Advanced Dynamics
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Module No # 03
Lecture No # 14
Impulse- Momentum Relation – II

We will carry forward our discussion on impulse momentum relation.

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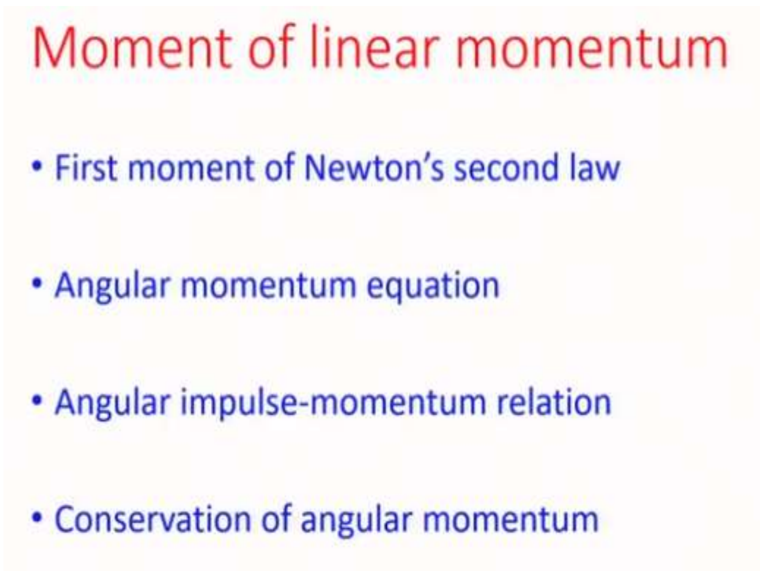

We are going to look at the first momentum of the Newton's second law and define a new quantity called the angular momentum. Then we will derive the angular momentum relation and see how angular impulse is related to the angular momentum. Finally, we will discuss the conservation of angular momentum.

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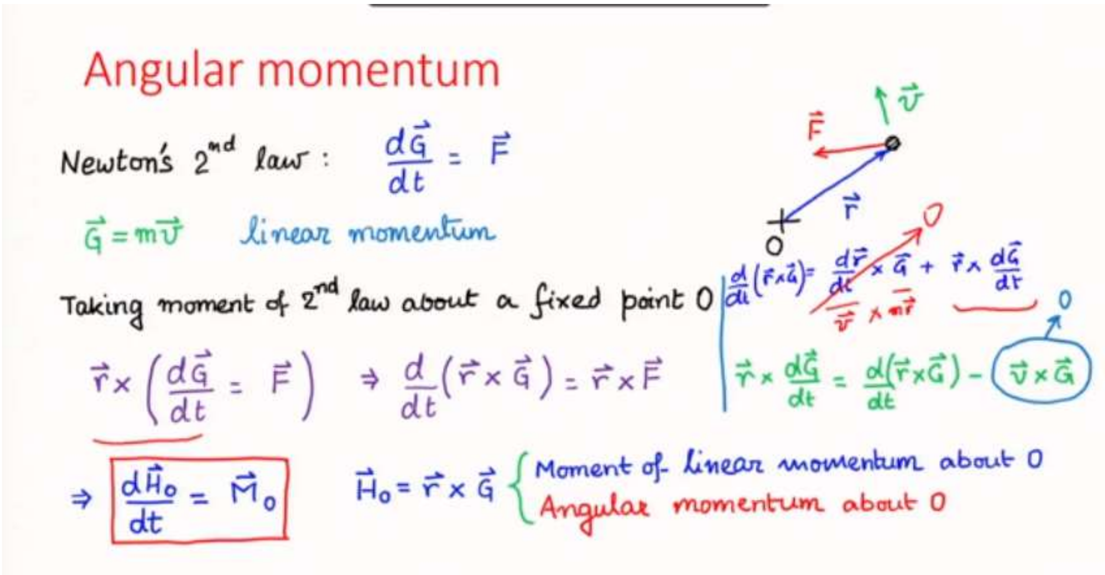

