Engineering Fracture Mechanics

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Module No. # 03

Lecture No. # 13

Displacement and Stress Formulations

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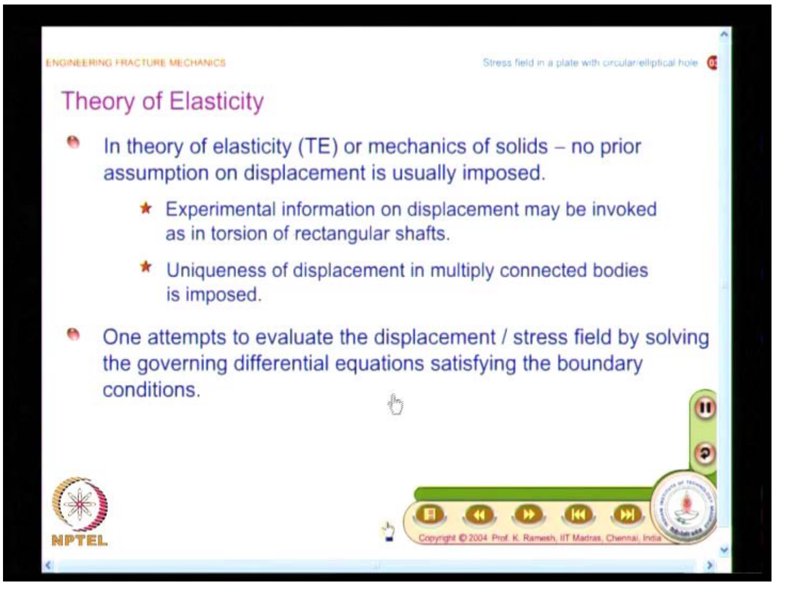

Let us now look at review of theory of elasticity. What is the fundamental difference between theory of elasticity and strength of materials? In theory of elasticity, no prior assumption on displacement is usually imposed. What we did in strength of materials? We imposed plane sections remain plane before and after loading. With that kind of an assumption, for a class of slender members, we were able to arrive at the stress fields.

In fact, if I take a circular disk like this, you cannot solve it from strength of material approach; in fact, you need to go to theory of elasticity, because in this, because of the diameter compression, plane sections do not remain plane before and after loading.

So, you have to necessarily go for theory of elasticity, and fortunately from theory of elasticity, it is possible for us to find out a closed form expression, that means, if you specify x y, you will get this kind of value for all points in the domain, except the load application points and you should also recognize once you come to theory of elasticity, you will classify the objects as simply connected and multiply connected; this is a simply connected object.

Though, in general, you do not impose the displacement, you may try to use experimental information on displacement as part of your solution approach. In fact, this is invoked in the case of torsion of rectangular cross section. I need a volunteer to come and take the specimen and then twist it.

It is rectangular shaft and this is being twisted; you have lines drawn, after this twisted, you find these lines become curved; in fact, that is the projection and the access of the shaft, if you take that as z axes, you have a projection here and if you look at very closely, these lines are similar when z is varied.

So, what you gain is there is warping and you say this warping is function of only the cross section, whatever the cross sectional plane, it is same as every section of z. You use this as a basis to solve this problem. In fact, when we look at solution of stress field for the case of mode 3, we would go through this displacement formulation; we would take this kind of an approach and evaluate the stress field, and once you come to multiply connected bodies, you have to invoked uniqueness of displacement and what are multiply connected bodies?

Suppose I take a ring like this, what you have here is I have a hole in between, in fact, you have solved this as a Lami's problem; you have internal pressure and this is a thick cylinder; so you can find out what are the stresses developed because of internal pressure. In fact, if you go and look at the way that you have solved the solution, you would have made one of the coefficients defining the problem to go to 0 mainly, because when I go through this complete circle once, you want to have at any cross section, the displacement remains same; in order to ensure that you make one of the coefficients of the stress fraction to go to 0.

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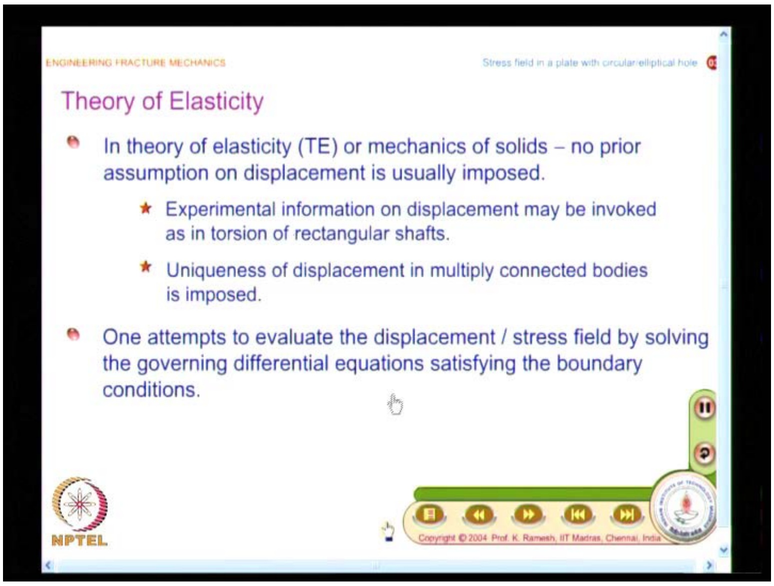

So, when you go to a multiply connected object like this, you have to bring in uniqueness of displacement and how do you identify, whether the object is simply connected or multiply connected? Now, I say I have a ring, this is multiply connected; you all accept.

The test is you take a contour; suppose I take an arbitrary contour like this, I can shrink that contour to 0 without encountering a boundary in the case of a disk, whichever type of arbitrary contour I take on this; I can always shrink that contour to 0 without encountering a boundary. On the other hand when I take a ring like this, suppose I take a contour like this, when I shrink it to 0, I have necessarily crossed this boundary.

Suppose I cut this horizontally, I have only a specimen like this; this is open like this, like a c specimen, then I cannot take a circular contour like this; I will have to take different type of contour. So, one of the tests is, if I am able to take an arbitrary contour and if I shrink it 0, if I do not go out of the region, then that problem is simply connected; otherwise, the domain is multiply connected. One simple thumb rule is, if I have cutouts like this, I have many cutouts like this; this is the multiply connected object.

I have another specimen like this; this is a very interesting specimen; this is appearing like Greek letter theta so it is called a theta specimen and when you apply compression what happens here is on the horizontal web, it experiences uniaxial tension. This was developed by Professor Durelli for a verifying some aspects of moirae and this is a multiply connected object.

So, in multiply connected objects, you have to invoke uniqueness of displacement in theory of elasticity. A general dictum is no prior assumption on displacement is usually imposed so that is what is summarized here. One attempts to evaluate the displacement or stress field by solving the governing differential equations satisfying the boundary conditions.

So, the fundamental shift is, you solve differential equations in theory of elasticity. But even that solution procedure you developed your simpler methods to handle; you never go and solve the complicated differentially equations, you have a simplified procedure. We will have a look at it.

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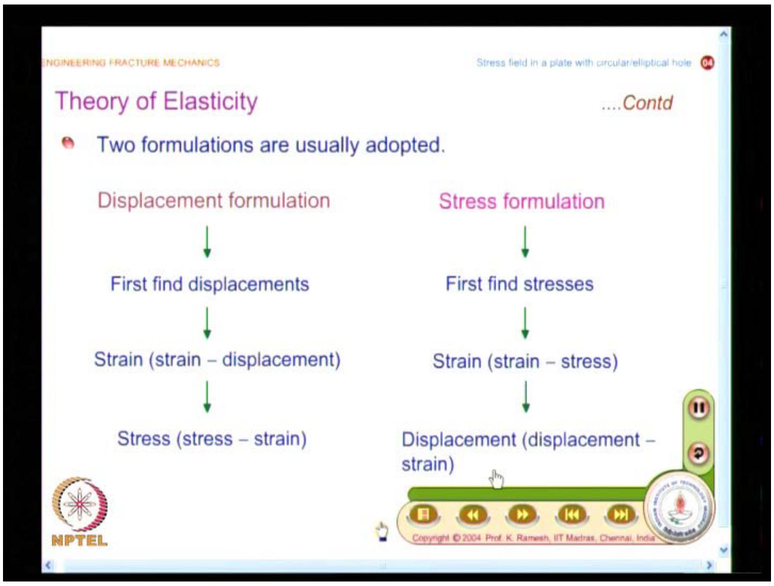

Usually two formulations are adopted; you have a displacement formulation; in this, you find the displacement first; from displacements you move on to determining strain by invoking strain displacement equations, then finally, you get the stress components by invoking stress strain conditions and the other formulation is stress formulation in fact, this is more popular for analytical development.

