Engineering Fracture Mechanics
Prof. K. Ramesh
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module No. # 02
Lecture No. # 12
Pop-in Phenomenon

In the last class, we have looked at the elegance of energy release rate approach to explain certain aspects that is seen in the experiments. And we have also looked at, by conceptualizing resistance, we have been able to extend what are all the concepts developed for brittle materials to ductile solids. In the case of brittle materials, the resistance was constant, whereas when you go to ductile solids, you have plastic energy dissipation, and in view of that, as the crack length increases, the plastic zone size also increases. So, the resistance increases as the crack also increases in length.

And we have looked at two extreme cases of specimen thicknesses. In the case of a plane strain where the thickness is very large, the variation of R was slightly shallow.

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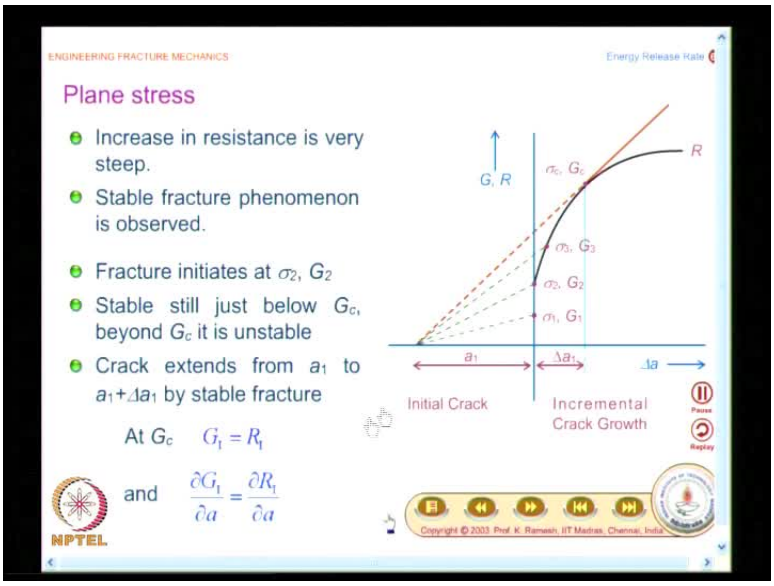

The moment you come for a thin specimen which is in plane stress, we saw that the R curve was very steep like this. This has helped in understanding a very important concept that would have what is known as stable fracture. And what do you see in this graph? When the stress level is increased from sigma 1 to sigma 2, you have at this point, G equal to R. So, fracture initiates at sigma 2 and the corresponding energy release rate is G; when I increase the stress further to sigma 3, then the crack advances by a small amount and stops. Until I increase the stress further, it would remain there. When the stress becomes sigma c, you find G equal to R as well as the slopes of these two curves are equal. So, beyond this, you will have fracture instability. So, what this graph shows is a crack of initial length a 1 has advanced by delta a 1. For this extension, the fracture was stable; beyond this, you have unstable fracture (Refer Slide Time: 3:00), and now you can appreciate.

When we said what are the necessary and sufficient conditions for fracture, we were able to see the necessary condition is G 1 should be equal to R 1 because we are looking at the mode1 scenario as well dou G 1 by dou a should be equal to dou R 1 by dou a. This becomes a sufficient condition. Only when these two conditions are satisfied, you will have catastrophic moment of crack.

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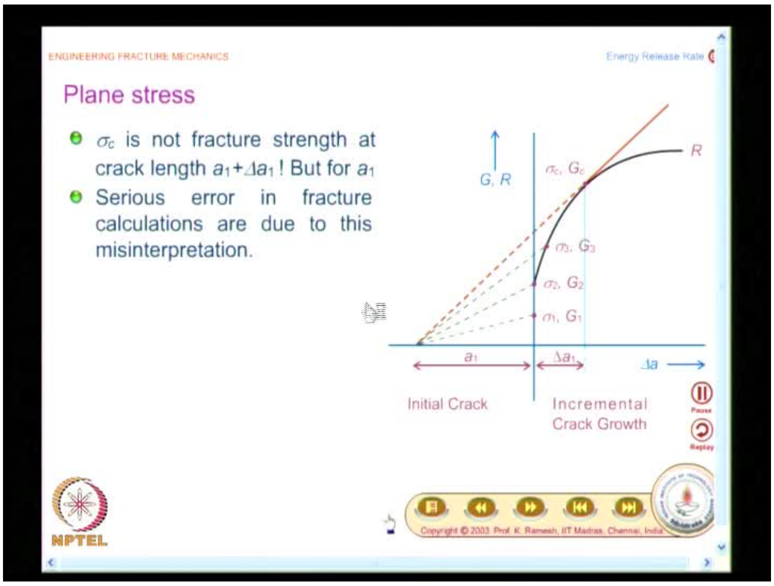

And another important aspect also we looked at. When you say sigma c, what is the corresponding critical crack length? Is it a 1 or a 1 plus delta a? You have to keep in mind when you have the stress as sigma c, a crack of size a 1 will have a stable fracture extension, followed by unstable fracture. So, you should not mistake that a crack length of a 1 plus delta a 1 is a critical crack length for the stress sigma c. In fact, many fracture calculations where erroneous because of this misinterpretation. So, these are all subtle things.

So, what you will have to look at is, without getting into much of mathematics, we have been able to explain the phenomena of stable fracture that is seen in thin plates. The stable fracture would be followed by unstable fracture, but you are able to explain with the strength of R curve concept. And we will also look at other subtle issues.

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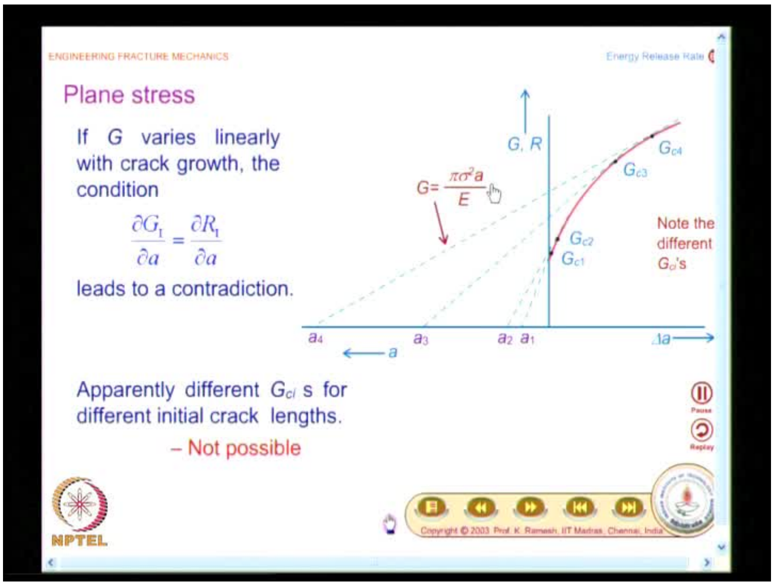

When you have the energy release rate as a straight line, it leads to another contradiction. We have the expression for G as pi sigma square a divided by E. So, they are all straight lines. Now, R is a curve and when I have cracks of different lengths a 1, a 2, a 3, a 4, I think I would redo the animation; have a look at it how the different crack lengths indicate the critical value of G. For the crack length of a 1, you will have the value of critical energy release rate as G c1; for crack length of a 2 you will have this as G c2; for crack length of a 3 you will have this as G c3.

