Engineering Fracture Mechanics
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Module No. # 03
Lecture No. # 14
Forms of Stress Functions

Let us continue our discussion on review of theory of elasticity. See in this review, I will try to highlight certain important aspects of how we solve the problem in theory of elasticity, rather than getting into detailed derivations. Whatever the discussion that we do, it will help us to develop the crack-tip stress and displacement fields in an indirect manner. This knowledge basis is essential for you to develop the solution for the crack problems.

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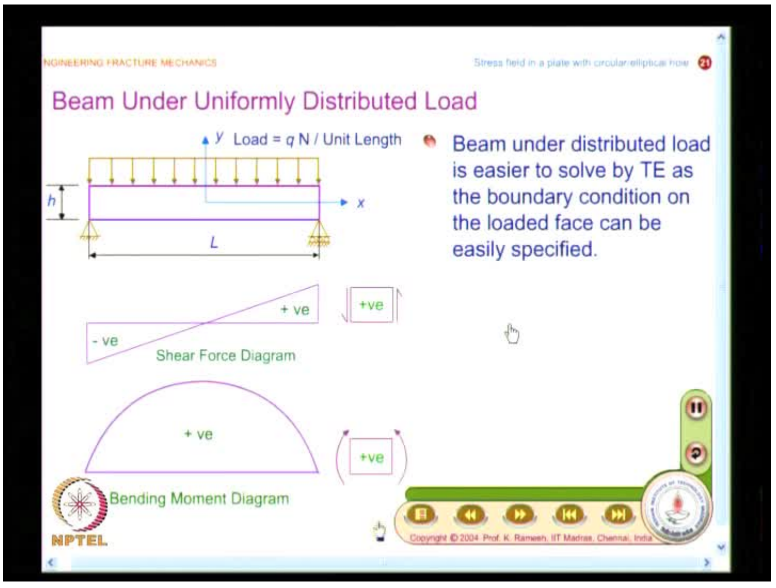

And in the last class, we had started discussing the solution for the case of a beam under uniformly distributed load. I drew your attention to the fact, when I have a uniformly distributed load shear force varies along the length of the beam.

It is not remaining constant; it is varying along the length of the beam. When shear varies along the length of the beam, your flexure formula is no longer value. Flexure formula is valid, only for the case of constant bending moment. We saw as an exception, if you have a constant shear force, you could use it as an exception. And if you have deep beams, then again there is a shear coupling, and in this case, what you find is the shear force varies linearly over the length of the beam. So, definitely, the flexure formula is not applicable. The plane sections do not remain plane before and after loading, so you need to expect a remedy by solving it through theory of elasticity.

And another aspect, is while you learned how to draw a bending moment diagram, you first take up the case of concentrated forces. You may have a three point bend. I may have concentrated force rather than a distributed loading. But when you solve the problem by theory of elasticity, the boundary conditions could be easily specified when the loading is distributed like this.

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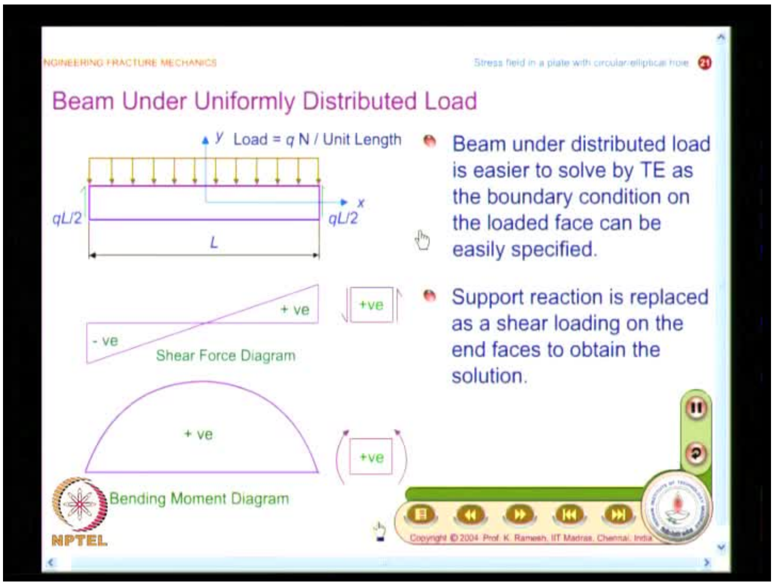

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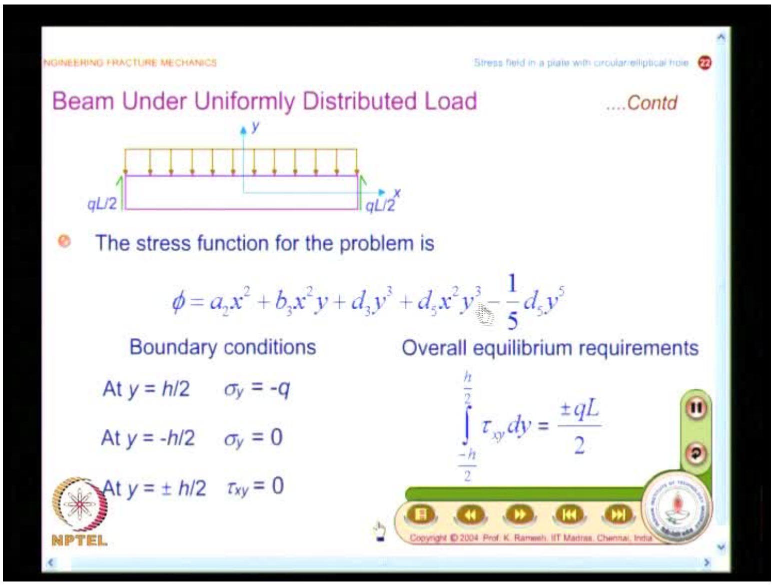

So, this is one of the simplest problems taken up while developing concepts in theory of elasticity. The reason is, the boundary conditions could be specified. One of the first things, you do is, in order to write the boundary condition, you replace the support reactions, by the shear loading, on the end faces. And this is what this indicated here for clarity, and what is the stress function for this problem? The stress function for this problem, is given as phi equal to a 2 x square plus b 3 x squared y plus d 3 y cube plus d 5 x square y cube minus 1 by 5 d 5 y power 5.

If you really look at, you have certain elements of polynomial of degree 2, polynomial of degree 3, and polynomial of degree 5. These are combined in a suitable fashion to be a valid stressfunction for a problem that we have taken up. How the people arrive it there, are no set procedures, by trial and error, people arrive at some logical reasoning, they would be able to give it. But once the stress function is specified, the entire problem can be solved. And if you look at the stress function, I have the coefficients a 2, b 3, d 3, d 4. These have to be determined. I have 1, 2, 3, 4 coefficients. In these, 4 coefficients need to be determined. And you look at the boundary conditions, they provide you sufficient number of equations to get the coefficients.

And what are the boundary conditions? The origin is taken as the center of the beam. So you have plus h by 2, minus h by 2, at h by 2. What you have is you have the distributed load. We are considering a unit thickness, so sigma y itself becomes minus q. So on the top surface, you specify the bulk boundary condition. On the bottom surface, and also you have specified the boundary conditions, sigma y equal to 0.

And we also have on the top and bottom surfaces that is y equal to plus or minus h by 2, the shear stress is 0. And what other conditions that you can write? You could write what happens on this edge, as well as this edge, and that is covered by your overall equilibrium requirements. Because what we know here is, only the sheer force. We do not know, what is the distribution?

The distribution has to be obtained as part of your solution procedure. So I can only write on this edge, integral minus h by 2, to plus h by 2, tau xydy, equal to, qL by 2.That is only thing, which I can write. I would not be able to specify individual magnitudes of tau xy like, what I had done on the top phase, and the bottom phase. I know only the integral equal to a quantity like this.

