Engineering Drawing and Computer Graphics
Prof. Rajaram Lakkaraju
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Module - 07
Lecture – 53
Overview of Computer Graphics - III

Hello everyone, welcome to our NPTEL online certification courses on Engineering Drawing and Computer Graphics. We are in module number 7, learning about how to construct surface models and further we are learning a little bit about Overview on Computer Graphics.

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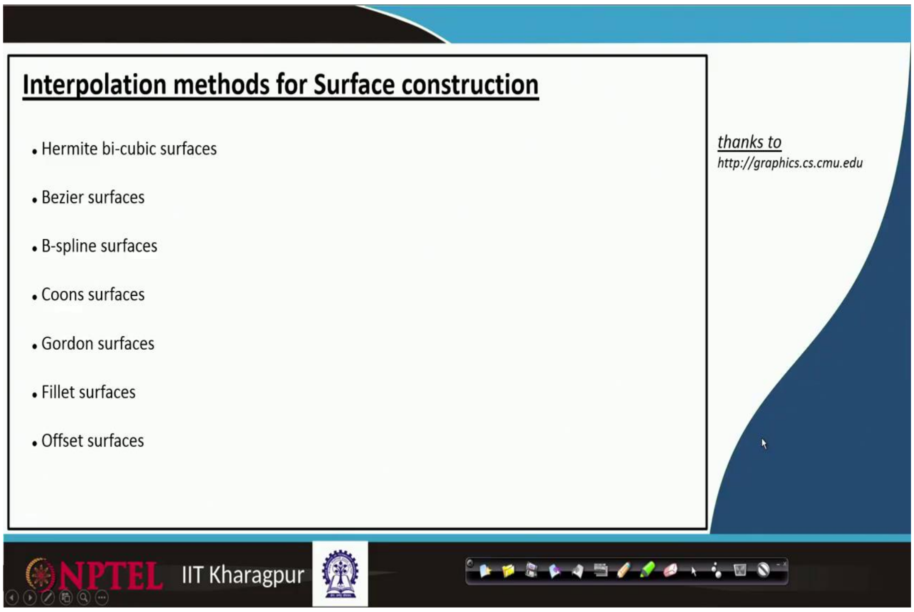

So, in today's class, we will learn about various surface construction techniques. In all these surface construction techniques, we mainly use cubic polynomials as the basis functions.

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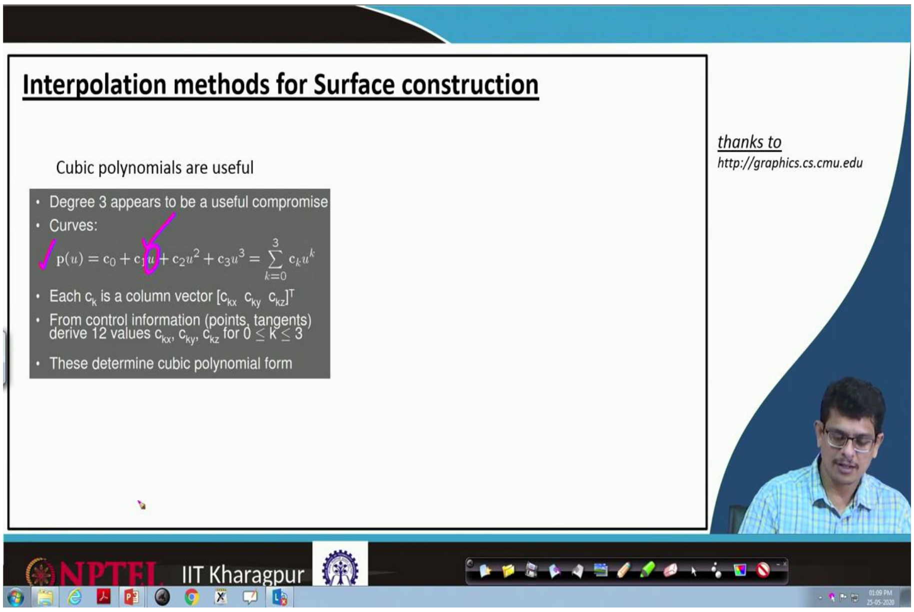

For example, we have a data points something like u 1, u 2, u 3 and so on. And if you want to, if you are interested in constructing a curve first, then we will impose that, it satisfies a cubic relation where c0, c 1, c 2, c 3 are the coefficients and u is a parameter which we might be in a position to say it like, what are the x y z coordinates, different x y z coordinates we are going to plug it. So, that one will we will be in a position to get a parameter curve. Here each of this cs c0, c 1, c 2, c 3 basically represents the weight functions of that polynomial curve. They can be 1 1 1 1 kind of thing or 1 0 1 0 kinds of thing 1 0.5 0.1 that kind of thing. So, these weights we have carefully shown it so that a nice curve really pass through these points. Once we construct these cubic polynomials, then we will interpolate apply the same technique to construct the surfaces.

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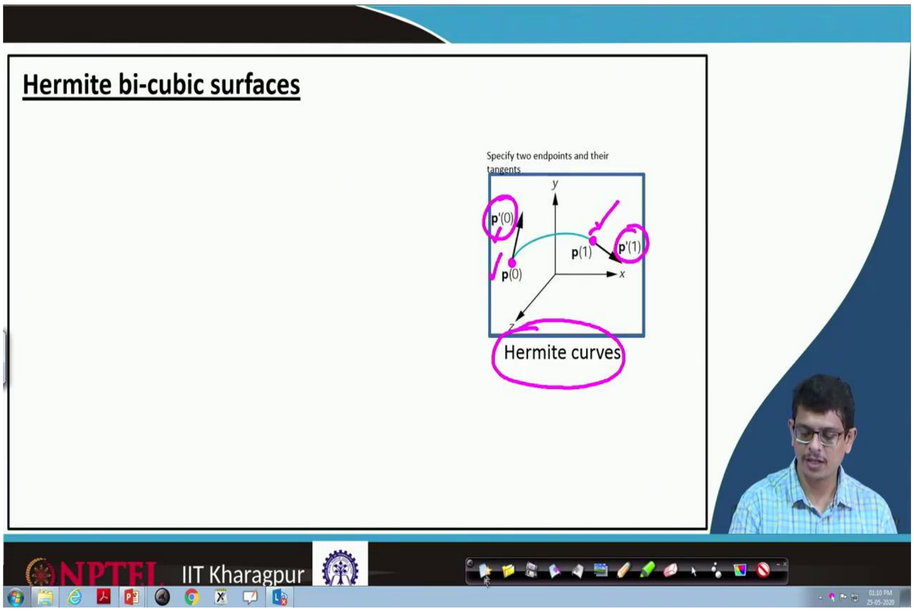

The first one, in terms of Hermite bicubic surfaces, what we do is we construct a curve between points. Those points, if we are joining if we are getting a specialized curve, for example, the point p0, p 1 we know these are any x y coordinates of that system.