I would like you to make neat sketches of this. This is very important. It is illustrating a very subtle concept. The animation is very helpful to understand this. For a 4, you have this as G c4. So, what you eventually get is different critical energy release rates for different initial crack lengths.

This cannot be possible because when we say the resistance is inherent in the material, the material has to behave in a similar fashion for any lengths of cracks. The material property should remain more or less same when you are looking at for different values of crack lengths. So, what is a mistake that is possibly happening here? See, one of the issues here is you have taken an infinite plate with a central crack. In practice, everything is a finite plate; nothing is infinite; not only this, you start with the particular crack length, then the crack advances.

So, when the crack advances, no longer the original calculation of energy release rate would remain so, because the crack length, as it increases, the edge of the crack faces the edge of the specimen, the crack-tip becoming closer to the specimen edge. So, this would definitely alter the expression for energy release rate. In fact, when we develop the stress intensity factor concept, we will also involve this kind of a parameter which is a function of the a by w ratio. You will have a basic expression for stress intensity factor and you will have a multiplying factor.

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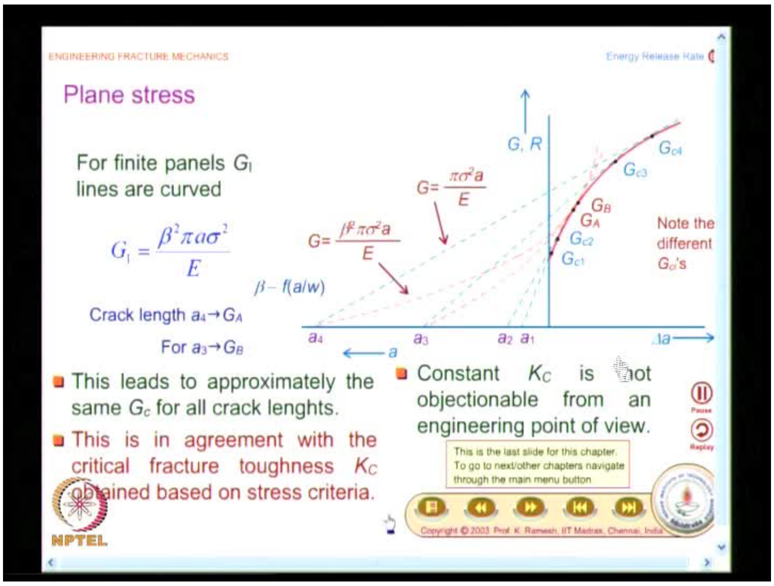

On similar lines, what you can also think of is, as the length of the crack increases, I cannot have this as the expression. This expression has to be different. That would also be a non-linear and you could label that as beta square pi a sigma square.

This is a factor function of the problem on hand; it is a function of a by w ratio. And what you get here, this makes the change that you no longer have a linear variation because beta is not constant; that itself is changing as a function of a by w and you will have, in essence, a non-linear curve depicting the fracture. And I have for the crack length a 4, the corresponding failure energy release rate is G A, and for a crack length of a 3, the corresponding critical energy release rate is G B; they are very close to each other whereas, if we had the energy release rate as straight lines, you are having a large variation from G c1 to G c4. So, what you gain here is, making the energy release rate as a non-linear function leads to approximately the same G c for all crack lengths. So, this explains what you would expect in actual practice.

In actual practice, the material parameter will dictate and material parameter will remain same, irrespective of whatever the cracks lengths you have. And when you go to stress intensity factor literature, you would have what is known as a fracture toughness which is given a symbol K suffix C and fracture toughness for specific thicknesses, or if you look at the plane strain fracture toughness, it is considered as a material property and constant K C is not objectionable from an engineering point of view because there again you will see, if you go for thin panels, the value of K C increases. That is a separate aspect, but for that particular panel, if I have several crack lengths, the failure also should be dictated by one critical value of K or one critical value of G. That is what is attempted here.

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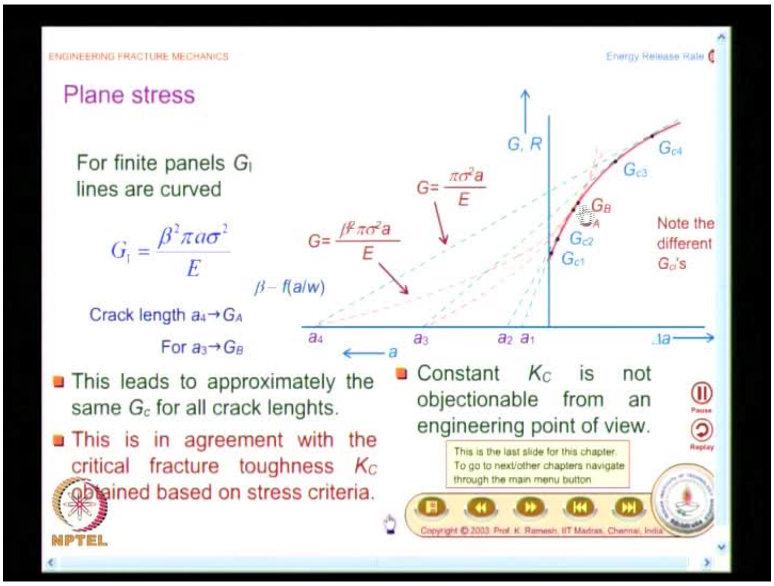

So, what you will have to look at is the energy release rate also has to be a non-linear function. I think I would redo this animation so that you will be able to appreciate how the graphs have been drawn. Now, you have the line drawn for the crack a 4. You have a non-linear curve which is touching this at this point at this point G equal to R and the slopes of G and R curves are matching. So, fracture instability will occur here and now we look at for a shorter crack length, and the line is like this.

