

Engineering Fracture Mechanics
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Module No. # 02

Lecture No. # 11

Utility of Energy Release Rate

Let us continue our discussion on Energy Release Rate. It is a very useful concept that we have primarily developed it for a brittle solid. In fact, in this class, we would extend this concept for high strength ductile alloys, but before we go into that, let us examine a few aspects of what we have discussed as energy release rate and how we have used it for understanding certain phenomena. The first and foremost is, if you actually look at, the fracture strength that we have calculated was for a central crack in an infinite plate.

While we want to answer the question of a size effect, we looked at the experiment on glass fibers and we used the result developed for the infinite plate, applied to glass and then estimated what is the minimum crack size that was causing the problem of bulk glass having a very low strength. The thicknesses that you come across in glass fiber are far different than what you have developed as a solution. It was actually for infinite plate with a central crack. We have accepted that kind of an approach because the results, whatever that we have got, were in tune with the kind of developments that we have seen.

So, the approximation is reasonably alright because you are really looking at order of magnitude than exact values, but people also have raised certain other objections on the development of the Griffith's theory. Let us look at and find out what people have arrived out of it because when you are looking at a new phenomenon, you will have to do a mathematical modeling, and mathematical modeling has to capture the essential features. We will not be able to capture all aspects of it because if you try it to capture all aspects, then the whole mathematical analysis would become extremely complex. So, we need to make certain engineering approximations. Some of these were questioned by people.

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Difficulties in Griffith Theory

Goodier in 1968 has pointed out that Griffith has neglected the stresses due to the surface tension while calculating the strain energy in the plate.

Existence of surface energy implies the existence of surface tension.

One has to include in the boundary value problem formulation, a normal traction due to surface tension.

In fact such an exercise was carried out by Rajapakse in 1975 who finally concluded that the contribution of the surface energy term is negligible in comparison to the applied stresses.

And let us see, what are the difficulties in Griffith theory? Goodier in 1968 has pointed out that Griffith has neglected the stresses due to the surface tension while calculating the strain energy in the plate.


See, the whole of Griffith theory hinges upon recognizing, you need energy for the formation of surfaces and we have also looked at the surface energy was very, very small, but if you look at from a mathematical point of view, once you say there is surface energy, we have looked at surface energies very similar to surface tension. So, you have tension on the surfaces. So, this has to be accounted for the analysis. It may be very small; whether it is relevant or not, we will see, but this is a very valid objection and that is what is written here - existence of surface energy implies the existence of surface tension. And this has to be included in the boundary value problem. And this would appear as a normal traction due to surface tension.

In fact people have tried to accommodate this and did their kind of exercise, and it was done by Rajapakse in 1975. He finally concluded that the contribution of the surface energy term is negligible in comparison to the applied stresses. So, it is fair enough. You find that there is an objection raised, and finally, the conclusion is, it is not really affecting much; let us carry forward.

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Relevance of Griffith Theory

Kanninen in 1985 has extended the work of Rajapakse and determined the fracture strength with improved specification of the boundary value problem and the fracture strength is found to be

$$\sigma_f = 0.52 \left(\frac{E\gamma_s}{a} \right)^{\frac{1}{2}} \quad \text{Plane Stress}$$
$$\sigma_f = \left(\frac{2 E\gamma_s}{\pi a} \right)^{\frac{1}{2}} = 0.80 \left(\frac{E\gamma_s}{a} \right)^{\frac{1}{2}}$$


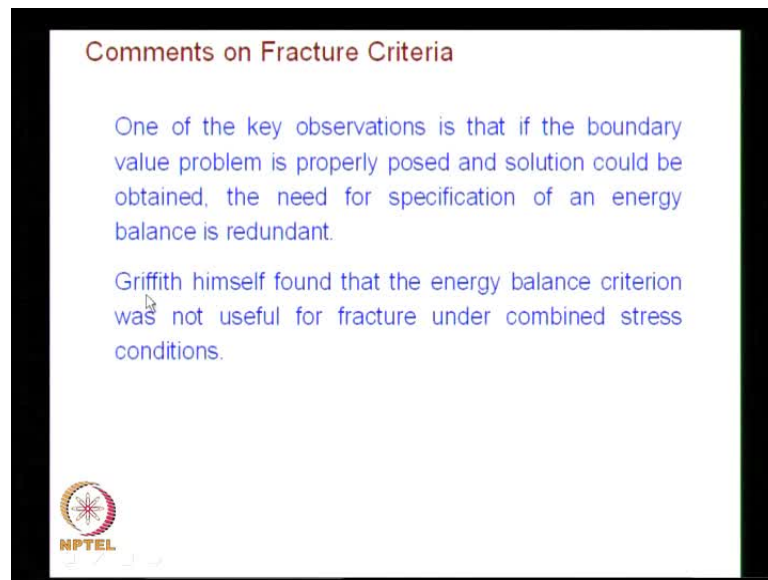
And later Kanninen in 1985 had extended the work of Rajapakse and determined the fracture strength. See Rajapakse stopped at some stage. We have actually calculated the fracture strength to convince our self why the theoretical strength is different from the fracture strength. We were trying to find out a reason. The reason was presence of inherent flaws. So, it is prudent to check by the new formulation, how does the fracture strength look like? Is there very great difference between the two? And this was actually done by Kanninen and he found the fracture strength expression to be of this magnitude sigma f equal to 0.52 into E into gamma s divided by a whole power half.

In fact, this is cast in a slightly different fashion than the way we have a calculated the fracture strength. You would also look at again how this could be recast. We have looked at the expression as 2 by pi E gamma s divided by a whole power half. And if you take out this 2 by pi, the value is 0.80. So, that means it is very close to this 0.52; it is not the way of the order of magnitude is similar. So, neglecting the normal traction due to surface tension is permissible because we are looking at a very complex problem.

And in fact, if you go and look at your hoop stress problem, you are actually having p R by t and p R by 2 t as the hoop stress and the axial stress, and if you go and try to satisfy the equilibrium condition in polar coordinates. It will not be satisfied unless you consider sigma R which is very small because it is of the order of the pressure p. So, the point that is raised here is a discussion something similar to that. It is desirable that you

incorporate, but if you incorporate, it does not make much of a difference, and that is comfortable for us, but it also threw up another important observation.