We have information about that point, we have information about p 1 and similarly, we have information about the tangent at p0, which we are representing p0 prime. Similarly, the tangent at p 1 which we are calling p 1 prime. If we have this information, a curve in the direction if we can interpolate based on those tangent information's and point information, a smooth curve if we are passing through that we call that as Hermite curves.

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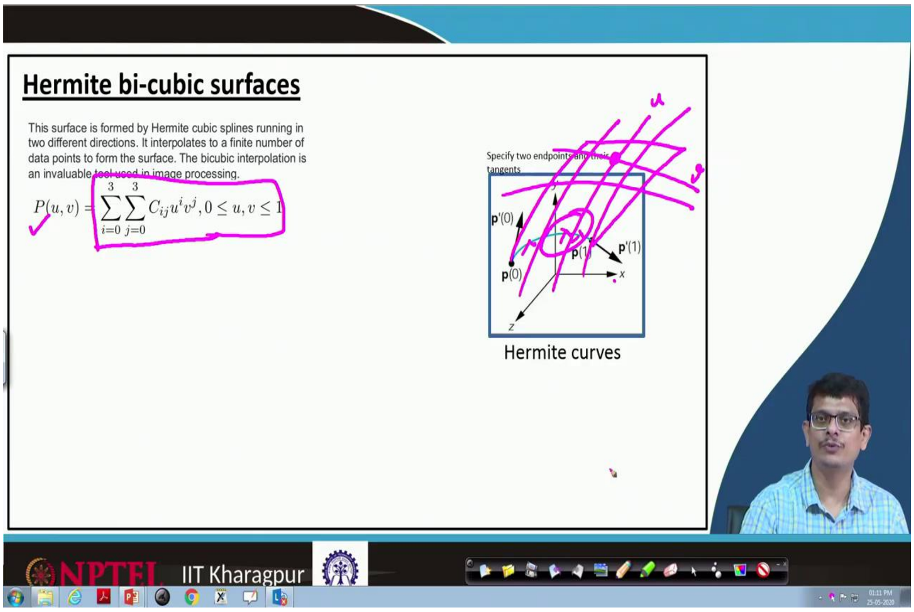

They satisfy one particular relation, where your P any point p on that curve, supposed to satisfy this relation. Hermite curve, which we are trying to locate even on the surface. So, you pick any Hermite curve, which is passing on that surface pick that point, where u v information we have parametric variation.

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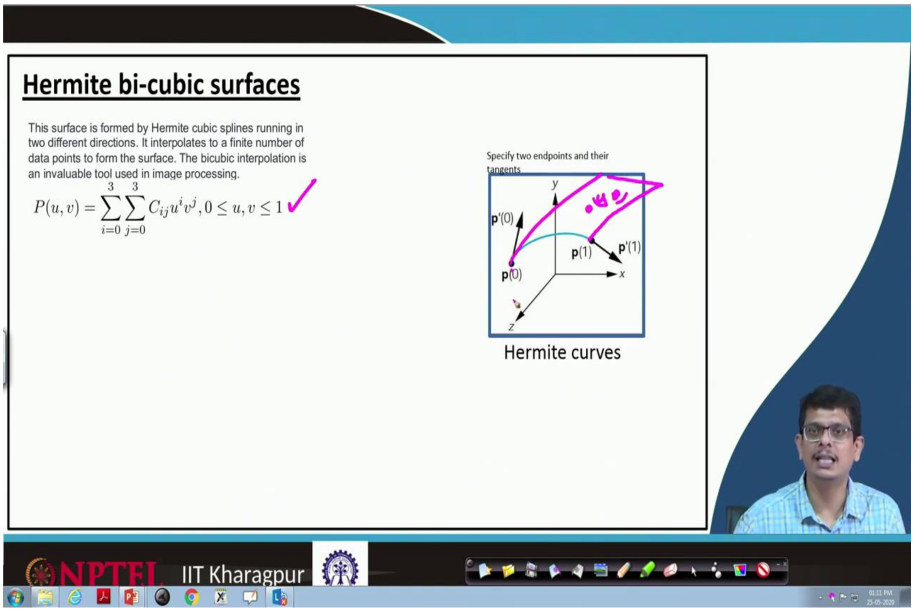

On that surface the point in the direction of u v, supposed to satisfy this relation. Based on the points and the tangent information what we have.

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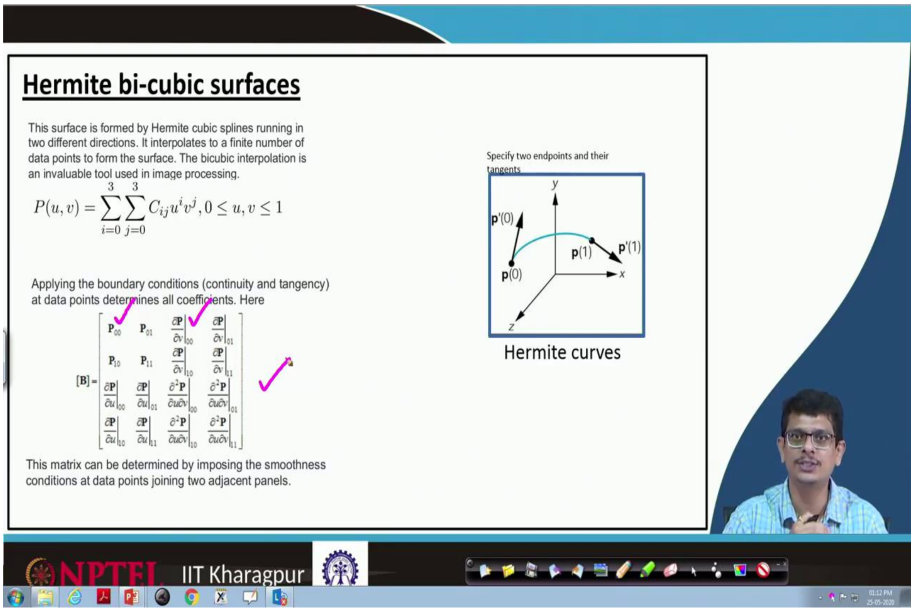

And when we are constructing that, we try to minimize certain issues like weights and other things. For that purpose, we try to use the function values, we use the function values and also their derivatives try to impose it different points to get that surface and that matrix based on how smooth that is how regularly it is conditioned or ill-conditioned based on that we will be going to construct these surfaces.