So, this shows that you would have a constant value of G for different crack lengths. If you recognize G also varies nonlinearly. See, now we have looked at thin plate. We have also looked at, earlier, a thick plate. What happens in intermediate plates? See, what you will have to keep in mind is we have developed the concept of energy release rate and resistance, which has successfully explained how failure occurs in brittle materials, how failure occurs in high strength ductile solids. We have seen it for a thin specimen and a thick specimen. In intermediate thickness specimens, a very nice phenomenon is observed; the phenomenon is called pop-in.

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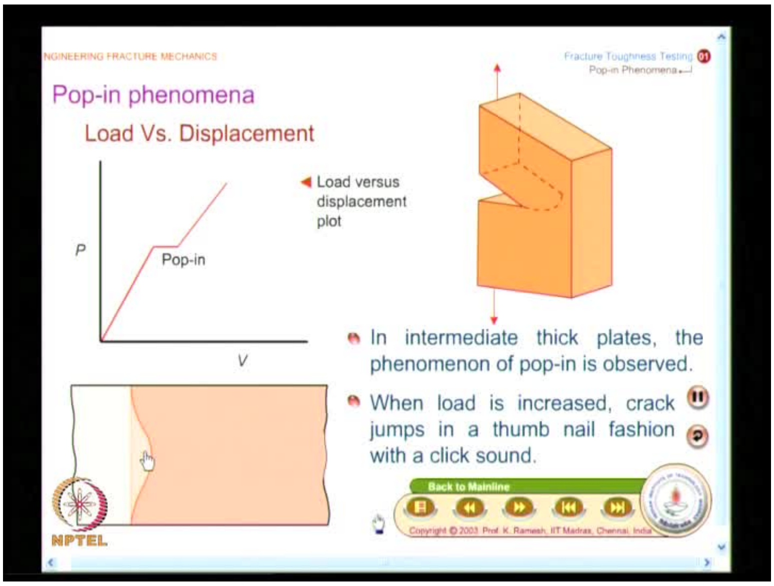

And we would see, what is this phenomenon? You observe very carefully and listen very carefully. I will do the animation again. So, this is for an intermediate thickness plate and cross section is shown; on the crack face it is shown; so, watch what happens. This is the initial crack front and you have the load versus displacement; make neat sketches of this; you do not have to worry about three-dimensional representation; you will have to make this graph as well as what happens on the crack front (Refer Slide Time: 13:08).

And also listen very carefully. I would repeat the animation. Have you heard a click sound? You know, what you find is the crack as advanced as a thumbnail and this is observed in intermediate thick plates, and we need to find an explanation whether such a thing is possible with our understanding of G and R.

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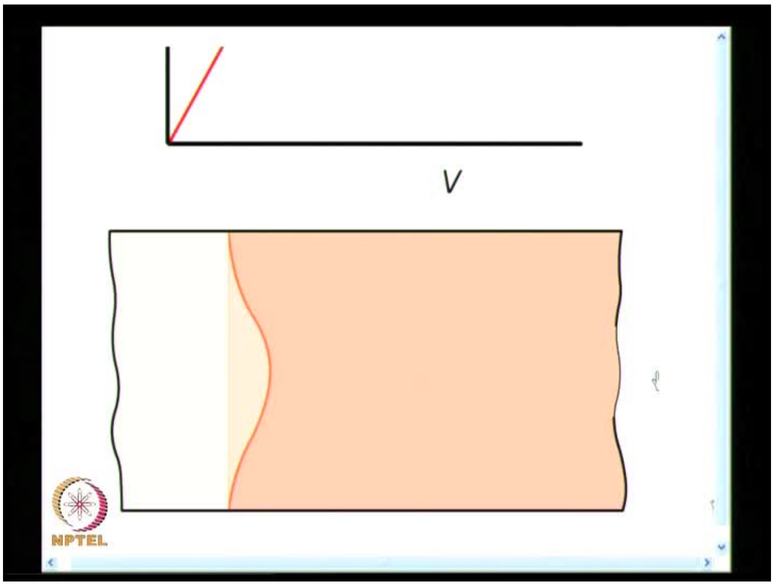

I would repeat the animation. You listen again carefully. Suddenly it jumps and stops there. It does not proceed further. The crack stops that it does not proceed further and you could see the enlarge picture of the thumbnail crack; that is what you see here.

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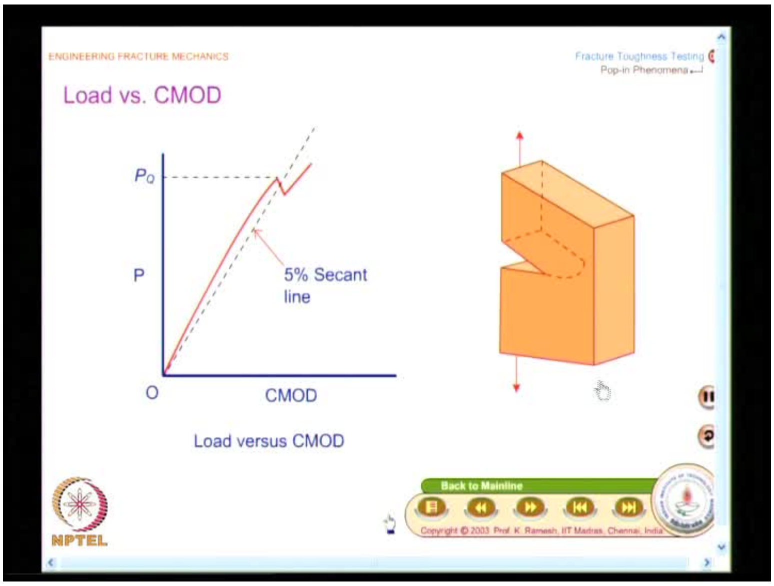

In fracture toughness experiment, what you do is you measure what is known as crack mouth opening displacement. You would measure the displacement in these two faces (Refer Slide Time: 14:36).

So, in such a graph, you will have a step here (Refer Slide Time: 14:42) and then you have a methodology, how to find out the load at which the crack has initiated; that will be done. Again I will repeat the animation; have a look at it and also sketch it; it is very important how the graph look looks like P versus CMOD.

So, this is an experimentally observed phenomenon and if you look at, this is an animation; so, the click sound is loud enough for you to hear; it is very small magnitude sound. And in fact, in one of the nondestructive testing technique called as acoustic emission, they capture this sound. So, when you have a structure, when crack advances, these small sounds would be captured by the probe and they would be in a position to locate where the crack is in this structure, what orientation, what length, so on and so forth. So, new branch of nondestructive methodology in monitoring cracks has grown, but it is very difficult to process the data also because it is like searching a needle in a hay stack because you could have noise from various sources. So, you find engineers have successfully found out what kind of an approach they should use for data processing.