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The other observation was if the boundary value problem is properly posed and solution could be obtained, the need for specification of an energy balance is redundant. In fact, you could have solved the problem in all its completeness, if we have posed the boundary value problem properly. Because by bringing an energy balance, you are looking at a global picture rather than a comprehensive picture.

In fact, when you are learning a course in strength of materials, you would not have come across discussions like this. What we would do in the class is we would develop the theoretical method and then we would say, we have got the result, and carry on with applications. We never go back and investigate whether the theoretical development was comprehensive enough for the problem on hand. Such a discussion was not there in earlier courses. In fact, this kind of a discussion, we continue to do this in a course in fracture mechanics because the phenomena what you are looking at is very complex.

In fact, when I develop the stress field equations using Westergaard stress function, we would specify boundary conditions. You would also get the stress field equation which is seen in every book, but we will again go back and investigate whether the boundary conditions were properly handled. So, there is something very unique to fracture

mechanics. So, you have to learn to live with that. And what is the consequence of this kind of a statement is that specification of energy balance is redundant. That means, some aspect of the problem was not fully captured by the energy balance and it will hit some aspect of the fracture theory. And what you find is, Griffith himself found that the energy balance criterion was not useful for fracture under combined stress conditions.

See, when you develop the theory, you develop it individually for mode 1, mode 2, and mode 3. But in reality, you may find the combination of these at various proportions depending on the problem on hand. So, when you developed a fracture theory, it must help you to solve combined modes of loading also. So, this kind of a restriction was noticed even by Griffith, but one of the major objection against Griffith theory was, it was primarily meant for brittle materials; the other aspects what we discussed were more of nuances. These are like subtle aspects; nevertheless, energy release rate is a very, very useful and convenient concept. In fact, in one of the classes I raised - when you complete this course, what are the questions that fracture mechanics need to answer? Some of those questions, we will try to answer to them if time permits.

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ENGINEERING FRACTURE MECHANICS Energy Release Rate €

Necessary and Sufficient Conditions for Fracture

$G_I = R_I \rightarrow$ Necessary Condition ①

For a brittle material Eq.(1) is also a sufficient condition.

Fracture instability occurs when

$\frac{\partial G}{\partial a} = \frac{\partial R}{\partial a} \rightarrow$ Sufficient Condition ②

Fracture of high strength ductile material is possible only when both the conditions are satisfied.

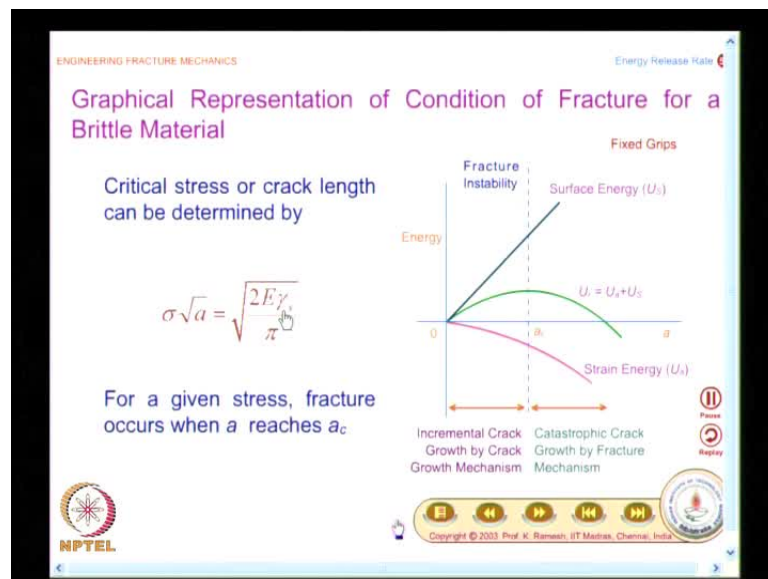
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Now, we look at the utility of Griffith's analysis and we rise up the question - what are the necessary and sufficient conditions for fracture? We have all along developed what is energy release rate and we have also said, there is resistant inherent in the material and the point of instability occurs when this energy release rate is equal to resistance. And

this equation is written for a mode IGI equal to R1. You could write similar ones for other modes of loading as well. Whenever you develop a mathematical concept, you have to realize - when you state a condition, you have to investigate whether the condition is a necessary condition as well as a sufficient condition; see, this is very important; just by arriving at a condition, the problem statement is not complete; you have to investigate that.

And if you really look at the case of brittle material, it so happens that whatever that we have got is also a sufficient condition, but this is not so when we extend the Griffith's analysis for high strength ductile material. You will also have to look at another condition, which is $\frac{dG}{da} = \frac{dR}{da}$. This slope also should match. So, this specifies the sufficient condition. And this, we would look at why this is so when we modify the Griffith theory for high strength ductile material, but before we get into this, we would look up on this identity (Refer Slide Time: 13:06) and also try to find out for what class of questions you would be able to get answers.

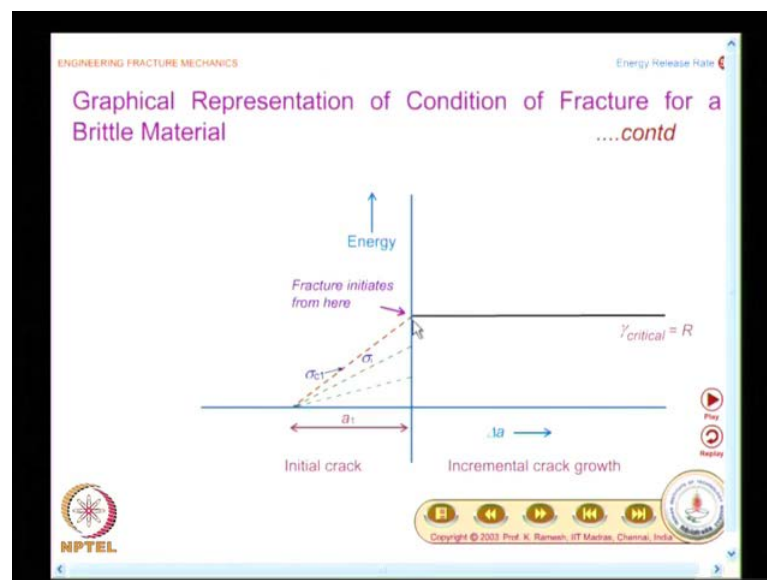
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We have really looked at a graphical representation earlier. We will look that again. We have looked at in terms of individually plotting surface energy, strain energy, and the total energy is also drawn. Then we say that you had you can identify a critical crack length and we say fracture instability occurs.