So, in this you first find the stresses; from stresses you find out the strain components; for strain components, you use the strain-stress relations; from strain, you find out the displacement; you invoke the displacement strain conditions.

Here you have a catch. See if you look at, the strain components are six; the displacement components are three. So, when you want to find out displacement from strain, unless you bring in compatibility conditions, the displacement will not be correct. So, in stress formulation, you will have to always look at compatibility conditions, they go with the solution procedure.

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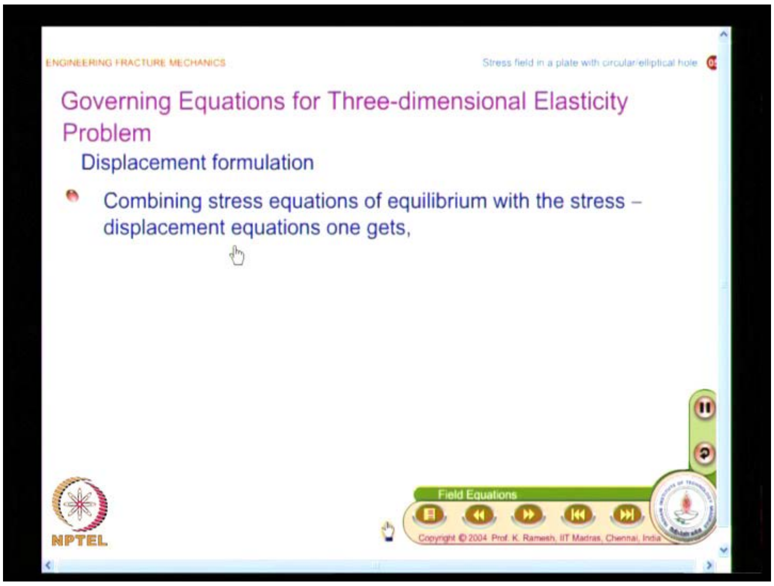

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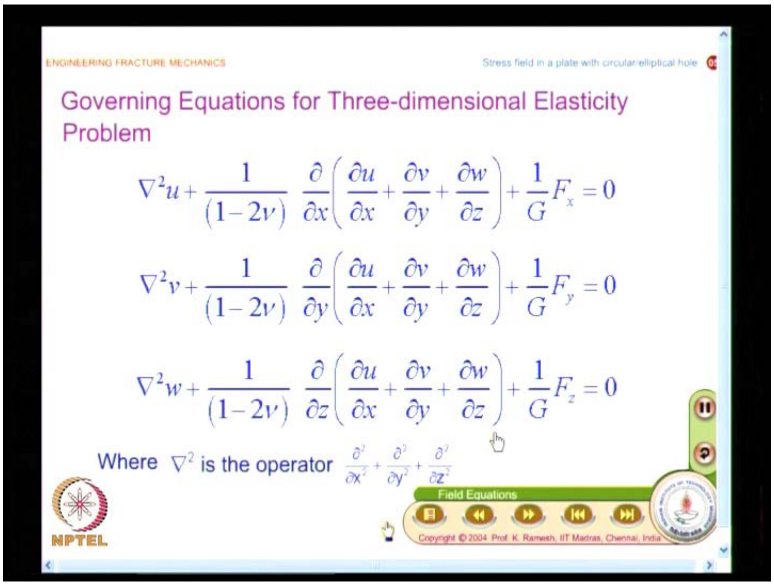

And we look at each of these formulations what kind of governing equations you have? We would first develop it for a three dimensional scenario, then simplify it for planar problems. So, in the case of displacement formulation, you can get the governing equations by combining stress equations of equilibrium with the stress displacement. So, this gives you, they are in cyclical order, we will look at them.

So, you have this as del squared u plus 1 by 1 minus 2 nu dou by dou x of dou u by dou x plus dou v dou y plus dou w by dou z plus 1 by G F suffix x equal to 0. For each of these directions, you will have cyclically this is formed; for the u displacement, you have dou by dou x; for v displacement, you will have dou by dou y and for w displacement, you will have dou by dou z.

The basic form of the equation is similar and here you have F x, F y and F z, and del is the very famous operator dou squared by del squared is nothing but dou squared by dou x squared plus dou squared by dou y squared plus dou squared by dou z squared. And what you will have to look at is, from mathematical point of view, I have to find out three unknowns u, v and w.

I have three equations; so I am in a position to identify by solving this, get the solution for u v and w; once I get displacement, go to strain and then, finally go to stresses. So, the procedure is fairly straight forward in the case of displacement formulation.

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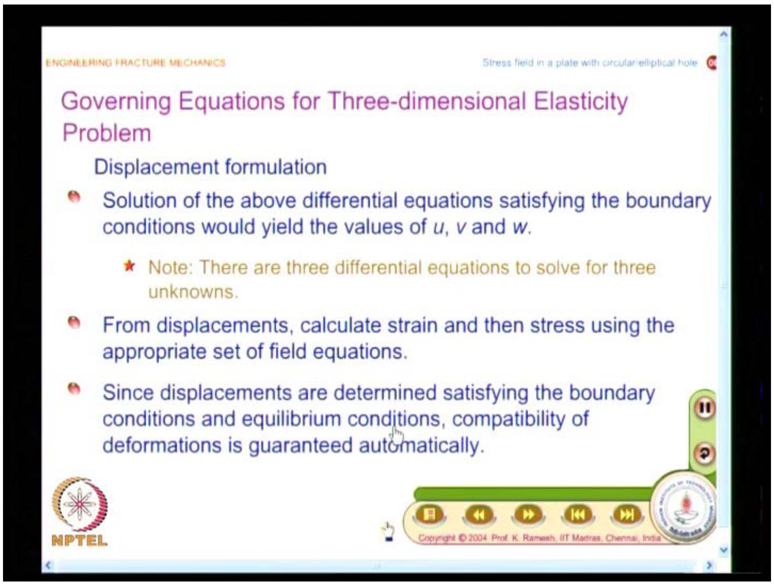

So, this is what is summarized here; from displacement calculate strain and then, stress using the appropriate set of field equations and what you do here is you evaluate the displacements satisfying the boundary conditions and equilibrium conditions.

So, automatically the compatibility of displacement is guaranteed. In fact, the very famous finite element course that we have, that is based on displacement formulation. One of the questions that could be asked an interview is where do you see compatibility infinite element formulation? You will have to simply say it is the displacement based formulation. So, compatibility is automatically satisfied.

