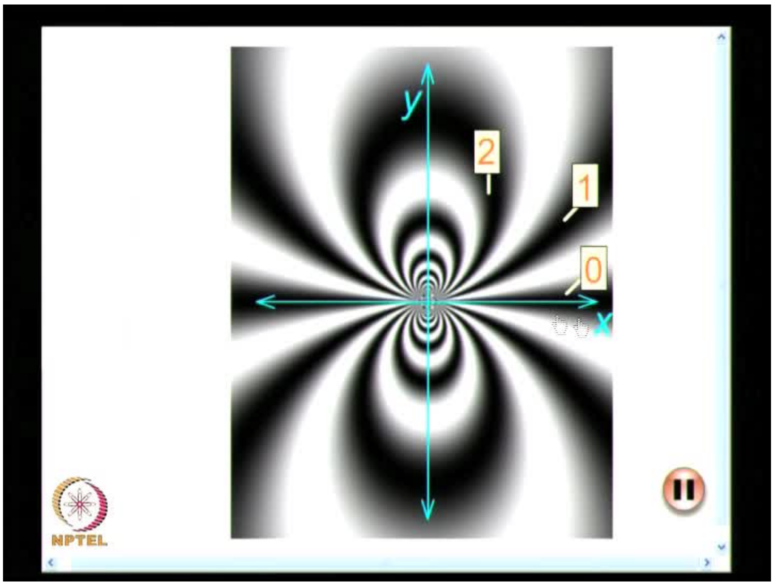

Now, I have this stress field equation available; as I mentioned, you could calculate maximum shear stress and make it as a plot and this is a numerical plot. Probably, I could have magnified for you; what, why I find here? The fringes are symmetrical about x-axis, fringes are also symmetrical about y-axis; and, along the crack axis, you have a constant fringe order 0.

See, we did not want this, we did not impose maximum shear stress is 0 along the crack axis. We have not proceeded from that perspective at all; but we are finding now, when you say tau xy equal to 0, automatically tau max is also 0 along the crack axis. This is the source of confusion; not only the source of confusion, but the source by which people looked at the procedure, and they came out with a very interesting observation. See, none of the problems, in theory of elasticity, you really question the stress function, you take a stress function and then look at sigma x sigma y tau xy, and let satisfy the boundary condition. In these cases, we have done all that we have taken as a stress function, which satisfy the bi-harmonic equation, and we have looked at the boundary condition; it satisfy all the boundary conditions.

So, we have not done any mistake in the mathematical analysis, but finally, we are getting a solution. I am sure you have already seen the photoelastic fringes; we would see them again. It is no way compared to that, and another important observation is, along the crack axis, you find the maximum shear stress is also 0.I would appreciate you make a sketch of this; you need to have this information available in your notes.

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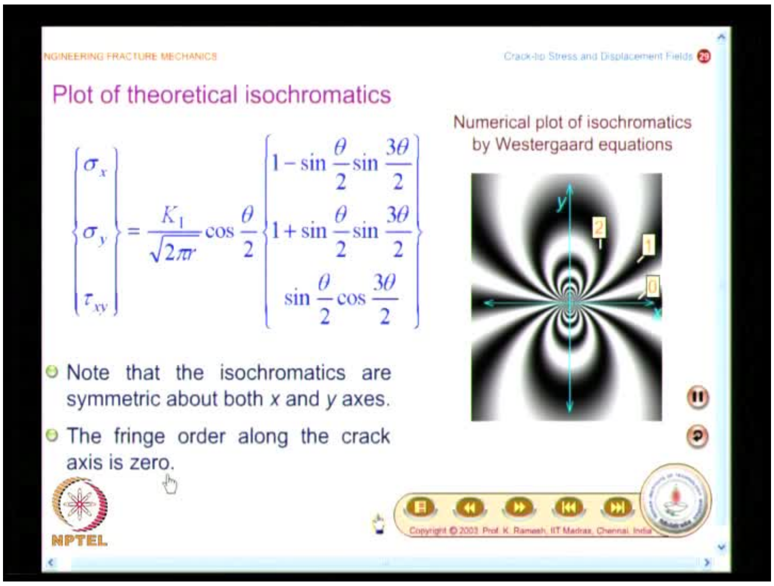

And, whatever I have said is summarized here. The isochromatics are symmetric about both x and y axes; the fringe order along the crack axis is 0, you know, for analytical; people were not exposed to experiments; it is better to say the maximum shear stress is also 0 along the crack axis, which we did not intend to be; it is accidental, the solution says maximum shear stress is also 0, but you go to the experiment, you see a different picture.