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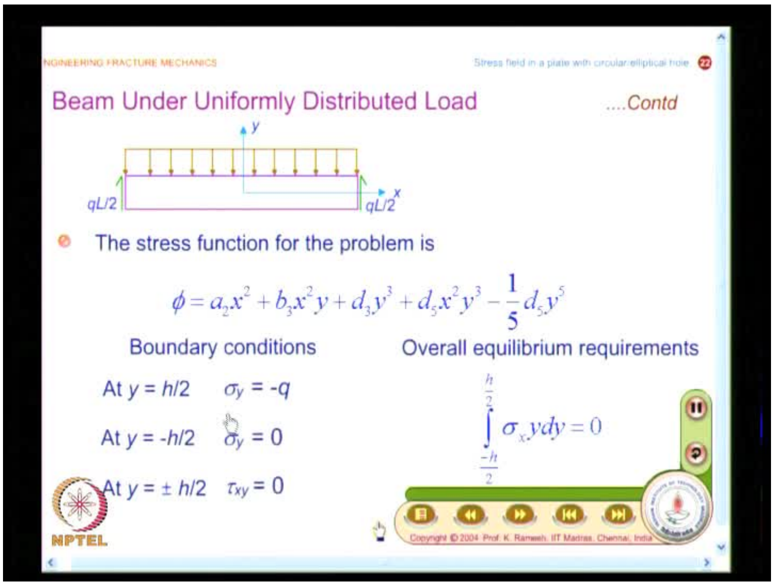

And what other things that you have here? There is no force in the x direction, there is no moment and they also appear as relevant conditions. So I have integral, minus h by 2 to h by 2 sigma x dy, equal to 0. The net force is 0. And you also have, minus h by 2 to plus h by 2 sigma xydy, equal to 0. So this gives you the moment. And if you look at the boundary conditions, it is possible for you to evaluate these coefficients. I is not getting into those details, and once you know what is phi, you can easily write the expression for sigma x, sigma y and tau xy.

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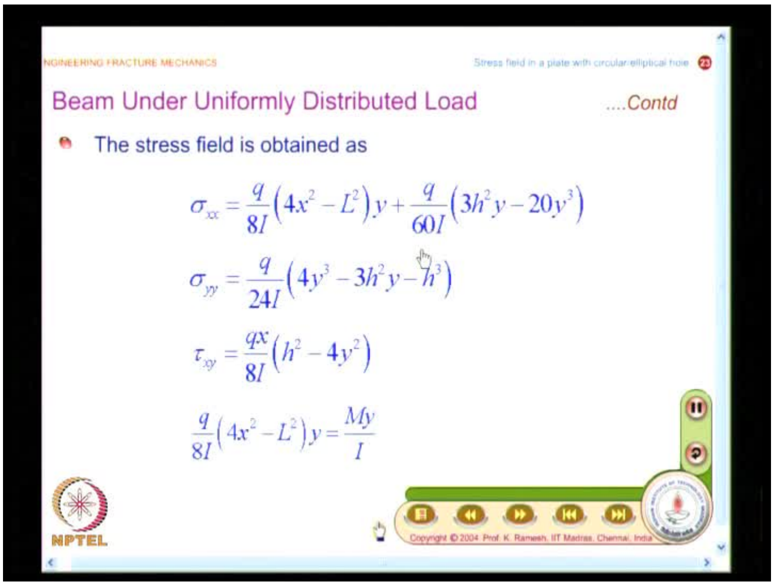

So, we will directly look up what are these stress quantities, and I get sigma xx as q by 8I into 4x squared, minus L squared y. If you look at like this, it looks as if it is something different. In a sense, this is nothing. But what you get from your flexure formula, this term is nothing, but My by I.

So, I have a part of the solution is same as what we have seen in the strength of materials approach. But there is an additional term, which is actually a correction factor which is given as q by 60I, 3h squared y minus 20y cube. So theory of elasticity recognizes the deformation of the planes, and because we are satisfying the overall equilibrium condition as well as the boundary conditions and compatibility. And finally we get a solution, because we have not made any assumption on the displacement picture displacement is evaluated as part of the solution procedure. So in a sense it has expanded. That the scope of problems that you could solve by a theory of elasticity approach. That is the way you have to look at it.

Then you have a surprise. That I have an expression for sigma yy, which we never saw in strength of material solution

You would not even recognize that sigma yy exists, and that is given as q by 24I, into 4y cube, minus 3h squared y, minus h cube

Then you have tau xy, which is given as qx, divided by 8I, into h square, minus 4y square. This is same as what you get from strength of material solution. Because shear force, varies as a position of x. You have the term qx here, and you will essentially get the shear force distribution as a parabola.

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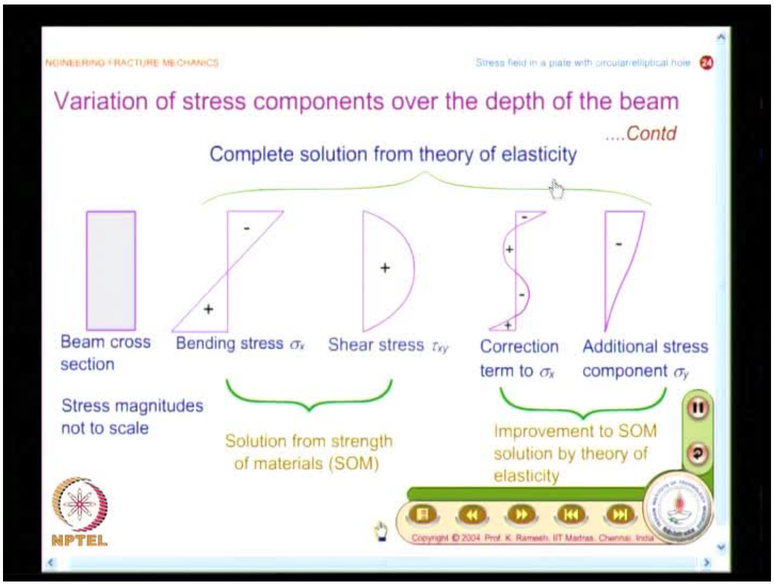

So, what you find is there are two quantities, which are different. You have additional term for sigma xx, and you also have another stress components sigma yy, because if you have to satisfy the equilibrium condition, on the top surface, unless you substitute these quantities however small, it will not go to 0. And this is pictorially represented in this slide. So you have the cross section of the beam, and you have the bending stress which varies linearly as in strength of materials. You have the shear stress. This is also the variation, is parabolic top and bottom surface shear is a 0 and you have the correction term to sigma x.

See, this is only a schematic representation. You should not compare the magnitudes of this stress, and this may be quite small. In order to visualize the variation, it is drawn big. Similarly is the case for sigma y. Also you get two additional quantities, as part of here theory of elasticity procedure.

So, if you take the problem of beam under uniformly distributed load, complete solution encompasses all these components. And this is only the correct solution. What you have got in strength of materials was only in approximation good enough, for engineering analysis. We are not making serious error, by neglect the correction term to sigma x, or not incorporating sigma y, and another aspect also you can keep in mind.

See, I said, you have been able to solve comfortably a problem, of distributed load on the beam. Because I could specify the boundary condition easily, and my stress function of was also very simple. I had only combination of polynomials. Certain terms of the polynomials were selected, and put. We were able to get the solution, and it coincided with strength of material solution on few aspects. So that way you find that we on the right direction, so on and so forth. Suppose I want to solve the problem of a beam, with the concentrated load at the center, that means 3 point bending.

We have seen already, you could have Fourier series, as a candidate for constructing stress functions. You would in fact have to do by taking several harmonics, and when you solve the problem, you will come out with a very interesting aspect.

See in this case, we have seen shear varies parabolically, over the depth of the beam. In the case of a concentrated load, as you go closer to the concentrated load, the shear will not vary parabolically. Because you have a concentrated load on the top, surface shear just below the surface would be very high and it will diminish to 0.

The overall shear force would remain same, that there is no confusion, but the sheer force distribution will be drastically different, and you can look at book by Timoshenko where he has written, pages after pages, how to solve the problem of a beam under 3 point bending. And one of the significant result, there is shear variation is different, as you go close to the points of loading. And unless you take care of that in your design scenario by providing extra reinforcements, your beam will fail.

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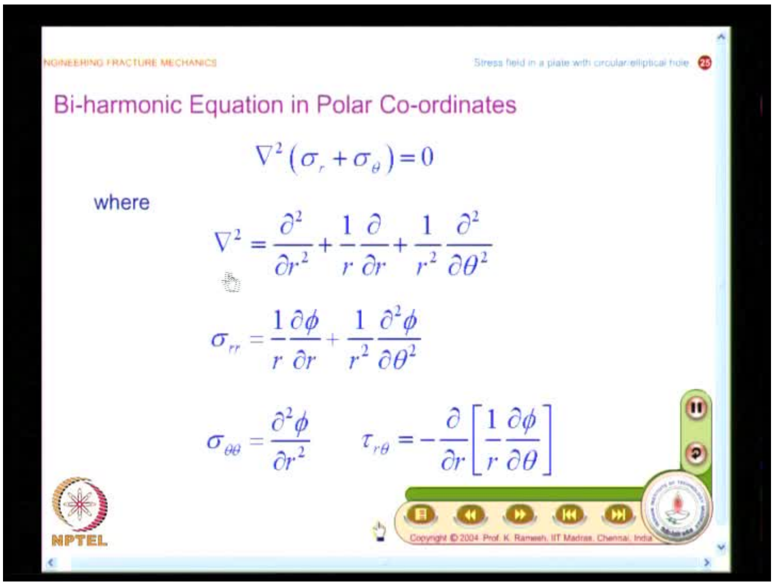

So, what you will have to keep in mind is, strength of material has provided a certain level of knowledge. But if you want to proceed further, you need to improve your method of solution, and that has been reasonably achieved by theory of elasticity. And you know we have looked at the problem in Cartesian coordinates earlier. Now we look at the problem in polar co-ordinates. What is the nature of bi-harmonic equation polar co-ordinates? And we will define what del square is.