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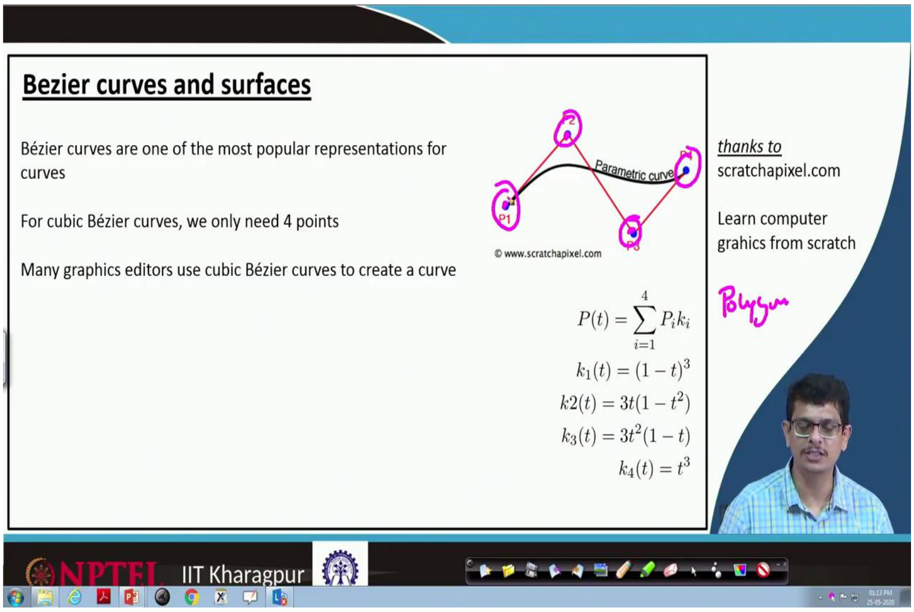

There are other ways of constructing surfaces also, those are called Bezier curves and Bezier surfaces. In that, we have particular data points like P 1, P 2, P 3, P 4. We do not straight away join these data points, what we do is we construct based on certain, polygons fit in between these data points. And when we are fitting these polygons, whatever the curve which passes through that crossing these polygons that is what we call Bezier curves.

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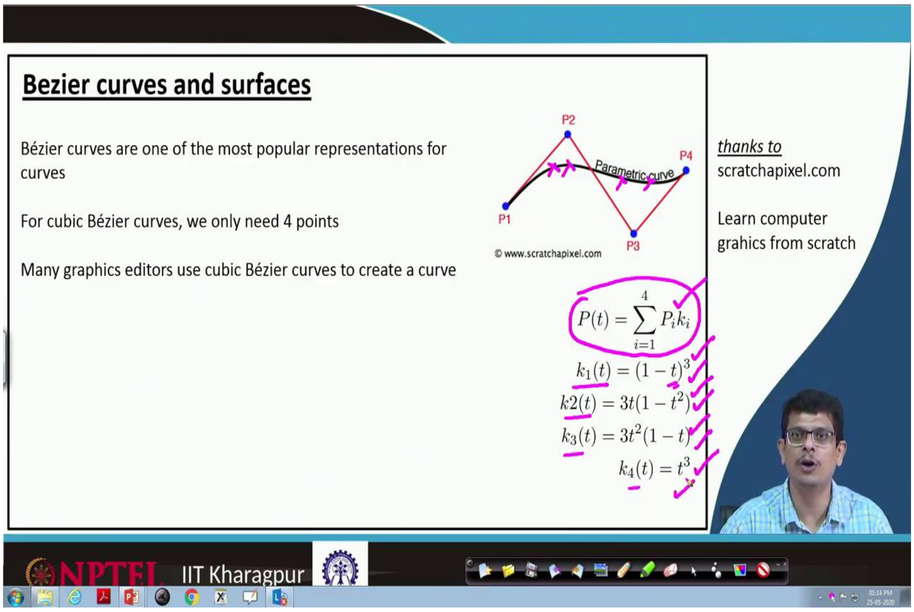

If I am representing it in terms of parametric variation the P any point P, on that curve the parametric curve supposed to be expanded in terms of P i k i where k 1 k 2 k 3 k 4s have these cubic interpolations. So, t is our parametric variable, like our u, a cubic polynomial, a cubic, a cubic, a cubic in different kind of formats we are going to carefully adjust. So, that we will be in a position to construct polygons, through which this particular polynomial curve really passes.

If we are going to do that, that kind of curves what we call as Bezier curves and these are the most popular representation curves.

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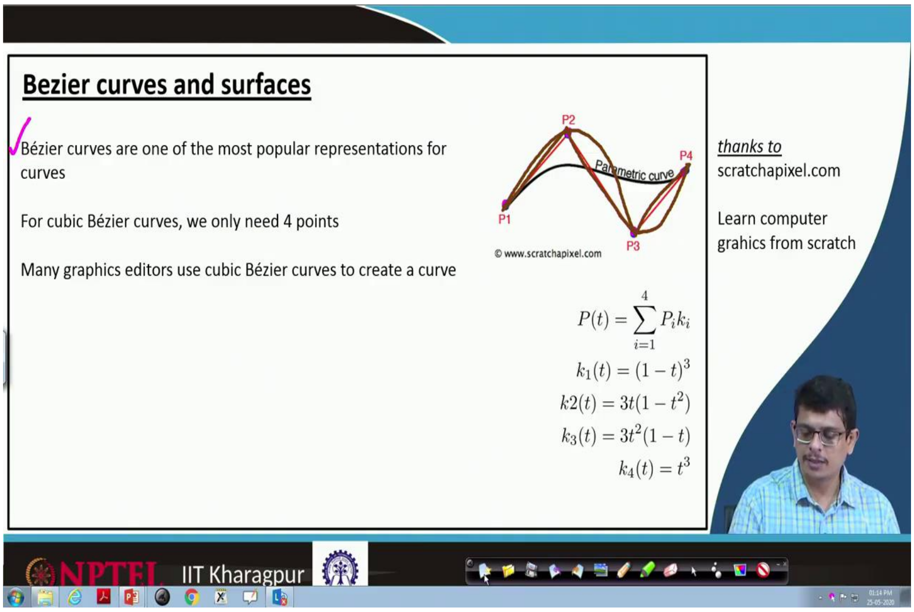

And when we are drawing such kind of curves, we require only 4 points to construct that smooth curve. For example, we have point P 1, P 2, P 3, P 4. One way of representing this one is maybe, I will be just going to construct like a line or perhaps, I just construct something like a spline.

The best representation is to go with this Bezier representation, whereby minimizing these weights we will be in a position to construct a curve. Most of the graphics editors, the software whatever we are going to really draw the curves and render the things works on these Bezier curves.