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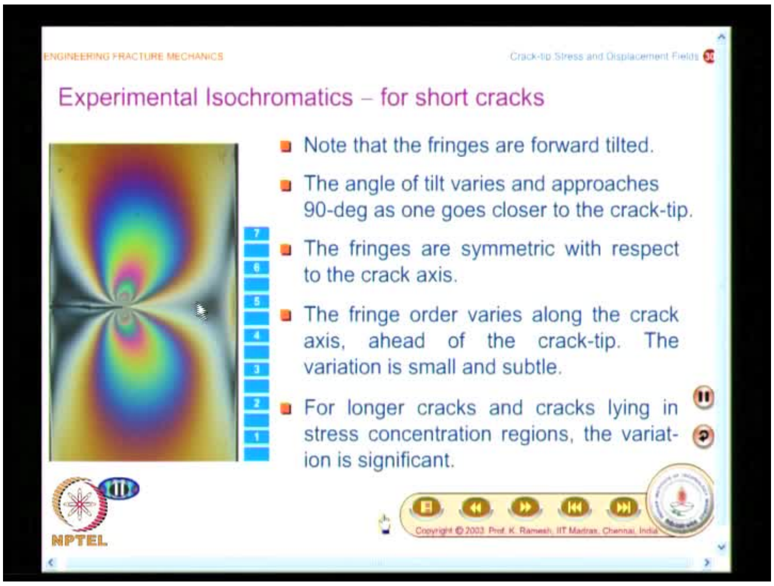

These experimental fringe patterns you had seen earlier, and what are the features that we have noted? First observation is that, the fringes are forward tilted; this is one observation; the second observation is subtle, you have to view it properly. I can find out the maximum point of this and this angle, the angle of tilt varies and approaches 90 degree as one goes closer to the crack-tip; that means, the fringes tilt like this, and you found in Westergaard solution, the fringe is exactly symmetrical about they-axis, whereas here, it is forward tilted; it may appear as backward tilted on the screen, so, I should put the hand like this.

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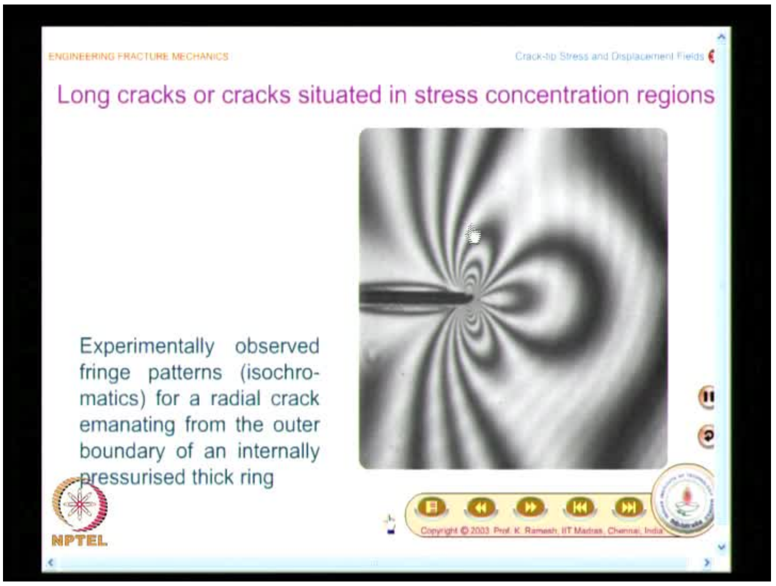

So, this is forward tilted, and another certain observation is, you find the color along the crack axis, gradually changes from some shades of gray to finally becoming more and more black, which indicates that there is a variation of maximum shear stress along the crack axes. It is not 0, that you have to keep in mind; which is not very clear in this fringe pattern. We would see another fringe pattern, we will look at long cracks, and in this, you see very clearly, I have a loop here, I have another loop here, so, there is variation of maximum shear stress along the crack axis.

