Engineering Drawing and Computer Graphics
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Lecture – 19
Conic Sections - XI

Hello everyone, welcome to our NPTEL online certification courses on Engineering Drawing and Computer Graphics. We are in module number 2 and lecture number 19, and we are covering Conic Sections.

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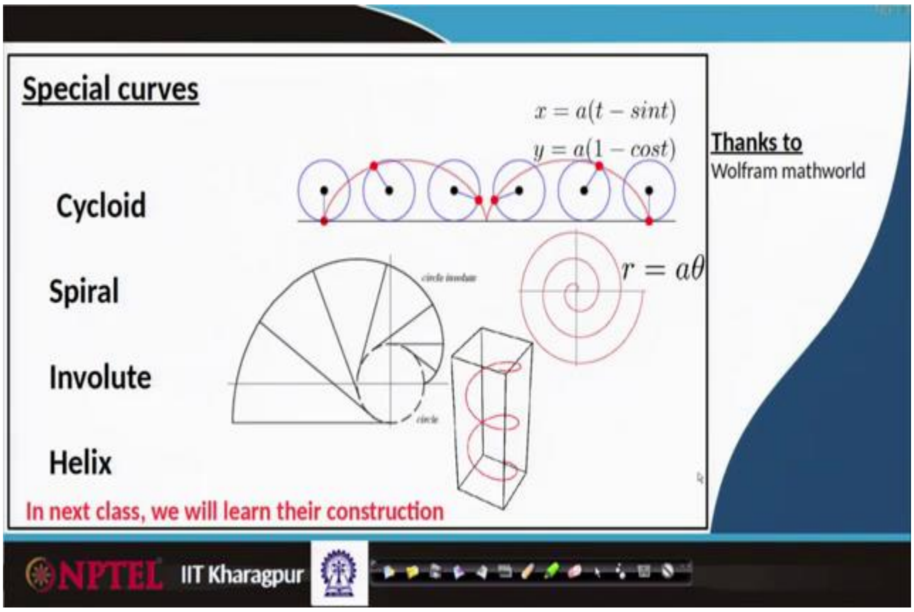

In that conic sections, in the last class, we have stopped at the cycloid curve. In today's class, we are going to look at involute and spirals.

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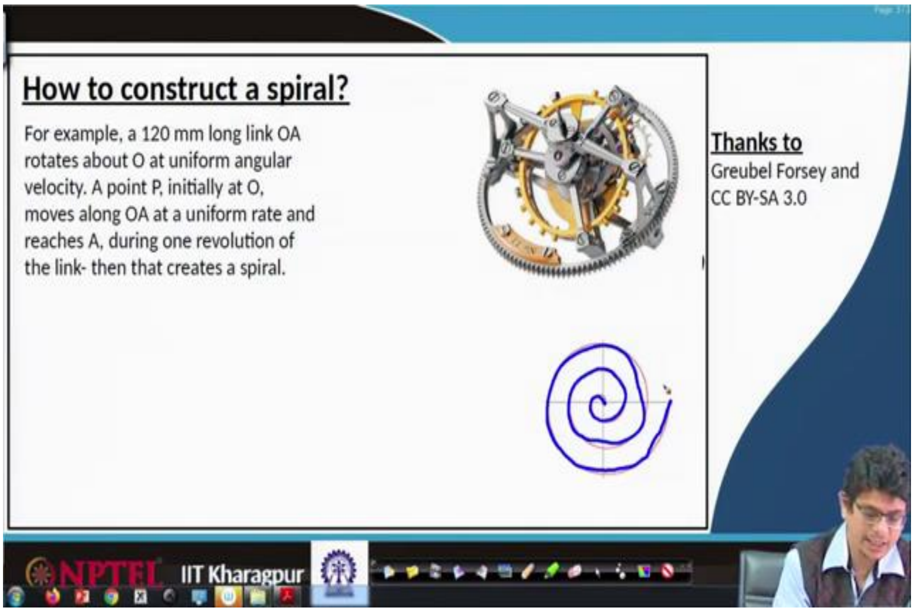

So, where do we see the first question spiral. For example, if we are taking any gear mechanisms like a watch to have the synchronization motion, we have a spring. The spring is wounded into a spiral shape and many torque restoring kind of mechanisms.