So del square sigma r, plus sigma theta, equal to 0, is the compatibility condition. And del square, is given as dou square, by dou r square plus 1, by r dou, by dou r plus 1 by r square dou square, by dou theta square. See in fact, when we take up the problem of a crack in an infinite plate, we would first solve by Westergaard's approach, in arriving at the stress field equations. The Westergaard stress field equation is constructed, based on Cartesian coordinate system, and you would use essentially analytic functions for arriving at the stress and displacement fields

Later, we would also look atWilliams methodology for crack problems. That approach is actually done in a polar coordinate system. So learning this is equally important. Another reason, why I cover this polar coordinates, is in fact, many problems of common interest, you find solution from polar coordinate system. The moment you define what is phi, the expression for sigma rr sigma theta theta, and tau r theta, are specified like this. And sigma rr is given as 1 by r dou phi by dou r, plus 1 by r square, dou squared phi by dou theta square, sigma theta theta, as dou square phi, by dou r square tau r theta equal to, minus dou by dou r, of 1, by r dou phi, by dou theta. In fact you should be able to remember this expression, by looking at the mnemonic way of understanding them.

If you look at Cartesian and polar by looking at them again and again, you will know how to remember this, because they are very, very fundamental quantities. So the idea here is whatever we have learnt in Cartesian coordinate, the knowledge would help here. Also you have this del square, is given in this fashion. Once phi is specified, you could find sigma r sigma theta dou r theta.

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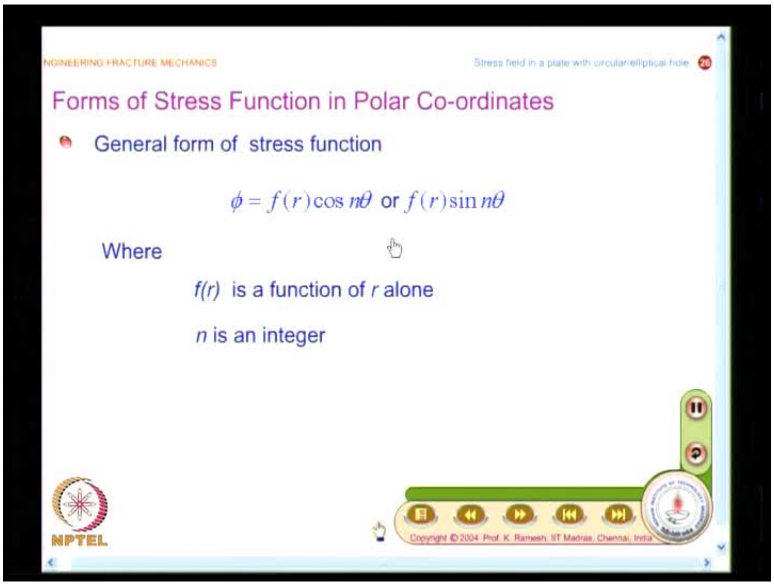

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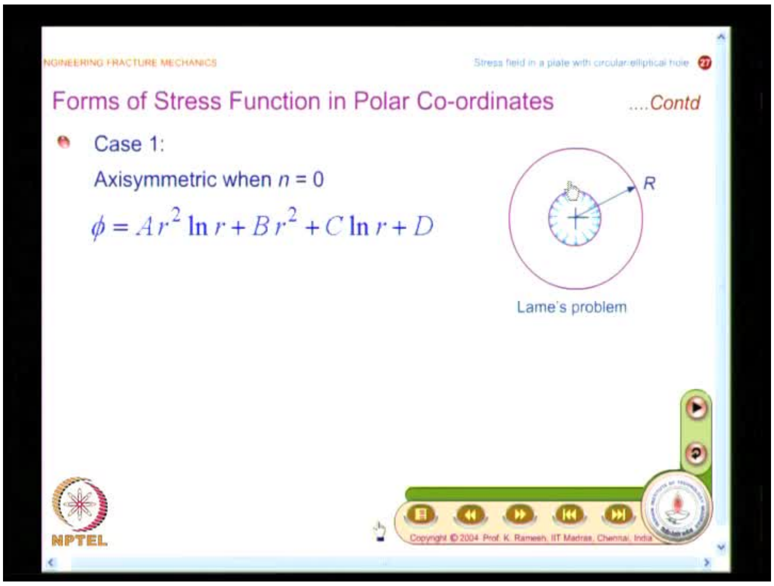

Now let us look, at what are the forms of stress function in polar coordinates. It is very similar, to what we have look that in Cartesian coordinate system. We have look that in Cartesian coordinate systems. I could have polynomial functions, I could have Fourier series, I could have analytic functions, as candidates for coining stress function. In the case of polar coordinates, the generic form is, phi equal to function of r, multiplied by cos n theta, or sin n theta. You are not taking a very generic expression. In terms of theta you could have a general function in terms of r, and this kind of a combination is found to solve a variety of practically important problems, and we look at each of them case by case.

Now you have the case 1, where you are talking about axis symmetric problem and here n equal to 0. One of the very famous problems in this category is the problem of a thick cylinder for thin cylinder. You knows p r by t p r by 2 t, you are able to develop. But when I have a thick cylinder, you have to go by theory of elasticity. And you have the stress function. For this is given as, A r square natural logarithm r, plus B r square plus, C natural logarithm r, plus D. And you know in theory of elasticity, if you come for each of this problem, a name is attached. He was the person who was to solve this problem first, and this is known as a Lame's problem. And you have a thick cylinder. The wall thickness is considerable and if you have to find out, you will have to go by theory of elasticity. And what is the kind of domain here, we have discussed. What are simply connected and multiply connected objects? It is actually a multiply connected object.

And in this case, just satisfying the bi-harmonic equation and boundary condition alone is not sufficient, you should also look at whether the displacement is unique, when you go round the ring, once that kind of argument is necessary to make the coefficients A0. So if you recall how you have solved the Lame's problem, you would say a coefficient A has to be 0, by looking at the displacement picture. It is very important. See in theory of elasticity, we keep saying we are not making any assumption, about the displacement. Certain aspects of displacement information, need to be used for the solution procedure also

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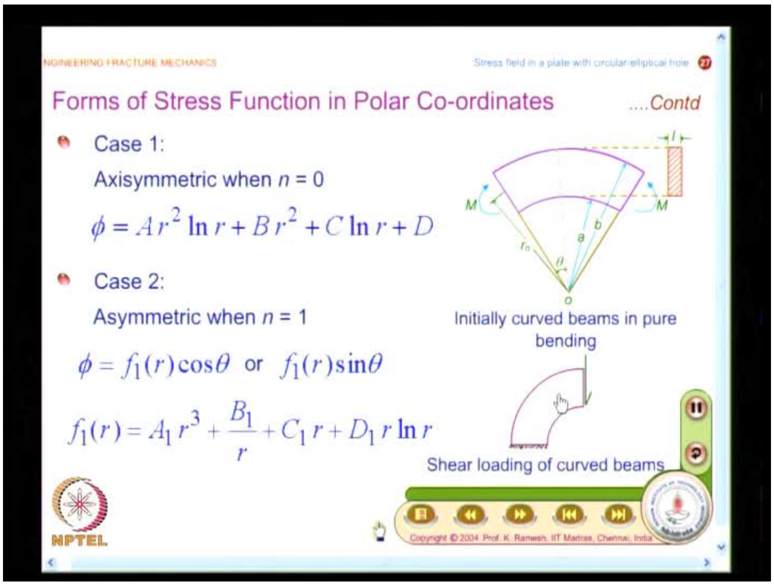

So, this is one category of problem, you can solve using this. The other category is please try to make neat sketch of this. The same stress function could also be used for initially curved beam in pure bending. You have very many practical applications, where initially the beam is bent. Suppose you want to analyze a crane hook, it is subjected to shear as well as bending moment. So if it is subjected to shear, we have to use another stress function to solve it, then linear superposition is possible, because we are living in the linear elasticity domain. So superposition is possible. It is a very useful technique. In fact superposition is a useful concept, when you have linear elasticity. The same idea we would also extend it, for linear elastic fracture mechanics. So principle of superposition is one of the very important tools.