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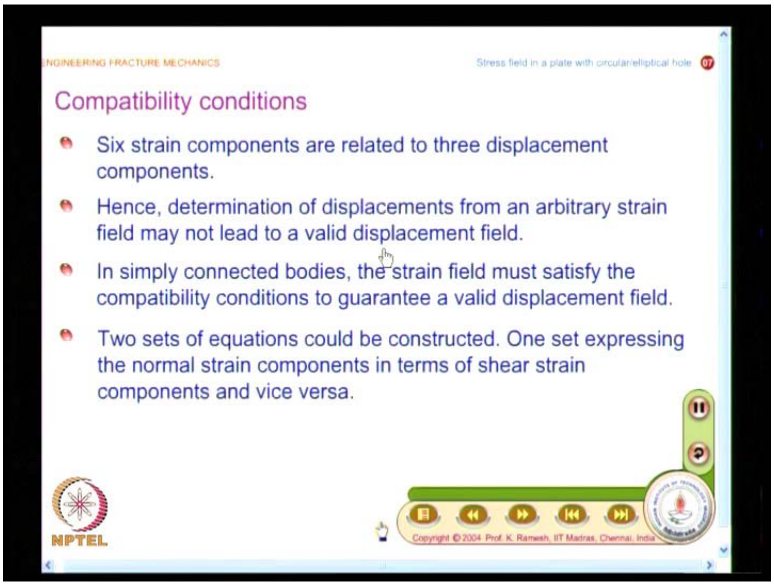

Now, we move on to what are compatibility conditions. I have already pointed out that you have six strain components, which are related to three displacement components and what is the problem because of this?

Determination of displacements from an arbitrary strain field may not lead to a valid displacement field. See the problem is you have more number of equations than number of unknowns; that is also a problem. You should always have equal number of unknowns and equal number of equations.

In either case, whether the number of equations is more or number of equations is less, you have a problem. And what is mentioned here is in simply connected bodies, if I satisfy the compatibility conditions, I am guaranteed to get the displacement field from the strain field; if I invoke the compatibility conditions. On the other hand, if I go to multiply connected bodies, I will still have to look at uniqueness of displacements, compatibility conditions alone are not sufficient; I will also have to look at uniqueness of displacement field.

And if you look the compatibility conditions, two sets of equations could be constructed, which you may not have noticed it earlier; you can actually have one set expressing the normal strain components in terms of shear strain components and vice versa.

So, on the left hand side, you will have normal strain components; on right hand side, you will have suitable combination of shear strain and you will have on the left hand side, shear strain components and suitable combinations of normal strain.

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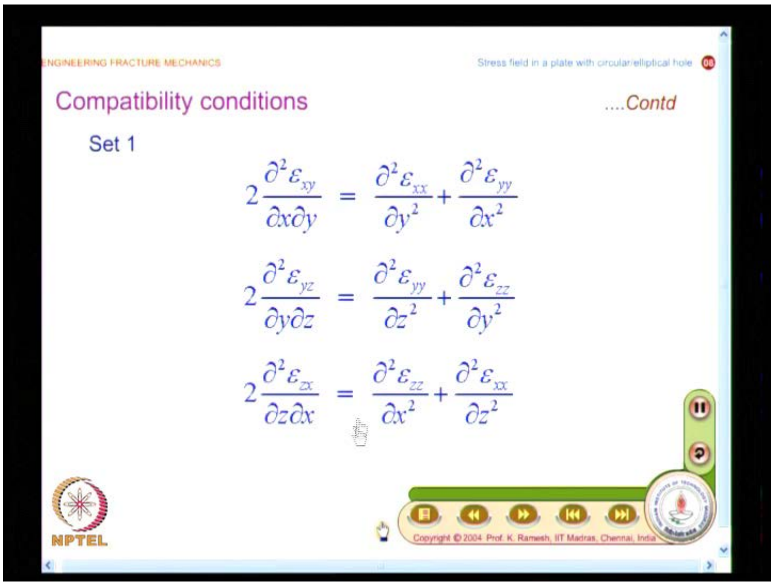

We would look at them. In fact, you would have derived it in your earlier classes and the set 1 looks like this and this is also in cyclical order; on the left hand side you have shear strain, which are related to normal strain components and you have strain epsilon xy epsilon yz and epsilon zx.

And what is the condition that you have? 2 times dou squared epsilon x y divided by dou x dou y equal to dou squared epsilon xx divided by dou y squared plus dou squared epsilon yy divided by dou x squared. I would like you to have these equations in your notes. And this is in a cyclic order; if you write one such expression, following that example you could write for the other.

So, if I have epsilon yz, I will have epsilon yy and epsilon zz coming in this equation and you differentiate it with respect to z and then, differentiate with respected to y here. So, when I go to epsilon zx, I have dou squared epsilon zz divided by dou x squared plus dou squared epsilon xx divided by dou z squared and these are the conditions relating strain components.

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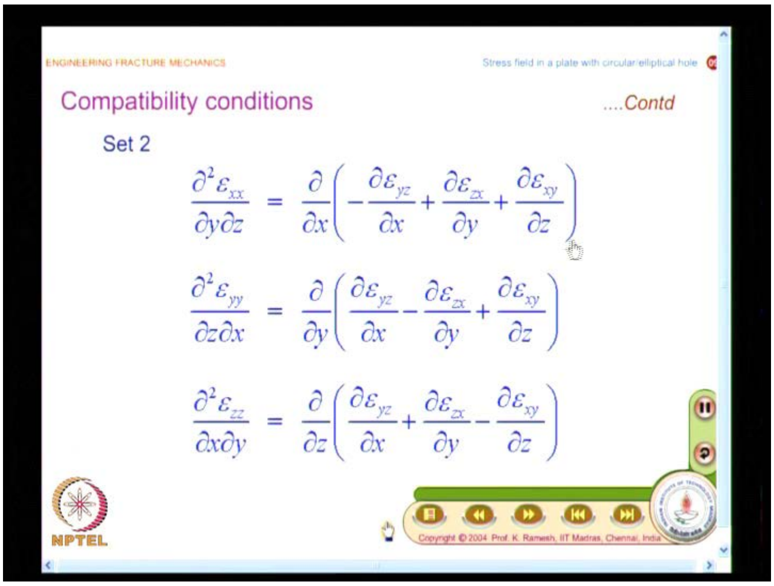

Because each strain component is related to a particular displacement component based on that only, to emphasize compatibility of displacement field, these strain components have to be inter related; they are not totally independent. And the other set is I have on the left hand side normal strain component.

On the right hand side, you have a combination of shear strain components and what I have here? I have dou squared epsilon xx divided by dou by dou z equal to dou by dou x of minus dou epsilon y z divided by dou x plus dou epsilon z x divided by dou y plus dou epsilon x y divided by dou z and this equation is repeated cyclically and you have to recognized that minus sign is related to whatever the differential here, If I differentiate with respect to dou y dou x, you have minus sign attach to that.

In the second expression I differentiate with respect to dou by dou y. So minus sign comes to the term involving dou by dou y; a similar thing you can see when you have the differential with respect to dou by dou z, you have the minus sign attach to dou by dou z. And how the components are… see if you look at epsilon xx, I have epsilon yz, zx and xy; they are repeated yz, zx, xy are repeated; the minus sign changes depending on the position. And you differentiate with respect to x in the first case; the second case, you differentiate with respect to y and finally, you differentiate with respect to dou z. These are very standard expressions.

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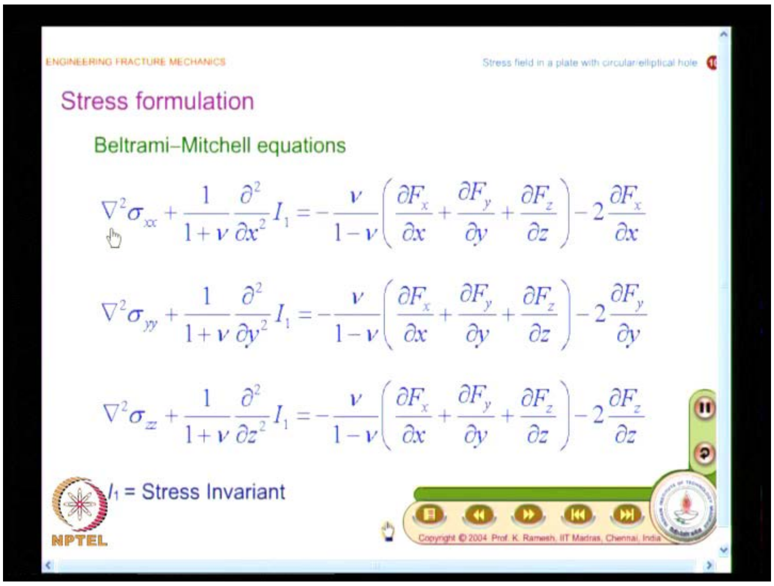

Now, we what we want to look at them is, we want to look at them in terms of stress components. You know by putting the stress components and solving it, it is a bit involved process; we would nevertheless look at the final expression and these are famously known as Beltrami-Mitchell equations.

