Engineering Fracture Mechanics

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Module No.# 04

Lecture No. # 15

Airy's Stress Function for Mode - 1

Let us proceed towards developing crack-tips stress field equations in this class. I had already mentioned for solving crack problems, analytic functions are used as stress functions. You have read what is analytic function in your earlier courses in mathematics? Nevertheless, we will look at some of the important aspects of it.

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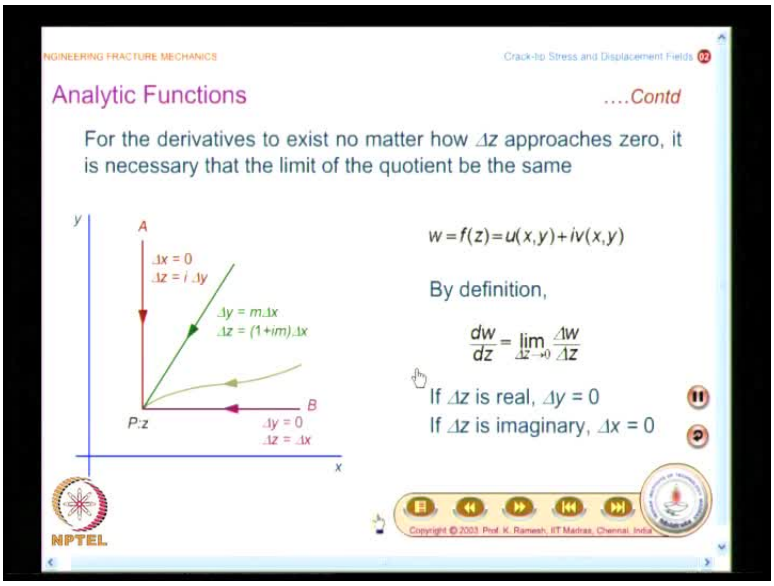

When you say a function is analytic, where you have a function w given as u of x,y defining the real path plus i into v of x,y. And z is your x plus i y and when you say that function w is differentiable, we are defining this, dw by dz equal to limit, delta z tends to 0. You have to evaluate the quotient, delta w by delta z. What we will have to look at, is for the derivatives to exist, no matter how delta z approaches 0, it is necessary that the limit of the quotient be the same.

And we have looked at two possibilities; one is if delta z is real delta y equal to 0. If delta z is imaginary, delta x equal to 0, and you have a graph that illustrates this. When delta z is imaginary, will proceed like this. When delta z is real, you will proceed like this, or it could be any generic path, either as a straight line, or as a curve. In whichever way delta z tends to 0,I must have a unique value for delta w by delta z, in the limit delta z tends to 0.

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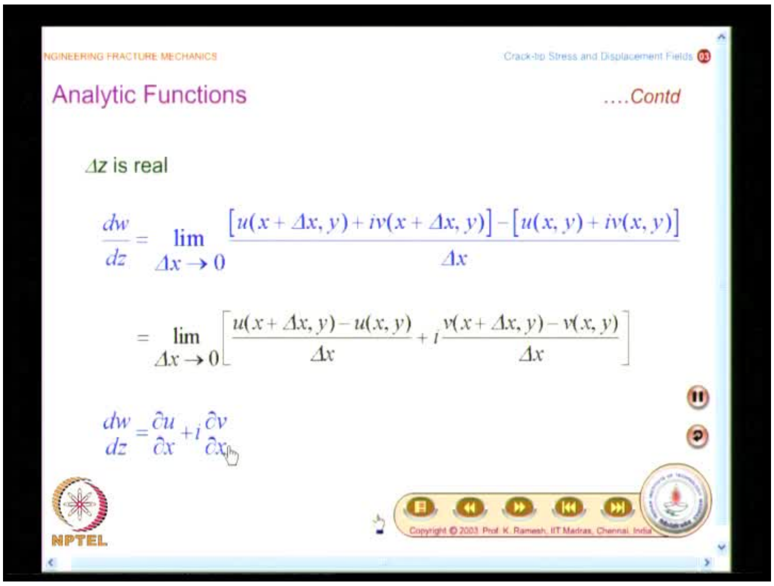

And what we had looked at in the last class, we have taken up two cases, one is when delta z is real, and we group the terms appropriately, and we found dw by dz equal to dou u by dou x, plus i dou v by dou x.

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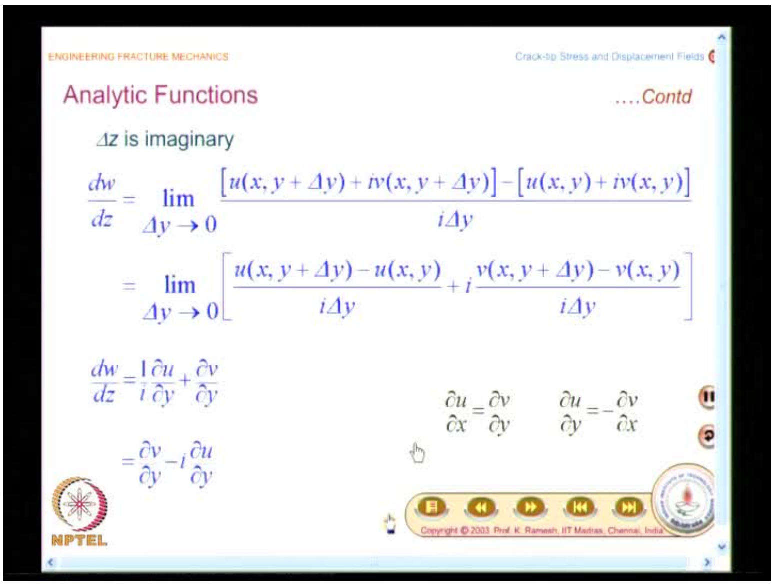

Then we considered what will happen, when delta z is imaginary. Then again you can group the terms, and the final result turns out to be dw by dz equal, to dou v by dou y minus i dou u by dou y. In fact in both the cases, we were only evaluating dw by dz.

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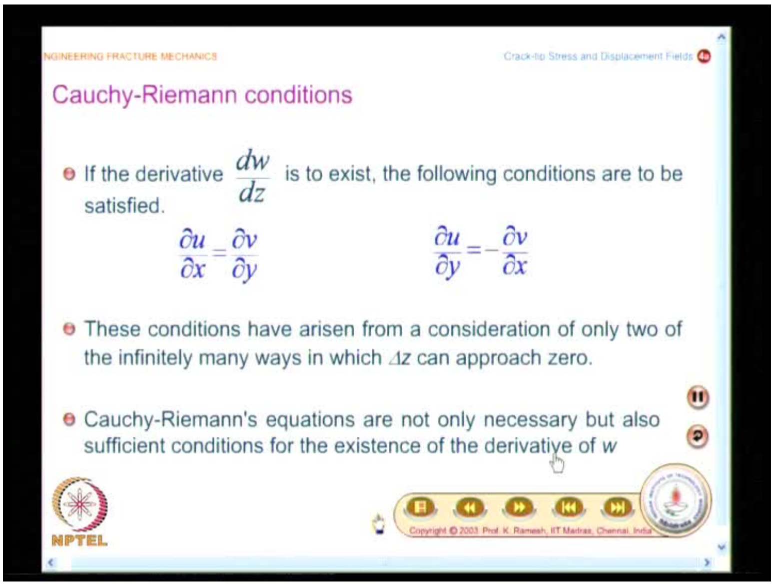

So, there has to be some kind of a interrelationship between the partial derivatives, and this is what is summarized in the next slide. These are famously known as Cauchy-Riemann conditions. If the derivative dw by dz, is to exist, the following conditions are to be satisfied. dou u by dou x, where u is the real path of the function w, v is the imaginary path of the function w, dou u by dou x should be equal to, dou v by dou y dou u by dou y should be equal to, minus dou v by dou x. Only if these conditions are satisfied, then the function w is differential that is very important.