For example, like a door damper also contains a kind of spring, spirally wounded and by applying forces it gets stretched and when it wants to close or perhaps to restore the torque it again comes back to it is the original thing. So, in terms of a spiral spring, we come across in machine elements. So, the first question is how to construct such kind of spirals on sheets.

For example, let us pick a problem. A 120 mm long link OA rotates about O at uniform angular velocity. So, it is a machine design kind of problem where there is a link which is connected to this spiral. One of the points is rotating at a constant angular velocity. A point P initially at O moves along OA at a uniform rate and reaches A.

During one revolution of the link, the shape what it forms is a spiral. With this kind of data let us construct a spiral for point P if it is moving in uniform angular velocity and also moves along a particular direction. For example, let us pick a point P. This point P though initially, it might be at the centre as it moves with the constant angular velocity and moving out radially then it tracks a shape like a spiral.

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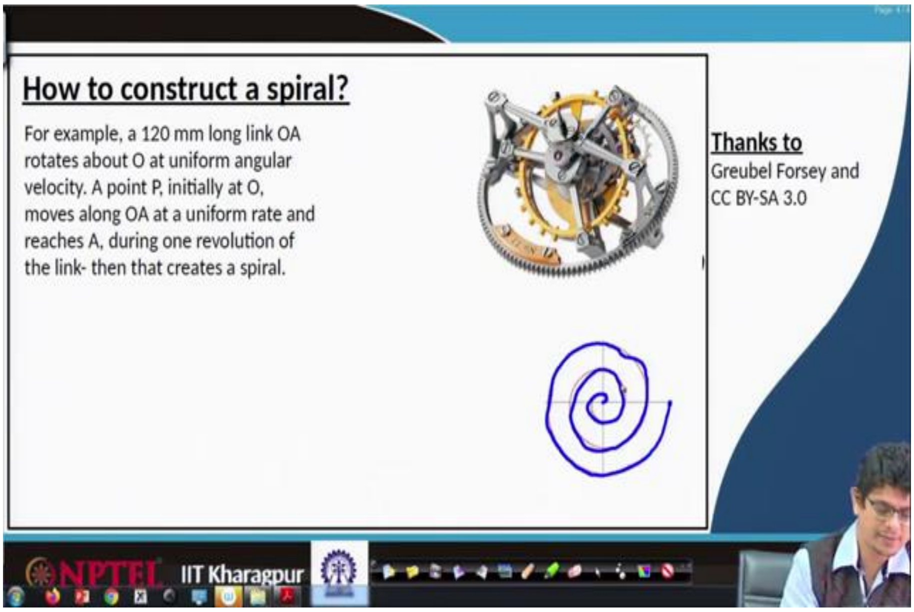

The other way is the point is here. It goes with uniform angular velocity. As it is going inside the radius continuously decreases. Finally, it reaches at 0. Such kind of shapes is curved what we call spirals. How to construct such kind of spirals?

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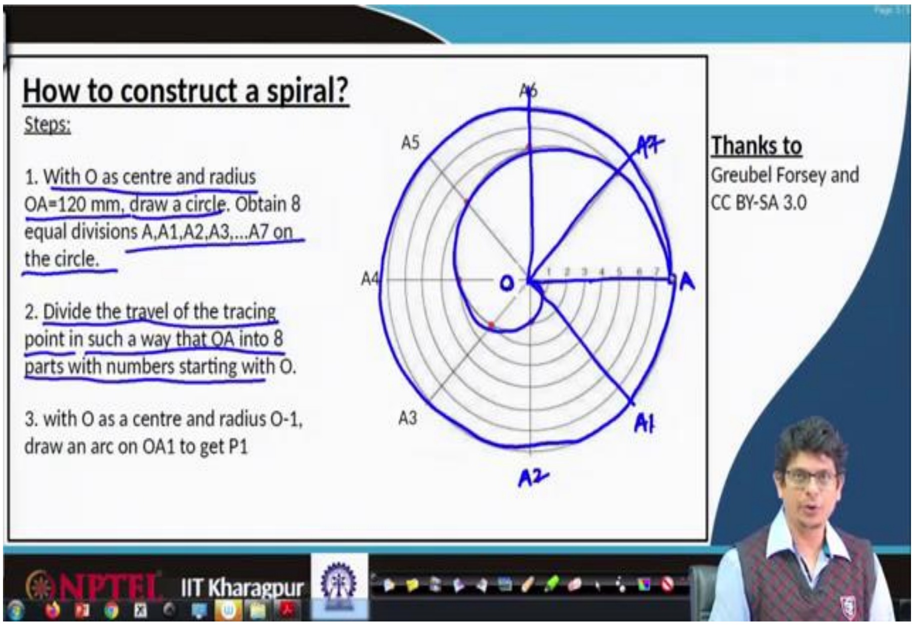