So, if I say that I have understood the stress field in the vicinity of the crack tip, my analytical solution should also explain, what is this? Only then, the analytical solution is correct. See, for the case of experimental is, we cannot go very close to the crack-tip, and then, take readings, because in the very near vicinity of the crack tip, you will have unelastic deformation, and no mathematical solution is available for this zone; I can only collect from this zone. On the other hand, people were working on numerical methodologies, because, they force how the crack-tip to behave; they can go very close to the crack-tip.

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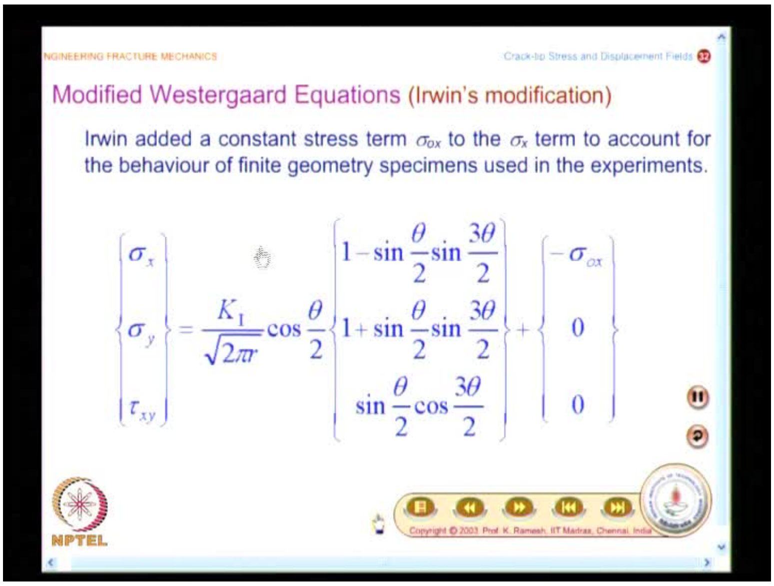

So, they were really not worried about necessity for higher order terms; they were living comfortably with singular solution; whatever the solution that we have got, which has the term 1 by root r, is called a singular solution. So, they were comfortable with the singular solution; only experimental is felt that, this is not so. And, we have to do something about it, and if you look at the historical development in 1952, Wells and post perform the first set of dynamic crack propagation experiment using photo-elasticity.

So, they captured crack travels in a plate; they saw the fringe pattern, and those fringe patterns had a forward tilt. These fringe patterns need to be analyzed; Irwin came up with intelligent argument, that you should add a term sigma naught x to the sigma x stress term. He simply added, you have to have minus sigma naught x 0 0, and this you argued, because you have this as a bi-axial solution. So, in order to compensate for whatever the experiment that they did, which was the uniaxial loading he added? This, based on simple arguments, not with great mathematical development; what he did was, he simply added a second term in the series.

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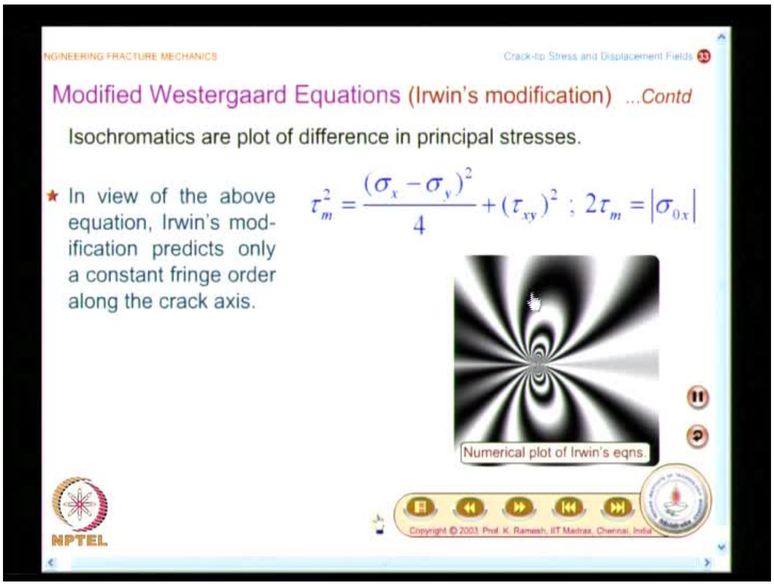

The second term contained only correction to the sigma x stress term, which is the constant value. The other two values are 0; how this was justified? If you plot the fringe pattern, which are nothing but contours of maximum shear stress; they appeared like this.