And this was useful when you are really focusing between what is this surface energy and when you are also calculating the theoretical strength. That was this kind of looking at the expression in terms of surface energy, helped to answer the questions related to theoretical strength. That was also developed in terms of surface energy, but if you want to generalize it, it is rather, wise to look at in terms of G and R . R is supposed to be inherent property of the material, and for the case of brittle material, R is linked to surface energy. When you say, the surface energy, you tend to have that as constant and we will develop another kind of a graphical representation. You have to look at how these axes are labeled.

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On the positive x axis, you have incremental crack growth. On the negative x axis, you have the initial crack. This is something different. This kind of graphs you would not have used it in other disciplines. When you come to fracture, you have different types of graphs are being presented; on the y axis, energy is plotted.

I would like you to make a neat sketch of this and focus on R ; do not pay more attention on $\gamma_{critical}$. We are not really investigating what is the actual value; you look at what is R ; R is a resistance, and in this case, we have taken this as a constant. So, this will be like a horizontal line and let us look at the problem of infinite plate with a central crack. We have looked at both the cases of constant load and constant displacement, and there was a difference. In the case of a constant load, the energy requirement for the

formation of two new surfaces came from the external load. Part of it went in increasing the strain energy of the system and part of it came for formation of two new surfaces. In the case of a constant displacement or fixed grips, it was coming from the strain energy. So, the strain energy of the system was decreasing as the crack was going and all our earlier discussion confined our attention only up to the point of instability.

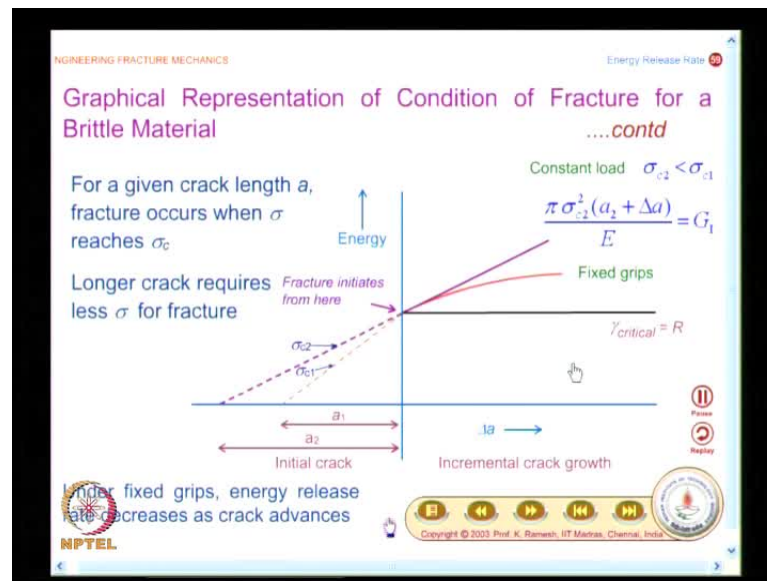
We have not looked at, after G equal to R what happens? We would look at a subtle difference in these two and what is the expression that we have got for an infinite plate? It was something like $\pi^2 \sigma^2 a^3$ divided by a . So, it is a linear function of crack length. So, I am going to have a linear curve and this depends on what is the value of stress. Suppose, I apply a stress which is sufficiently small when you plot that energy as a function of the crack length, it would only hit here (Refer Slide Time: 17:15).

So, whatever, the energy availability is far below the resistance; so, the crack would remain stationary; it would not grow. Now, I increase the stress. The stress magnitude is something like σ_1 ; still the stress is not critical. For a given crack length, all our earlier discussion have shown that you can find out a corresponding critical stress. This is where you find Griffith theory is useful at the backdrop of Inglis solution. Inglis solution alerted crack is dangerous; it also gave a very inconvenient answer; even very small crack, even for very small load, will have very high stress developed; if you really look at that, you will only have powders; you will not have any solid form.

It was only Griffith's analysis which categorically showed that, if there is something called a critical crack length or for a given crack length, there has to be a critical stress; we had looked at that kind of an expression. So, what is plotted here is, when the stress levels are below the critical stress, the energy graph would look like this. This is for energy release rate.

Now, let us increase the stress level. Now, I have the stress level as, taken as, σ_c ; the c denotes it is a critical stress. And what happens? For this critical stress, the value of G is equal to R . So, this indicates the point of instability. And we have looked at great length, the role of constant load as well as constant displacement, and said, in both the cases, the energy availability is same as long as the incremental values are small. So, both the cases of constant load and constant displacement would satisfy this, but beyond this, these two will be different. We will look at that also.

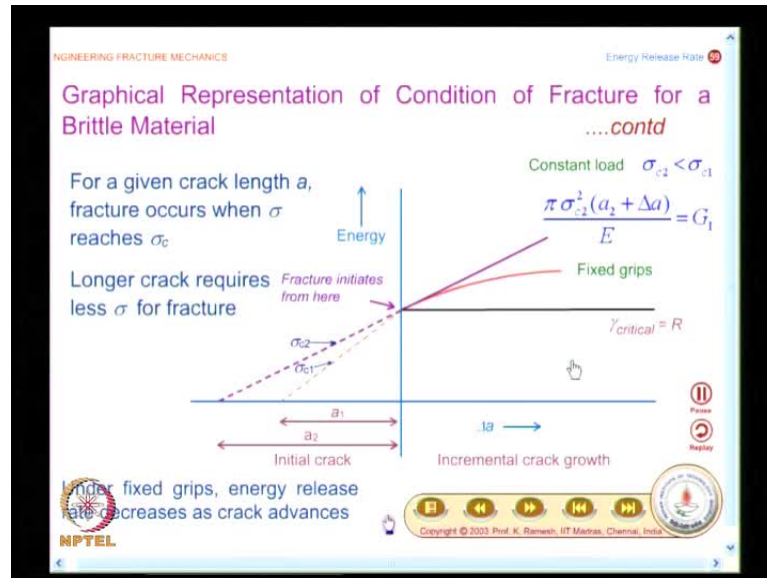
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And this is what you look at here. For the case of constant load, the energy availability will keep on increasing as the crack grows in length; as the crack grows, the energy will keep on increasing because the energy for the formation of crack surfaces was coming from external work done. And as the crack is advancing, you will have more and more work done by the external force and this would be available in the case of fixed grips; whatever the strain energy stored, that gets depleted. So, this will take a shape like this.