So, what is interesting is, if you find any small clue, engineers are ready to exploit it to the extent possible. Now, our concern is, we have seen what is pop-in and it is also used in one of the nondestructive testing methodology, and we should be able to explain it from the point of view of R curve concept.

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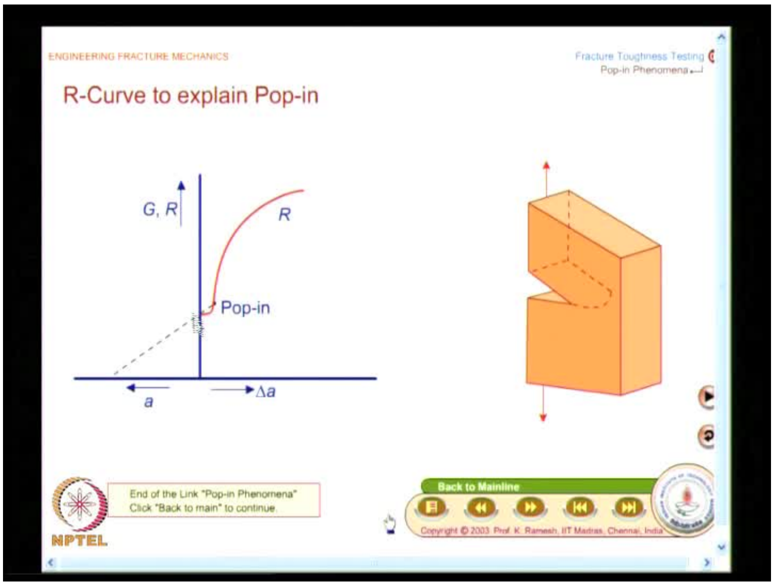

So, if we do that, it would give us a confidence that we are proceeding in the right direction. So, this is what happens and this could be easily explained. So, what you find here is, look at the shape of the R curve; see, in the case of brittle materials, we had R as constant because it is only the surface energy which is really causing the resistance; so, it remained constant and you were able to explain, when G equal to R, fracture not only initiates but it is also an instability.

When you go for a plane strain specimen, you had a shallow R; when we went for a plane stress specimen, you had a steep R; when you go for an intermediate thickness specimen, you could have the R such that you have a horizontal phase and then the curve starts. So, what happens is the fracture will initiate and it will jump. So, this is what happens. So, with the R curve concept, we are in a position to explain what is observed in actual practice. So, that means we are in the right direction. We have been able to see for thin specimens, thick specimens as well as intermediate specimens.

I would again repeat the animation just for your clarity. So, this is what happens (Refer Slide Time: 18:20). So, when the stress level is increased, fracture instability will follow. So, this is another example which shows we are on the right direction. This brings to our conclusion, all the concept that we wanted to look at in an energy release rate. You would come back to this chapter after developing crack tips, stress and displacement fields because we need to find out an identity between stress intensity factors in energy release rate.

And we have also said, whatever the calculation of energy release rate we have done for the infinite plate with a certain crack is only approximate because we have directly taken the value or looked at the analogy. We have not actually derived it. We will do that derivation at that time.

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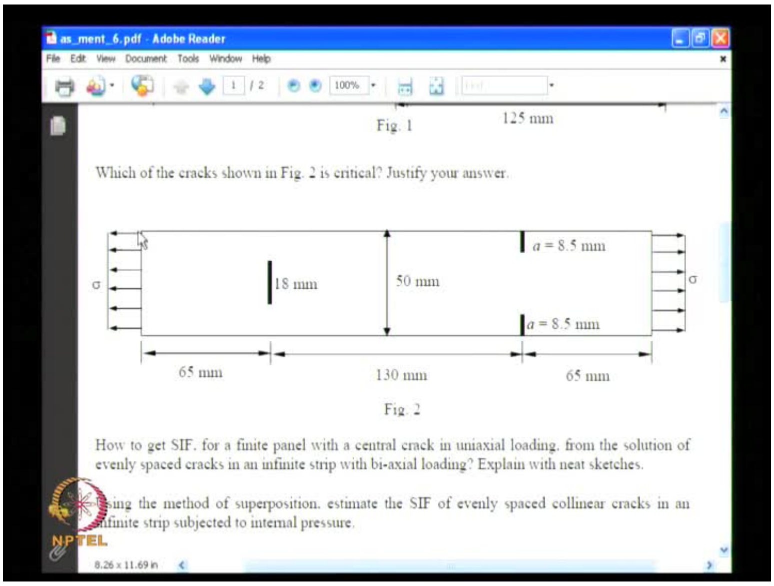

Now, what we will look at is, we have looked at in the last class, a problem for which you need to bring specimens and see, in a multiple crack scenario, how the specimen behaves? Let us look at the specimen first and the specimen was something like this. What you had was, you had a thin strip of paper because it is easy far us to bring it as a specimen and you had one central crack. This is shown as a thick line just for clarity.

Crack will be invisible and you have edge cracks far away from the central crack and these are shown as a equal to 8.5 millimeter; they total to 17 millimeter less than the central crack. But what I had asked to you was I had asked you to bring 3 specimens: one specimen will have the edge crack of the size of 9.5 millimeter, totally 19 millimeter; that is, the edge cracks, if you add them together, it is longer than longer than the central crack; then, you had another specimen with 9 millimeter; if you add them, the length of the crack is same as a central crack; the third case is the double edge crack; total length is shorter than the central crack.

Now, we will take up the specimen with 9.5 millimeter double edge crack. I would like you to take the specimen and insert the pencil and carefully load it. Carefully load it and I would like to see, how the failure occurs. Are you all ready? Now, I think you keep the specimen straight and apply it uniformly; apply the load uniformly. Fine. I think you can you can take the specimen now. We just break the specimen; break the specimen; break the specimen. Can you tell me how many people have got the double edge crack failed?

Raise your hands. So, I find 1, 2, 3, 4, 5, 6; so, it is obvious because in this example, we had the longest crack was the double edge crack. And some people have not got the result as double edge crack mainly because you know you are doing the experiment with hand and you may not be in a position to apply the load uniformly.

See, in experiment, you have to be very careful at every stage of making the specimen. You cannot be less alert while making the crack and be very careful in applying the load because when you are making the crack itself, you should have taken a very sharp blade and then make it very very carefully because crack-tip is very important. Later on, we are going to see if crack-tip blunts, it is helpful in fracture mechanics. That is what has happened to them because if the crack tip is not made properly, the specimen behavior would be different. So, it is obvious to expect, when I have double edge crack longer than the central crack, fracture would initiate there.