And we would observe how the equations look like. These equations are very long nevertheless take pains to write them down, I would read them for you. del squared sigma xx plus 1 by 1 plus nu dou squared by dou x squared I 1, where I 1 is the first invariant that is equal to minus nu by 1 minus nu dou F x by dou x plus dou F y by dou y plus dou F z by dou z minus 2 times dou F x divided by dou x. So, what you find here is this term gets repeated for all the expressions; only the last term changes. The first equation it is F x; second equation it is F y and third equation it is F z.

And this changes as sigma xx, sigma yy, sigma zz and the middle expression everything remains similar, except the first expression is dou squared by dou x squared; second expression is dou squared by dou y squared and third expression it is dou squared by dou z squared.

And another aspect you have to look at, see what you have this is as F x, F y and F z are body forces. See one of the important class of problems in solid mechanics is when the body force either remain constant or goes to 0; whether it is constant or goes to 0 what way this expressions look like? If you look at very closely, suppose I take the case when body force is constant, none of these differential can exists they will all go to 0.

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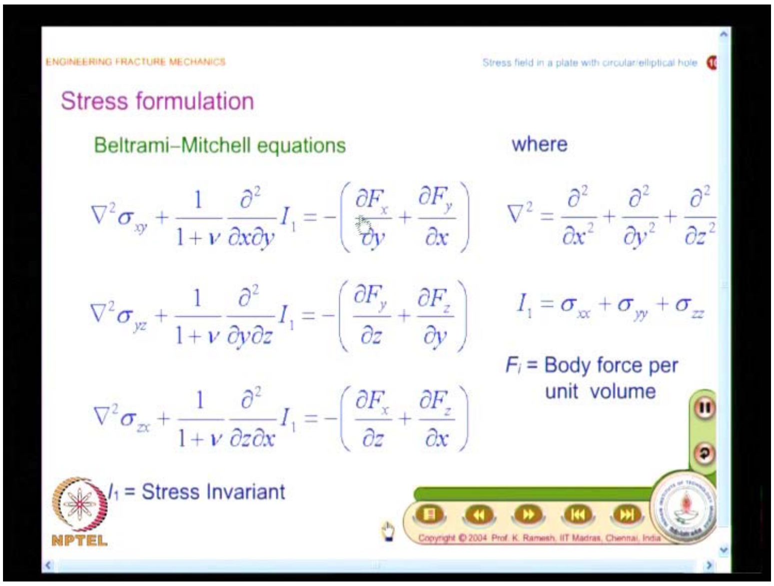

So, the right hand side of these expressions will go to 0; if you look at the left hand side, the equation is a function of the Poisson's ratio; you have to keep this in mind. So, the elastic constant of the material comes into play when you are looking at a three dimensional problem in the absence of body forces. So, I have three such expressions; I will have three more such expressions involving the shear stress quantities, they are del squared sigma xy plus 1 by 1 plus nu dou squared by dou x dou y into I 1 equal to minus of dou F x by dou y plus dou F y by dou x.

And these are cyclically repeated and it is mentioned for clarity, del squared is nothing but dou squared by dou x squared plus dou squared by dou y squared plus dou squared by dou z square and I 1 is the first invariant sigma x plus sigma y plus sigma z and F suffix I are body force per unit volume.

Here again, you can notice when the body force remain constant or 0, the right hand side goes to 0, nevertheless the expression contains the Poisson ratio; it appears as 1 by 1 plus nu. So, when you look at the compatibility conditions in three dimensions, material constant also influences the stress field.

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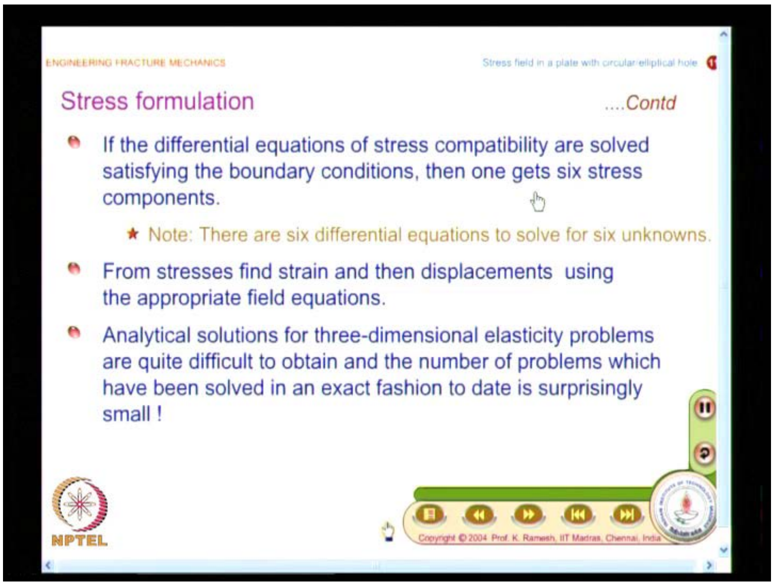

So, very important aspect and I hope you had sufficient time to complete writing these expressions. So, what you have is if the differentially equations of stress compatibility are solved such that they satisfy the boundary conditions, then one gets six stress components. You obviously have six differential equations to solve for six unknowns.

So, now, you have stress components; from stress components, find out strain, then displacements using the appropriate field equations. See, although the governing differential equations are formulated; if you really look at the analytical approach, the number of three dimensional problems that are solvable are very less. And only a very few problems people have solved in all its totality, that is the reason why numerical and experimental techniques are really necessary for solving practical problems.

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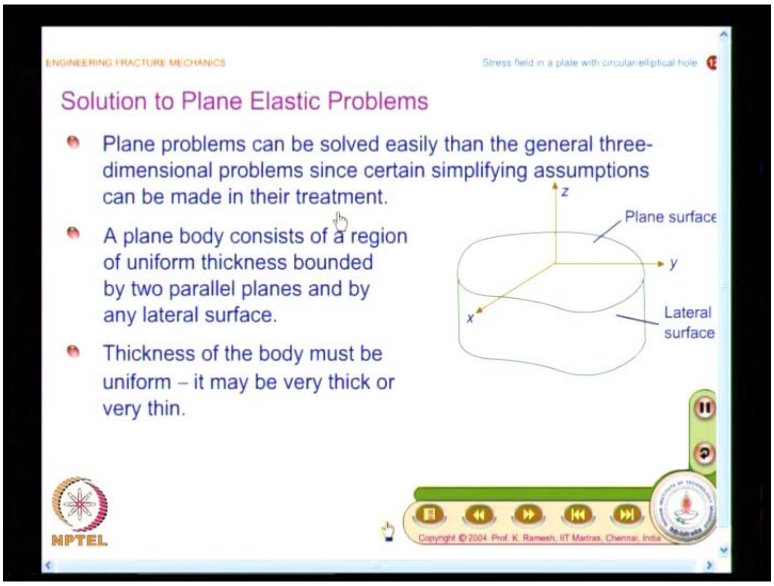

But what we would do is, we would look at solution to plane elastic problems. In fact, in this slide just observe the animation, you do not have to try to draw this; we would spend time on each of these special cases separately, at that time if you draw the animation, fine. Plane problems can be solved easily than the general three-dimensional problems since certain simplifying assumptions can be made in their treatment, that makes your life lot more simpler.