So, for all other future discussions, we might just keep this constant load and what it implies? Suppose I have more energy than what is required for the formation of crack surfaces, this would propel the crack at increasing velocities. So what would happen is the crack would initiate and it would acquire the velocity. People have studied all that using photo elastic experiments.

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Now, let us look at another case. Here, I have drawn the graph for initial crack length of a 1. Suppose, I have a longer crack, how I can use the graphical representation? And we have already noted, on the negative x axis I put the initial crack. So, I would start my graph from this point. If I have the crack length is this much; so, that is away I would start. So, let us start plotting the energy release rate from that.

So, what it shows? These two graphs are parallel. This itself shows the stress levels are same and at this stress level for a smaller crack, there was no fracture. For the same stress level, it becomes a critical stress for a longer crack. And after the fracture initiation, the graphs would be different for constant load and constant displacement. So, that is what you see here (Refer Slide Time: 22:38), and what is mentioned here is that the critical stress σ_{c2} is less than σ_{c1} and this appeals to our commonsense. You know, a longer crack would require a smaller stress for fracture to get initiated. We have also looked at that in the residual strength diagram and this is a new form of representing the energies involving the fracture processes. We are looking at the energy release rate and the resistance; all these discussions confine to brittle materials; that is the reason why we have kept this R as a constant.

We will also look at certain other issues. Now, in the case of a constant load, you have more energy availability. So, that means the crack would acquire velocity and whatever I

have mentioned, these are summarized as points here. We will again look it up and then go to the next slide.

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ENGINEERING FRACTURE MECHANICS

Energy Release Rate

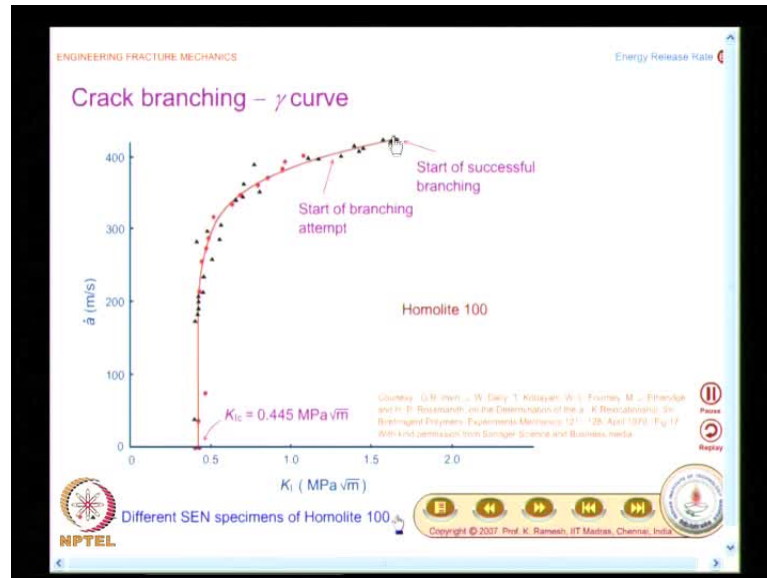
- For both constant load and fixed grips, the condition for onset of crack growth is the same.
- At the point of crack instability, the energy release rate is same in both the cases.
- In fixed grips, since external load does no work, the energy release rate decreases as the crack advances.
- For further discussion, let us consider the constant load case.
 - ★ Increase of energy availability leads to increase in crack velocity.
 - ★ Beyond a certain velocity crack branching can occur.

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For both constant load and fixed grips, the condition for onset of crack growth is the same. We have really convinced our self by looking at the graph. At the point of crack instability, the energy release rate is same in both the cases. As I pointed out, in fixed grips, since external load does no work, the energy release rate decreases as the crack advances.

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For further discussion, let us consider the constant load case. One of the key issues here is, increased energy availability leads to increase in crack velocity. And we have also noted earlier, cracks can attain only a particular maximum velocity in any material. That is dictated by the Raleigh wave speed in that material. Beyond that, cracks cannot travel faster; it will pick up speed, but the maximum speed would be limited by the Raleigh speed and we will again look at what was the example that we saw. And before that, we would also look at another interesting set of experiment on dynamic photo elasticity, and do not confuse this gamma to your surface energy.

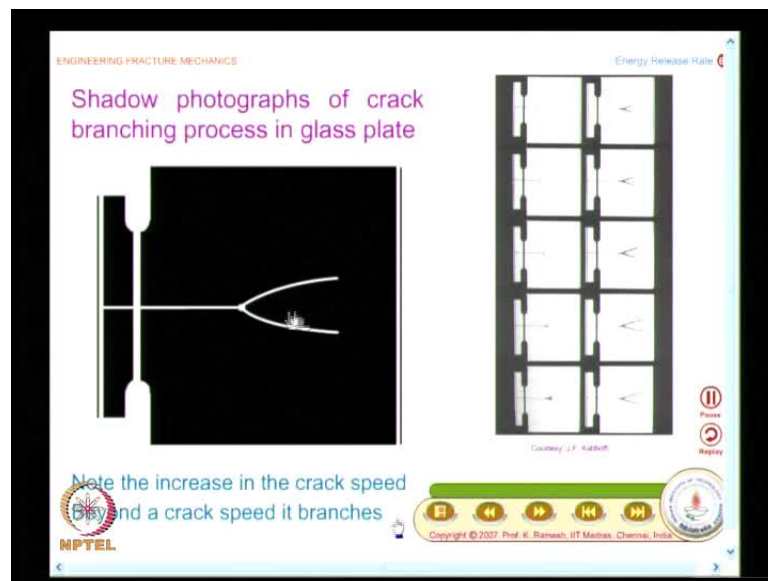
What this graph shows is - you have stress intensity factor on the x axis. On the y axis, crack speed is mentioned, a dot is mentioned, and you have a data from two different specimens. And what is done is - the crack is allowed to propagate. So, they were able to find out from the experimental measurement, both the value of K as well as the velocity. And when you plot those points as the function of K versus velocity, it takes a shape of this nature. This looks like gamma. So, do not confuse this gamma as surface energy because we have used gamma as surface energy, and in fracture literature, again the symbolism is sometimes confusing. See, for example, they use an unconventional capital B as the thickness; this we are not used to.