So you could go and get the solution for initially curved beams, with the same stress function. So what will happen is the coefficient will differ, it is depending on the loading and the boundary condition of the specific problem.

And you can also construct another set of stress functions. First we saw the case n equal to 0, we label that as an axis symmetric situation.

Now we go the situation where n equal to 1. We have already seen a generic form of stress function. So that would be rewritten as phi equal to, f 1 r cos theta, instead of cos n theta n, as has become 1 here. So it is cos theta or you can also have f 1 r sin theta function of r into sin theta, and the function of r. Why we put this as 1? It's because we have also written in the generic situation f suffix n, we are now talking about n equal to 1 so it becomes f suffix 1. So the function of r is taken as, A 1 r cube, plus B1 by r, plus C1 r, plus D1 r ln r, and this is the useful stress function, to find out these tresses due to shear loading of curved beams.

You know these are all problems that we come across. You cannot avoid them in actual engineering practice and some of the solutions we have been using it all along and actually it comes from theory of elasticity analysis.

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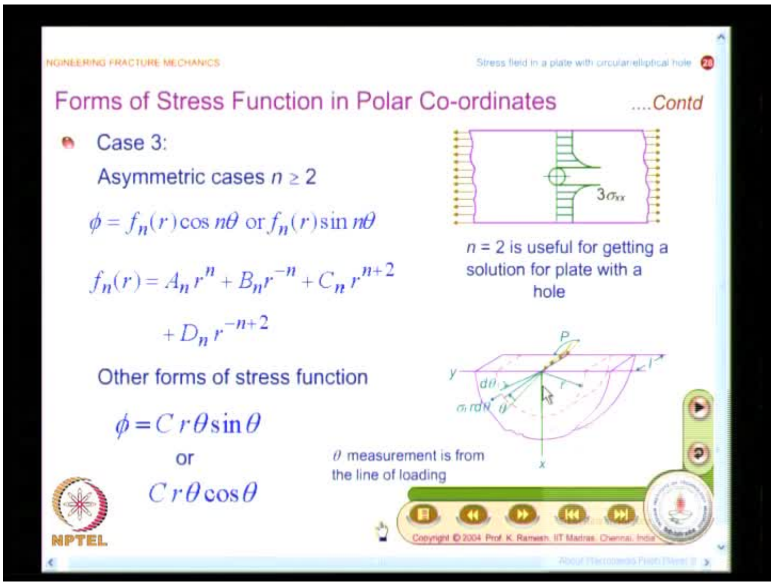

We have seen n equal to 0 n equal to 1, the next step is n equal to 2. So what you find is n equal to greater than or equal to 2, you have a generic expression, where phi is defined as f n r cos n theta, or f n r sin n theta, and the generic function f of r, is given as A nr power n plus B n r power minus n plus C n r power n plus 2 plus D n r power minus n plus 2.

You know this is a useful stress function, to solve the problem of a plate with a small whole infinite plate, with a small hole. See I am discussing this problem using a finite screen, so I have to draw a finite diagram. So when n equal to 2 the stress function is useful to solve the problem of stress distribution. In the case of a small hole in infinite plate subjected to uniaxial tension, you would also see some aspects of the solution more from the point of viewof appreciating the principle of superposition.

You know quite a good variety of problems. You have been able to solve by taking stress function in this nature, there are also other forms of stress function. You have phi defined as C r theta sin theta.

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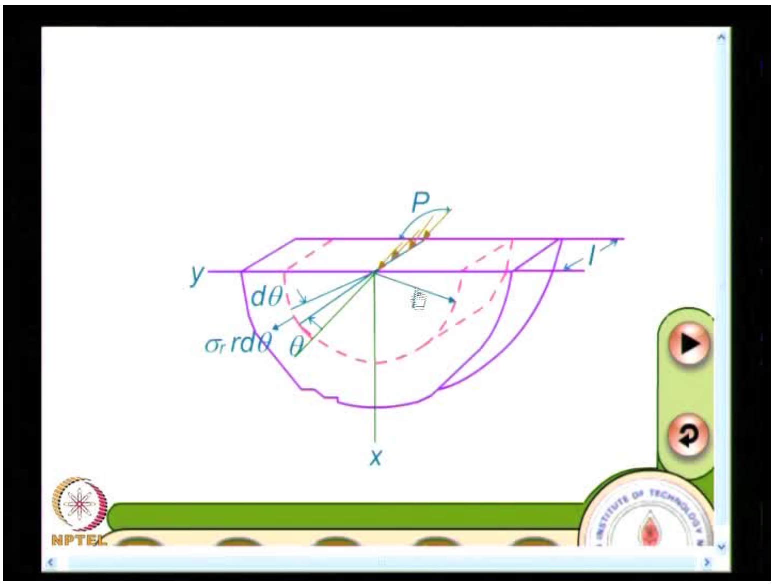

You know the function of r is simply r, but the function of theta is slightly involve and you could have another stress function which is defined as C r theta cos theta. It is a very useful stress function. You know many problems, which are of practical interest could be solved I think, I would enlarge this figure for you. What you have here is, I have a concentrated load on a semi-infinite plate, you have infinite boundary here.

I have a concentrated load, I have a load at an inclination, and I have a load which is tangential to the surface. And if you look at the diagram carefully, the same stress function C r theta sin theta or C r theta cos theta, could be invoked for all these three cases, by carefully defining theta. theta is defined from the reference point as the loading.

Loading is horizontal. You measure theta from this loading, is vertical measure theta from this loading, and is inclined measure theta from the axis.

So, if you recognize this you get a family of solutions. For important problems, just by redefining, how you measure theta? For all this cases, you get solution and if you look at the concentrated load, this is like your Boussinesq's problem.

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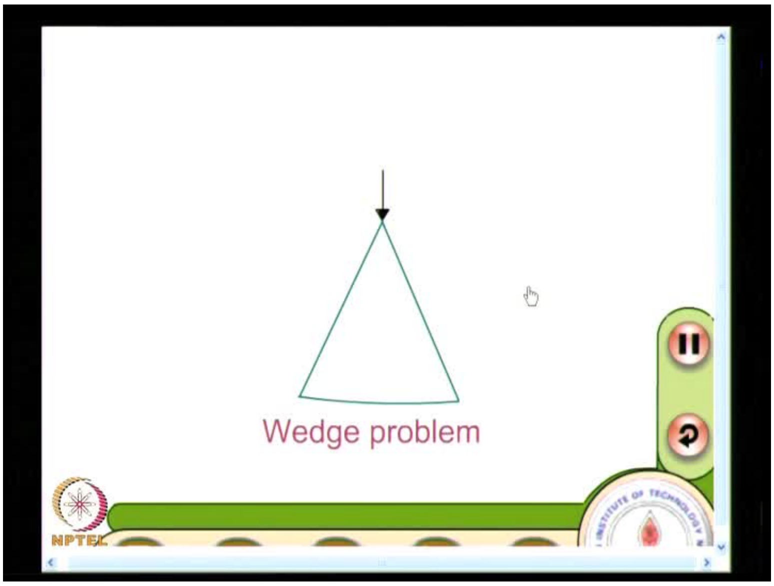

Then you have extension, of this. If you look at the contact stress analysis, it all started from this basic solution of Boussinesq's, which is extended by hertz. And you have Harshian contact problem, and you have also look at how to find the solution for a disk under diameter compression. That also comes from this basic solution. From this as the basic solution; you could find the solution, for the case of a disk under diameter compression. So it is a very useful solution and you would also be able to find out for the case of a wedge problem, the same stress function is useful.