Let us start with Newton's second law: the rate of change of linear momentum G is equal to the force F, as shown above. Consider a fixed point O as shown above about which we take moment of Newton's 2nd law to write

$$\vec{r}\times\left(\frac{d\vec{G}}{dt}=\vec{F}\right) \Rightarrow \frac{d}{dt}\left(\vec{r}\times\vec{G}\right)=\vec{r}\times\vec{F}$$

$$\Rightarrow \boxed{\frac{d\vec{H}_o}{dt}=\vec{M}_o} \qquad \vec{H}_o=\vec{r}\times\vec{G}\begin{cases}\text{Moment of linear momentum about O}\\ \text{Angular momentum about O}\end{cases}$$

H_o is the new quantity which we have now obtained by taking the moment of Newton's second law about the fixed point O. This is called the moment of linear momentum vector, or also known as angular momentum vector. It is very important now to attach this point about which we have taken the moment.

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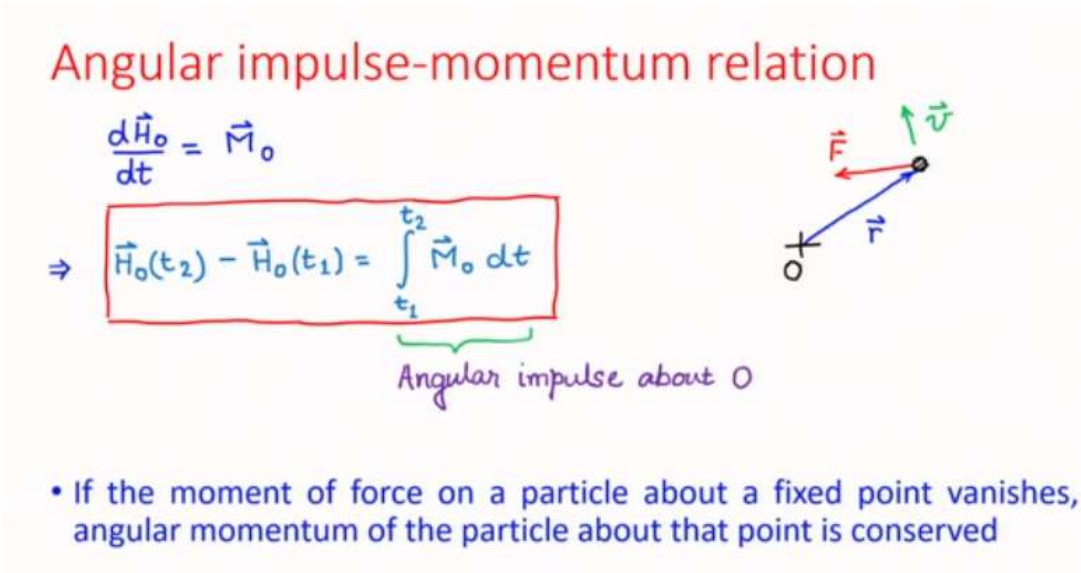

Thus we have this relation: rate of change of angular momentum of a particle about the fixed point O is equal to the moment about O of the net force acting on the particle. Now if we just integrate over time, we get the difference in angular momentum about O at 2 time instants is equal to the integral of the moment about O over this time interval. This is the angular impulse momentum relation. Now, it straight forwardly follows that if the moment of the force on a particle about a fixed point vanishes, then angular momentum of the particle about that point is conserved.