And the other aspect is, if you look at fatigue, the symbol R is used for the stress ratio. In fact, fatigue and fracture are closely interrelated; despite that, they have coined the

resistance as capital R. So, you will have to understand based on the context what does the symbol indicate? Because if you look at the symbol, out of context, you may interpret it differently; so, what is shown here is when you put those points, and it is a difficult experiment. You know you have to calculate the value of K if time permits as part of this course I would also tell you how to find out K from photo elastic data. So, from dynamic photo elastic experiments, they were able to get the measurement of velocity as well as K and what does this show?

The crack has to propagate when the value reaches critical value of K_{Ic} . Though it is the energy release rate chapter, we cannot avoid talking about stress intensity factor. The graph is represented in terms of stress intensity factor and you find the crack starts to branch at a particular value of K, and successful branching is seen after sometime.

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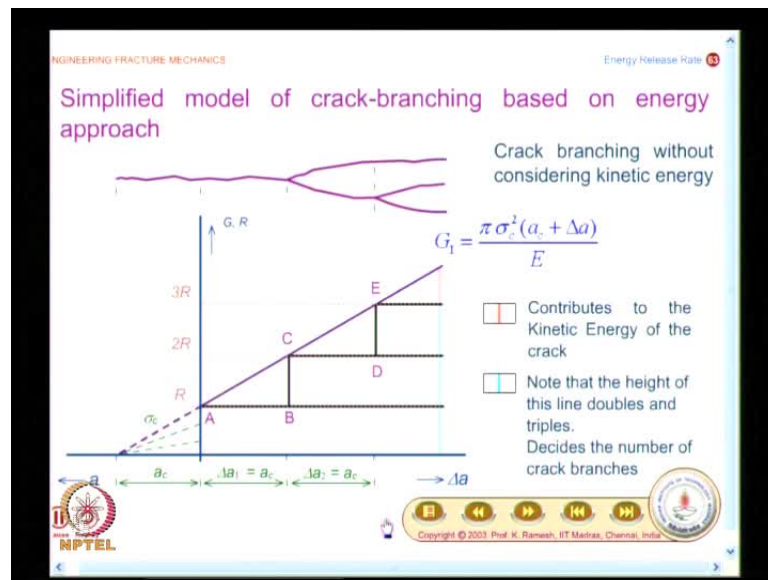


So, people have conducted experiments and verified that the crack can attain only a maximum velocity. Beyond a velocity, it has to branch out; so, this is observed. And we will again look at the experiment done by Kalthoff which was shown as the simulation here. The crack moves with certain velocity; the velocity increases; after that, crack branches off; this, we have seen this earlier.

Now, with our knowledge of whatever you have understood as energy release rate and resistance, is it possible to give a plausible explanation? We must also be able to find

out, are we in a position to explain the crack branching based on whatever the graphs that we have seen?

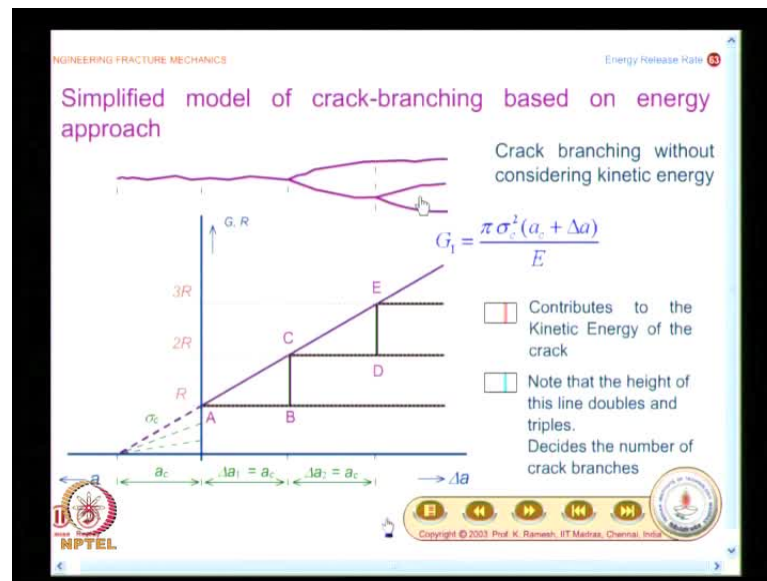
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Look at this animation. What I want to show is I have a crack, and on the y axis I have drawn a horizontal line corresponding to resistance R and 2 times R and 3 times R. Let the initial crack be a, and on the positive x axis, you plot delta a. So, the crack will initiate only when the stress levels are sufficiently high. So, what you will have to look at is, I would be showing the crack growth in this place and you would be seeing the energy availability here (Refer Slide Time: 29:52). So, you should see these two; then you would be able to understand quite well.

Now, what happens is, the stationary crack starts growing. That is what you see here. The crack was initially stationary. Until the stress was increased to sigma c, the crack will not initiate and this graph will go and hit this graph (Refer Slide Time: 30:25). When it becomes 2R, let us see what happens? So, what do you find beyond 2R? You find that crack branches off. You have two branches; each branch is providing a resistance R. And whatever the discussion that we do, it is very simplistic; we are not considering the role of kinetic energy; that is quite alright. I mean it gives you a conceptual appreciation why crack-branching is possible; how it is explainable from the concepts that we have already looked at, say, first approximation; it is reasonably good enough to convince our self that crack-branching can take place.

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Now, let us look, as the energy keeps on increasing, the crack is growing. So, beyond that, you find three cracks grow and you have this as $\pi \sigma_c^2 (a_c + \Delta a)$ divided by E . So, this gives you a possible explanation, why crack-branching is possible. So, what I will do now is I will put this animation; again, you just observe, the animation is illustrative. You look at what is the energy availability and what is the resistance, and parallelly look at how the crack grows, and it is schematic. You know you have to take it as a first order approximation in explaining crack branching.