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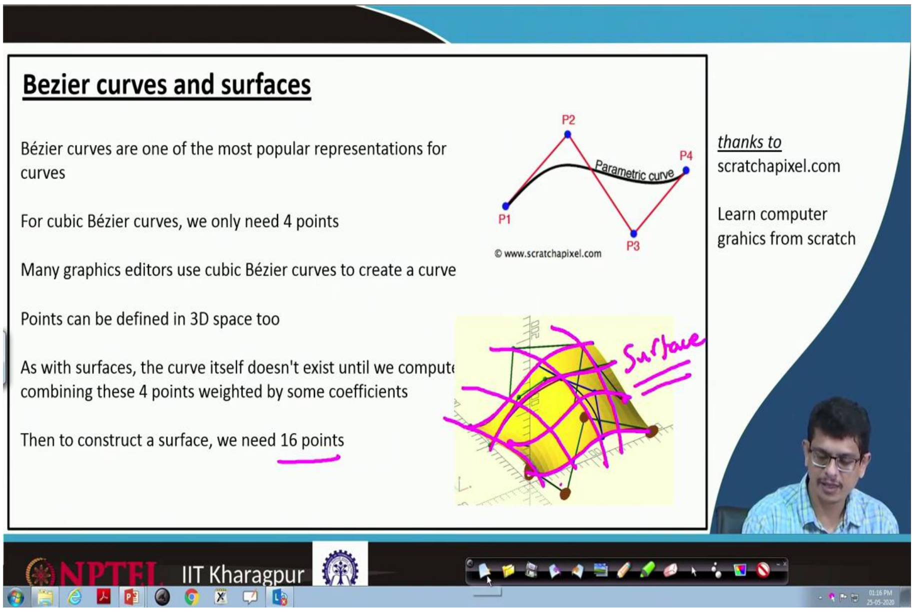

If you are looking that in 3 dimensions. For example, this is the first data these are the 4 data points, we have. Then first, we will construct a Bezier curve in that. Similarly, we will be having other data points like one perhaps, two at bottom three and four then again we will be in a position to pass another Bezier curve. Similarly, another Bezier curve, another Bezier curve similarly on the other directions we have 4 data points. So, pass a surface a Bezier curve in that way.

So, that we will be in a position to construct, a free surface on that object. By just knowing 16 data points we will be in a position to construct a very smooth surface.

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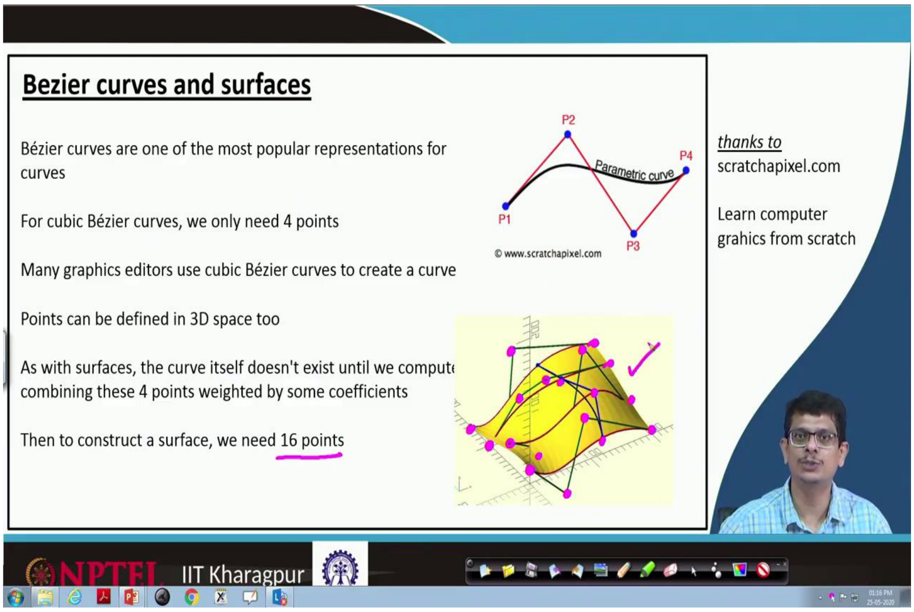

Let us look at carefully, the data points what we have is 1, 2, 3, 4. Similarly, 1 maybe, 2, 3, 4, a bottom 5 and so on. These are the data points what we have.

Now, one has to construct a surface, because these data points are scattered. What is the best way one can really have that surface? For that purpose what we require is, if we have minimum 16 points by using these Bezier curves principles, one can construct this one a smooth surface. As with the surfaces, the curve itself does not exist, until we compute combining these 4 points weighted by some coefficients. So, what is that curve does not exist what we have these discrete data points, from there we are trying to construct it.

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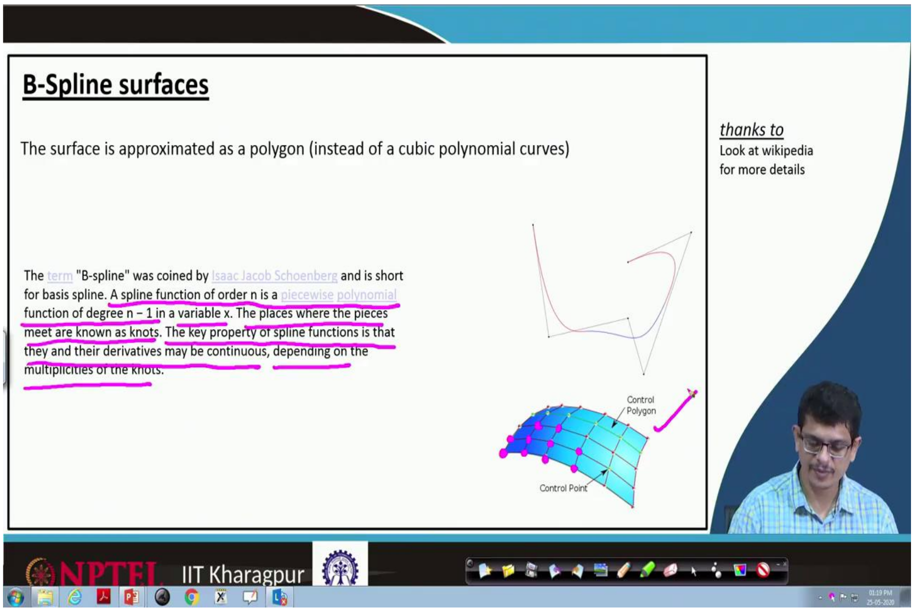

The other way of representation, these data points trying to construct surfaces called B-splines, which is also called as basis splines technique. Because it is a drawing course we are not going too many details into the mathematical formulation of these curves, but just to have an overall idea about what kind of curves are good for what kind of applications, that is the thing what we are going to learn about it.

In basis spline techniques, we are going to use polygons instead of any cubic polynomials. So, instead of using any cubic polynomials, if we are going to have these polygons, we call that as basis splines. The term B-spline was coined by Isaac Jacob Schoenberg and it is a short representation of saying basis spline. A spline function of order n is a piecewise polynomial function of degree n minus 1 in a variable index x. The places where the pieces meet are known as knots.

The key property of spline functions is that they and their derivatives may be continuous depending on the multiplicity of knots, how complicated these knots we might be having smooth curves. And we do not use anything like cubic polynomials, like what we have used for Bezier curves, but here we use polygons. By carefully adjusting these polygons, we will be in a position to construct surface.

For example, we have these data points. The easiest polygon what I can really fix is something like a tail, a rectangular tail I can put maybe a rhombus I can really construct or perhaps a triangle I can really construct. So, varying these kinds of objects I will be in a position to construct a surface. So, based on controlling those polygons, it can be hexagon, dodecahedrons and so on things, we will be in a position to construct a nice surface.

