

Advanced Dynamics
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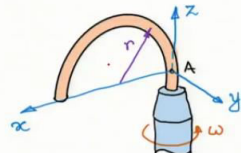
Lecture - 37
Spatial Kinetics of Rigid Bodies - III

We will continue our discussion on spatial kinetics of rigid bodies, and look at some further problems. We consider the following problem.


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Problem 1:

Determine the bending moment M at the tangency point A in the semi-circular rod of radius r and mass m as it rotates about the tangent axis with a constant and large angular velocity ω . Neglect the moment mgr produced by the weight of the rod.



Source: Dynamics, Meriam and Kraige



The detailed solution is presented in the following 2 slides.

(Refer Slide Time: 01:40)

Coordinate frame $x-y-z$

$$I_A = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} \quad \vec{\omega} = \omega \hat{k} = \begin{Bmatrix} 0 \\ 0 \\ \omega \end{Bmatrix}$$

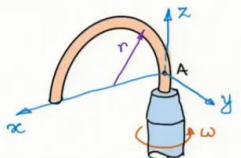
$$\vec{H}_A = -I_{xz} \omega \hat{i} + I_{zz} \omega \hat{k}$$

$$I_{xz} = \int xz \, dm = \int_0^\pi r(1-\cos\theta) r \sin\theta \, \rho r d\theta \quad \left(\rho = \frac{m}{\pi r} \right)$$


$$= 2 \rho r^3 = \frac{2}{\pi} m r^2$$

$$I_{zz} = \int (x^2 + y^2) \, dm = \int_0^\pi r^2 (1-\cos\theta)^2 \rho r d\theta$$

$$= \frac{3}{2} \pi \rho r^3 = \frac{3}{2} m r^2$$



$$\left. \begin{aligned} I_{xz} &= \frac{2}{\pi} m r^2 \\ I_{zz} &= \frac{3}{2} m r^2 \end{aligned} \right\} \vec{H}_A = -\frac{2}{\pi} m r^2 \omega \hat{i} + \frac{3}{2} m r^2 \omega \hat{k}$$



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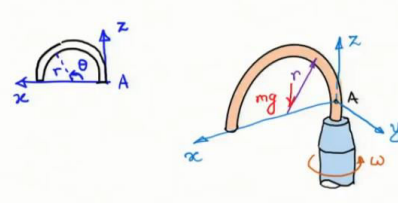

Coordinate frame $x-y-z$

$$\vec{H}_A = -\frac{2}{\pi} m r^2 \omega \hat{i} + \frac{3}{2} m r^2 \omega \hat{k}$$

$$\vec{\omega} = \omega \hat{k}$$

Moment on the semi-circular rod

$$\vec{M}_A = \dot{\vec{H}}_A = \frac{\partial \vec{H}_A}{\partial t} + \vec{\omega} \times \vec{H}_A$$

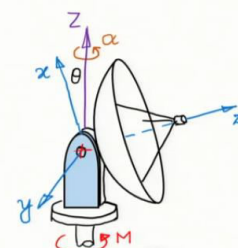
$$= -\frac{2}{\pi} m r^2 \omega^2 \hat{j} \quad (\text{Neglecting gravity moment})$$



Next, we consider the following problem.


(Refer Slide Time: 09:58)

Problem 2:

A large balanced antenna has a moment of inertia I about its z -axis of symmetry and a moment of inertia I_0 about each of x and y axes. Determine the angular acceleration α of the antenna about the vertical Z -axis caused by a torque M applied about Z by the drive mechanism at a given orientation θ .



Source: Dynamics, Meriam and Kraige



The solution of the above problem is presented in detail in the following slide.

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Coordinate frame x-y-z

$$\dot{\vec{H}}_0 = \vec{M}_0 \quad \vec{H}_0 = [I_0] \vec{\omega}$$

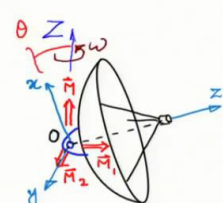

$$[I_0] = \begin{bmatrix} I_0 & 0 & 0 \\ 0 & I_0 & 0 \\ 0 & 0 & I \end{bmatrix} \quad \vec{\omega} = \omega (\cos\theta \hat{i} + \sin\theta \hat{k}) = \vec{\Omega}_f$$

$$\Rightarrow \vec{H}_0 = I_0 \omega \cos\theta \hat{i} + I \omega \sin\theta \hat{k}$$

$$\dot{\vec{H}}_0 = \frac{\partial \vec{H}_0}{\partial t} + \vec{\Omega}_f \times \vec{H}_0$$

$$= I_0 \alpha \cos\theta \hat{i} + I \alpha \sin\theta \hat{k} + (I_0 - I) \omega^2 \cos\theta \sin\theta \hat{j} = M(\cos\theta \hat{i} + \sin\theta \hat{k}) + M_1(-\sin\theta \hat{i} + \cos\theta \hat{k}) + M_2 \hat{j} \quad (\text{balanced antenna})$$