And we have got these conditions, by considering only two of the infinitely many ways in which delta z can approach 0. And what is the final conclusion is though I have looked at only two of the many possible ways, whatever the conditions that we have arrived at, are not only necessary, but also sufficient conditions for the existence of the derivative of w. I have always been mentioning, whenever you develop a condition mathematically, you must always qualify and see whether those conditions are necessary, as well as sufficient. If they are not sufficient, what are the sufficient conditions?

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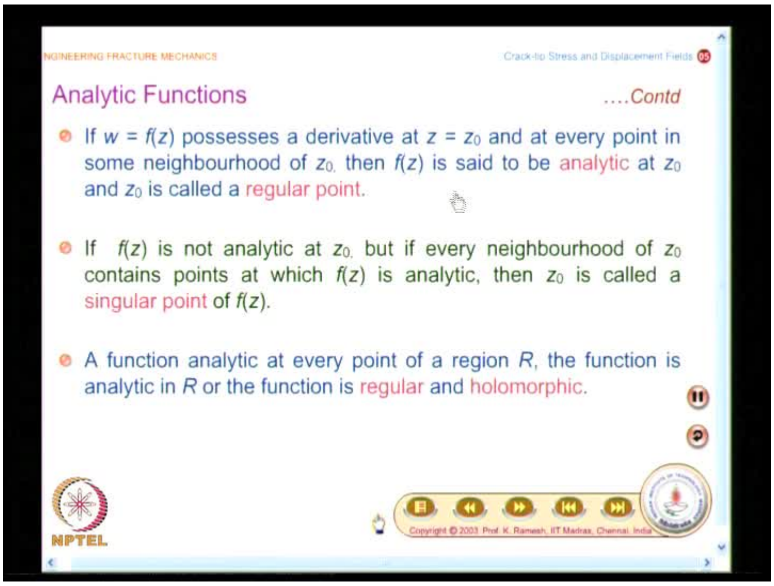

This is very important, and what you find is for a complex function, w to be differentiable, the Cauchy-Riemann conditions are given like this. If these two conditions are satisfied, the function w is differentiable and its analytic function; and we also have certain generic definitions. We look these aspects.

If w equal to f of z, possesses a derivative, at z equal to z naught, and at every point in some neighborhood of z naught, then function f is said to be analytic at z naught, and z naught is called a regular point. If you take any book in mathematics they would define some of these quantities. And if you take up a research paper, they might mention some of these aspects. So you should not feel uncomfortable, while reading research paper. So it is better, that you get to know these terminologies. So you will have to know, when you have an analytic function what is a regular point.

If the function f is not analytic at z naught, but if every neighbourhood of z naught contains points at which function, f is analytic, then z naught is called a singular point of f of z.

So you define, what is a regular point? And you define, what is a singular point? And you also have one more definition. Suppose you find a function is analytic, at every point of a region R, the function is analytic in R, or the function is regular and holomorphic. And this terminology will come across in research papers; they will say it is an holomorphic function. So that means it is an analytic function, where it is has a differential available and you can differentiate it comfortably. If you look at, we have to satisfy the bi-harmonic equation. Unless the function is differentiable, we would not be able to satisfy the bi-hormonic equation.

So we proceed towards solving problems using analytic functions, and we have to have a complete understanding on what aspects of analytic functions are important in this context. So you have to remember the Cauchy-Riemann condition, and also some of these definitions.

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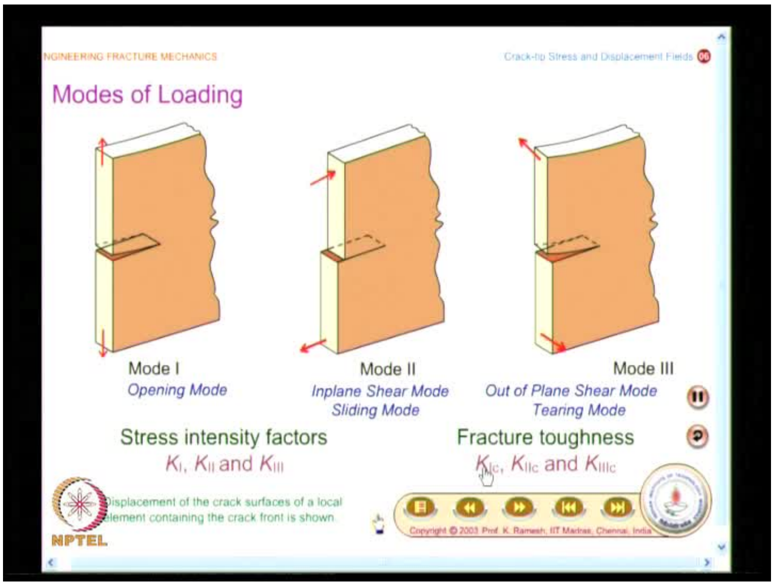

This is just to recapitulate, the what kind of problems, that we are going to solve in this chapter. We have already looked at, what is the mode 1 loading, what is the mode 2 loading, what is the mode 3 loading. And in each of these cases, our interest is to find out the stress intensity factors K1 K2 and K3, and from a material testing point of view. You would get K1c, k 2c, and k 3c. As long as these stress intensity factors, are below this crack will not propagate.

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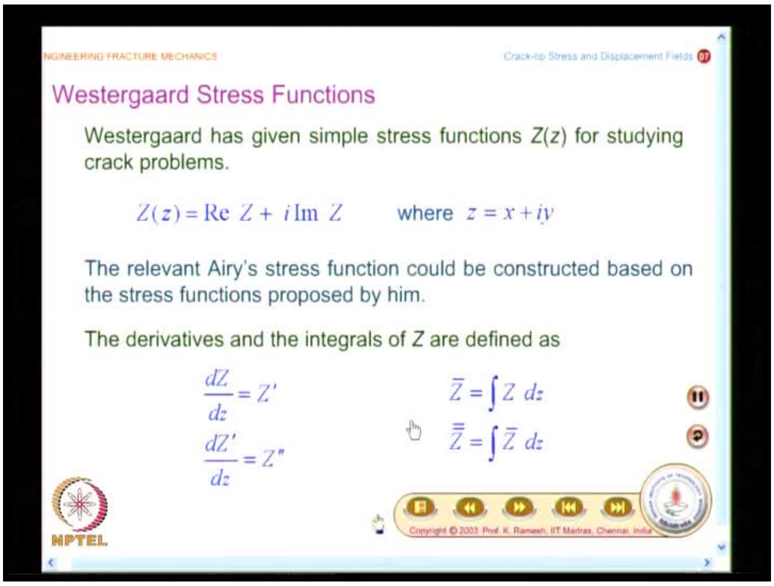

Now our interest is, to find out how to get the stress field, for these modes of loading.