In fact, a variety of practical problems could be simplified either as the .plane stress or a plane strain situation and what is a plane body? A plane body consists of a region of uniform thickness that is very important; the thickness has to be uniform bounded by two parallel planes and by any lateral surface.

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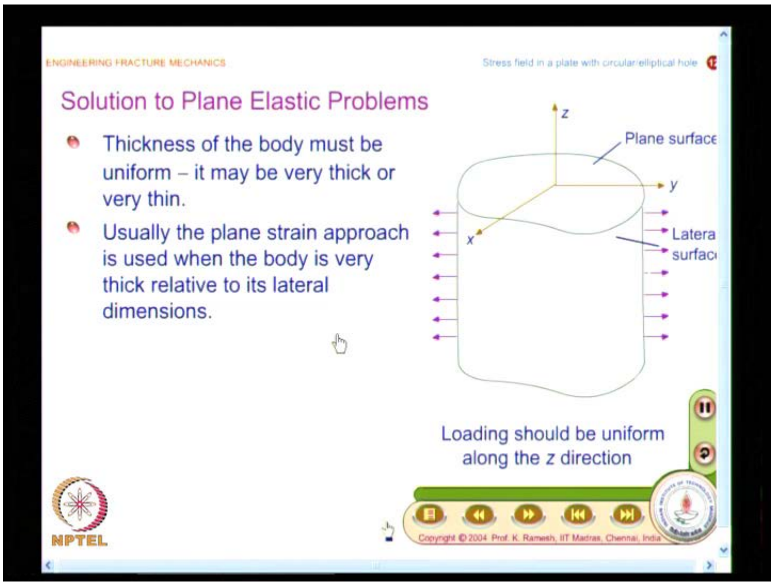

So, what is emphasized here is the top surface and bottom surface have to be plane; they have to be parallel; it could be connected by any lateral surface, the shape could be anything; the thickness of the body must be uniform; it may be very thick or very thin, both these cases you could identify as a simpler methodology.

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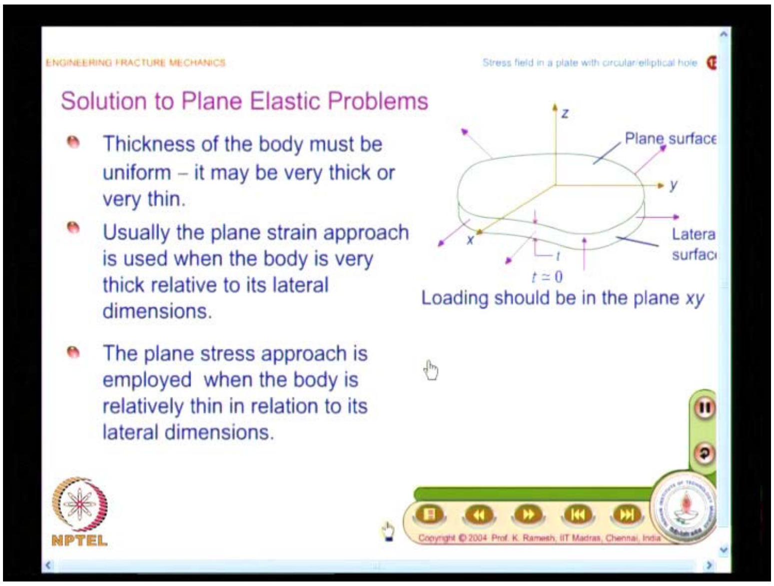

When the thickness is long, very high usually, the plane strain approach is used; on the other hand, if the thickness is small, plane stress approach is invoked. The key point here is loading should be in the plane x y.

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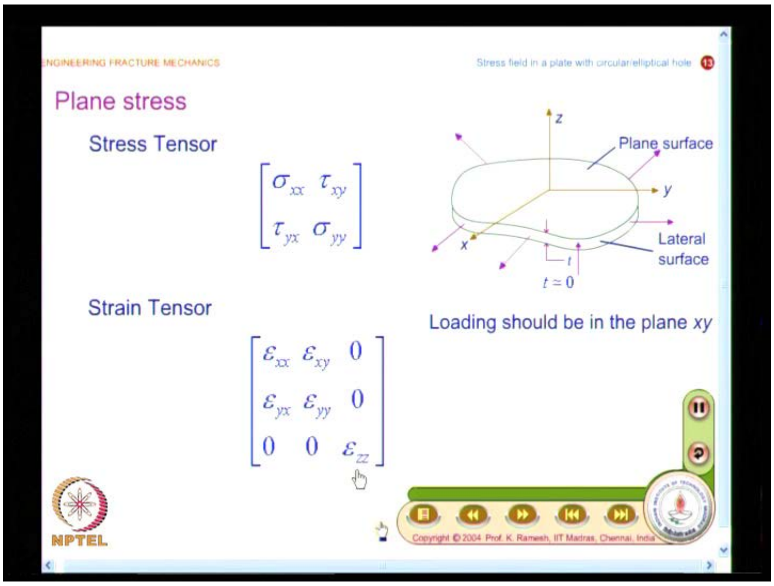

Now, we will take a plane stress as well as plane strain case by case. You could now sketch the diagram; the key point here is loading should be in the plane x y and the thickness is almost close to 0; you know if only for convenience, you show a finite thickness; in reality, it just a plane.

In the moment you plane stress, everybody will ask what is the plane stress situation define. Most of the times you would come out with the stress tensor and you state the stress tensor as sigma xx tau xy, tau yx sigma yy.

But it does not stop here; see the moment you say plane stress and planar problems, people think everything related to the problem remains in plane, that is, the two stronger conclusion; you should not do like that, when you look at plane stress, the stress tensor is only in the xy plane.

Suppose you take that as the plane stress situation, you will have sigma xx sigma yy and tau x y, but you look at the strain quantity, it is no longer two- dimensional; it will be three-dimensional and what is the strain tensor? The strain tensor will appear like this; I have epsilon xx epsilon xy, epsilon yx epsilon yy as well as epsilon zz.

You know, if you take a tension strip and then pull it, you would really not recognize the change in thickness that much, because the stress levels are reasonably small; the strain is going to be much smaller. On the other hand, if I have a specimen, which has a crack because of very high local stress, you will find near the crack tip, a dimple would be formed.

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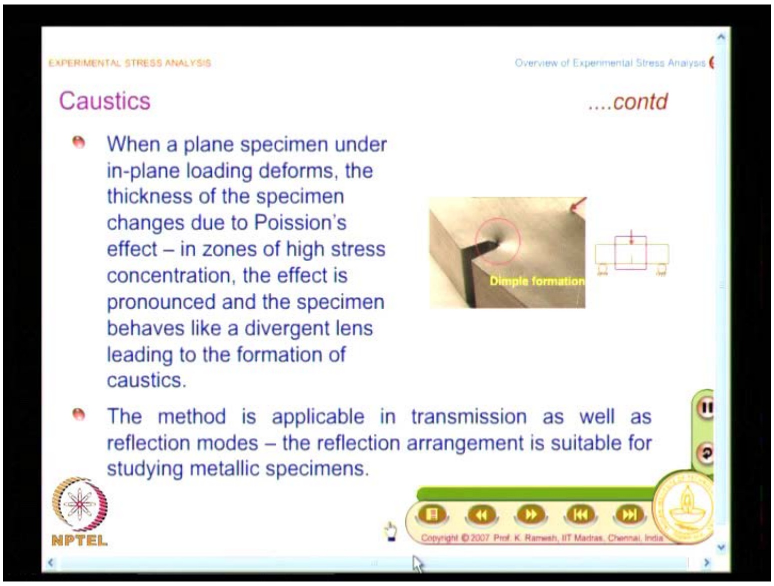

That is because of the lateral strain lateral strain cannot be ignored and you have the dimple formation in the zone near the crack tip. In fact, I would show you another diagram; this is taken for an aluminum specimen, you could see here; you could see very clearly, I will enlarge large this picture.