Now, let us look at the second specimen. Please take the second specimen where the center crack and some of the double edge cracks are of equal magnitudes. Now, hold them straight, or if you are fine, you can apply uniform load horizontally; try that; it is all trial and error; Fine. And then, you would report which crack has broken. Can you raise the hands where the double edge crack has broken? How many of you have got it? 1, 2, 3; out of 6 or 7 students, 3 have got specimens broken; others, they have not been able to break; double edge crack or center crack? Double edge. And what about any one has got the center crack breaking? Fine.

So, that kind of scatter should be there. Then only you are doing a proper experimentation. In fact, by enlarge if you look at even in this case, it is only the double edge crack has got separated first. So, at this stage, you can understand one thing. Even though we have multiple cracks in a structure, it is the crack that is critical, is going to affect the performance of the structure. So, we are justified in taking a single crack and develop our mathematics so that we understand what happens in the presence of a crack. Now, we will take up the last specimen. Unless you have made the specimen very, very carefully and also applied the load very carefully, the specimen will not behave the way it should behave.

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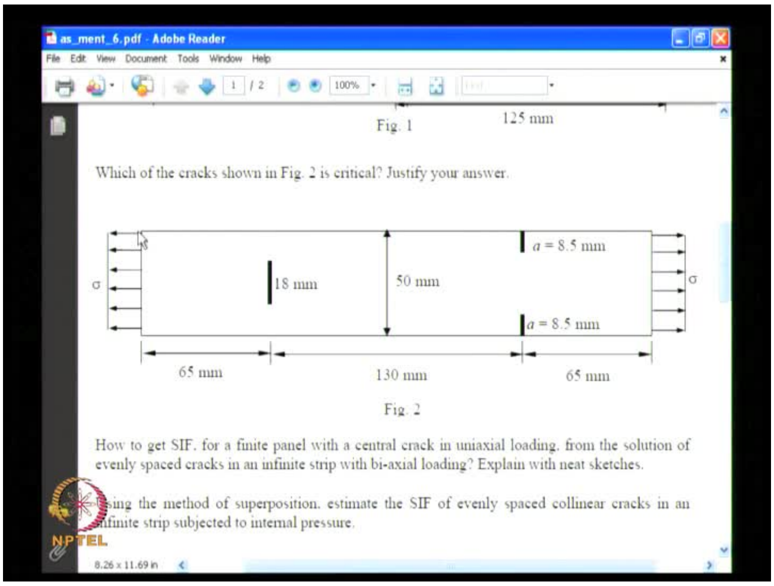

So, let me see, we start breaking the specimen; start breaking the specimen where have you got the specimen broke? That is good. So, which one? How many have got the central crack? How many have got the double edge crack? Central crack - you raise your hands; so, I have 1, 2, 3, 4; double edge crack - 1, 2, 3. So, that means only three students have got very good specimen made for this case. See, later on, we are going to have methodologies to calculate the stress intensity factor. And if you look at, these are all finite specimens; I have not supplied the batch of paper to them; each one had brought their own quality of paper; so, there would be variation and specimens were made by individual students.

There could also be variation and unless you perform the experiment carefully, even in this case, only the double edge crack would fail because the stress intensity factor would be slightly larger than the central crack. So, when I keep increasing the load, the stress intensity factor would become critical at the double edge crack. So, common sense fails.

What you find is, a crack can be shorter physically, but what configuration it is available on the actual structure also matters. And this also gives you a comfort that even though I have multiple cracks, if they are far away, we have already seen another example; we have seen several cracks in a pressurized thick cylinder; there also we found there is no major interaction even when they were few millimeters away. So, we are justified in learning fracture mechanics for a single crack, for us to develop the mathematics comfortably.

And now, what we will do is, before we get into the chapter on the crack-tip stress in displacement fields, it is better that we review concepts in strength of materials as well as theory of elasticity and get into the solution for the crack-tip stress fields.

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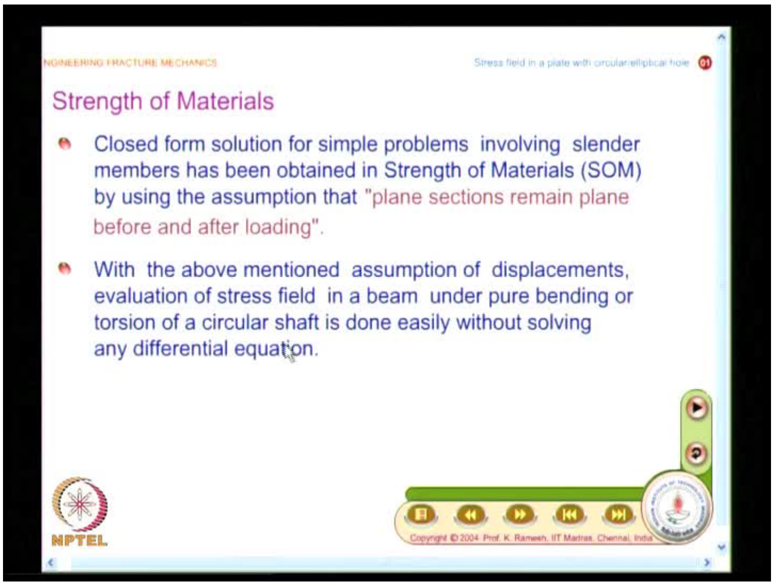

Now, let us look at what we have learnt in strength of materials. Every statement here is important. First thing what you get is, you get what is known as closed form solution for simple problems. And what is the closed form solution? You get the values of stress magnitudes at every point in the domain. That is how you have got a solution.

Suppose I take a tension strip. The problem is so simple, away from the points of loading, you get the normal stresses, sigma is constant everywhere. If the cross section is constant, if the load is uniform, then it is constant everywhere. So, at any xy position, you have the answer from the solution. Similarly, when you go for bending, away from the points of loading, if you look at the central zone, you will be in a position to get what is the variation of stress over the depth of the beam.

If I specify any value of x y z, here, not x y z, only x y, I would be in a position to find out the complete magnitudes of stresses. No problem. For all that, what is the kind of assumption that you have done? A very important assumption you make in strength of materials; you make an assumption that plane sections remain plane before and after loading. And what I have achieved out of it?

See, if you look at the whole course of strength of materials, you avoid solving differential equations. It is a first level course in solid mechanics. You are able to avoid solving differential equations by making a very important assumption about the displacements. Plane sections remain plane before and after loading. And what you have done? You have been able to solve problems related to slender members, and when you have looked at the slender members, you have also carefully looked at what kind of problems you would solve.