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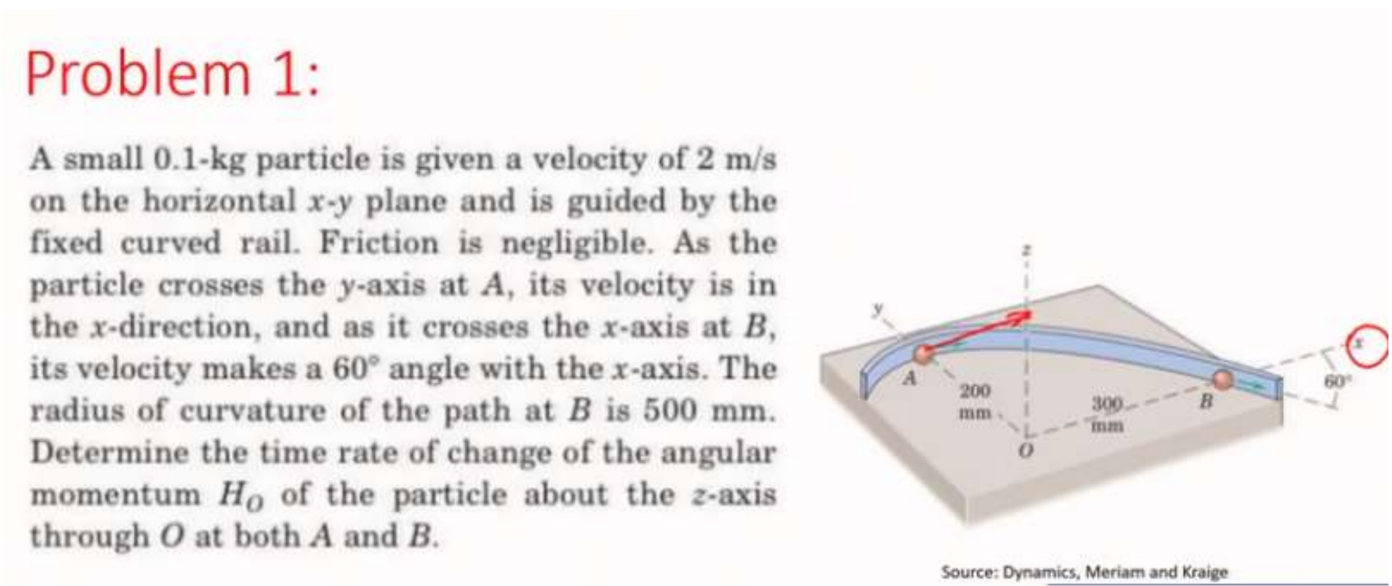

We consider the above example.

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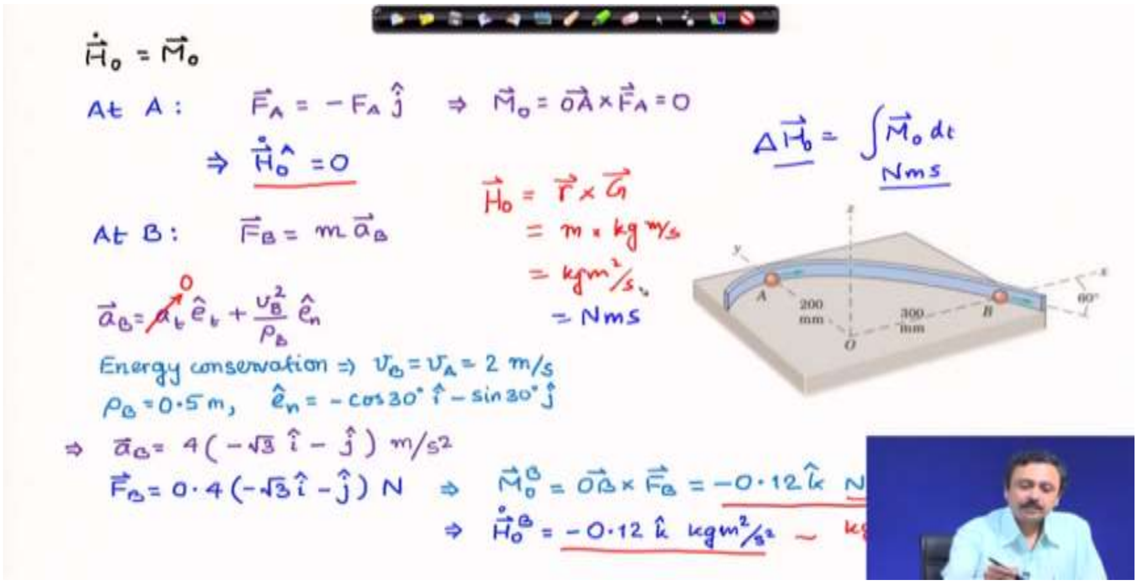

The detailed solution is presented in the slide above. It is important to note of the units of angular momentum as detailed above.