I have a crack and you find a significant change near the tip of the crack and this is how the model has been loaded and this is a dimple; this is primarily because you have lateral strain; this cannot be ignored that becomes very significant when I have very high stress concentration.

In fact, people have utilized this as a physical feature to develop a new experimental technical called method of caustics. You know normally plane stress, you look at the stress tensor and carry on with, you do not even look at how the strain tensor looks like; that is wrong. You have to know both this stress tensor as well as the strain tensor.

Not only this, I would appreciate you go and look at what is the orientation of maximum shear stress plane in the case of plane stress situation. Because the maximum shear plane orientations are different in the case of plane stress as well as plane strain; when we take up modeling of plastic deformation in the crack tip, slip would be dictated by that plane only. So, you have to understand how slipping of planes can happen in the case of plane strain and plane stress. So, it is worthwhile to go and look at the more circle for these cases.

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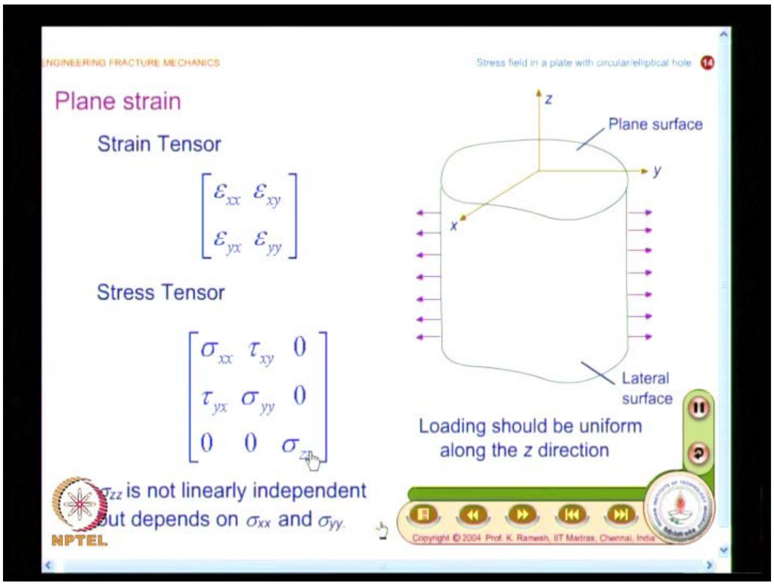

Now, we move on to plane strain; the moment you come to plane strain, you all note that you have to look at the thickness is sufficiently long and another important aspect is loading should be uniform in the thickness direction. Here it is shown as direction z; if the loading is not uniform, you cannot idealize the situation as plane strain.

Normally you would take the case of a roller bearing the rollers in the roller bearing; you consider that as the plane strain problem or if you have the dam, the dam you consider the in civil engineering away from the edges as a plane strain situation; it is uniformly loaded and it is very long and plane strain assumption is valid.

Here again, you should look at also the stress tensor; you should not stop with the strain tensor and if you look at the stress tensor, that will be fully populated; you will also has sigma z z; it is not in a plane; it is 3 by 3 matrix and what is the implication of this?

See in a plane strain situation, you would take out a slice and you want the edges to remain straight- the meaning epsilon z suppose I have this as a z direction epsilon z remains 0 means, these two surfaces should remain straight; in order to keep them straight you will have stresses developed in the z direction only because of that the planes remain straight. In fact, we would look at this when we want to extend the solution for a finite body crack problem. We would study for series of collinear cracks in an infinite body from that solution, if you want to take out a single edged crack or double edged crack, we would pass a plane and we would argue what kind of situation that should be maintained on the surfaces. So, when you look at the plane strain situation, you should recognize and understand physically the meaning of sigma zz. So, here again, you go and look at what is the orientation of maximum shear stress. You must handle this problem as a three-dimensional one.

Similarly, the plane stress also looks as a three-dimensional one. In this space, what is the plane that is very important. Right now you go and look at and clarify your understanding; that understanding will help you to rationalize what kind of deformation takes place in the case of plane strain and plane stress situation, when we are looking at the plastic deformation.

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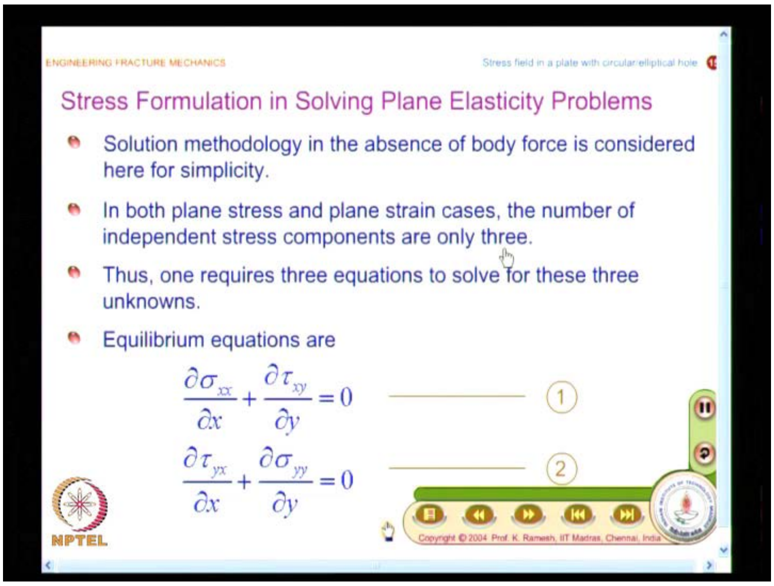

And we are going to look at the solution methodology in the absence of body force for simplicity. And we have already seen what constitute the stress tensor as well as strain tensor in the cases of plane stress and plane strain; in both the cases the number of independent stress components is only three.

Because you could calculate epsilon z in the case of plane stress; you could also calculate sigma z in the case of plane strain; they are dependent on other quantities. So, the independent quantities are only three; so you require three equations to solve for these three unknowns and where do the three equations come from?

You have equilibrium equations, they are very famous and mind you, we have already said that we are not going to consider body forces. So, the equation reduces to dou sigma xx divided by dou x plus dou tau x y divided by dou y equal to 0; dou tau y x divided by dou x plus dou sigma y by divided by dou y equal to 0.

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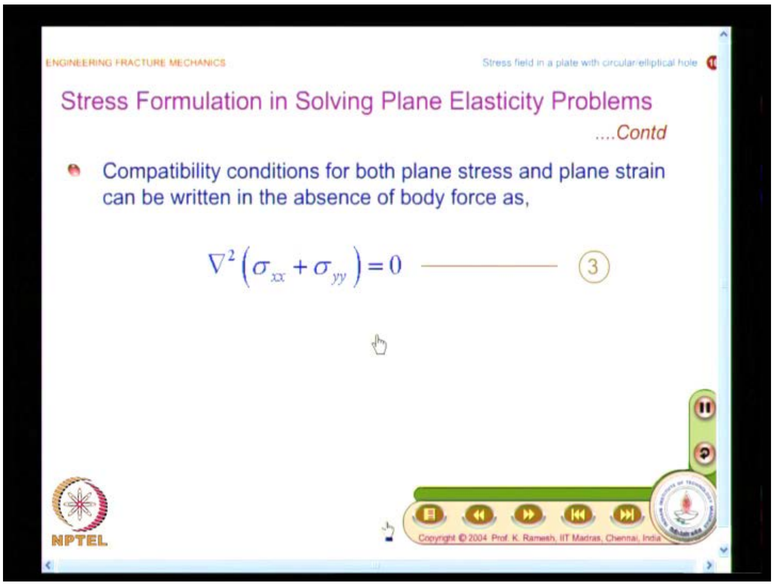

And we can also look at the equation of compatibility. We have already said compatibility conditions are necessary when you are having a stress formulation and this reduces for both plane stress and plane strain in this fashion.