So, until you reach a critical stress, the crack will not initiate. When the energy levels are sufficiently high, it branches out. When further energy is pumped, it branches for them, and mind you, you have to consider I have taken a really infinite plate with a small crack; the crack has not seen the other edge of it; all that happens; it is a fictitious experiment, but, gives you a reasonable appreciation that crack-branching is explainable on the basis of energy release rate and resistance.

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The slide is titled "Irwin-Orowan Extension of Griffith's Analysis" and is part of an "ENGINEERING FRACTURE MECHANICS" presentation. It contains three bullet points:

- In brittle materials, advancing cracks require small energies of the order of surface energies, and therefore, once a crack starts advancing, it runs through the body easily causing catastrophic failure.
- In most engineering materials (metals, plastics, etc..) energy much larger than the surface energy is required to grow a crack.
- Therefore, besides surface energy of the solids, some other mechanisms are operating which involve large amounts of energy.

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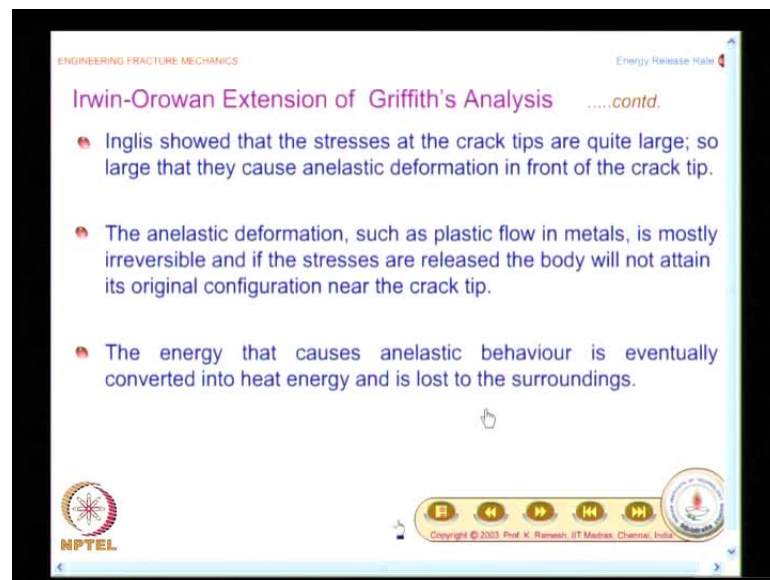
Now, let us look at what was the contribution of Irwin and Orowan. See, fracture mechanics has to be useful to society and we have many structures made of ductile materials. Unless I extend whatever the concepts developed to ductile materials, fracture mechanics will not be useful, and this was done by Irwin and Orowan. They have extended the Griffith's analysis and what is the difference in the case of brittle materials? Advancing cracks require small energies of the order of surface energies. So, once a crack starts, it advances through leading to catastrophic failure and this is not the way brittle material behaves.

See one of the key assumptions in the case of brittle material was R remained constant. We have already looked at, in one of the earlier classes, what is the value of surface energy and what is the value of energy due to plastic deformation. That was several orders of magnitude greater than the surface energy. So, that is what is mentioned here. So, in most engineering materials, energy much larger than the surface energy is required to grow a crack - this is one aspect.

Another aspect is, it does not remain constant; the resistance does not remain constant. And what people questioned was, now we know fracture mechanics; so, I directly jump on to say that it is energy because of plastic deformation; while thinking in that direction, they argued, besides surface energy of the solids, some other mechanisms are operating which involve large amounts of energy. What is that physical phenomena in the case of

ductile materials? It is the plastic deformation, and the moment you come to plastic deformation, what happens? It is irreversible; energy is lost; it is not like a reversible system. Whole of Griffith theory, he developed this energy balance based on the premise that the system is reversible.

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ENGINEERING FRACTURE MECHANICS Energy Release Rate

Irwin-Orowan Extension of Griffith's Analysiscontd.

- Inglis showed that the stresses at the crack tips are quite large; so large that they cause anelastic deformation in front of the crack tip.
- The anelastic deformation, such as plastic flow in metals, is mostly irreversible and if the stresses are released the body will not attain its original configuration near the crack tip.
- The energy that causes anelastic behaviour is eventually converted into heat energy and is lost to the surroundings.

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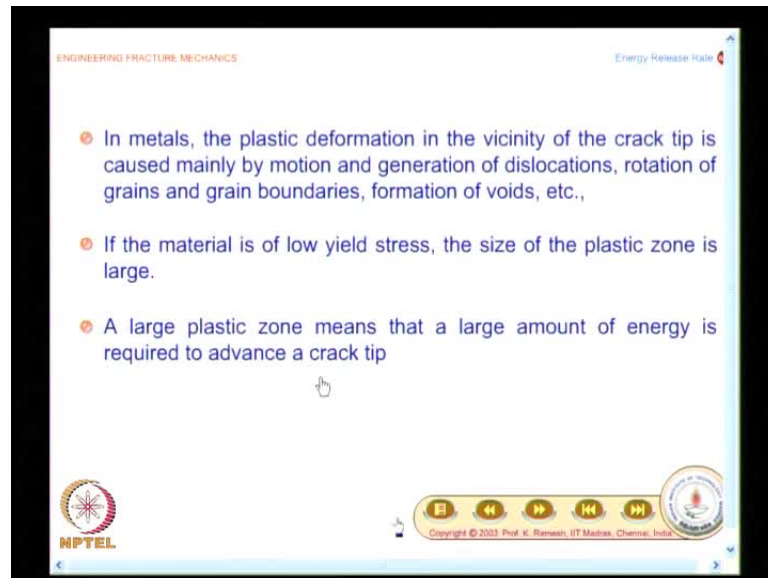
So, now, we have to make a very important engineering step for us to apply fracture mechanics to ductile solids. It was done by a very simple modification. And while justifying their modification, they again bring in the contribution by Inglis. Inglis showed that the stresses at the crack tips are quite large; so large that they cause an elastic deformation in front of the crack tip.