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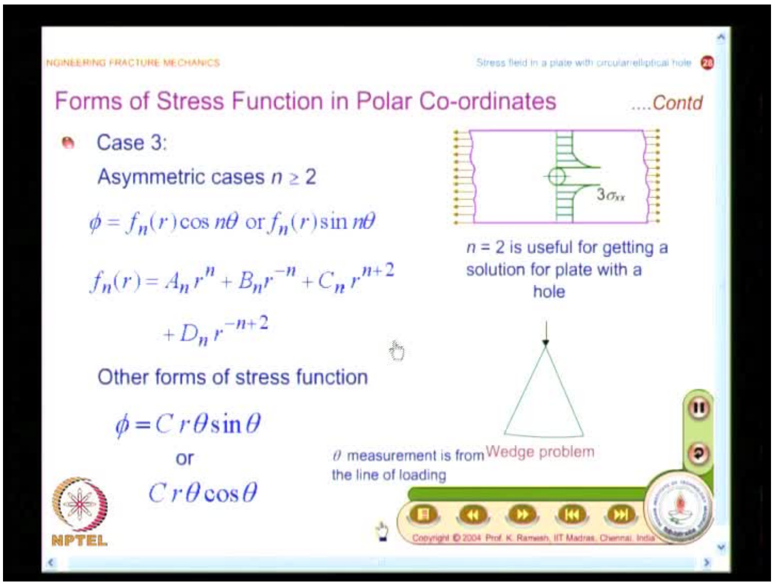

And where do you find the application of wedge problem? Suppose I want to analyze a cutting tool. The cutting tool is a removing metal. While you do the measuring operation if you want to analyze the cutting tool, the wedge problem could be invoked. So from a practical point of view, quite a nice set of problems could be solved by looking at the stress function in polar co-ordinates.

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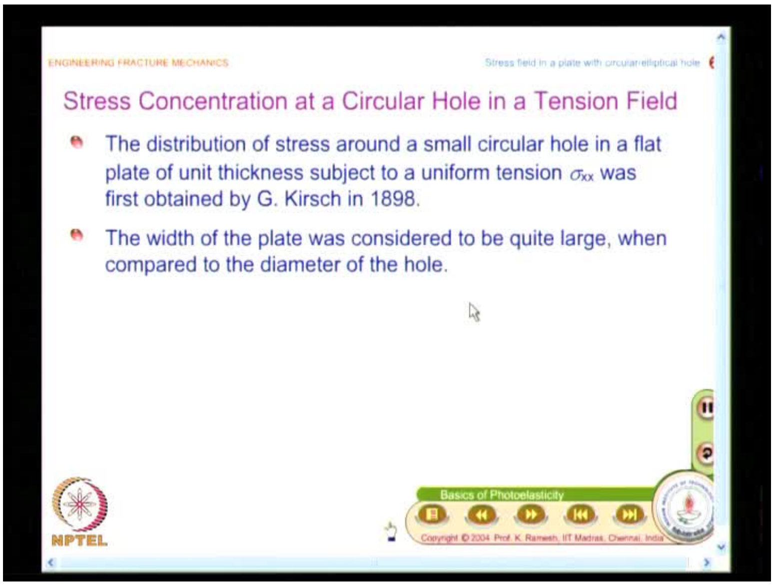

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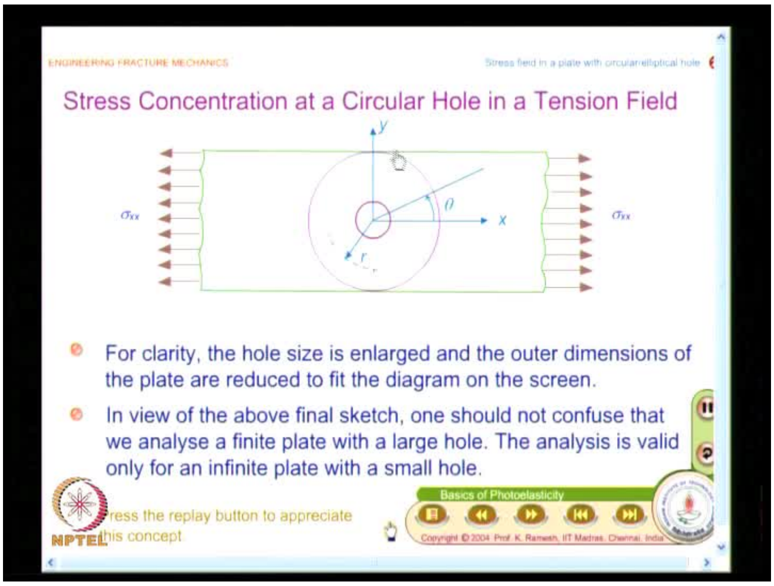

So polar co-ordinate is very useful. You find very many practical problems. You could solve from stress function expressed in polar coordinates. You will now look at the problem of infinite plate with a small circular hole, just the highlight of it. I will not get into the complete solution, and as I mention for any of the problem in theory of elasticity a name is associated. For this problem of a circular hole, it was kirsch in 1898, who solved it, and this is known as a Kirsch's problem, and since we have a nice facility for animation for illustration I have taken it like this. You have an infinite plate with a small circular hole that could be visually appealing. But when I want to do it in a finite domain I have to take a finite diagram, and in order to see what kind of things I represent in the hole I make it bigger. It was one of the common confusions, I have come across in students, is they do not recognize the theory of elasticity provides solution only for an infinite plate with a small hole. Because you are accustomed to seeing a finite diagram like this in the books.

The students immediately jump when they actually solve a problem of a plate, with the hole which is finite in dimension. They think that the solution is directly applicable because of space constrain you have this.

You have to imagine that the boundaries, are at far away distances from the hole dimension, and there are other things that also we do. Though the problem is like infinite plate rectangular plate, with a small circular hole. One of the methods of solution is look at this as a boundary which is in circle in nature at infinity, and rewrites the boundary condition. You have only sigma x axis. These stress components could be rewritten as suitable boundary condition, for a circular plate and why we take up this in polar co-ordinates. I have the circular hole and I also have the outer boundary a circle. It is convenient for me to specify the boundary condition. If I look at the problem in polar coordinates and this is what I mention, when people wanted to solve for the problem of elliptical hole. They needed to develop elliptic coordinate system, because they have to specify the boundary condition on the boundary of the elliptical hole

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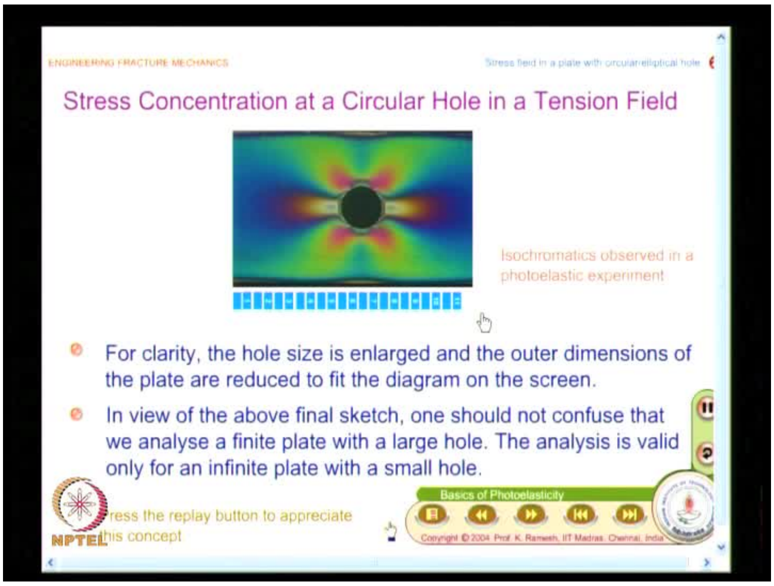

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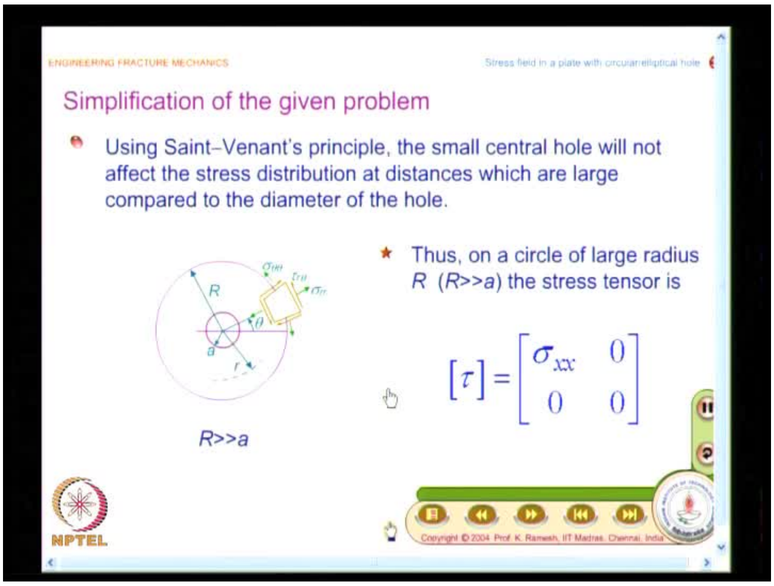

And polar coordinates system is lot more simpler. That is the reason you find infinite plate, with a circular hole was solved first. And we will also have a brief look at the principle of superposition employ, and before I get into that you have nice set of fringe patterns that are form, because of photo elasticity. And this is for a finite plate. Do not confuse this this is for a finite plate with the hole, not for an infinite plate with the small hole.