So, let us look at the picture. The steps involved in constructing the spiral are with O as centre and radius OA first of all, we have to draw a circle. So, locate a point O another point A with O to A a 120 mm mechanical element is present and construct a circle with OA as the radius. After that, divide the circle into 8 equal parts. First one, the second one and so on to eighth one name it like A, A1, A2 and so on to A7. On the circle first of all note it down. We have divided a circle into 8 equal parts. After that, divide the travel of the tracing point; there is a point P which moves on that spiral or circular direction in such a way that OA into 8 parts with numbers.So, this A point perhaps moving either inside or perhaps O point which is going out in the direction outwards. Whatever might be the end of the day for one revolution, this entire spiral covers 360° and reaches the point OA. It begins at O goes in angular velocity direction radially out finally, it comes to A.

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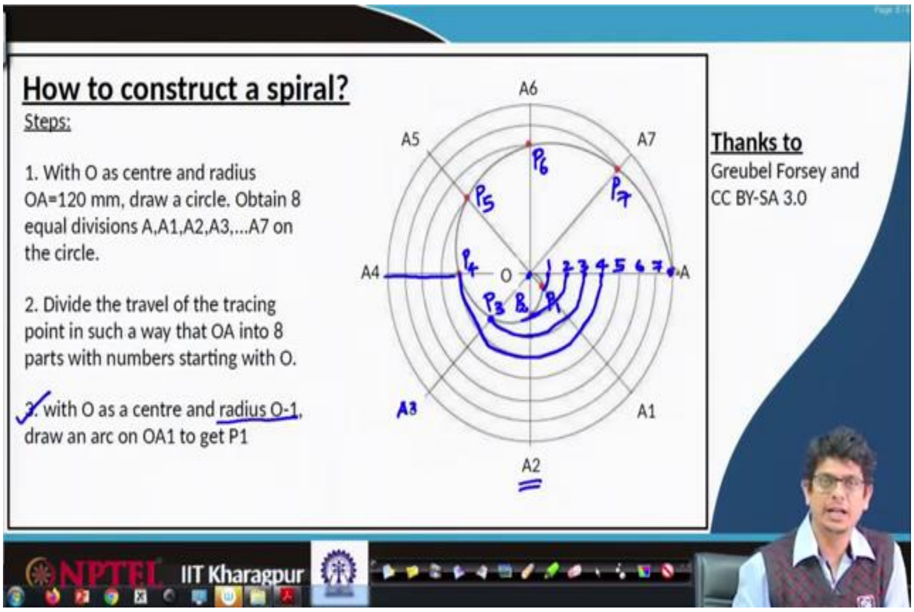

In that case, what we are going to do is divide the distance the radial distance this point locus point which moves up to this point A divide that into 8 equal points 1, 2, 3, 4 and so on all the way there name it into different points. Now, with O as centre and radius O1. This is O centre, and this is one radius draw an arc from O to that name this point as P1. This point moves with the angular velocity and also radial direction.

So, from O to 2 draws one more curve to make a point P2 which intersects this A2 line. From O to 3 make an arc such that is going to intersect A3 line here. From 3 goes intersect there. Similarly, from 4 draws an arc which goes intersect A4 line. In that way make points P1, P2, P3, P4, P5, P6, P7 and P8 point.