See, you have to look at the statement from the context of the time while it was developed. People found ductile structures failing in a brittle fashion in practice. So, for them, they have to re convince - yes, there is plastic deformation near the crack tip; whereas, the structure as the whole still remains in elastic state; so, in order to convince that, they have to bring in the analysis of Inglis, and then show that you will have defiantly an elastic deformation at crack tip.

And as I mention, it is written here that the an elastic deformation is irreversible. So, when the stresses are released, the body will not attain its original configuration near the

crack tip. And what happens? The energy that causes an elastic behavior is eventually converted into heat energy and is lost to the surroundings; that is quite alright.

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Not with standing all this, what Irwin suggested was, he simply said, anyway that is not seen in this slide; anyway I will find out what are the other aspects. So, in the case of metals, the plastic deformation in the vicinity of the crack tip is caused mainly by motion and generation of dislocations, rotation of grains and grain boundaries, formation of voids, etcetera. If the material is of low yield stress, the size of the plastic zone is large. A large plastic zone means that a large amount of energy is required to advance a crack tip.

So, these are all re convincing statements. For them to formulate the methodology, they have to convince that there is large value of plastic deformation, and he made a very simple modification to Griffith's analysis. It is a very intelligent step.

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ENGINEERING FRACTURE MECHANICS

Energy Release Rate

The overall surface energy can be written as

$$\gamma = \gamma_s + \gamma_p$$

Where, γ_s – surface energy possessed by surface even if it has not been subjected to any plastic deformation.

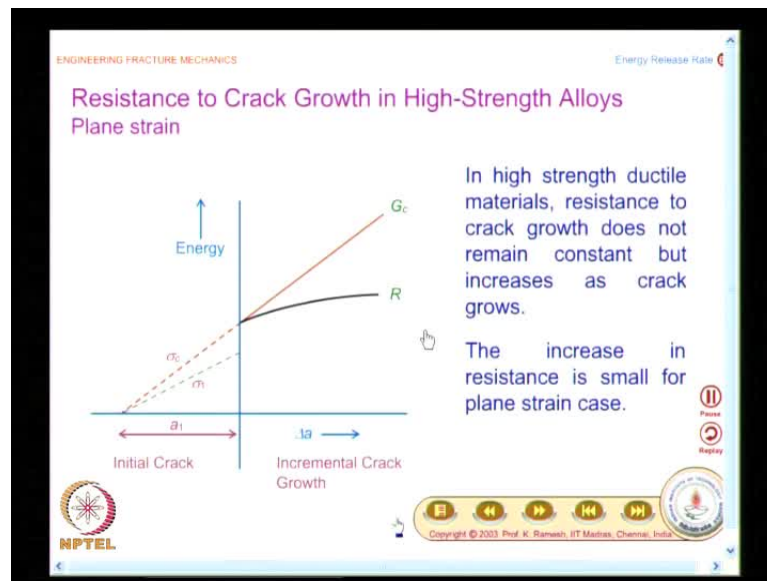
γ_p – surface energy caused by the plastic deformation near the cracked surfaces.

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So, what he did was, he said, instead of just writing gamma s, take the sum of gamma s and gamma p, and whatever you have discussed as energy release rate, we will still use it; absolutely no problem, but the resistance behavior is different. Resistance behavior changes as the crack advances; not only that, you need large amount of energy for the crack even to initiate. And this appeals when you compare it with actual experimentation; in the case of ductile materials, you need to apply sufficient amount of stresses for the crack to grow. So, he has really tweaked the concept of resistance; a resistance he had simply modified it as gamma s plus gamma p.

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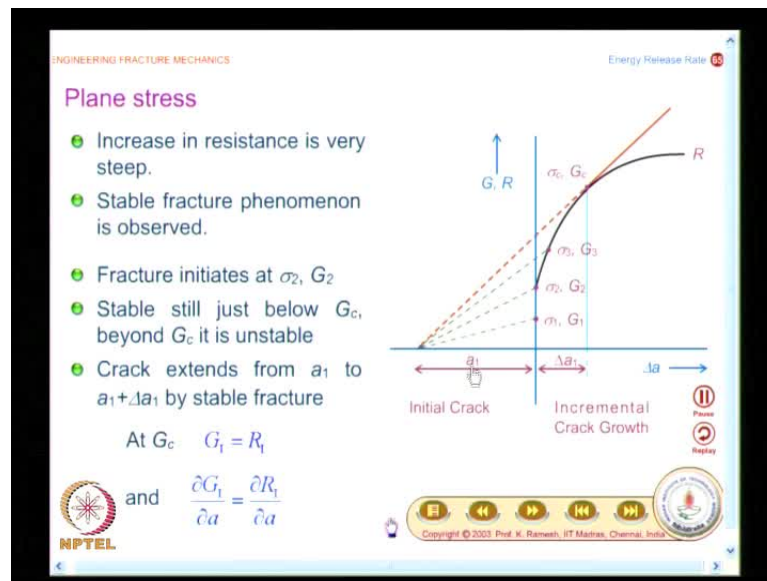


And we will look at, how does the resistance look like in a graph and here you will have to classify this for a plane stress case separately and plane strain case separately. This is again a similar graph that we had drawn for a brittle material. On the x axis, you put the delta a, incremental crack growth; on the negative x axis, put the initial crack on the y axis; you have the energy and you look at what way the resistance is depicted.

In the case of high strength ductile materials, resistance to crack growth does not remain constant, but increases as crack grows. But this increase is reasonably small in the case of a plane strain case. We will complete the picture and you would see, totally a different type of picture when you look for plane stress case. I have already mentioned, in the case of plane stress problems, people have observed stable fracture followed by unstable fracture.

So, now we are moving from constant R to variable R, and people call it as R curve, and people also devise methodologies to establish R curve for different materials. And now with the R curve concept, it is possible for us to explain the phenomenon of stable fracture. If I am able to explain that and you will get convinced, then what we are doing is the right direction. See, without getting into much mathematics, we are in a position to understand certain key concepts in fracture mechanics and we have a via media now. In the case of a plane strain case the R curve looks like this; in the case of a plane stress case, how does it look like.

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Increase in resistance is very steep. See again, these are all schematic figures; these are not drawn to scale. You have a shape something like this. R increases rapidly and I would like you to make a neat sketch of this. And we also look at what is the value of sigma 1 what is the value of G1, and the graph is like this. By looking at this graph, you would really appreciate the necessary and sufficient conditions for fracture. We have stated that earlier, but we are not understood it completely.