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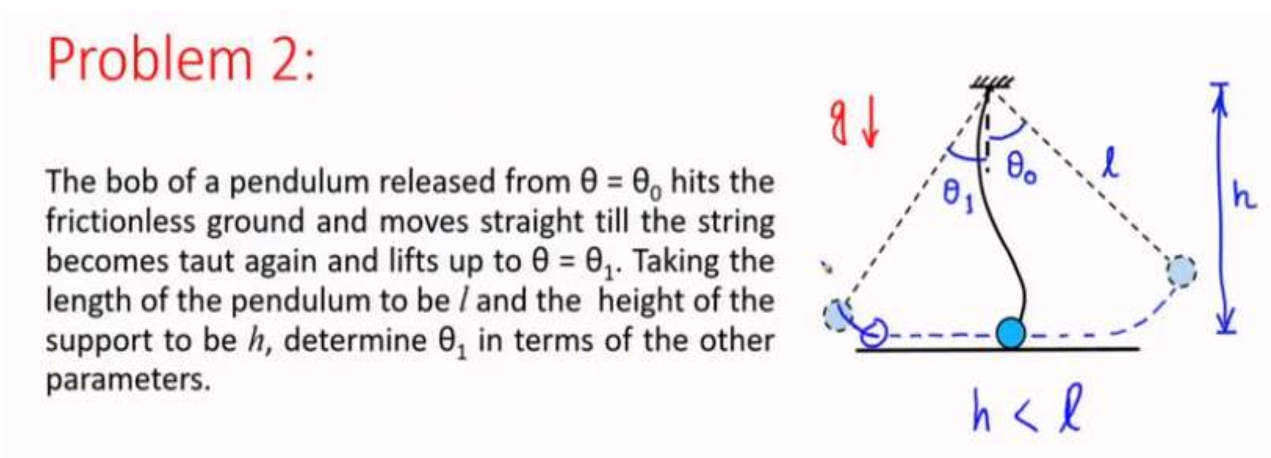

We consider the above problem next.

The problem is solved in 3 parts: (1) motion on a circular arc from the point of release to the first impact with the ground at A and subsequent horizontal frictionless motion on the ground, and (2) impulse from the string at B, and (3) motion on a circular arc from B to the highest point reached. These are shown in the following 2 slides.

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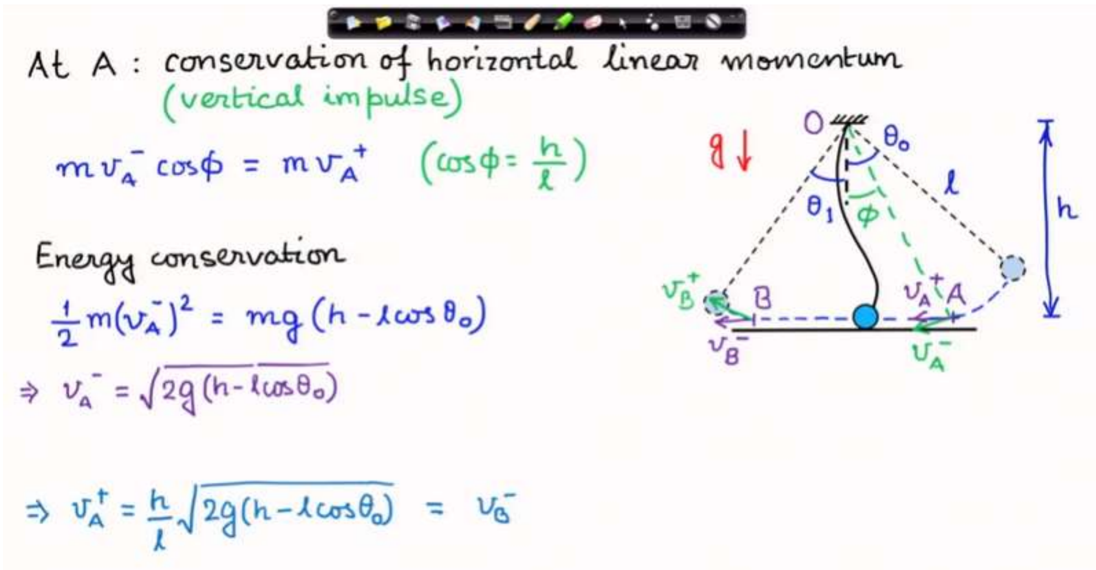