In the absence of body force it appears like this del squared sigma xx plus sigma yy equal to 0. You know many times you look at this equation, you will carry on method you do not reflect what the equation conveys to you. See I had pointed out when we looked at a three -dimensional scenario, even in the absence of body forces, the governing equation had a material constant appearing in the form of Poisson’s ratio.

If you look at in this case, the governing equation does not contain any material constant; this is a very added advantage. The observation is very important, because I have been saying I am going to illustrate many phenomena in fracture mechanics by looking at results from photo elasticity. People have done crack propagation studies in photo elasticity; people also have done static experiments on mode 1, mode 2 and mode 3.

As long as the problem is plane stress or plane strain, the elastic constant do not play a role. So, whatever I see in a plastic is same as what I would see in a metallic specimen; definitely the deformations will be different, we are talking about deformation; as long as you are concerning your attention only to stresses, for the same load both will have same levels of stress. Usually you invoke model to prototype relations and you apply a smaller load on a plastic model than what is actually happening in the prototype, but the stresses developed for that value of load is same in a metallic specimen.

So, this gives the mathematical justification of validity of photoelasticity for solving two-dimensional problems. So, you should also look at that as part of this.

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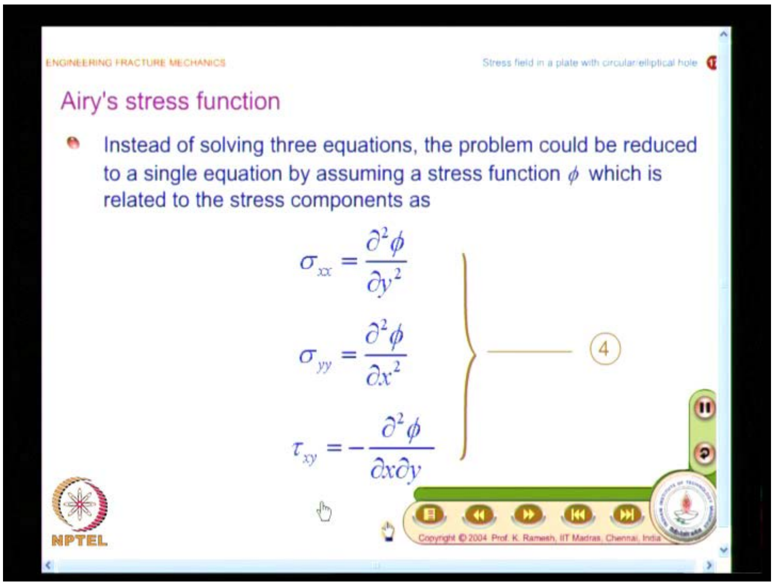

Now, what we will have to do is we will have to solve these equations and how people have approached? You know people wanted to make our life simple; they have looked at a stress function approach.

You know this was introduced by Airy and this is known as Airy's stress function approach. We have to get the solution by solving the three equations, instead of doing that, the problem could be reduce to a single equation by assuming a stress function phi, which is related to the stress components in the following fashion.

So, you have sigma xx is related to phi as dou squared phi by dou y squared; this is for cartesian frame of reference; we are looking at the cartesian stress components; sigma yy equal to rho squared phi by dou x squared and tau xy equal to minus dou square phi by dou x dou y.

So, what we would try to show is, by just finding out phi, we would immediately get to know what is the sigma x stress component, sigma y stress component, dou x stress components. So, instead of three unknowns, we would modify the set of equations to determine only one function phi.

The moment phi is known, the problem is solved; once I get the stresses, go to strain, then go to displacement and the equations are given like this sigma xx equal to dou squared phi by dou y squared; sigma yy equal to dou squared phi by dou x squared; tau xy equal to minus dou squared phi by dou x dou y.

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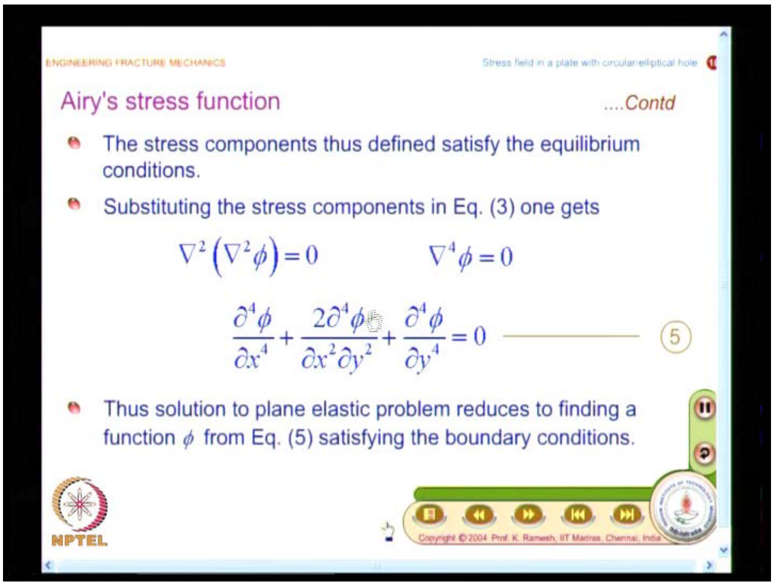

We have already seen the compatibility condition as del squared sigma x plus sigma y; if I substitute the definition of sigma x and sigma y, how does the equation change?

The equation changes to del square of del squared phi equal to 0; it reduces to del power 4 phi equal to 0 and in an expanded form, this is nothing but dou power 4 phi divided by dou x power 4 plus 2 dou power 4 phi by dou x squared by dou y squared plus dou power 4 phi divided by dou y power 4 equal to 0.

So, what we have achieved is, for plane elastic problem, the solution reduces to finding a function phi satisfying the boundary conditions. We have to go and look at; do we solve the problem like this? See we said in strength of materials, you assume the displacement field and solve it for a class of slender members.

In theory of elasticity, we said we do not assume the displacement; we evaluate as path of solvent differential equations, then we qualify; in some cases we will have to look at uniqueness of displacement. So, multiply connected bodies are difficult to handle, then we said we will look at experiments and look at how the displacement is we will invoke it; we said all that finally, we are not looking at as a mathematical exercise. Ideally I have to get phi satisfying this equation and the boundary conditions and solve the problem.

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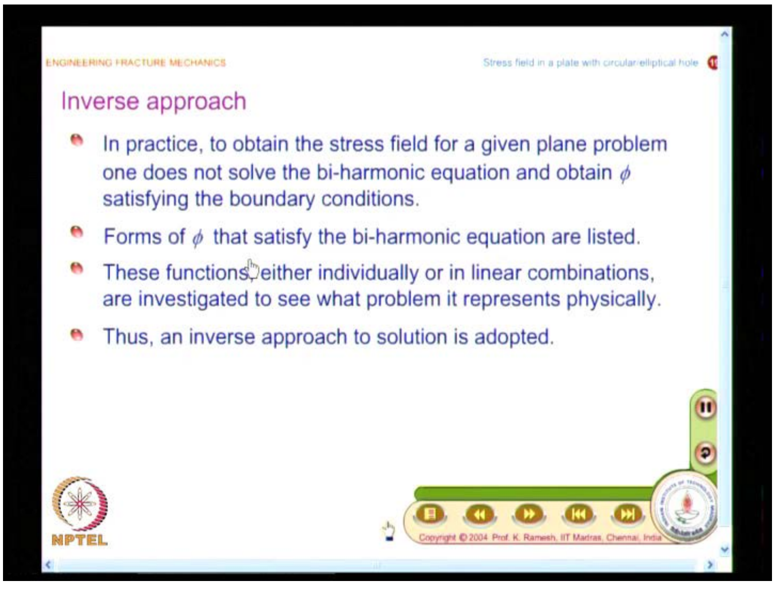

So, I have to define the problem by specifying the boundary and evaluate phi from that, but we will not do that, what we will do is, we will follow what is known as an inverse approach.