So what we do is we are using Saint-Venant's principle? The small central hole will not affect the stress distribution at distances, which are large compared to the diameter of the hole. So I take r is much greater than a, but pictorially it is represented in this fashion.

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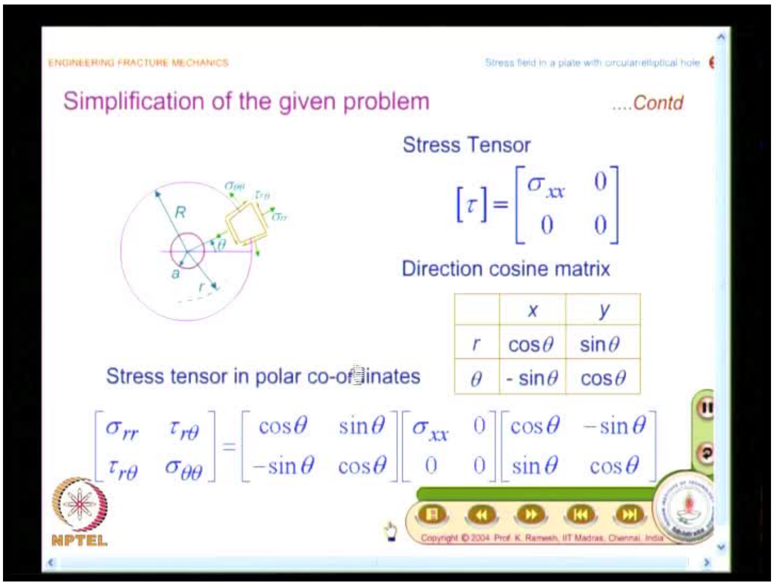

But mathematically recognize R is much larger than, and this is the kind of stress field that you have, and we would get that from the stress tensor. Stress tensor is given as sigma xx 0 0 0. It is a two-dimensional problem, and what we will do is, we will replace this; whatever the far field stress as appropriate boundary condition on the boundary of the hole, and which you could do it by suitable stress transformation. So I have the direction cosine matrix. I am taking a generate section at theta I between r and x, it is cos theta between r and y, it is sin theta between theta and x direction, it's minus sin theta between theta and y it is cos theta.

And we know, what is the stress tensor? I can find out the stress tensor in polarco-ordinates by simply multiplying the direction cosine matrix. So you have this as a and this as a transpose. So you have this as, cos theta minus sin theta sin theta cos theta. So what we are doing in this step is, we want to recast the problem as the outer boundary being circular for facilitating a solution procedure.

See there are also other methods of solution. People have taken the outer boundary, as rectangle and they have also proceeded. That is also another way of doing at it and this is all another method, which is far more simpler, and my interest is to show that how we use the principle of superposition to arrive at the solution. My focus is only on that because if I have to go to the level of superposition, I must look at the stress components on the boundary of the hole and then recast the problem.

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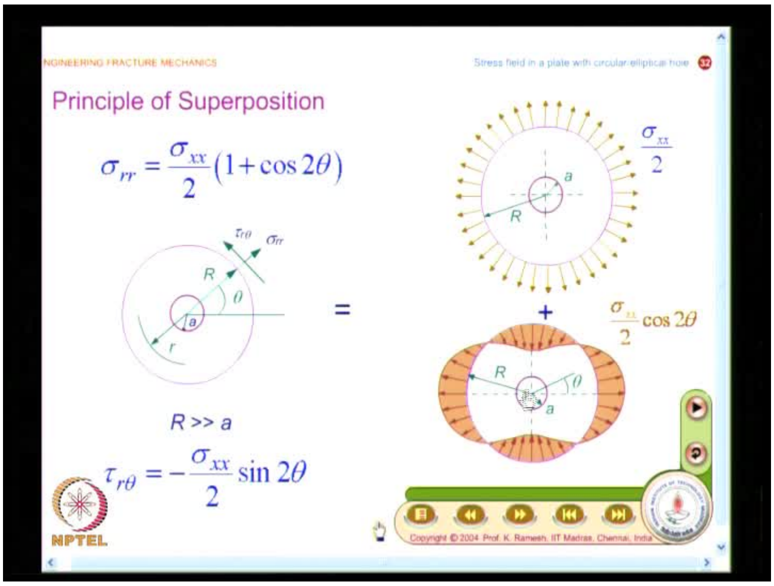

So from the expression, we would be getting sigma RR, as sigma xx divided by 2 into 1 plus cos 2 theta, and then tau r theta, as minus sigma xx by 2 sin 2 theta. I think you can take a sometime to plot this nice picture, so whatever you have as the boundary condition on this hole could be thought of as superposition of two problems.

Here I have sigma xx by 2. Look at this as an annular plate, with sigma xx by 2 acting uniformly all over the boundary. This is one problem. For this you already have a solution, this is nothing. but your Lame's problem and you have another problem I have shown only one loading I have shown the loading of sigma xx by 2 into cos 2 theta make a sketch of it

The pictorial sketch is very interesting. You may not find in many books. In addition to this you also have the shear stress tau r theta that also we will show after you have drawn this. We will look at how the variation of minus sigma xx by 2 into sin 2 theta appears may be.

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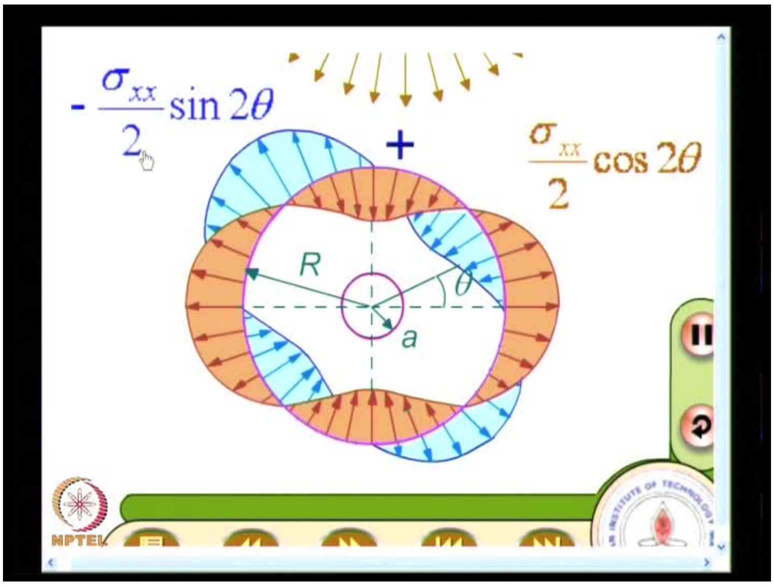

If you want, I can make this big. So this is what you have. You can draw it on the boundary, so this denotes positive values, this denote negative values. That is the symbol is that is used and we will also look at the shear stress.

The shear stress variation will be something like this, and this is minus sigma xx by 2 sin 2 theta, so you have both these kind of loading exist. In this problem you have contribution from sigma xx by 2 cos 2 theta and minus sigma xx divided by 2 sin 2 theta

So what we have looked at is, we have looked at the problem of a small hole in an infinite rectangular plate is recast as small hole in a circular plate. And we have suitably looked at the boundary condition, by looking at the boundary condition; we were able to split this into two sub problems

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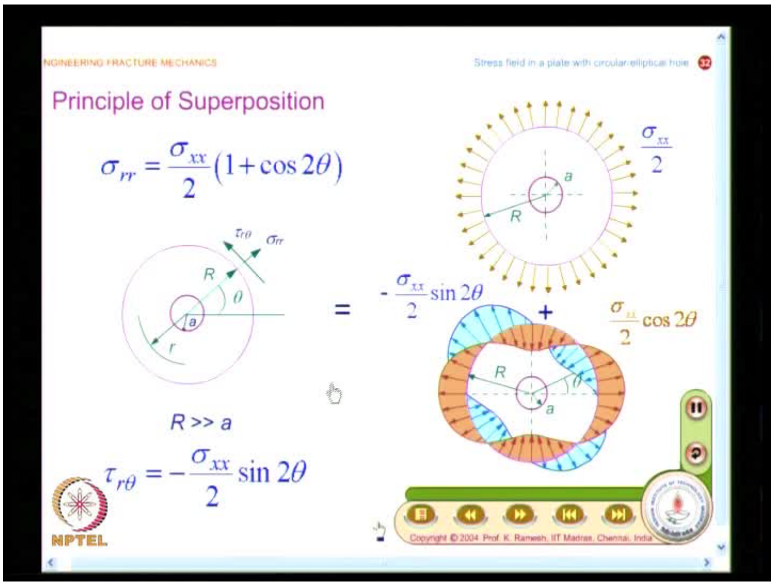

One problem is your conventional Lame's problem, with a specified loading. The loading is different here. That loading we have seen the other problem is you have the stress components sigma xx by 2 coos 2 theta, and sigma xx by 2 sin 2 theta. And if you look at the nature of the stress function, it is in the form of sin 2 theta and cos 2 theta.