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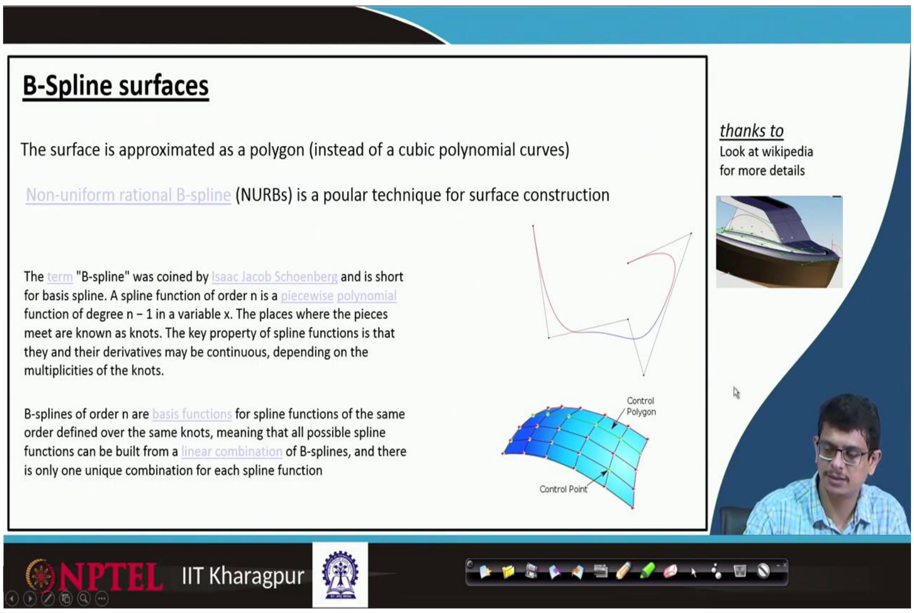

B-splines of order n, basis functions for spline functions for the same order defined over same knots, meaning that all possible spline function can be built from a linear combination of B-splines. The advantage is you can really add it up as a linear function, that is the advantage what we will get with this B-splines, and there is only one unique combination for each spline function. When you are using that, there will not be any multiplicities involved in that surface construction, you will be having that unique representation.

For example, like ship hulls, you want to construct and so on, usually, these B-spline surfaces are quite popular. If you open any CAD software may be solid works, AutoCAD these options you will have or Open GL. These are kind of software where you will have these kinds of options, uh like whether you would like to really interpolate it like through B-splines or Bezier curves or Hermite basis functions and so on.

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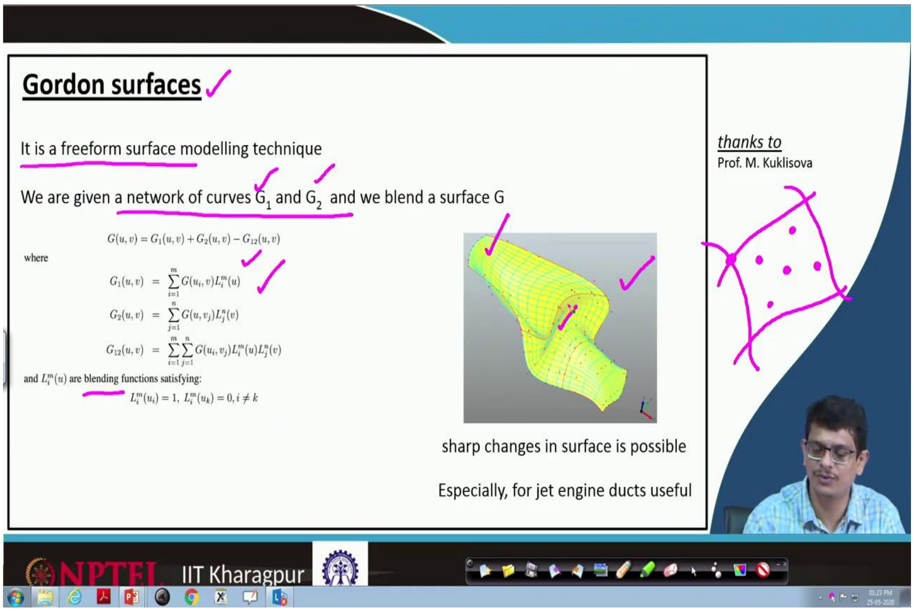

The other kind of representation is by Gordon surfaces. So, these kinds of Gordon surface is called freeform surface modelling, that means, you have data points you can really drag-drop, so that surface is naturally adjusted on that CAD software.

So, you will be having a network of G 1, G 2 curves. For example, as if I am saying something like this is one curve, you have another curve, you have another curve and you have isolated data points here and there.

Now, using your CAD software, you can just pick one of the data points to move your mouse here and there. So, those surface variations you start seeing and you carefully pull your surface in all these points, nicely mapped to get a smooth kind of surface. That kind of surfaces what we call Gordon surfaces. It satisfies certain kind of relations like your G 1 curve, G 2 curves and the intersection of these G 1, G 2 curves supposed to satisfy a certain blending function.

For example, like you have a drastic variation on these surfaces, like you are making an automobile engine cylinder where mufflers will be there, the sudden change in this pipeline structures will be there. Then the easiest way what you can really construct is this Gordon surfaces. Here such kind of object is shown, a something like a cylindrical nice surface comes suddenly, there is a bump happens there a kind of dent also there.

So, you would like to really fit a nice curve, through that or perhaps you are working on the biomedical field your femurs or bone structures. These are not just described by very smooth curves, there might be sudden variation happens on these bones, you want to really reconstruct such kind of objects. Then the best way of looking at that is using these Gordon surfaces. It is not necessary that the hip bone joint kind of thing might be circular, it might be having some specialized curve.

So, when you are actually taking pictures through cameras, you will be having isolated data points now you want to reconstruct it and transfer that entire data to your 3D printing object. Then what you do is, you use those pixel information's try to construct these Gordon surfaces and teach it to the computer or to that machine element print it in this way. For that purpose, these kinds of Gordon surfaces will be used.

Especially, for jet engine ducts and other things where certain abruption happens, you use this Gordon surfaces.