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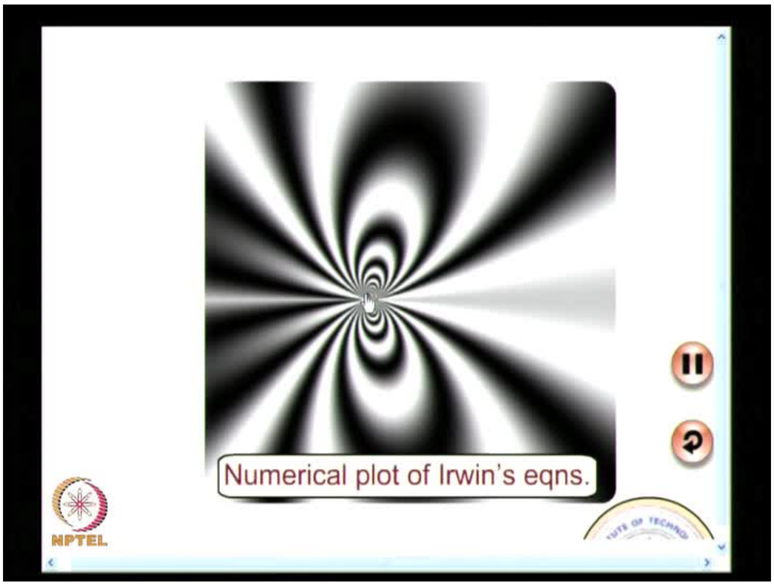

It was a bonanza; just by add one single term, the fringes got tilted forward; not only this, as you go close to the crack-tip, I think, I could enlarge this; the fringes also become straight, as that was observed in experiments. So, in 1 stroke, two important observations of experiments were satisfied. The fringes were forward, tilted, then the tilt angle is also becoming straight; as you go to the crack-tip, I mean, I should put it this way, forward tilted; and then, it is becoming straight at the crack-tip, we would also see the experimental fringe pattern.

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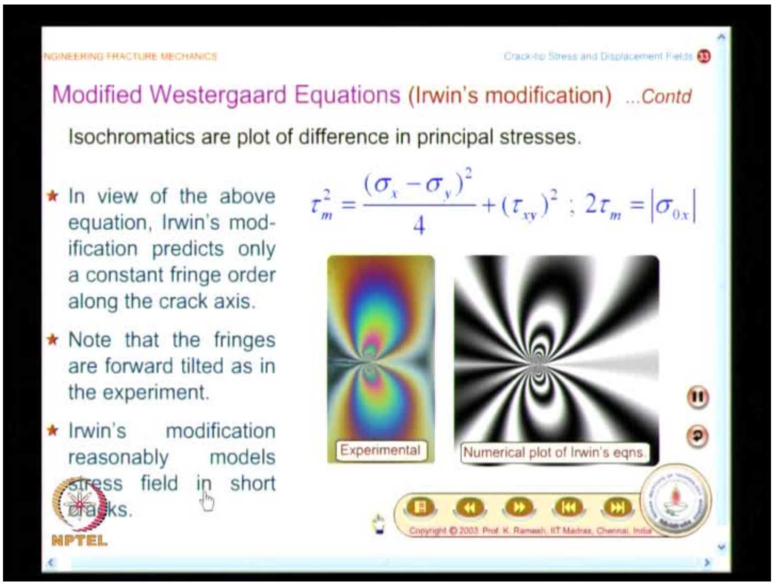

So, this matched very well with the analytical equation, by just adding a term sigma naught x; but if you look at what happens along the crack axes, when you look at the expression for maximum shear stress, tau squared maximum shear stress gives equal to sigma x minus sigma y whole squared divided by 4 plus tau xy whole squared; this gives you a constant value of maximum shear stress. In the case of a Westergaard solution, the maximum shear stress was 0.

Now, if it is the constant value in an experiment, involving longer cracks, we saw that this is varying; but for the time being, you should be happy when the crack is shorter. Now, we have captured a solution, which, as a second term, which is known as sigma naught x in experimental mechanics literature; in numerical literature, it is known as a t-stress. In fact, if you look at the historical development, it was only in 1977,F.T. Subramanian and Leibovitz came out with analytical argument on the existence of, atleast the second term to be considered; and, now, people are focusing on influence of t stress; and also, they are using this to find out even criteria for fracture.