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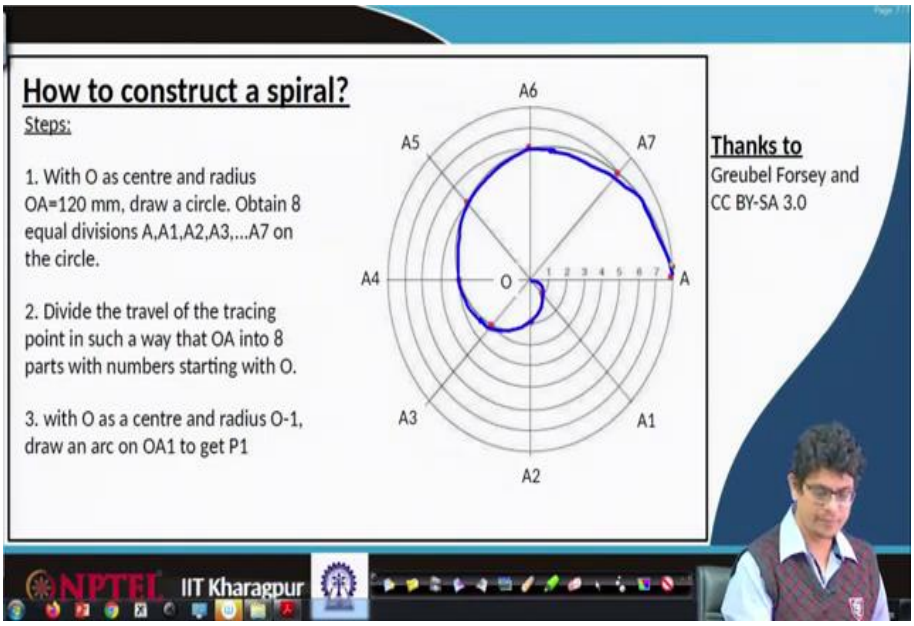

Once these points are noted down draw a smooth curve which passes through these points. That way, we will be in a position to construct a spiral. So, let us do that on the graph sheet. Use a scale. OA supposed to be 120 mm.

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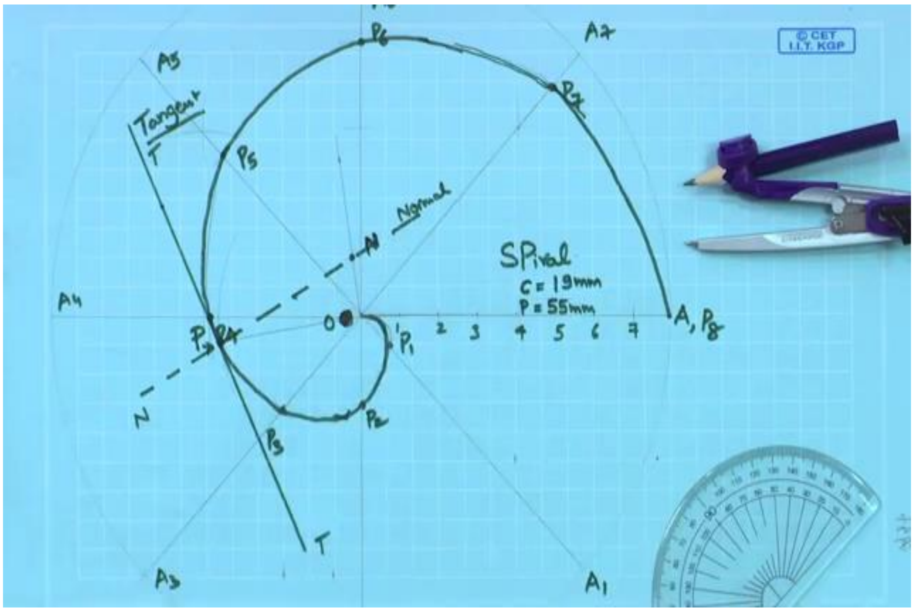

So, located on the graph sheet with that we have to draw a circle. It will be a very large circle on the sheet. It is going all the way up. So, let me draw that part of the circle first of all. This is the last circle what we are going to have. Now, divide that into 8 equal parts.

So, let us connect the lines which are going all the way up then we have to bisect these angles into 2 equal parts. So, if this is the one make an arc or 45° line, also we can do that. Now, join these lines using a scale. So, 8 equal parts. Now, line 120 mm that we are going to divide into 8 equal parts. So, 120 by 8 we have to use 15 mm locate it 1, 30, 45, 60, 90, 105 and 12 this is the way we locate these points.