So we will look at what are Westergaard stress functions. See the moment, you take up theory of elasticity Airy's stress functions are very popular. We had seen quite a variety of problems that have been solved by that approach. And in 1939, Westergaard published a papers baring pressures and cracks, in the general of applied mechanics, where he proposed a simple stress function capital Z, and mind you this symbolism is slightly confusing.

Since these slides are prepared I am able to show the stress function as capital Z. This is a function of small z, which is equal to x plus iy. So when you write it, using a software tool, you are able to show the capital Z and small z with sufficient difference. So while you, take down the notes. you must find out a way of representing the capital Z and small z appropriately may be for the capital Z, you can put a horizontal line

So that will indicate not from the size of your letter, but by that line, you can identify that you are referring to the function capital Z. And he has also written the function, as real part of Z, plus i imaginary part of Z. Though conventionally, we write the real part as u and imaginary part as v, once you come to solid mechanics, you know u and v, are reserved as u displacement and v displacement.

So, in order to avoid that, he has retained this as real part of Z, and imaginary part of Z. And you will have to live with this kind of notational scheme. There is no escape from it, so even the Cauchy-Riemann conditions; we would look at in terms of real parts of Z and imaginary part of Z. It will be very clumsy, while writing and also reading. But that is a way, the paper has been written in those early days. It is a very good paper get solved a variety of problems in just five pages. Lots of information available in the paper.

So what you have is, we now take up an analytic function capital Z, which is represented as the real part of Z, plus i imaginary part of Z. And based on Z, we would construct a relevant Airy's stress function for a given problem. And there are also other relationships that are needed for us to carry on. For there, we have to look at how to represent the derivatives and the integrals of Z. They are defined as follows.

So d of capital Z, divided by d of small z equal to c prime. Similar notation when you have a prime, you say, that this is a first derivative. When you have two primes, you have this as a second derivative. So you have d of capital Z prime, divided dz equal to z of double prime. These are for differentials, you also have a notational scheme, for representing the integrals, z bar denotes integral capital Z dz.

See normally in complex numbers usual convention, is when you put a bar, it is a conjugate function. It is not refers in that context here, so you should look at the difference in symbolism and based on that interpret, whatever the equation that you come across.

So I have Z bar is defined as integral Z dz similar to your prime. You have a single prime and you have a double prime. You have a Z double bar that is equal to integral z bar dz and you know in all our mathematical development, we would do it only as capital Z. We would not look at the details of the stress function right away. The advantage in the approach is, once we get the final set of expressions by changing capital Z, you could get solution for a variety of problems. We will make very generic mathematical calculations. We will not reduce it to a particular problem; we will develop in a very generic sense.

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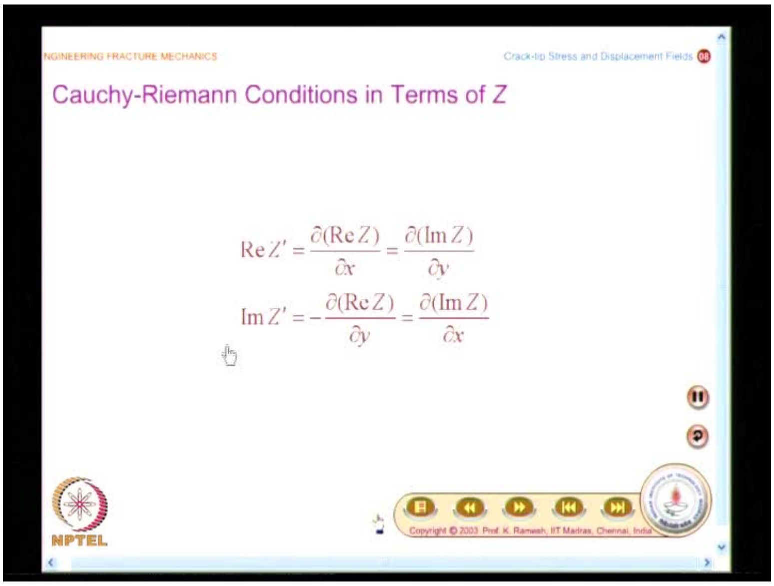

And what are the Cauchy-Riemann conditions? In terms of the function Z, we have already seen dou u by dou x equal to dou v by dou y. So which is written as dou of real part of Z, divided by dou x equal to dou of imaginary part of Z, divided by dou y.

So, that is written straight away from your basic definition of Cauchy-Riemann conditions, and you have to remember these conditions for simplification later. And what is written on the left hand side, we have also written. What you get by differentiating the real part of Z with respect to x, gives you the real part of Z prime. In fact we would use this identity, for simplification. On the other hand, if we look at the imaginary part when I differentiate with respect to y, the left hand side, it is real part of Z prime. But you have imaginary part. When I do it, with differentiation with respect to x, the real part remains real. Whereas when you differentiate with respect to y, there is a flip over imaginary part turns to real. And the next equation, if you look at the real part, will turn to imaginary this. You have to keep in mind, this is nothing, but the same Cauchy-Riemann conditions you are written in terms of u and v. This is written in terms of the capital Z, the function itself.

So the next equation, gives minus of dou real part of Z, divided by dou y equal to dou imaginary part of Z, divided by dou x. And this is equal to imaginary part of Z prime. You know you will realize its importance only, when you want to satisfy the bi-harmonic equation, for the given Airy's stress function. Then you will have to find out the derivatives and substitute, so all those, when you find the derivatives. You have to use these conditions in fact, you will do that development as part of this class.