But, if you are looking at the literature in 1957, Williams came out with an Eigen value solution. In fact, it had several terms in the series, and the second term had this constant stress value; in fact, people did not pay attention to it; that is the surprise, because, Westergaard solution was so popular, it was introduced in1939; and not only that, for a variety of problems, he could get analytical expression.

People paid more attention to his contribution, and Williams' contribution was not noted mainly, because, the analytical and numerical people were happy with singular solution; because, they could go very close to the crack-tip experimentally; it is, we are not happy with it. It is only experimental is, who emphasized initially, the need and requirement for considering higher order terms. The Irwin's modification is at least, you should have a correction to sigma x stress term.

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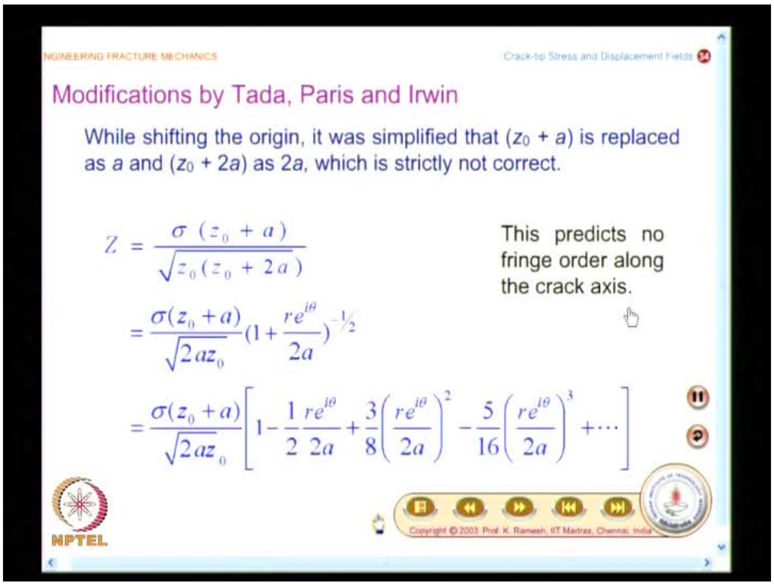

So, as I had mentioned, the fringes are forward tilted as in the experiment, and you can say, Irwin's modification reasonably, Models stress field in short cracks. So, that is a comfort, so, the Westergaard solution was useful; we were able to get the stress field; we have looked at only the first term, in fact, you should look at this as a series of terms, and people attempted what kind of attempts that they looked at, what kind of gross simplification, we attempted to do? If you look at the solution development, in origin shifting, what we did? We simply said z naught is much smaller compared to a, that could be relaxed.

And, this was done by Tada, Paris and Irwin; so, people tried all tricks that are available to them, for them to correct; you have simply modified z naught plus a as a, and z naught plus 2a as 2a and so on; instead of taking it that way, you could express the denominator in terms of a binomial series, and you are getting a series solution; atleast try to write the form of the solution, if not the complete solution, try to write the form of this binomial solution. You know, it gives the comfort to people who are analytically focus, that I have many number of terms in the series, so, if I take several terms, it would model larger and larger areas of zones near the crack-tip; it is not so.

This is the only place where we had done a gross simplification, you put in a better type of mathematics, then what you find is, though this is not shown here, the observation is, when you look at the maximum shear stress, this predicts no fringe order along the crack axis; that means, it is not useful. So, a modification in the mathematics alone has not resulted in arriving at an improved solution. So, you have to look at fundamentally, what is it that we have to do. So, only in this kind of a problem, we find, even we go back and question the stress function and modify the stress function, which I would postpone for another few classes; because, we would looked at other important results, then get into this.

So, in this class, what we have looked at was, we have developed the very near-tip stress field equations in Mode1 type of problems. We have looked at the definition of a stress intensity factor. We also looked at that the stress field, what we obtain is varied very close to the crack-tip; if you want to analyze even short cracks in experiments, I need, at least a second term, which was introduced by Irwin; and we have also noted that you need to look at much more fundamentally, whether the stress function we have taken for this problem is a valid one, to explain the features that you come across in photo-elastic experiments.

Thank you.