So that is the reason, why you are able to take that kind of a stress function? We have said n equal to 2, is the good candidate for solving the problem of a plate with an infinite hole.

So you solve this problem using that stress function, and add these two, when I find out sigma xx component 1 here, component 2 here. You could add them, because we are living in the domain of linear elasticity. In linear elasticity, one of the very powerful methodology is principle of superposition.

In fact when we look at certain solutions for finding out, stress intensity factor in fracture mechanics problems, we would employ principle of superposition and evaluate the stress intensity factors. Because there again, we are going in for a linear elastic fracture mechanics.

So that is the reason, why I thought, that I should emphasize principle of superposition has been used even, in your conventional theory of elasticity approach, it is not something new.

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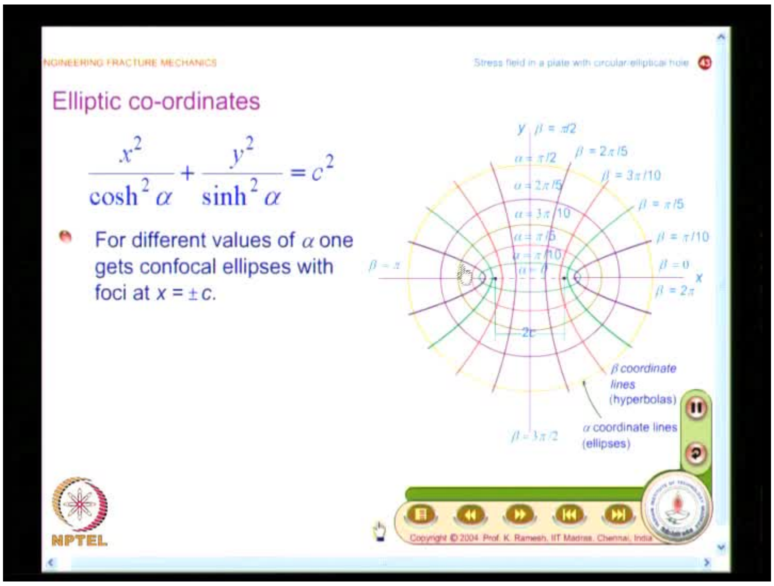

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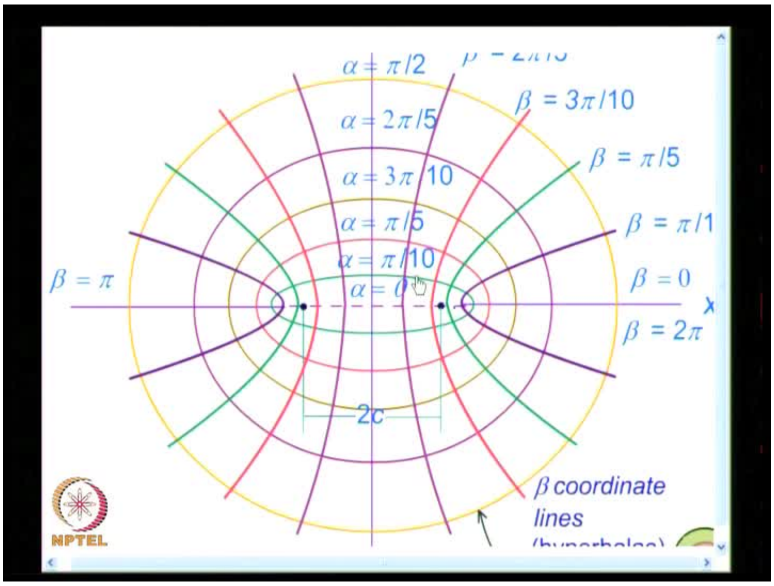

The strength comes from your linear elasticity that is what is important. Then you could also have a look at the elliptic coordinates. I will just show the elliptic coordinates and you have to recognize, that these were necessary for solving the problem of a plate, with an elliptical hole. And this comes from your basic expression, x square by cos h squared alpha, plus y square, by sin h squared alpha equal to c square. And for different values of alpha, one gets confocal ellipses. These are shown here. I will also zoom in and show you have confocal ellipses.

So when you specify, what is alpha? You define the boundary of the elliptical hole. So it becomes convenient, to specify the boundary condition, and this is what, I emphasize. In the early development of theory of elasticity, when they have to solved a problem parallely they had to develop suitable coordinate system. Also skewed plates are used in aerospace structures.

So they develop skewed coordinate system, and they if they have to analyze swear, they had developed spherical coordinate system and spherical bipolar coordinate system. And these were all important stress contributions, in those days. So without suitable coordinate system, they were unable to attach problems that come across in actual practice. And this is where your finite element solutions, which are basically numerical in approach, provided a via media, to handle complex boundaries at least approximately. So that is the advantage of your numerical techniques. So if I have to solve a problem by analytical approach, I need to develop suitable coordinate system as well.

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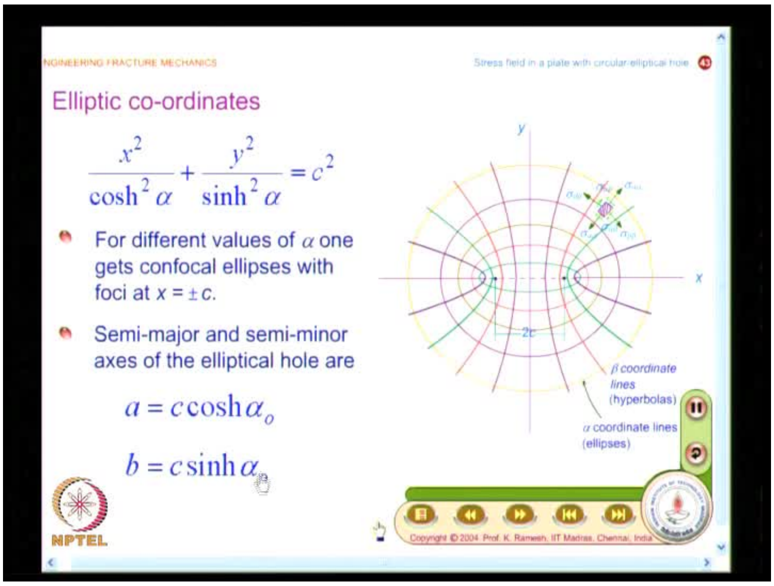

So you specify alpha equal to a constant, and then a find out what is the inner boundary of the ellipse, and you have alpha and beta. You essentially get sigma all alpha, alpha and sigma beta beta that you interpret as suitable quantities for your data interpretation.

So what you have is semi-major and semi-minor axes of the elliptical hole, are given as a equal to c cosh alpha naught and be equal to c sin h alpha naught.

So this provides you a comprehensive over view, on certain aspect of theory of elasticity. We have look at the Cartesian coordinate system, as well as the polar co-ordinate system, and the moment you get the stress function you are in a position to arrive at the solution. So the starting point, is a stress function. How people get the stress function is a very challenging aspect.