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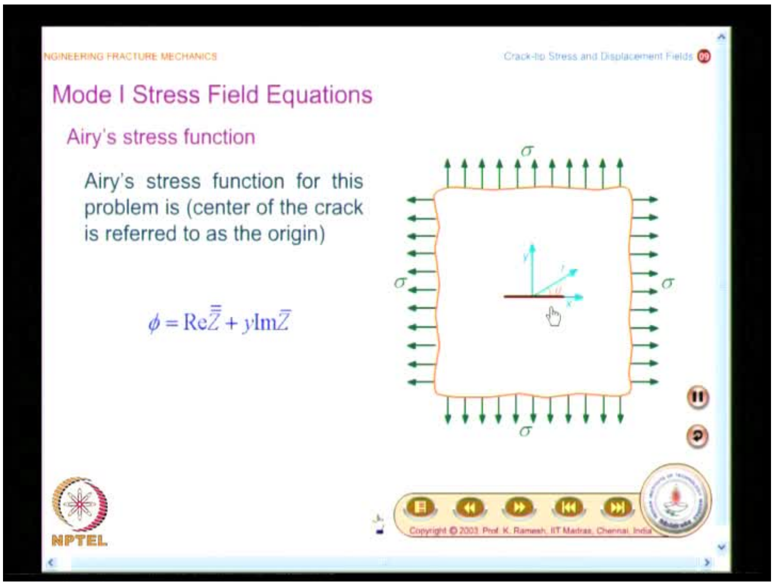

Make a neat sketch of this, and what I have here is, I have a central crack in an infinite plate, subjected to what there is a deviation. I have very clearly shown the infinite plate with a central crack, is subjected to biaxial loading. It is very, very important. See if you look at what people had developed the solution, for the plate of a small hole in a tension strip or an elliptical hole in a tension strip, they had only considered uniaxial loading. When even when the energy release rate was developed by Griffith, he considered a problem of a central crack, subjected to uniaxial loading, but what you find here is Westergaard, as actually considered, only the problem of a central crack, in a biaxial loading.

Just because people were accustomed to solving these problems, for uniaxial loading rightly or wrongly, whatever the solution which Westergaard, provided they simply used it for a uniaxial situation. Then later researchers as pointed out, no this is not for uniaxial loading, it is for a biaxial loading. And if you look at the original paper, the diagram does not show a biaxial loading, a sentences show that he has done it for a biaxial loading. You know this is a misnomer you have to be very careful about it.

So, you have to recognize that this is subjected to biaxial loading, and another aspect that you have to keep in mind, you know I have shown the central crack. And the origin is fixed at the center of the crack, I have this as x axis coinciding with the crack axis and y axis is perpendicular to there. And when I refer the point in the domain, you know we are not referring it in terms of Cartesian coordinates x comma y.

We are referring it in terms of r comma theta, why do I do, when I have complex numbers. It is easier to represent in terms of r e power I theta. So I will have only Cartesian stress components, expressed in terms of r n theta. This is again something new. You know you have a function. The function is expressed in terms of r comma theta, but it is actually giving Cartesian stress components. And what is a stress function, which was used, which is constructed based on the Westergaard stress function, was phi equal to real part of Z double bar plus y imaginary part of Z bar.

See in all of a development what we have looked at. If I have a problem once the stress function is specified, everything about the problem is known. In fact in our earlier discussion, on review of theory of elasticity, I just focused on certain salient aspects. I have not gone into the details of it.

Now, when we take the problem of a crack, we would investigate whether the function phi satisfy the bi-harmonic equation or not. In all its completeness, we would see whether it satisfy the bi-harmonic equation. Then I have to do all those derivations. Then later on I would take up the stress function, and ensure whether the boundary conditions of the problem are fully satisfied.

So, all that we will do systematically. So at this stage, what you will have to keep in mind is, I am having a central crack, where the center of the crack is taken as the origin. I had already pointed out, Irwin said do not focus on the crack focus, at the crack-tip he shifted the attention to the crack-tip.

So later, we would also shift the origin to the crack-tip and re-modify the equations and that would really bring in, or we looking at a close formed solution, or are we looking at a near field solution. So all those approximations, you have to keep track off. And in the case of fracture mechanics, when some equations is specified, you should know whether they are referred with respect to the crack center, as the origin or crack tip, as the origin, so keep this at the back of your mind.

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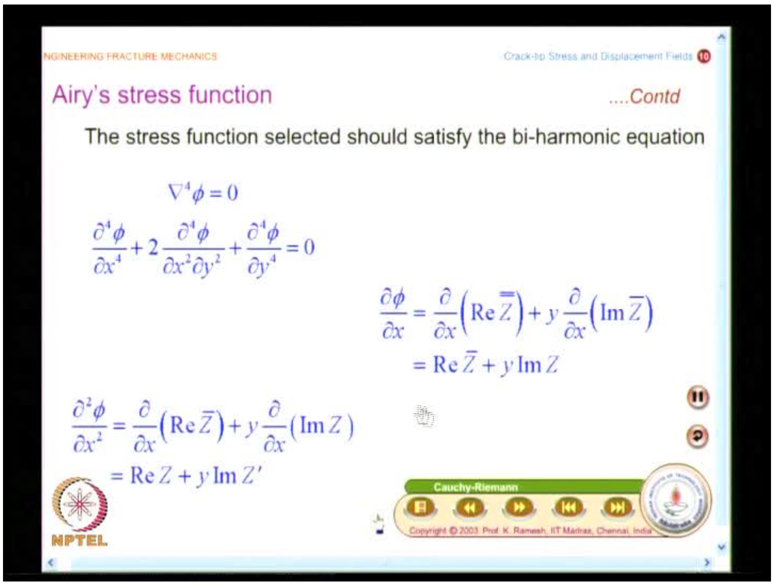

So the first test that would what we have to do is, when we have defined phi, we have to ensure whether its satisfies the bi-harmonic equation. And what is the bi-harmonic equation I have this, as dou power 4 phi, by dou x power 4, plus 2 times, dou power 4 phi, by dou x square dou y square, plus dou power 4 phi, divided by dou y power 4 equal to 0. The first thing I have to do is I have to get dou phi by dou x. Then go on with it, until I find out dou power 4 phi by dou x power 4. And when I have to do this we have already defined phi.

So what we are really looking at is, we have to get the differential of real part of Z double bar. It is dou by dou x of real part of Z, double bar plus y into dou by dou xof imaginary part of Z bar. You know I would like you to take a 2 minutes of your time, and write the expression. If you write for one of this, rest all the derivation is simple and straight forward. Let me see how you take time to develop this, I will go round the class and see.

(No audio from 24:28 to 25:03) I am happy to see, some of you have got it correctly. This is nothing but real part of Z bar plus y imaginary part of Z. If you know how to write this, rest of the derivation is simple and straight forward. In fact you would do this for all the derivatives now. I would give you sufficient time now, we will have to get dou square phi by dou x square.

So when I have dou square phi by dou x square, I will have to differentiate this and that is what is given here. So I have this as dou by dou x of real part of Z bar plus y, into dou by dou x of imaginary part of Z. From whatever you have done for this case, what this amounts to this is nothing, but real part of Z plus, y imaginary part of Z prime.