You have evaluated stress field in a beam under pure bending or torsion of a circular shaft. See, I have brought this specimen; this is rectangular; I can also twist it. But you do not solve a twisting of a rectangular shaft in a first level course; you postponed it. You never even asked a question when you were learning torsion, why I am not considering torsion of a rectangular shaft?

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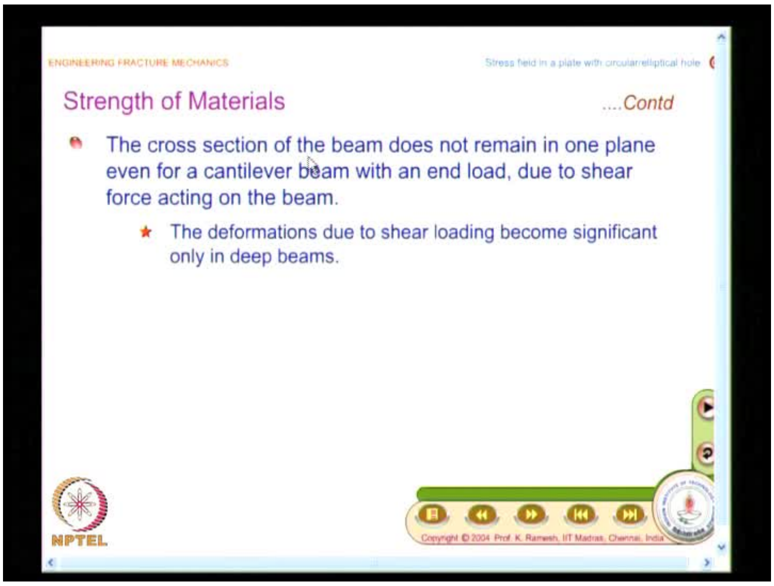

Suppose I take it and then bend it, you can look at it; if you look at very closely here if when I bend it, you would be able to see how the lines (Refer Slide Time: 30:24), the lines they remain vertical. They get rotated; the plane sections remain plane; they do not change; they just do not change. So, that is the advantage in the case of strength of materials. You have made this important assumption and you are very careful in selecting the kind of cross section; that also help in solving your problem. And many may not know that the cross section of the beam does not remain in one plane, even for a cantilever beam with an end load. What is the difference?

See, when I say beam under pure bending, if you really go back and see your strength of material course development, you would have developed the flexure formula only for the case of a beam under fore point bending, which is giving only constant bending moment in the length of the beam, for which you want to find out the stresses. The moment I go to a cantilever, when I have an edge load, what happens? You have the bending moment varies along the length, but the shear force remains constant.

So, what you are having is you are having bending as well as shear and this is the simplest beam that you come across in many applications. And what you have done? You have used only flexure of formula for this also. You never questioned, you have not derived your equations for this, but you have still utilized flexure formula thinking the formula is fully applicable. How this was applicable and what way the sections change when you have shear? We will have a look at it.

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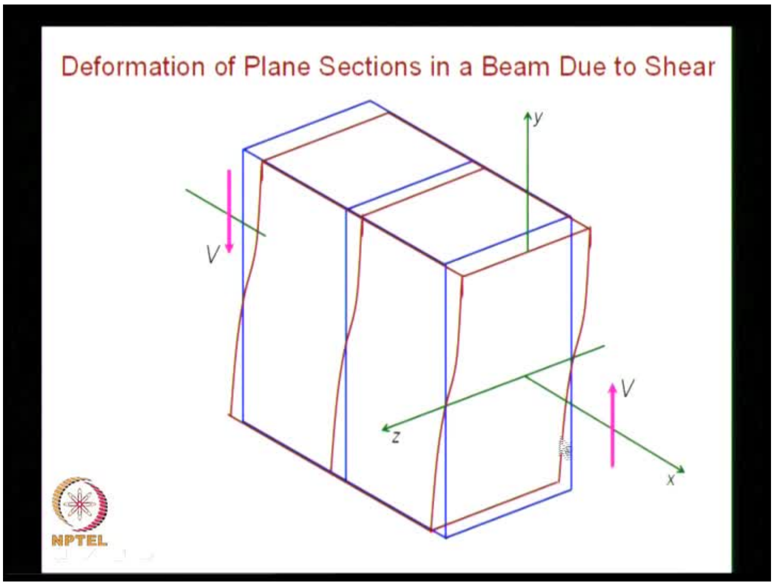

Now I am having a section of a rectangular beam and I have taken two sections for illustration and we are going to look at what way the plane sections change because of shear. This is subjected to shear loading like this, and mind you, the shear loading is constant; I am only looking at constant shear loading. Try to make a sketch of it because you have a feeling that you have learnt strength of materials very well. There are certain issues which you have not focused.

When I am developing a course in fracture mechanics, I am pointing out then and there, what is a kind of solution development? What are all the difficulties people have pointed out? How these difficulties were addressed? In fact, when we look at crack-tips, stress and displacement fields, even on the boundary condition, we are going to have an elaborate discussion. So, when we want to do that, let us look at in strength of materials, have we understood all aspects of it? You make an assumption in strength of materials plane sections remain plane before and after loading. That is violated even when I have a constant shear. How does this appear? When I have constant shear, you have these lines changed. It is not a straight line; it is a curve; this is a curve.

So, what you find is, your assumption is no longer valid in the case of even a simple situation of constant shear. How are we justified in utilizing that result? There is a comfort. Engineers always work like that; they will develop a solution and try to apply the practical problems and then bring in correction factors, if they are necessary.

If you are unable to solve the problem in all its totality by your analytical methodology, correction factors really help. In fact, when you are looking at design of a spare gear tooth that is consider as a cantilever beam and you would have done something like a Louie factor; that factor accommodates these kinds of issues; you may not be aware of it. So, in actual solution, you assume plane sections remain plane, but in reality, it is a curve.

We will again look at this animation. It was originally like this (Refer Slide Time: 32:25), then after the shear force is applied, you find that the sections have deformed like this. And if you look at the distance between two lines - two blue lines and two brown lines, they would remain same. See, whatever the effect of shear is not affecting the cross section. So, the shear and bending are decoupled; they are coupled. So, that is why when you want to calculate the bending stress, you still use the flexure formula.

The flexure formula is valid as long as you have constant shear, if it is a variable shear force along the length of the beam which is happening in many practical situations, when your analysis is only approximate; please make a note of it; it is very important .