$t = 0^+$

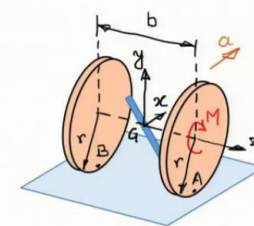
$$\begin{aligned} \hat{i} \Rightarrow I_0 \alpha \cos\theta &= M \cos\theta - M_1 \sin\theta & -(1) \\ \hat{j} \Rightarrow (I_0 - I) \omega^2 \cos\theta \sin\theta &= M_2 & -(2) \\ \hat{k} \Rightarrow I \alpha \sin\theta &= M \sin\theta + M_1 \cos\theta & -(3) \end{aligned} \quad \left. \begin{aligned} & \\ & \\ & \end{aligned} \right\} \begin{aligned} & (1) \times \cos\theta + (3) \times \sin\theta \\ & \Rightarrow \alpha = \frac{M}{I_0 \cos^2\theta + I \sin^2\theta} \end{aligned}$$



The next problem considered is presented below.


(Refer Slide Time: 20:09)

Problem 3:

Two identical circular discs of mass m are welded to the two ends of a rigid rod of mass m_0 at an angle such that the discs have a common z axis, and separated by a distance b as shown. A couple moment M , applied to one of the discs with the assembly initially at rest, gives the combined mass center G an acceleration a along the horizontal direction shown. Determine the normal reactions N_A and N_B on the discs from the ground in terms of the acceleration a .



Source: Dynamics, Meriam and Kraige



The next 2 slides present the solution in detail.

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Coordinate system $x-y-z$

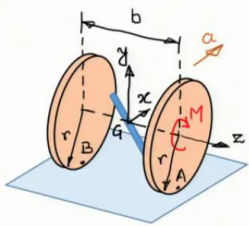
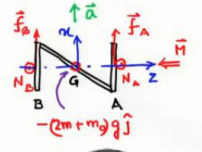

$\dot{\vec{H}}_G = \vec{M}_G$

$$[I_{G\hat{u}}] = \begin{bmatrix} 2m(\frac{r^2}{4} + \ell^2) & 0 & 0 \\ 0 & 2m(\frac{r^2}{4} + \ell^2) & 0 \\ 0 & 0 & mr^2 \end{bmatrix}$$

$$[I_{G\hat{u}}] = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} \quad \vec{\omega} = \omega \hat{k} = \vec{\Omega}_f$$

$$\vec{H}_G = [I_{G\hat{u}}] \vec{\omega} = -I_{xz} \omega \hat{i} + (I_{zz} + mr^2) \omega \hat{k}$$

$$\dot{\vec{H}}_G = -I_{xz} \alpha \hat{i} + (I_{zz} + mr^2) \alpha \hat{k} - I_{xz} \omega^2 \hat{j} \quad \leftarrow \vec{\Omega}_f \times \vec{H}_G$$

$$I_{xz} = \frac{1}{6} m_0 b r \quad I_{zz} = \frac{1}{3} m_0 r^2$$




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Coordinate system $x-y-z$

$\dot{\vec{H}}_G = \vec{M}_G$

$$\vec{H}_G = -I_{xz} \omega \hat{i} + (I_{zz} + mr^2) \omega \hat{k} \quad \left\{ \begin{array}{l} I_{xz} = \frac{1}{6} m_0 b r \\ I_{zz} = \frac{1}{3} m_0 r^2 \end{array} \right.$$

$$\dot{\vec{H}}_G = -I_{xz} \alpha \hat{i} + (I_{zz} + mr^2) \alpha \hat{k} - I_{xz} \omega^2 \hat{j} \quad (\alpha = \frac{a}{r})$$

$$= -M \hat{k} + r(f_A + f_B) \hat{k} + \frac{b}{2}(N_B - N_A) \hat{i} + \frac{b}{2}(f_A - f_B) \hat{j}$$

$$\Rightarrow N_B - N_A = \frac{2}{b}(-I_{xz} \alpha) = -\frac{1}{3} m_0 r \alpha = -\frac{1}{3} m_0 a \quad \text{--- (1)}$$

From translational dynamics (y-axis): $0 = N_A + N_B - (2m + m_0)g$

$$\Rightarrow N_B + N_A = (2m + m_0)g \quad \text{--- (2)}$$

From (1) and (2) $N_B = mg + \frac{m_0 g}{2} (1 - \frac{a}{3g})$ $N_A = mg + \frac{m_0 g}{2} (1 + \frac{a}{3g})$