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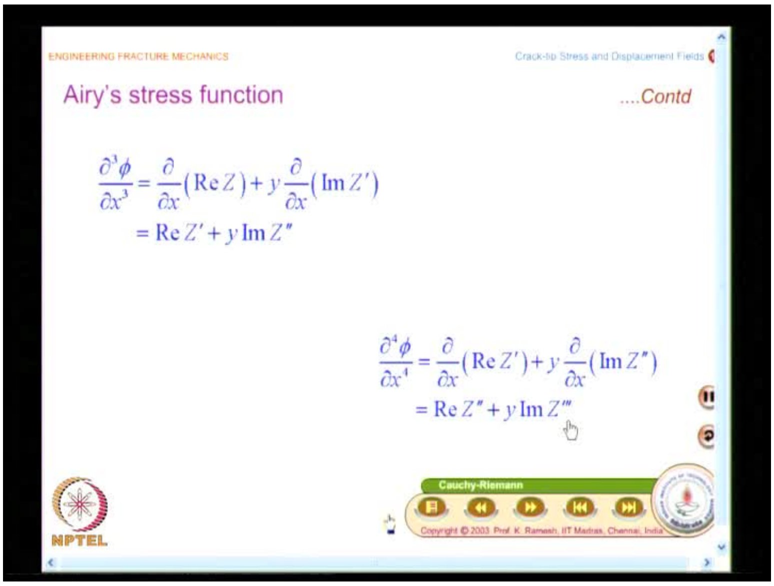

So there is no difficulty in arriving at this, and we would proceed further. We will have to get dou cube phi by dou x cube. So I will have to differentiate this and that is what I am going to do. You know when you are differentiating, with respect to x it is simple and straight forward, its quite easy for you to handle.

So when I do this, I get this as real part of Z prime plus y imaginary part of Z double prime, and mind you, these are all the function capital Z. I am not specifically saying that as capital Z, it is shown as capital Z in my slide. So when you are writing it put a horizontal bar, to differentiate with between capital Z and small z, in some way that should be a differentiation in your notes, in handwritten notes, you have to consciously do that.

If I have to get dou power 4 phi, by dou x power 4, I have to do dou by dou x of real part of Z prime, plus y into dou by dou x of imaginary part of Z double prime. This is again straight forward. You get this as real part of Z double prime plus y imaginary part of Z double prime.

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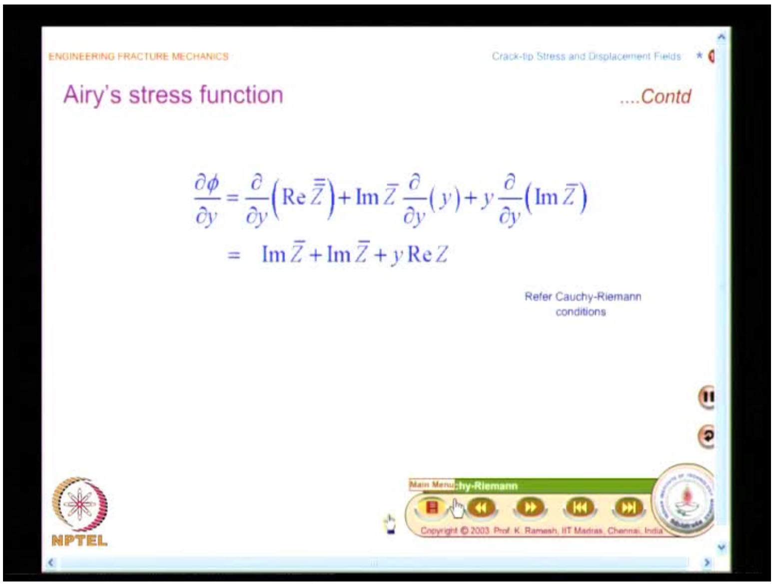

So now we move on to find the differential with respect to y. We want to get dou phi by dou you have to bring in all your understanding of your chain rule. And all that anytime if you do not use the mathematical approaches you tend to forget, because I have the function as y imaginary part of something is coming. So when I have to do that, I have recognized that y is also a function. I have to use the chain rule.

So I will have to have this, as imaginary part of Z, bar dou by dou y of y plus y, into dou by dou y of imaginary part of Z bar. See I have nice slide. I have written this, is it correct? Just look at, because you know while you are writing it, you tend to copy what is shown on the slides. You know you should reflect, whether what is written in the slide is correct or not. You know when something is written very neatly; even if it is wrong you tend to think that it is correct, what is the way stay here.

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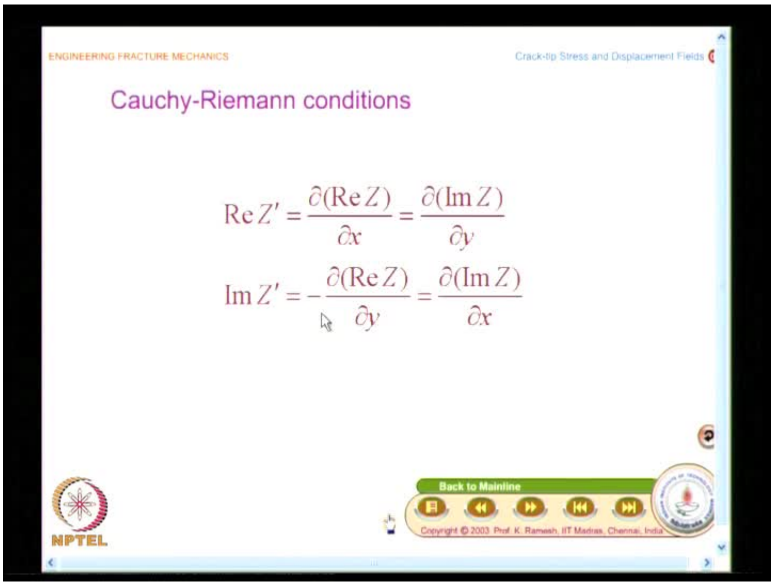

So you have to really look at the Cauchy-Riemann conditions. I have written the first term. I have this as dou by dou y of real part of Z double bar, and if I really look at that there is a minus sign sitting here.

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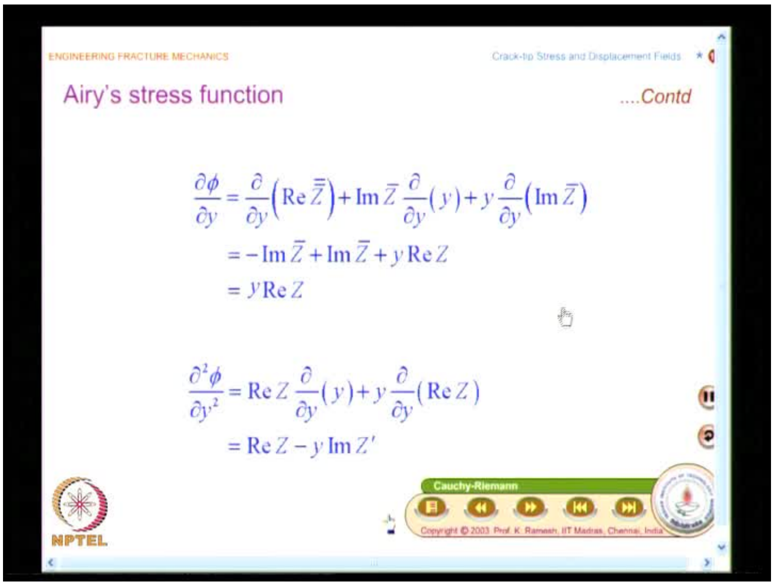

So I cannot forget that. So that is the problem that was there in that equation. So this is wrong. So when I refer the Cauchy-Riemann conditions, that becomes minus and what is the advantage? This gets cancel and your dou phi by dou y is simply y real part of Z as simple as that. So you have to be careful, and also look at. When I am differentiating the real part, it switches to imaginary part. When I do it with y, when I differentiate with to respect to y, this also you should recognize. This directly comes from your Cauchy-Riemann conditions. See the Cauchy-Riemann conditions are very important. You are just using them for finding out all this. There is nothing more or nothing less

So I have y real part of Z. So you should use the chain rule while doing this. So that is what is depicted carefully in this slide. So I have this as real part of Z dou by dou y of y plus y into dou by dou y, of real part of Z using Cauchy-Riemann conditions. You can write this terms out, to be real part of Z minus y imaginary part of Z prime. See right in the class, if you derive them with your involvement, rather than writing down from what is written in this slide, while you review the course, you also feel: Yes, I have derived all of them a sense of confidence. You will develop, these are not Greek and Latin, these were derived step by step.