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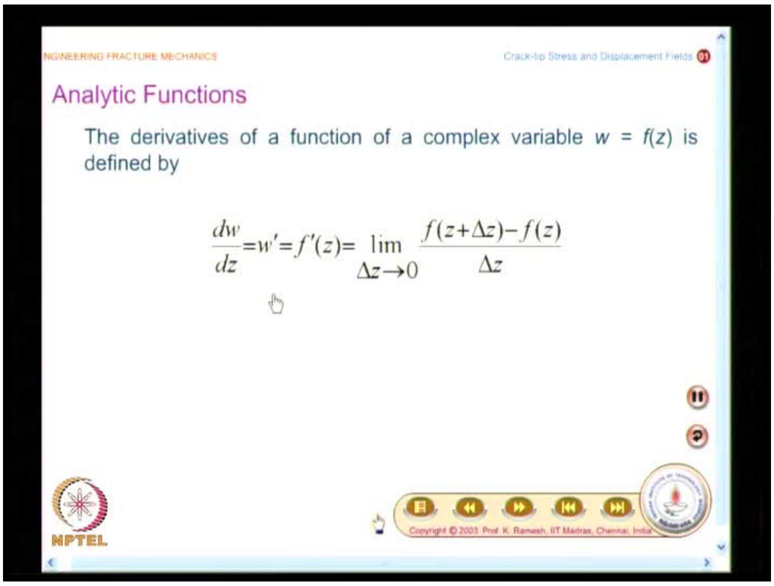

See, before we take up the issue of developing crack-tips stress and displacement fields, I said analytic functions are used to coin the stress function in fracture mechanics. That is what Westergaard has used. So we need to understand, what an analytic functions. You know if a function is analytic, when it is having a derivative. The derivatives of a function of a complex variable, w equal to f of z, is defined by dw by dz. You have the similar notation as linear interpretation, you put w prime, you put 1 prime it is, dw by dz equal to f prime z. It is given as limit delta z tends to 0. You evaluate the function at z plus delta z minus f, of z divided by delta z.

So now we will have to look at, for you to get this differential. What are the finer aspects that we will have to look at, and z is a complex number, .z equal to x plus i y

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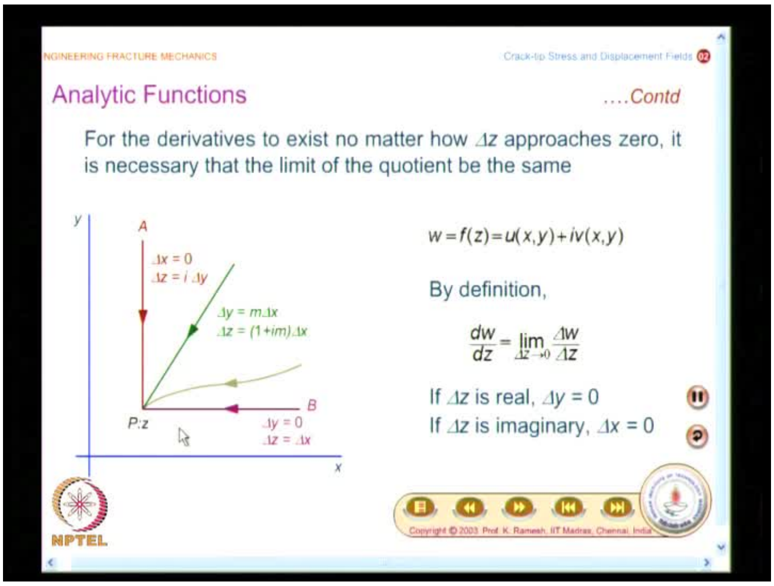

So what you will have to keep in mind is for the derivatives to exist, no matter how delta z approaches 0. It is necessary that the limit of the quotient be the same

So that has to be ensured. So you look at this function. W as a real part, u x comma y plus imaginary part, given as i, into v into v of x comma y. The real part is u x y, imaginary part is v x y.

And we have already said, we want to find out whether the derivative exists. So dw by dz the definition is limit, delta z tends to 0, delta w by delta z.

So you could have, delta z is real. In that case, delta y equal to 0. You could have, delta z is imaginary. In that case delta x equal to 0.

So no matter, how delta z approaches 0 that is depicted in the graph here. So you have in the first case, delta x equal to 0, delta z equal to i delta y, so it varies like this. The second case you have delta y equal to 0 delta z equal to delta x, so it varies like this.

In second case you have delta y equal to 0 delta z equal to delta x so it varies like this you could have a case where both delta y and delta z exchanges so there are related like this. It is a linear relationship or it could also move in a curve. Now what we want to ensure is whichever way delta z approaches 0I, must have a unique value of dw by dz which enforces certain conditions, between the real and imaginary part forming the complex function.

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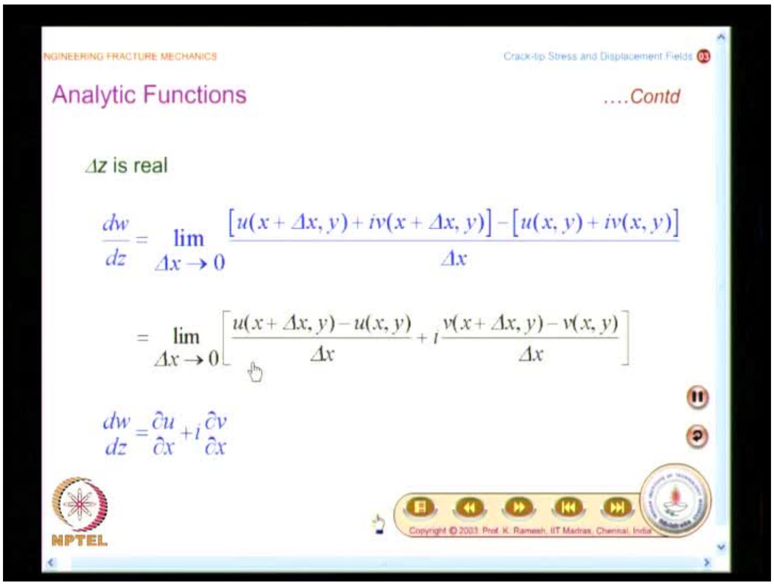

These conditions, you have developed, you have to know, what the reason behind it is. Because whichever way, delta z goes to 0 I must get one unique solution, and now we will take case by case. We will take the case that delta z is real, and we want to write what is dw by dz.

And what is dw by dz? It is nothing but in the limit delta x tends to 0. You have to calculate u of x, plus delta x comma y, plus iv, of x, plus delta x, comma y, minus u of x comma y plus iv, of x comma y, divided by delta x. And these could be grouped as real and imaginary part. That is what the way depicted in the next step. And these could be written comfortably, from the basic definition of how do you write differentials.

You are having this as, u of x plus delta x comma y, minus u of x comma y. So this I could write it, as when the satisfy the limit x tends to 0, I can simply write this as dou u by dou x. I can simply write this as dou v by dou x. So what you get here is dw by dz becomes equal to dou u by dou x plus i dou v by dou x. We are taking a case when delta z is real.

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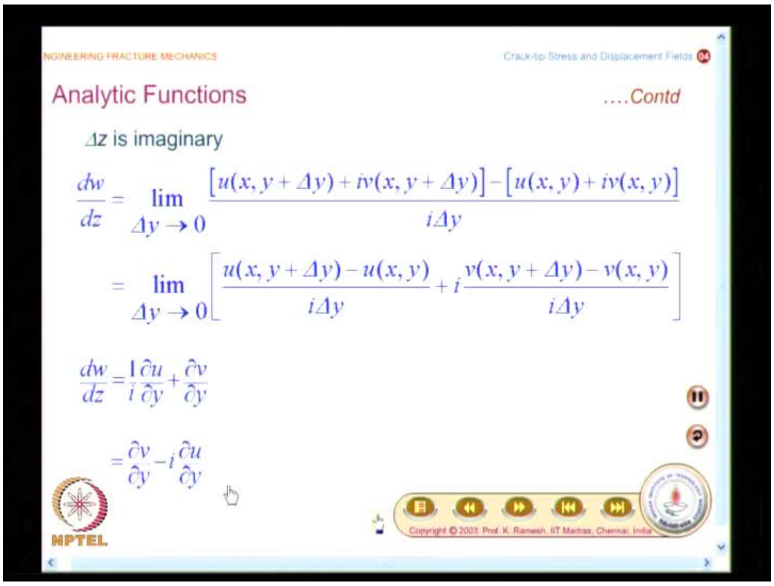

Now we will take the case when delta z is imaginary and we will write the same thing, let us see what we get. So here you have to carefully write this as, limit delta y tends to 0, so that means x plus i delta y, is what we are looking at and x is 0 here. So you have this, as i delta y coming to the picture, and suitably these expressions are written here again. You write the generic expression, then segregate them by real and imaginary parts and you get comfortably write this as, 1 by i dou u by dou y, and you will have this, as dou v by dou y.

So what I get here, I get this as dw by dz equal to 1, by i dou u by dou y, plus dou v by dou y. Or in other words, this actually dou v by dou y minus i dou u by dou y. So now I have two expressions, for DW by dz. They cannot exist like, that so there has to be some interrelationship between the real and imaginary points. We will continue that in the next class.

So in this class what we have done is, we have done a brief review of theory of elasticity and towards again we have looked at what are analytic functions. All this have been developed in your courses on mathematics. Because you have lost touch for a long time it is desirable that we review them, and take up evaluation of stress fields, in the case of a plate with a crack. Thank you.