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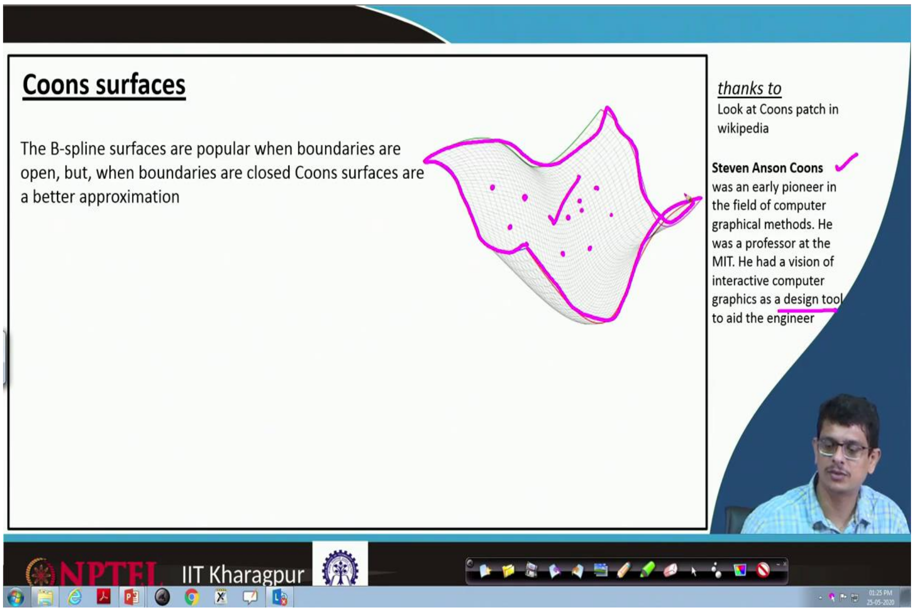

The other kind of surfaces is Coons surfaces. Especially, the B-splines the basis splines are popular when boundaries are open, you have discrete data points, but these discrete data points are not forming any boundary. Somewhere inside of that object you have it then you use B-splines, but if these data points on one particular kind of boundary. We employ these Coons surfaces and this Coons surfaces are better approximations.

So, Coons is a famous pioneer in the field of computer graphics. He was a professor at MIT, and he has spent a lot of time in terms of constructing these design tools for engineers. So, something like here, in an abstraction we have this curve or the data points on that particular curve and those forming boundaries. What is the best way of constructing these data points?

A surface inside of that which we do not have any information, what we have is only the boundary information's we have. What is the best way I can really construct that surface? It looks for visual very nice, but construction means it requires little bit mathematical skill set. So, any CAD software actually employs that background programming, to construct such kind of surfaces.

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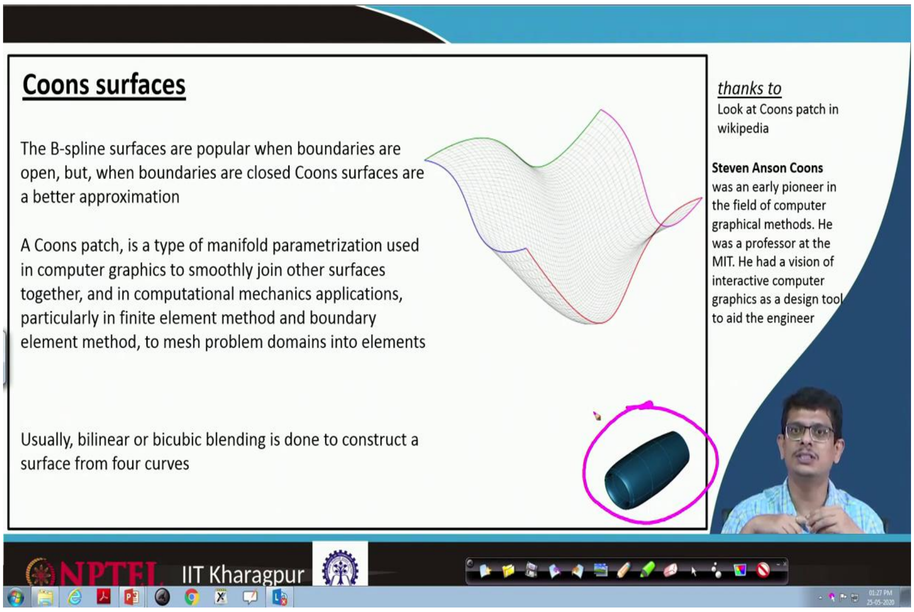

A Coons patch, is a type of manifold parameterization used in computer graphics to smoothly join other surfaces together, and in computational mechanics applications, particularly in finite element methods and boundary element methods, you would like to really mesh the problems of domains into elements and so on. These Coons surfaces are quite popular.

So, there are certain mathematical processes to solve differential equations to understand the fields, which we call finite element methods. You have boundary conditions on one of these particular things and maybe a surface you would like to really represent a smooth surface, then you employ this Coons surface. Even meshing may be locally zooming into that picture, this Coons surfaces always be helpful.

And further, on these Coons surfaces, people try to use a blending, try to smoothen these surfaces and even if you are zooming that pixel variation should not really vary, that kind of techniques what people use, for that they use bilinear and bicubic kind of interpolations also.

So, here what we are seeing is, basically the engine duct. The aeroplanes, the jet engines what you have on the wings, you will have something like jet engines which grab air put a fuel, burn it so that exhaust will come out and that can supply the thrust of that engine. So, these ducts always have some specialized kind of form, if you are looking at that they nicely turn and twist in that directions, for that kind of surface construction these Coons surfaces are quite popular.

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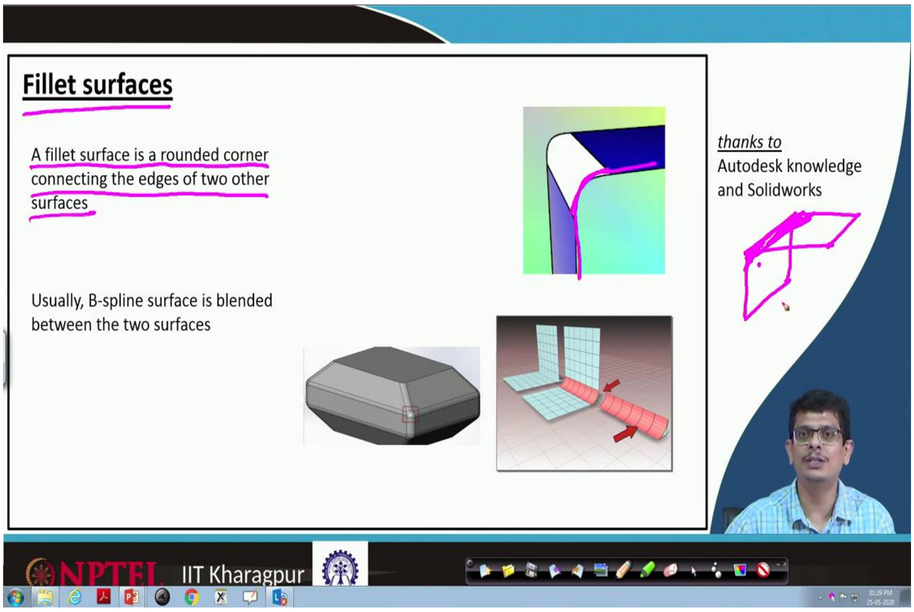

So, other than that, when you have these curves. Edges we do not want to really have very sharp things, making these sharp edges always be bit difficult also. We want to round it off, such kind of thing if we are going to do we call fillet of those surfaces. For example, we have a plate, a rectangular plate and again we want to join by another one, we have this sharp corners we do not want that sharp corners, what we do is we try to remove that surface make it something like smooth.