So, what you do in an inverse approach? We will not solve the bi-harmonic equation and obtain phi, but forms of phi that satisfy the bi-harmonic equation would be investigated. So, we do it in a reverse fashion. Ideally what I should do? For a given problem, specify the boundary condition and you know the governing differential equation, solve the governing differential equation and get phi we do not do that; we look at what forms of phi are permissible that they satisfy the bi-harmonic equation. Either the functions individually or in linear combinations are investigated to see what problem it represents physically.

So, you are actually adopting an inverse approach; you are not solving the problem up front; what you are actually doing is, this is the bi-harmonic equation, we would look at what functions of phi are valid candidates to satisfy the bi-harmonic equation, then we go and see which physical problem this stress function represents.

So, in fact in 1930's, the research was focused on determining phi for various types of problems. The moment phi is determined everything else is found; because once phi is known sigma x is given as dou square phi by dou y square and so on so forth.

So, the challenge is in finding out what is phi.

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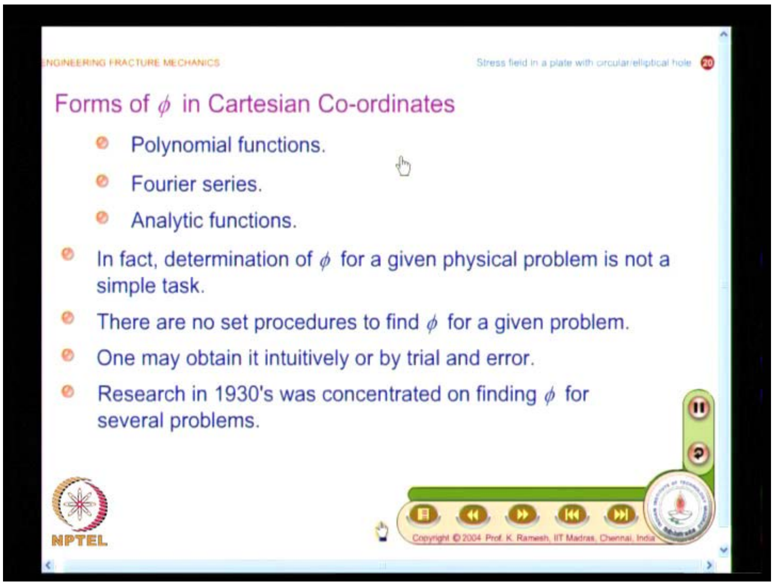

As I mentioned, we are actually looking at forms of phi in different coordinate system. Now, we take up the cartesian co-ordinates; we will also look at forms of phi in polar coordinates and phi could be a polynomial function that is a simplest think off; you can take different orders of polynomial that satisfy the bi-harmonic equation and investigate what physical problem it solves; you could also have phi represented in terms of Fourier series.

See in fact, when you are learning a first level course, you learn the bending moment diagram and shear force diagram to understand how the bending moment changes along the length of the beam. You normally take up concentrated loads, because they are easy to illustrate how a bending moment could be drawn. When you come to theory of elasticity, it is always convenient to handle it distributed loading; concentrated loads are difficult to handle.

So, when I have a concentrated load, I have to use a Fourier series and invoke several harmonics to obtain a concentrated load. So, you will find solution to three point bending is done in a much more complex fashion using Fourier series than a beam with a distributed loading. You learn it other way when you are learning the bending moments and you could also find out phi based on analytic functions.

In fact, in a course in fracture mechanics, we would look at analytic functions as candidates for defining the stress functions. The moment I strike a gold mine what is the stress function for a given problem, all other steps are completely known.

So, the challenge lies in finding out what is phi for a given physical problem and it is not a simple task and if you look at, are there set procedures to find phi? The answer is only no. One may obtain it intuitively or by trial and error.

So, the whole of analytical development hinges on what is the stress function phi. If you are able to find out a stress function for a given problem, the problem is completely solved and this is the reason why people also developed different coordinate system to comfortably specify the boundary conditions.

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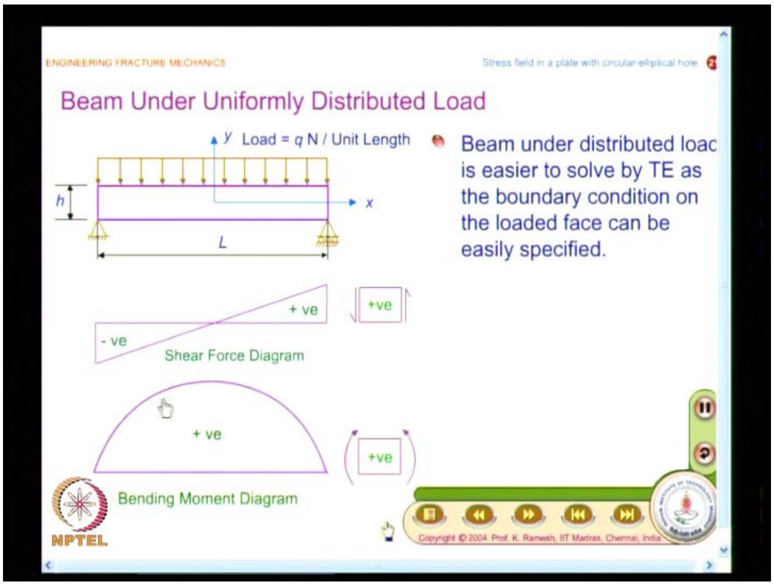

So, what we will look at is, we will take up a simple problem; we will just start the procedure and continue it in the next class.

So, what I am showing here is beam under uniformly distributed load and this is the problem usually solved in a course in theory of elasticity. We would just see the highlights, we will not get into the complete solution procedure and we would look at what is the shear force as well as the bending mode variation. And what is shown here is what is the sign convention used on a positive phase, positive shear is positive. Similarly, on a positive phase anticlockwise bending moment is positive and for a problem like this, you will have variation of shears force like this; variations of bending moment like this.

See I have already pointed out is flexure formula valid for this case? This question you would not have asked when you have done a course in strength of materials. You simply learn flexure formula for a beam under uniform bending moment and applied it to cantilever beam, simply supported beam and over hand beam.

So, many problems you had done; you have never looked at the no answers. In the last class, I had shown when I have constant shear force, then plane section do not remain plane, before and after loading in fact, it has a variation; fortunately it does not interfere with bending. So, bending and shear were uncoupled for normal cross section beams unless, you look at deep beams.

So, you are justified in extending flexure formula when shear force remain constant and what you see here? Shear force is not constant; so bending stress whatever you calculate from your strength of material will not be accurate from a mathematical stand point; the solution will not be accurate you have anticipate that.

See you should not say I have already solved this problem in strength of materials why should I solve it in theory of elasticity? You would solve it by theory of elasticity and find out what are the corrections that you get and you say for practical situation, these corrections are very small, I can still live with strength of material solution; that is different, that is ok and that is how engineers operate; the engineers always solve the problem and bring in correction factors.

In fact, we would look at from that perspective. Because while learning fracture mechanics, you should also have your fundamentals clear and that is the reason why I am spending some time on reviewing strength of materials as well as theory of elasticity to bring out the no answers. So that is the idea behind it. So, in this class, we have looked at basics of theory of elasticity; I have said that you determine displacement as part of the solution procedure.

So, essentially you attempt to solve differential equations. Even in that case, we have segregated class of problems involving simply connected bodies and multiply connected bodies. In simply connected bodies, compatibility conditions actually guarantee for you to get the final solution while using stress formulation; in multiply connected bodies, you have to bring in uniqueness of displacement until, then the solution is not complete, then we moved on to look at what is an Airy stress function approach; we said that it is an inverse approach, rather than finding out phi satisfying a given problem of the boundary conditions, we in fact look at candidates for phi, which satisfy the bi-harmonic equation, then qualify, which physical problem does this phi represent.

So, in that sense, it is a semi inverse method. Once you get the phi for a problem, everything about the problem is known, because that is embedded in the development of this semi inverse method.

Thank you.