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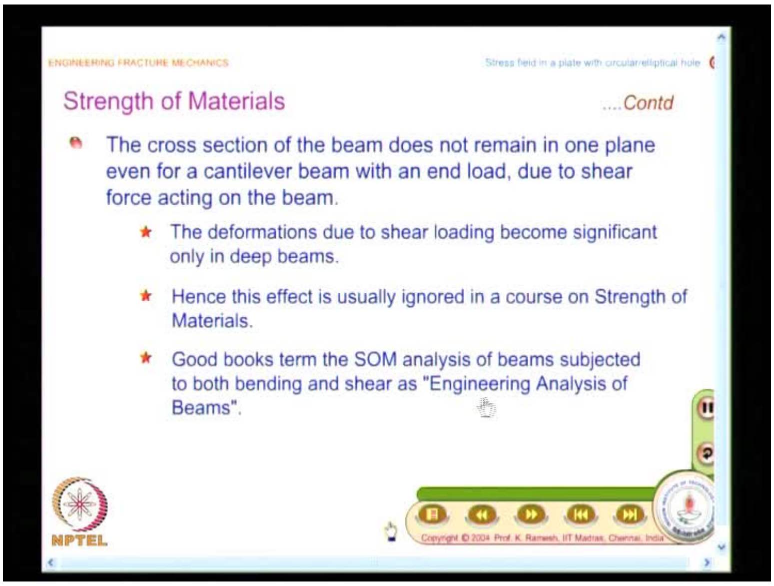

And that is what is summarized here: the deformations due to shear loading become significant only in deep beams. So, in deep beams, you accommodate that. So, usually this effect is ignored in a course on strength of materials. But if you look at good books, the term, the strength of materials analysis of beams subjected to both bending and shear as engineering analysis of beams is no longer called as analysis of beams; the moment you bring in engineering, you also understand there are approximations. So, you are looking at the approximations. So, without approximations, there is no engineering. So, this explains, even in simple bending, how we have compromised in the case of strength of materials.

Now, before I take up theory of elasticity, I would like to answer a common question raised by many students. In the first assignment sheet, we had a review of solid mechanics, and one of the problems asked was - what is the definition of a free surface?

The concept of free surface is very important both for numerical studies as well as experimental studies. Even for analytical development, you should know how to specify the boundary condition carefully and we would look at that. You know, what you will have to look at is, we have learnt a stress tensor; what is the utility of stress tensor? Stress tensor provides totality of the stress vectors at a point of interest.

When you go to a more circle, every point at the boundary of the more circle identifies particular plane passing through the point of interest. What is actually fundamental is you want to know the stress vector on all the possible planes passing through point of interest. Stress tensor is only a via media. But we have focused so much on stress tensor; we forget the importance of stress vector.

So, that is the reason why I said you have to go back and look at stress vector more closely.

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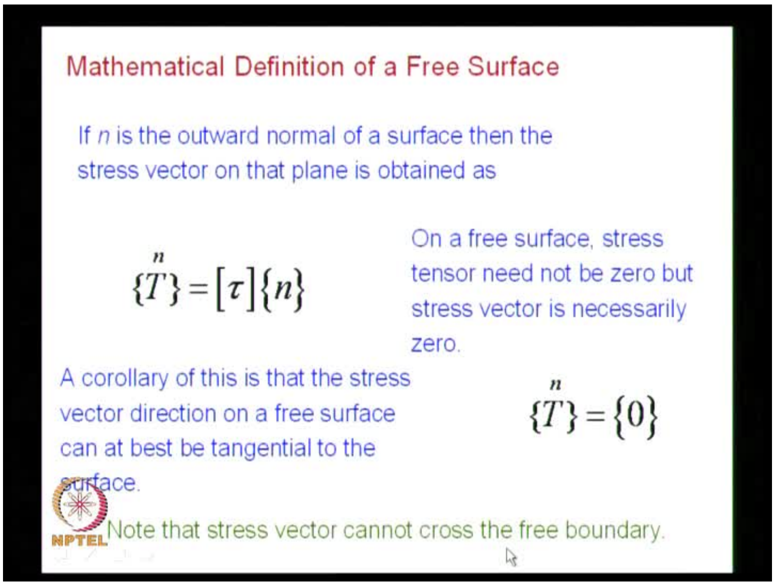

Suppose I have a stress tensor, how to find out the stress vector? Because I am going to find out stress vector on a plane of my choice and that is what is given here: If n is the outward normal of a surface, then the stress vector on that plane is what we want to find. I know the stress tensor at the point of interest. Can anyone recall what you have done in the course? It is nothing but your Cauchy's formula; as simple as that.

So, you have the symbolism. I have the stress vector on plane n is nothing but stress tensor multiplied by the direction cosines defining the outward normal of the plane. And before we look at from a solving an example, we would just make a statement, what is a definition of free surface.

On a free surface, stress tensor need not be 0, but stress vector is necessarily 0. In fact, in many of your problems like simple tension, bending, or torsion, you had free surfaces. So, some stress component may exist. That is a reason. When you are writing the boundary condition, you would be trained to see if it is a free surface; you cannot say everything is 0 there. So, what is said here is stress vector is necessarily 0; stress tensor need not be 0; that is what is mentioned here. We have to qualify the statement. We would go and qualify the statement. We will also look at other issues. So, when I say like this, mathematically I would write it like this - T n equal to 0. So, that is the mathematical definition of a free surface.

And another important aspect is, what is the direction of this vector? See, when you take a surface, the surface may have outward normal like this, but stress vector can be at any angle; it need not coincide with an outward normal direction. And at a point of interest, you will have infinite planes passing through it; one of the planes may be a free surface.

Suppose you are taking a point on a boundary of the specimen and it happens to be a free surface, you would be able to identify a plane that is defined as free, but you may have stresses on all other possible planes. So, in all other possible planes, you will have a direction of the stress vector. And what you have? Another important result is - the stress vector direction on a free surface can at best be tangential to the surface; this is another property. This you can go and verify.

I am going to take up one example problem which all of you know. We would qualify all these statements because this understanding is very fundamental and it is important. And another statement is stress vector cannot cross the free boundary. So, if I have a free boundary, if I find in any one of the planes passing through the point of interest, the stress vector, that direction cannot cross a free boundary. See, if you have looked at the development of shear, you would have learnt shear cannot cross a free body. That is the reason why you have on the top and bottom surfaces of the beam under bending, shear is 0. So, shear cannot cross a free boundary is only a particular case of a generic result; that is, stress vector cannot cross the free boundary.