For example, here that rectangular portion we have smooth corners. Because of the sharp corners, we try to avoid it because it might affect our productivity or harm or perhaps scratches forms are is difficult to maintain that surface particular thing. So, we always round it off, such kind of things what we call fillet surfaces. A fillet surface is a rounded corner, connecting the edges of two other surfaces.

The easiest way is, you pick a radius of curvature, a centre from there you draw a knock to join as a tangent to these curves and remove the remaining material and extrude that information so that, you get a fillet surface.

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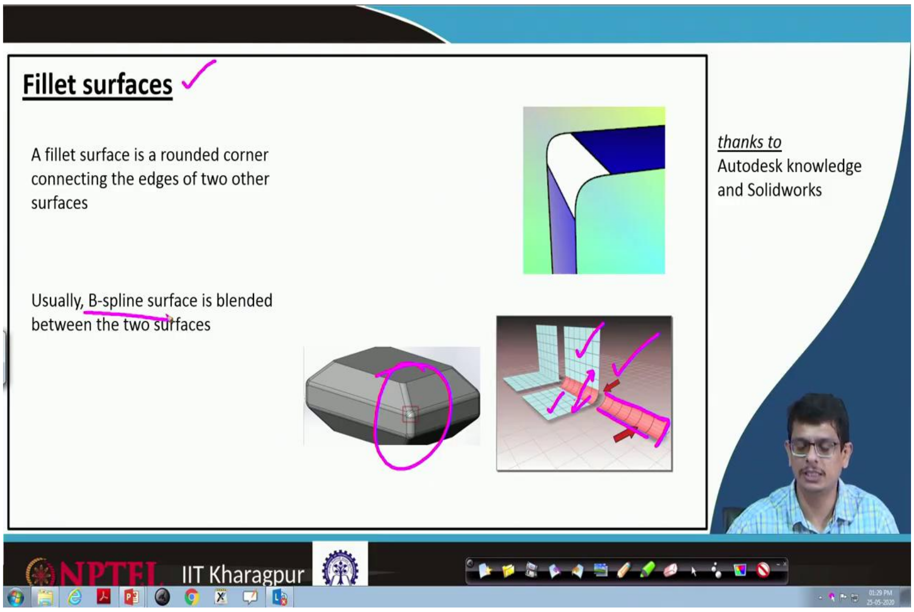

Here we have another kind of representation, you have this rectangular plate you try to construct this smooth surface and try to add to these portions, that is what we call fillet surface.

Similarly, let us look at this, maybe you have automobile car windshields have to be nicely aligned with the object. It should not be sharp variation, for that purpose also we use fillet surfaces. Whenever you are going to use these fillet surfaces, the popular way of using is B-spline surfaces one can really use it. They will be nicely mapped from one point to other points.

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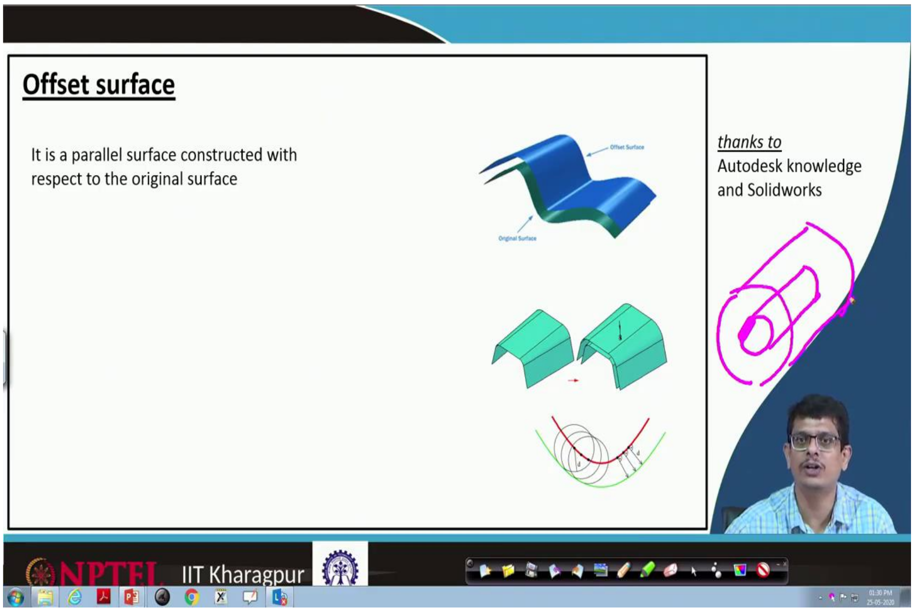

Similarly, in a CAD software, we use other variety of surfaces, named offset surfaces. You have one surface with respect to that one surface with an offset, similar kind of surface we would like to construct it. For example, I have a cylinder, let, for example, I have a cylinder. I would like to have one more concentric cylinder around that if I am going to construct such kind of thing that is what we call this offset surfaces.

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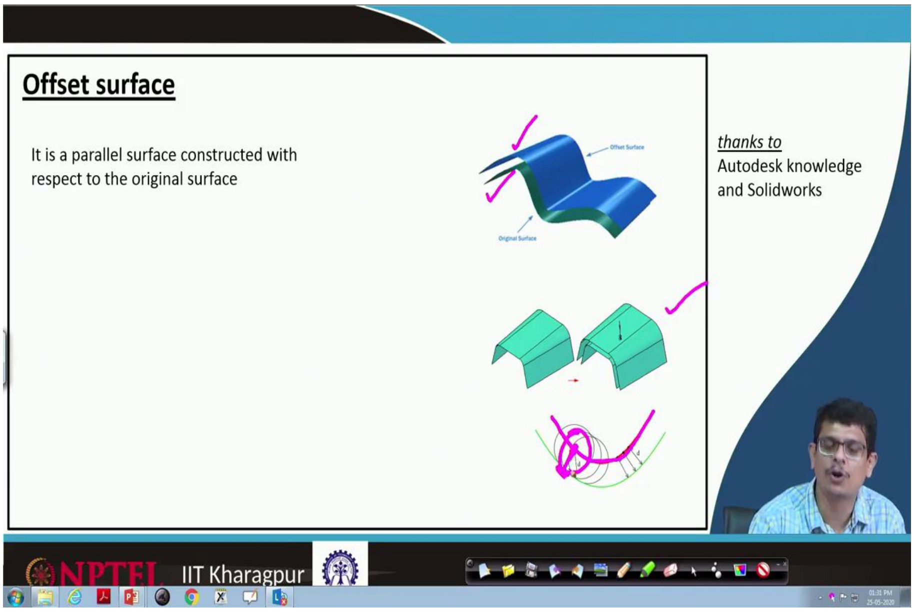

Here, we have some asbestos kind of sheet, the roof a parallel thing we would like to construct, that is what we call offset surfaces here also. One way of the easiest way of looking at that is, this is the curve or surface what we have first construct a point in that normal to that direction, draw a circle with one particular radius, wherever that surface is a tangential point pick it. Something like, we have a point here with an offset distance as radius draw a circle.