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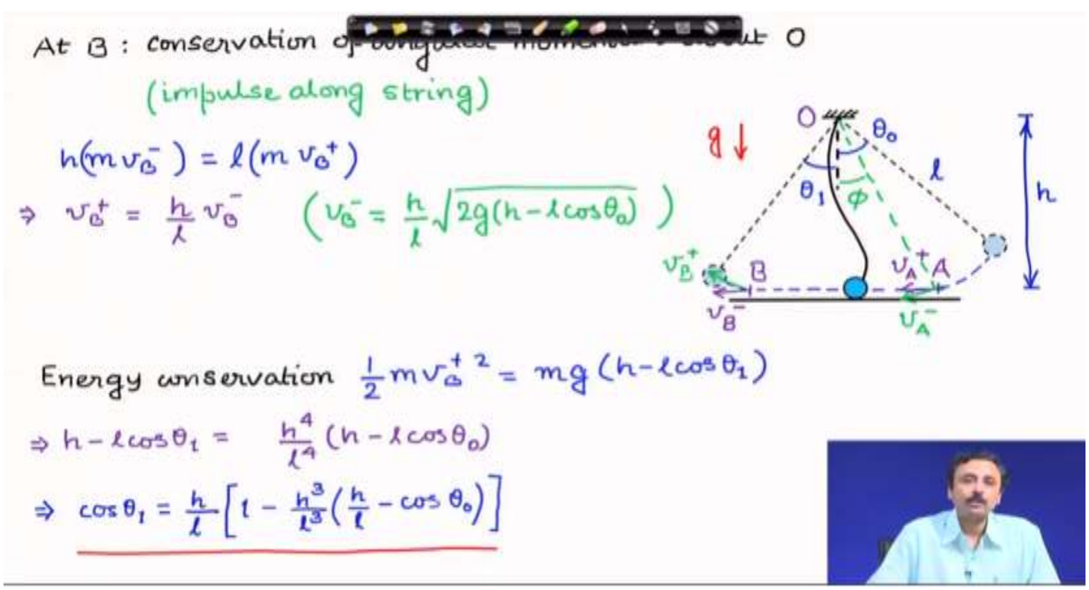

The final answer is shown above.

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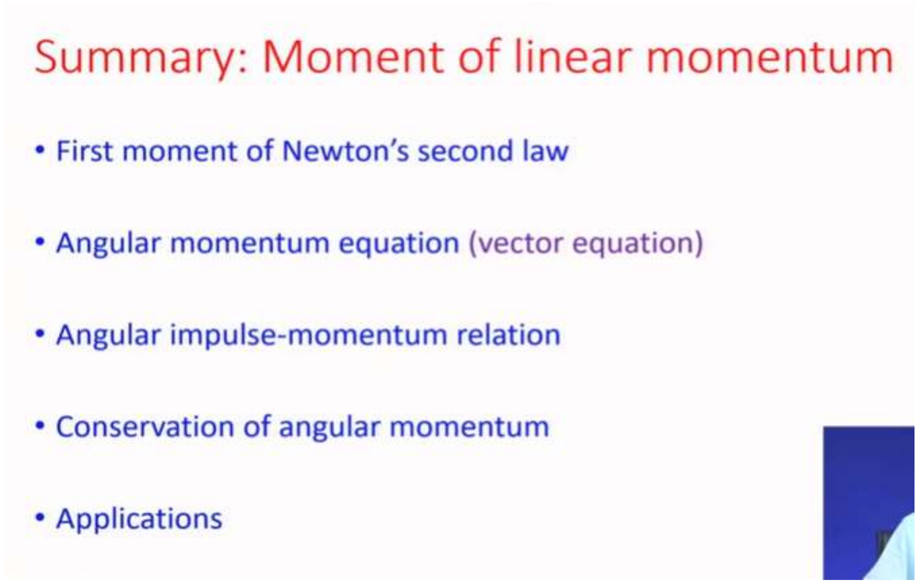

The summary of the discussions is provided above.