Now, name these points as O and this point as A. Now, write down this sectorial information A1, A2, A3, A4, 5, 6, 7. Once is done make arcs all the way beginning let us name these numbers also 1, 2, 3, 4, 5, 6, 7 and the 8th one. A measure, the distance makes an arc. Locate that point as P1.

Similarly, from A2 make an arc P2 point. From 3 make an arc which will be at P3. With O as centre 4 as radius O to 4 make an arc here to name it P4 point. Similarly, the fifth point makes an arc here. P6 on A6 goes there. P7 on A7 goes there. Once it is done mark these points, P5, P6, P7 and P8 are always at A.

Now, join these points all the way beginning with O. So, if we have many more divisions it comes nicely 1, 2, 5 to 6, 7, 8. This is of a free hand sketch one will be in a position to construct it. If we have more number of divisions, the curve will be very smooth to draw it. This is what we call spiral. Let us begin the step once again. First of all, using the radius O we have to construct a circle in that way then divide this circle into 8 equal parts, 16 equal parts, 24 equal parts and so on based on the smoothness how much we require.

Once it is done O to A, we are going to divide into the same equal number of parts what we have done for employed for the circle. Then O to 1 construct a radius; make a cut there. From O to 2; make a cut on A2 line. O to 3, 4 and so on; make arcs on respect to a line so that intersection points we can name it like P1, P2, P3 and so on to P8. Join a smooth curve on that we will be in a position to get spiral.

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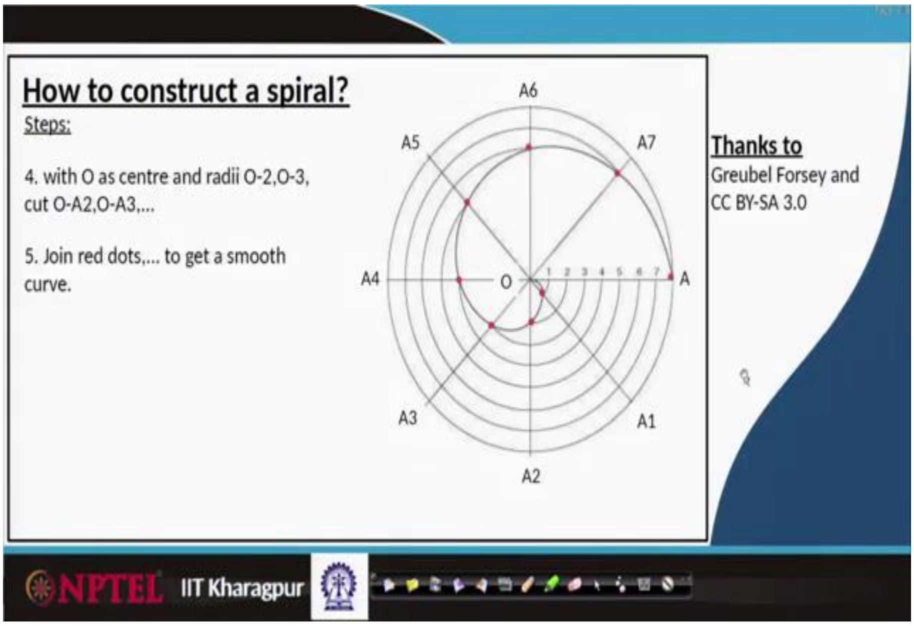