So whatever the final result, I get, I am confident of that and you feel certain level of closeness to the solution. So try to involve yourself, solve it mentally, and then agree with what is written in the slide. So do not take the slide as 100 percent correct. It is deliberately done, so that you are alert in the class.

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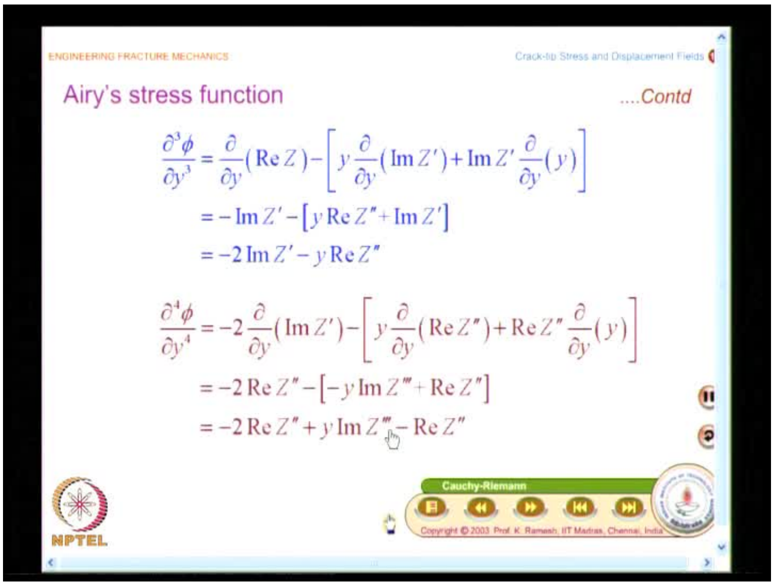

So now we have to find out, dou cube phi, by dou y cube, so that is dou by dou y of real part of Z minus y, into dou by dou y of imaginary part of Z prime, plus imaginary part of

Z prime dou by dou y of y. So when you simplify, you get an expression like this. I have this as minus imaginary part of Z prime, minus of y real part of Z double prime, plus imaginary part of Z prime. So these two add up, so I have this as minus 2 times imaginary part of Z prime, minus y real part of Z double prime.

See our ultimate objective is to see whether the bi-harmonic equation is satisfied. If the bi-harmonic equation is satisfied, what we would conclude phi is a valid stress function later on we will have to go on investigate, what problem it represents. So we are testing out each step in the procedure. Now we evaluate, dou power 4 phi by dou y power 4. So this is nothing, but minus 2 times dou by dou y of imaginary part of Z prime, minus of y dou by dou y of real part of Z double prime, plus real part of Z double prime dou by dou y into y.

So upon simplification, you get this as minus 2 times real part of Z double prime, or I could also put this. I could also have this as minus 3 times, real part of Z double prime plus y imaginary part of Z 3 prime.

See we have evaluated the dou power 4 phi, by dou x power 4, as well as dou power 4 phi divided by dou x power 4. Now we have to get dou power 4 phi, dou x square dou y square. I can start from anywhere. I can start from x, or I can start from y. It is easier because we have seen when you are differentiating with respect to y, you have to be careful.

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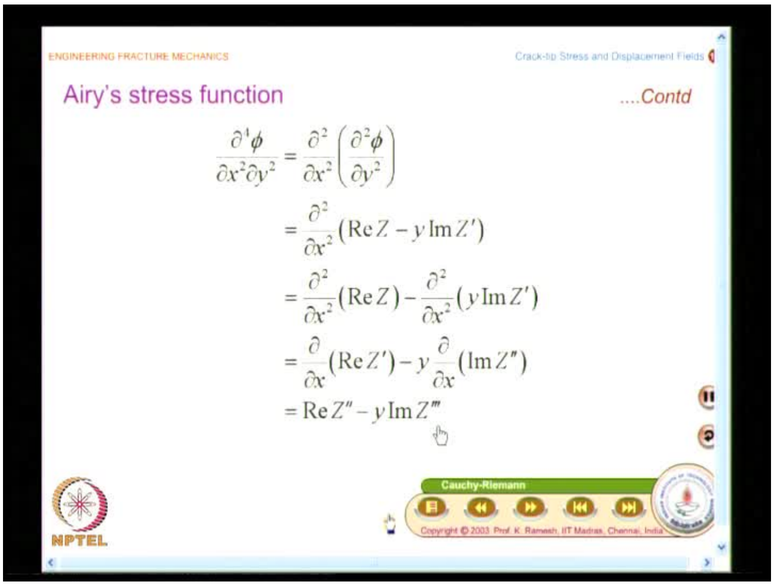

So I would directly start with dou square phi by dou y square, then I differentiate with respect to x. Once again differentiate with respect to x again, that makes my life simpler. So that is what is depicted here, so what I will do is, I would take the solution which we have already obtained for dou square phi by dou y square. Then writing dou square by dou x square, is not more simpler. It is a simple and straight forward. They are less chances of making any calculation error, and that is what is shown here.

So I have to get dou squared by dou x squared of real part of Z, minus y imaginary part of Z prime. And when you do, I have to differentiate dou by dou x real part of Z prime minus, y dou by dou x of imaginary part of Z double prime, which finally turns out to be real part of Z double prime, minus y imaginary part of Z triple prime.

Since you have all these expressions, now what we will have to do, we will have to go and substitute it in the bi-harmonic equation, and see what is it that we get. We have to get that should go to 0. Let us see whether it goes to 0.