When you look at an R curve, like this that provides you all the necessary explanation. We will spend some time on this graph and understand what it tries to tell you. Look at the animation. I keep increasing the stress and when sigma 1 reaches sigma 2, and the corresponding energy release rate is G2, fracture initiates at sigma 2. In the case of brittle materials, what happened? Once the crack has initiated, it will keep propagating more or less in the **case of** plain strain case also, the R curve is shallow; there again crack propagation is much simpler.

The moment you come to plane stress case, the R curve is lot more steeper. You have the energy release rate equal to R here; that is fine. When I increase the stress, what happens? There again, you find energy release rate equal to R. So, from this position to this position, if you look at the incremental crack growth, it is fracture, but the fracture is stable. That means if I remove the load, the crack would have grown and stopped; it would not have progressed beyond that. When I keep on increasing the load, crack also

will increase; crack will not become a catastrophic failure; it will grow incrementally until a stage, when I have shown this as a brown line where σ_c and G_c are shown where the curve is tangential to the R curve. So, what does this show? At this point $G = R$ is satisfied, but beyond this point, you will find... Because the curve is tangential, you will find the energy availability is much more than resistance. So, this explains the possibility of stable fracture followed by unstable crack growth in the case plane stress situation. So, that means we are on right track.

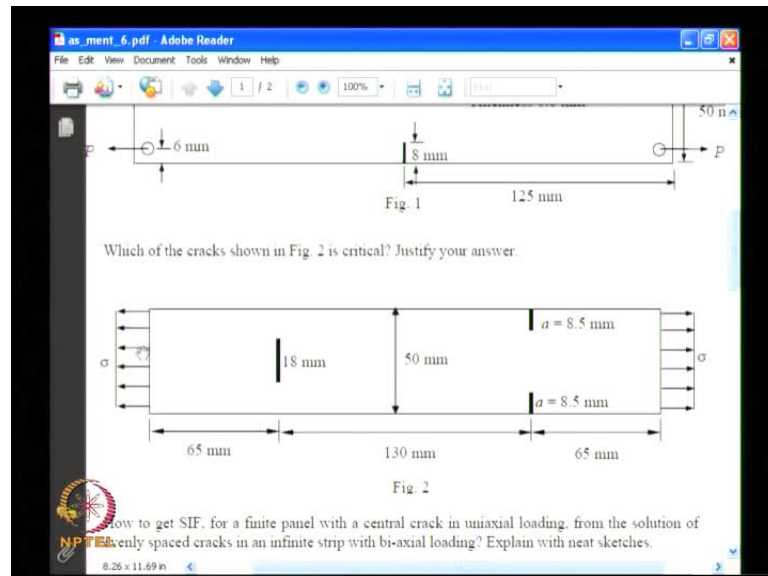
What Irwin did was he made only a simple step of changing γ_s to γ_s plus γ_p , and γ_p he justified why there is local plastic deformation. You have to look at the time he developed because the structure as the whole remind elastic, but near the crack alone, you will have plastic deformation he needs to convince. After doing that, you find you are able to develop the concept of R curve which provided explanations for difficult phenomena. So, look at this graph, I have an original crack a_1 which had two phases; there is a stable crack growth phase Δa_1 ; that means what if I increase from σ_2 the σ_3 , then σ_c , until or I reach σ_c , the crack would not become catastrophic, but you know actual practice; stable fracture is immediately followed by unstable fracture. Do not think that it is like a crack growth mechanism. We are not discussing crack growth mechanism. We are discussing only fracture, but we are trying to provide an explanation, fracture is possible; stable fracture is possible and your R curve concept gives you that comfort.

And what I have here is another important issue. When you say, when fracture occurs at σ_c , what is the corresponding critical crack length? Is it a_1 or $a_1 + \Delta a_1$? People would simply jump to $a_1 + \Delta a_1$; it is wrong. If I have σ_c , a crack of length a_1 would really become unstable; you would not even notice Δa_1 and this has been one of the issues in fracture mechanics. When they were initially developing the subject, they were reporting wrong lengths of crack lengths. When I have a stable fracture, I will have to go by what was the initial crack.

So, what we will do is we will further develop these concepts in the next class, but before I go into that, I want to show you an assignment problem, which I would like to bring it as a specimen to the next class. That would really clarify, if I have multiple cracks, what is it that I have to worry for, which crack I have worry for, and it is a very interesting experiment.

What you will do is, you will do the experiment. In the next class, after I develop crack tip and stress field equations, you would also be in a position to calculate the stress intensity factor and convince yourself, the experiment was indeed helpful in understanding which crack is more dangerous and why we consider only one crack for all our analysis. How this is justified? We will look at the problem here.

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Now, I have a problem for you to look at this is from your assignment sheet; and take a note of this picture. This picture has central crack of 18 millimeter and edge crack of 8.5 millimeter, here another edge crack of 8.5 millimeter here, and width the plate is 50 millimeter.

So, take a sheet of paper and make this specimen and come with this specimen in the next class. See, if you look at these dimensions, these dimensions are carefully chosen so that the cracks are far away; not only this, they are also away from the points of loading. And what you will have to do is you will have to provide an end loop so that you can insert a cylindrical pencil in that and you are going to do it by hand. So, when you are doing the experiment by hand, we will try to maintain as much as possible, there is uniform stress on the specimen

So, if your results are deviating, it may be because you are not in a position to apply uniform stress. So, what I would like is, I would like you to bring three specimens each

because whenever you do an experiment, you must make at least three samples. And I also want you to do one more thing here. I have shown a crack length of 8.5 millimeter; I want you to bring another specimen with 9 millimeter on either side; another specimen with 9.5 millimeter on either side.

So, you will have three different specimens and on each of them you provide three samples, and what you do is, you take a longer sheet of paper; you ensure a loop pasted properly, make sure that you are able to insert a cylindrical pencil here as well as the cylindrical pencil here, and we would indeed break that specimen in the class, and find out of the cracks I have shown, which crack do I have to worry about because one of the fundamental things in fracture mechanics is, we consider presence of inherent flaws. When you say inherent flaw, there may be many, but for all our theoretical development, we just use only one crack. We would get an understanding by performing that kind of an experiment.

Thank you.