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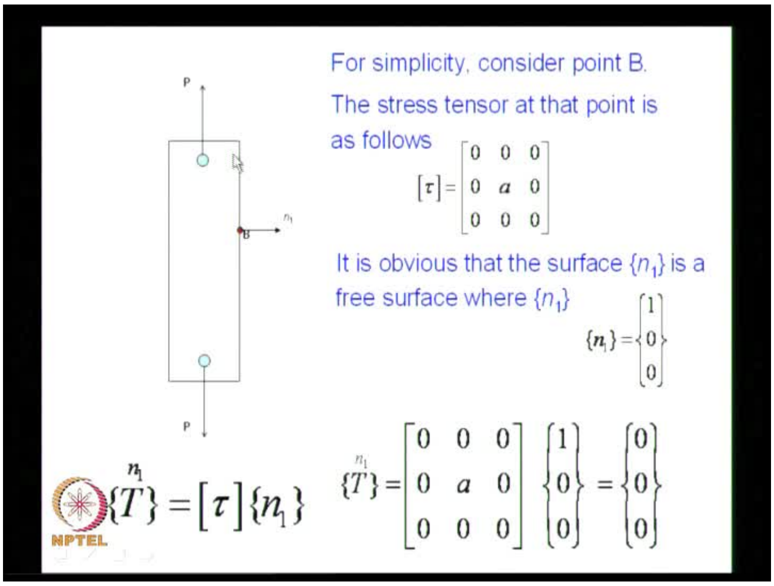

Now, what we will do is, we will take up a simple problem of a tension strip. I have taken (( )) to show the loading like this. I have not shown this as a uniform loading because I want to draw your attention that this is a free outward corner and this is where I have applied the load. And what happens to this surface (Refer Slide Time: 43:36)? This is a free surface, this is also a free surface, because I have applied load only here. I have put a pin here and I am pulling a load. So, at distances away, because of sign may not principle you will have the uniform stress solution.

Now, you take a point B somewhere on the free surface and you also imagine that I have x axis as horizontal and y axis as vertical. And first of all, we have to find out what is the stress tensor. And when I want to write the stress tensor, I will fill in all the nine components; it is a good practice to do that; it is a good practice to write the stress tensor in all its completeness and since I have considered this as a y direction, I would have some magnitude. If I know the cross section, I can find out that the stress magnitude in the y direction, sigma y is a.

No other stress component exists, because the problem is very simple one and this is again a close form solution. This is the case at every point in the domain, away from the points of loading because near the points of loading, there will be deviations. You could obtain that only from theory of elasticity, if at all the problem is solvable; otherwise, you have to go for a numerical and experimental analysis.

Now, I know the stress tensor and you also know that this surface is a free surface. So, free surface is defined by outward normal and that normal also we can find out. That is given as n 1 and the direction cosines are very simple to write. The direction cosine is nothing but 1 0 0. Now, I can find out what is the stress vector on plane n 1 because I define plane n 1 as the free surface.

So, I have the calculate T n 1, and when I do this, T n 1 goes to 0. So, we have indirectly proved, the mathematical definition of a free surface is T n equal to 0. In this particular case, the plane is n 1; so, T n 1 equal to 0. So, that is what you get from your Cauchy's formula and in an expanded form this is done. This you could go and verify you could also go and verify on this surface that it is a free surface; you could go and verify. And we would see another interesting problem; I have taken a corner here.

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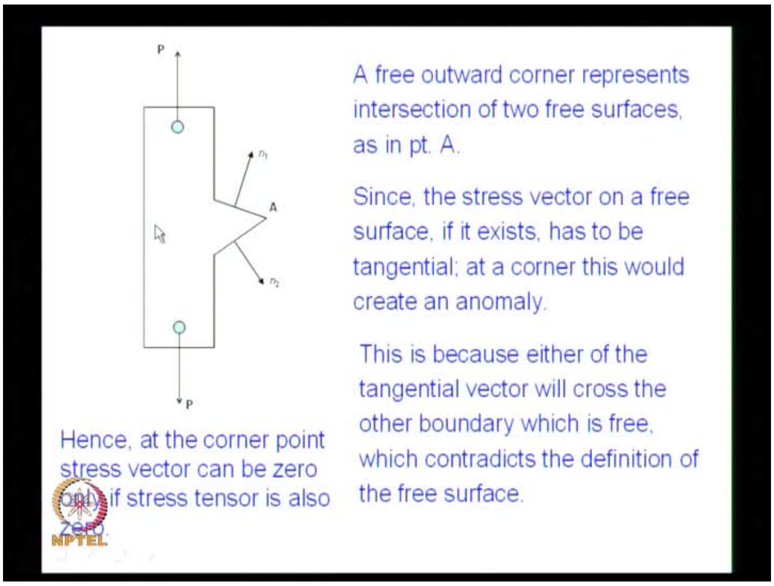

I will show another interesting picture. I have a protrusion that is out of the specimen. See, without performing a stress analysis, I want you to tell me, what is the value of stress at the corner. You can only make guess work. See, you will not be able to convincingly say, this is so. What I am going to do is, if I know what happens to the stress vector, if I understand what is a free surface, if I also understand what is the direction of stress vector on all the possible planes passing through the point of interest, I would be able to answer this question convincingly.

So, what you have here is, I have to look at what happens at point A, and mind you, this is a very sharp corner. It is a very sharp corner; there is no curvature there; that is very important. If there is curvature, the discussion is not valid. So, what you find here is, this is a free surface defined by outward normally n 1; this is a free surface defined by outward normal n 2 (Refer Slide Time: 48:07).

Suppose I have on this point A, if I consider point A forming the surface n 1, the stress vector would be along this direction. Suppose I consider point A belonging to surface n 2, the stress vector would be on this direction. So, this leads to a contradiction. How the contradiction can be resolved? The contradiction can be resolved only when stress tensor is 0. So, at the corner point, stress vector can be 0, only if stress tensor is also 0. So, you get a very important result. See, without performing stress analysis, you can go and say - now this is an outward corner. The same discussion you can do for corner 1, corner 2, corner 3, corner 4 - all of these corners' stress tensor is also 0. Stress vector is 0, stress tensor is also 0. And it is a very useful property when you do an experiment by photo elasticity because we would start the zeroth fringe order from there.

Now, if you look at, I have the instead of the protrusion, if I have a material removed here like a v groove, then this corner becomes singular. You will have very high stresses; they are called reentrant corners. When you are doing a course on fracture mechanics, you should also know about reentrant corners. In fact William's Eigenvalue solution was meant for that kind of a v groove when it is protruding out; when it is having a free surface, it is 0; when it is inside is totally different.

In this class, we have looked at the further concepts in energy release rate. We have been able to show convincingly, how to explain the phenomenon of pop-in that is observed in intermediate thick plates. Then we moved on to performing an experiment which showed among the multiple cracks, it is the critical crack that we have to pay attention; length of the crack is not the definition of identifying its criticality; its configuration is also very important. Then, we took up review of strength of materials followed by what is a definition of a free surface and its utility for getting quick results in certain situations.

Thank you.