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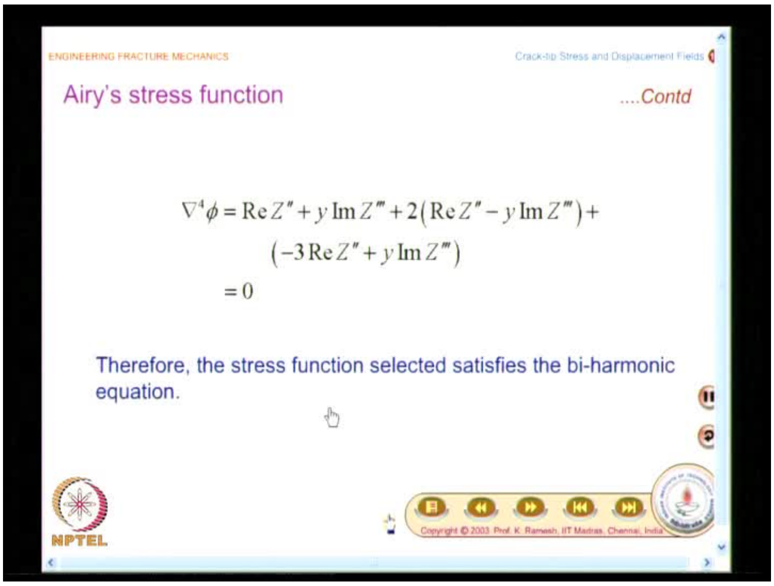

So when I substitute all of these quantities, I get an expression like this. Please take some time to write this down expression, is sufficiently long and it goes to 0.

So, what is the conclusion? The stress function selected satisfies the bi-harmonic equation. So that means our first step is correct, we are really looking at solving the problem. So we have to select a valid form of stress function, so the first condition is the bi-harmonic equation should be satisfied, that is satisfied.

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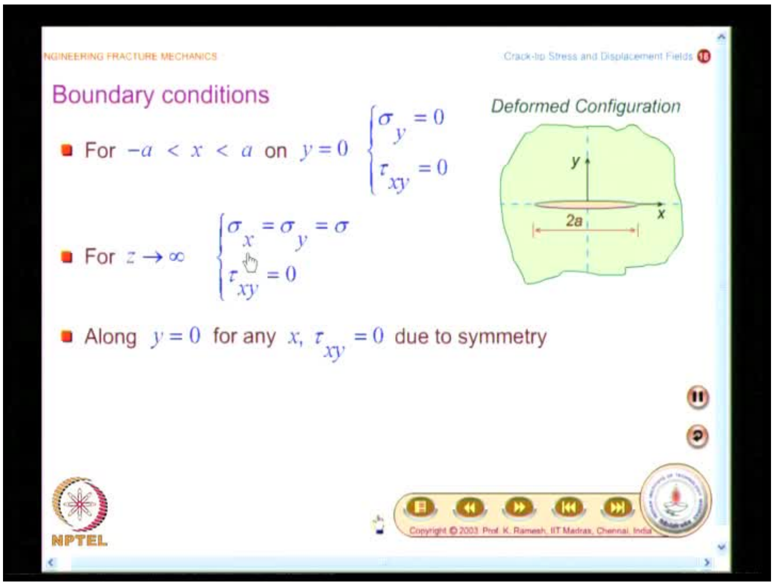

Now we have to look at what are the boundary conditions. See boundary conditions are very, very important. Only to write the boundary condition, I have trained you on the concept of what is a free surface, and if you know how to write the boundary condition carefully, and correctly the problem is solved. And in fact in this case, we would write the boundary condition we would solve. It would appear, as if we have satisfied all the boundary conditions satisfactorily. Nevertheless, the solution will not reflect what is seen in an experiment. It is something very peculiar in your strength of materials. You are not done something like this, because the theory is very well developed. So you do not have to do this. But in the case of crack problems, you will have to investigate whether the boundary conditions have been fully satisfied or you have over satisfied some condition. All this have to be looked at, but first let us look at the simple problem, and you should know how to write the boundary condition.

And what I have here, I have the crack that is opening up, and how do you classify the crack surfaces? The crack surfaces are free; you should recognize that you should recognize that it is a free surface. And you know if you are also looking at small deformation, you have to look at the deformed configuration as well. As undeformed configuration for writing out the boundary condition, but you will write the boundary condition only in the undeformed configuration, because you say the deformations are small.

So we have to write what happens on the crack surface. Crack is defined because we have taken the center of the crack as the origin. It is defined between minus a to plus a and y remain 0. So when I have this as a free surface, you have the access of reference given, which are the stress components should remain 0. See we have sigma x sigma y and tau x y of this. Whether only one stress component should remain 0, or all the stress component should remain 0 or only some stress component should remain 0, this is where you apply in your mind, because somebody has given you the stress function. Once stress function is given, the mathematical procedure is straight forward that there is nothing special about it. Only thing is you have to remember the mathematical steps, and do it, where you have the creative potential is used, is only in looking at the boundary conditions that is very, very important. I have this as a free surface

Shear cannot cross a free surface, so shear stress can be 0. That one condition, I can write between sigma x and sigma y, which stress component should go to 0 sigma y. So if you recognize that, then the first condition is completely specified. You know this is, what I have mentioned, that I am showing the deformed configuration for clarity. And another thing we observe whenever I show the deformed configuration, the crack is opened up like an ellipse. There is a reason behind it, you know we have not really looked at the displacement field. The moment you look at the displacement field, the crack faces would open up like an ellipse. Ellipse is very closely link to crack problem, in many ways. Only the problem of an elliptical hole, in a tension strip alerted the scientist, that cracks are very dangerous. It does not stop there, the ellipse goes with the crack.

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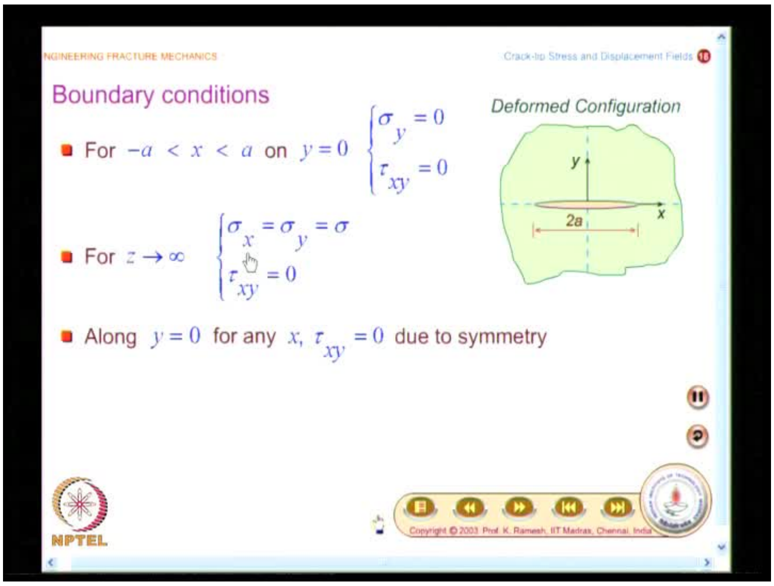

It opens up like an ellipse only we would see the equations and reconvene our self. So whatever the animation you have is, capturing a realistic feature. See what you should not confuse is, I have shown this as a curved surface. But I have written the equation for the boundary conditions, as minus a x between minus a plus a on y equal to 0 That means I am considering this still as a straight line, for writing the boundary condition. This we are justified, because we are looking at small deformation.

And what are the other boundary conditions, we know what happens at the far field. At the far field, we have taken up a problem of biaxial loading sigma x, equal to sigma y equal to sigma, and no shear stress exists tau xy equal to 0. This is where I alerted. See people have been used to looking at a small hole in a tension strip, without looking into the where the solution was developed. If you only look at the final solution there is every reason you wrongly interpret, for which problem the solution represents. See in the methodology, what we are adapting, we would develop the first term in the series solution that is called as a singular solution.