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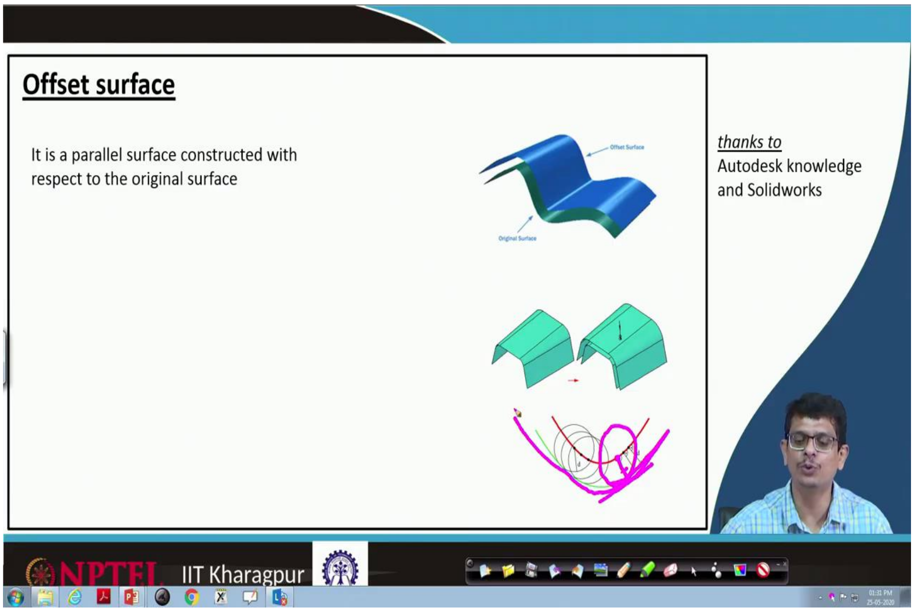

Now, we are going to draw a tangent there so that means, we have that point. So, now, join those points construct an offset surface. This is the easiest way one can really construct any offset, if we want to really improve the quality of that curve, we use specialized curves.

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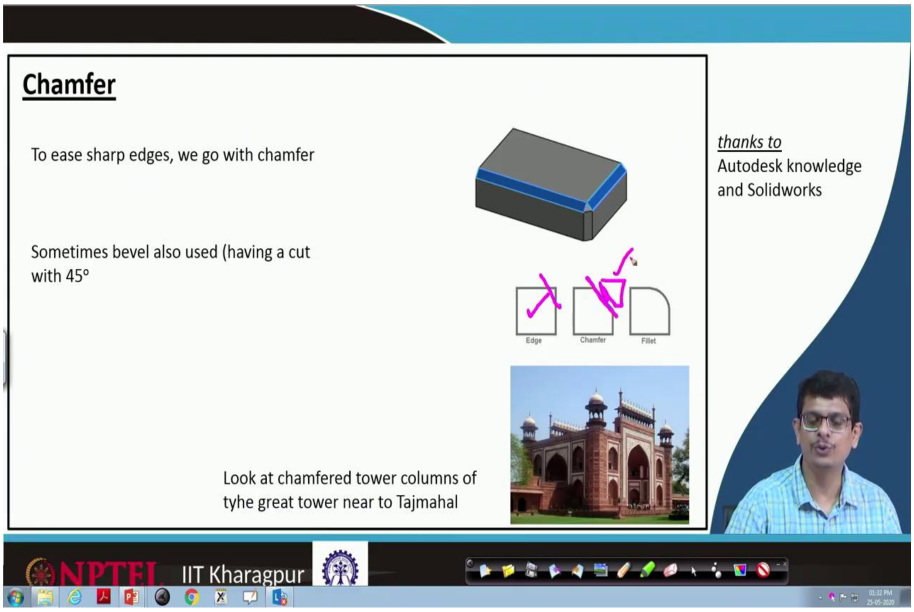

There is another terminology also we use when we are constructing these 3D objects using software, which we call chamfer.

A chamfer is basically removing material on a plane, we do not round it off, but we straight away chop or remove that material like a plane, a cut. For example, if we are looking at in the planar view, let us consider this is the object what we have in 2 dimensions, we want to really remove that material cut, remove the extra remaining material if we do that we call that one as a chamfer.

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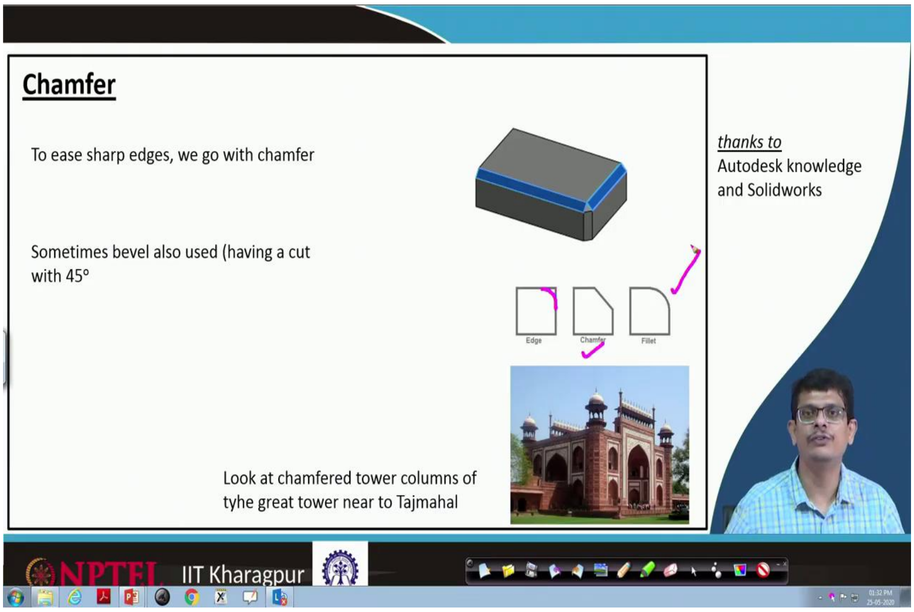

Now, we want to round it off then we call fillet. For example, here let us look at for architecture point of view, near Taj Mahal, there is an entrance gate the great tower, if you are looking at these columns these are not rounded kind of column towers. These are something like a plate, next plate attached with another plate that kind of sharp cuts if we have it on that we call chamfering.

Chamfering always is a nice cut we will be having, because of the architectural point of view you might be going to have that kind of thing. You want to stick some kind of pictures on that chamfer surfaces so you require flat surface rather than a curved surface, then you go with that chamfering.

So, in the next class, we will learn about solid works and practice a couple of examples of how to use that tool.

Thank you very much.