That solution is valid, but that solution is not sufficient to explain experimentally observed fringe patterns. And we will have to investigate the boundary conditions for that, so I have written two boundary conditions. Are there any more boundary conditions to have along the crack axis, you can write something about it along y equal to 0 for any x due to symmetry your shear stress has to be 0.

And you have to distinguish this is in-plane shear stress. You are talking about tau xy. See in this course, I said I am going to investigate whether the solution obtained from fracture mechanics is correct or not. Whether some modifications need to be looked at, I said I am going to use photo elasticity. In photo elasticity, you get contours of sigma 1 minus sigma 2. In fact their contours of maximum shear stress, see the maximum shear stress is different from in-plane shear stress remember that. And if you recall, I had mentioned along the crack axis, you see several fringe orders this. We had seen when the crack is very long, or when the crack is situated in a stress concentration 0 along the crack axis you see a frontal loop and fringe order varies.

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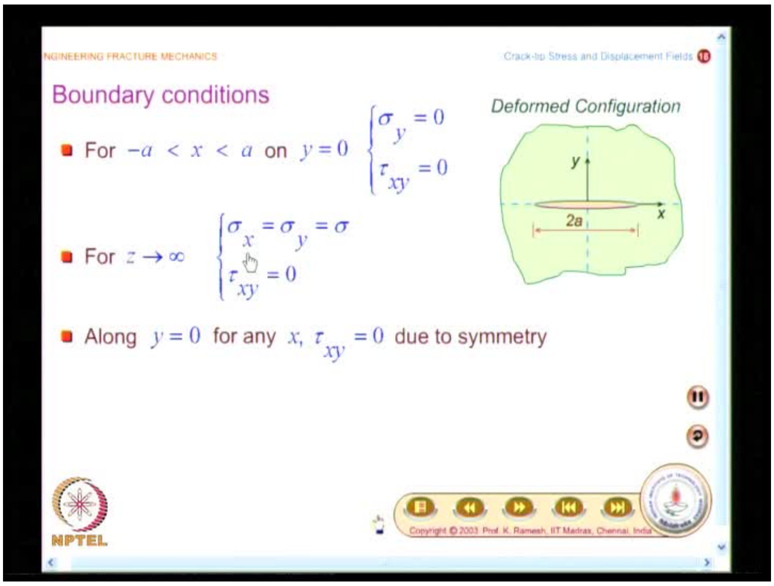

So that means along the crack axis there is variation of maximum shear stress; and here you are making a statement, along the crack axis in-plane shear stress is 0. This is correct, but what we should show later is by imposing tau xy equal to 0. Without our knowledge we have also impose maximum shear stress is 0. In the process of the solution development, this was not looked at by people in initial stages. This was discussed by Sanford way back in, after very long time from Westergaard gave the solution. He gave the solution in 1939 that only around 1980s Sanford identified that there is a deviation in fact. If you look at the history of fracture mechanics development in 1957, Williams came out with a different approach to finding the solution for the crack problem. He solved it from polar coordinates, and you have, what are known as a William corner functions that had infinite series solution.

Since the entire fracture mechanics community focused on the singular term, they have not looked at the advantages of the contribution by Williams. It was remained dormant so the analytical people woke up only around 1977. They said higher order terms also play a role in fracture mechanics, and if you look at the history, when Westergaard proposed the solution in 1939, Wells performed an experiment around 1952. Around that time, for interpreting the results of photoelastic fringes, Irwin recognize the role of higher order terms. So experimental is knew this 25 years earlier, than analytical people. So we will have a look at all of that in the subsequent classes.

So what I want to say is in a normal problem situation, if you write the boundary conditions, 1 boundary condition, 2 boundary condition, 3; and if you satisfy the solution and obtain based on all this you would be happy. That you have solved the problem, that is what people thought even in the initial stage of fracture mechanics. There is thought that they have solved the problem, only when they were unable to explain. What you find in experiment, they realized that such no answers have to be relooked, at you know it is very important.

And another aspect all along we have been saying, you know if you look at the boundary condition also. We are looking what happens close to the crack-tip, we are looking what happens at infinity at this stage of solution development. We are really looking at the complete domain say I said closed formed solutions are a luxury in solid mechanics only. Very few problems have closed form solutions, and I also alerted for the problems involve in crackwe would not be able to get a close form solution, but we would develop near field solution near the crack tip. How the stress field is, how the displacement field so on and so forth at this development, it gives you an impression that you are able to satisfy what happens near the crack-tip, what happensat distances away from the crack-tip.

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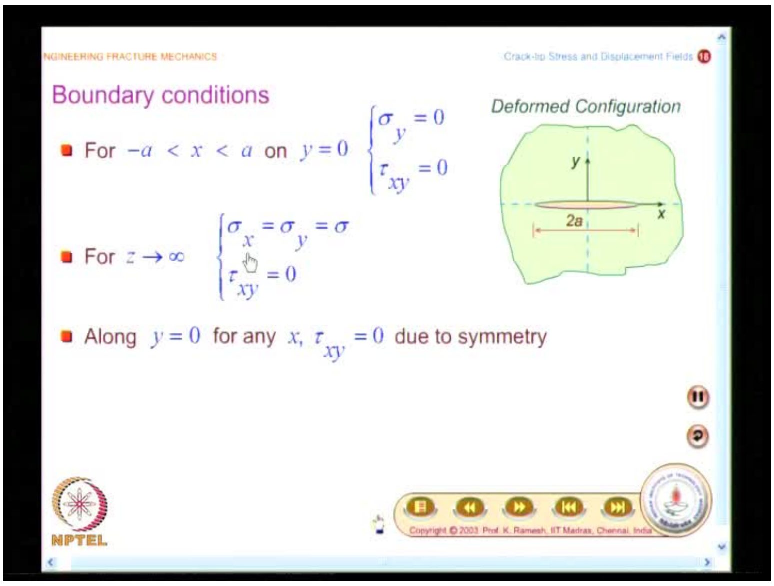

So in this class, what we had looked at was for crack problems, analytic functions are used as stress functions, and we need to know certain features of analytic functions. For us to proceed further, we looked at what are Cauchy-Riemann conditions, and these Cauchy-Riemann conditions are extensively used, when you want to investigate whether they given stress function is a valid stress function. That is what we had derived and I had also alerted whatever the problem that we have taken up, is a central crack subjected to biaxial loading you should never forget. That if you look at the solution and simply jump that for whole in a tension strip or elliptical hole in tension strip, we have looked at only for uniaxial loading on similar lines. Whatever the solution I get, for the crack problem, I will extra polite it, for uniaxial loading. Do not look at that, because the boundary conditions clearly show, you are really looking at what happens at infinity. And here we specify sigma x equal to sigma y, equal to sigma. Keep that in mind that is essential for you to look at the stress field development in the next class. Thank you