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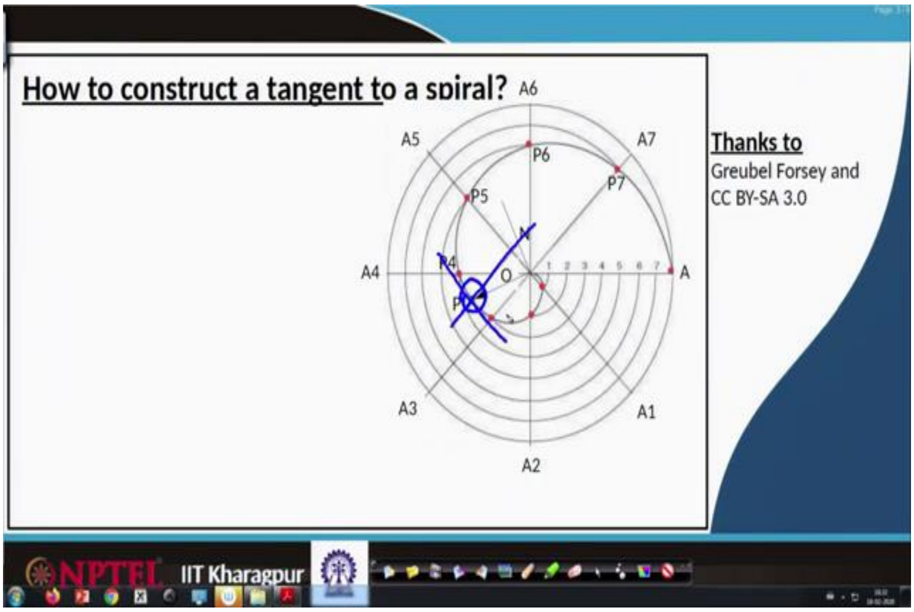

The next question is how to draw tangent a normal to a spiral. For example, if we are going to pick some point here in between P3 and P4. For example, here we would like to pick a point. There we would like to have a normal line and a tangent line which makes 90°. Let us look its construction.

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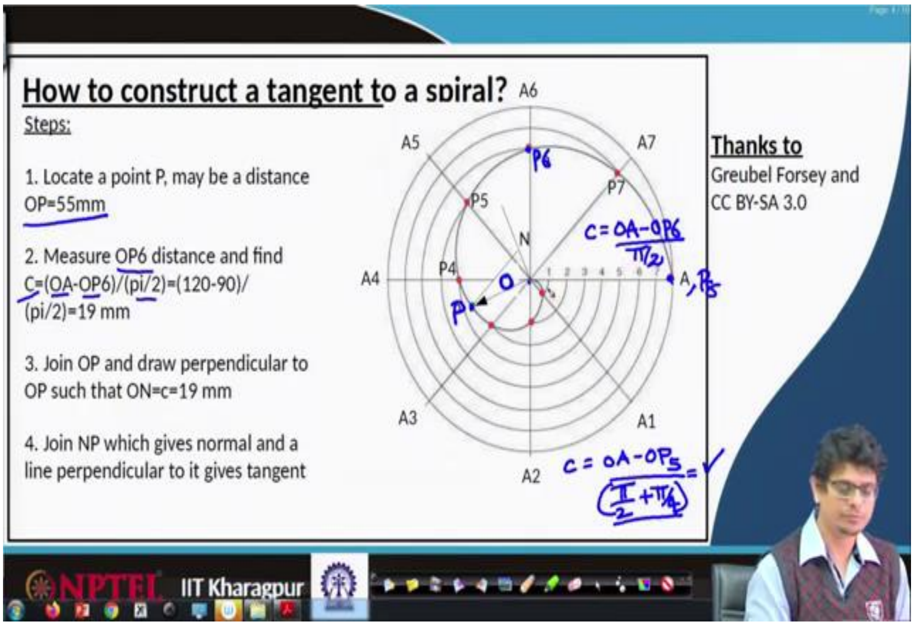

First of all, locate the point on the curve P. Name it may be at a distance O to P it might be 55 mm. Using a compass, we can make an arc of 55 mm where it is going to intersect the spiral name that point P.

Then measure O to P6 distance for a spiral we already know it moves equal distances in equal intervals of time. Continuously, the radius is changing increasing or decreasing proportionately. So, if point P6 to P8 if it is going to cover 90° angle, then naturally P6 to P7 proportionate angle we will be having something like P6 to P7 it takes pi by 4 lengths.

Using that strategy what we are going to construct is O to P6 we will measure the distance. Already we know O to A. So, we will subtract OA minus OP 6, O to A minus O to P6 by pi by 2 angles. So, the distance from O to A minus distance O to P6 covered in 90° pi by 2 angle which gives us a constant of the curve C.

For this spiral, there is a constant of the curve C is equal to OA minus OP6 by pi by 2 angles. This will remain same C can be OA minus OP5 by 90 plus pi plus pi by 4 plus this extra angle. So, pi by 2 plus pi by 4 angles. To cover this much angle the distance in terms of OA minus OP 5 it will cover, and this ratio always remains the same for a particular spiral what we have drawn.

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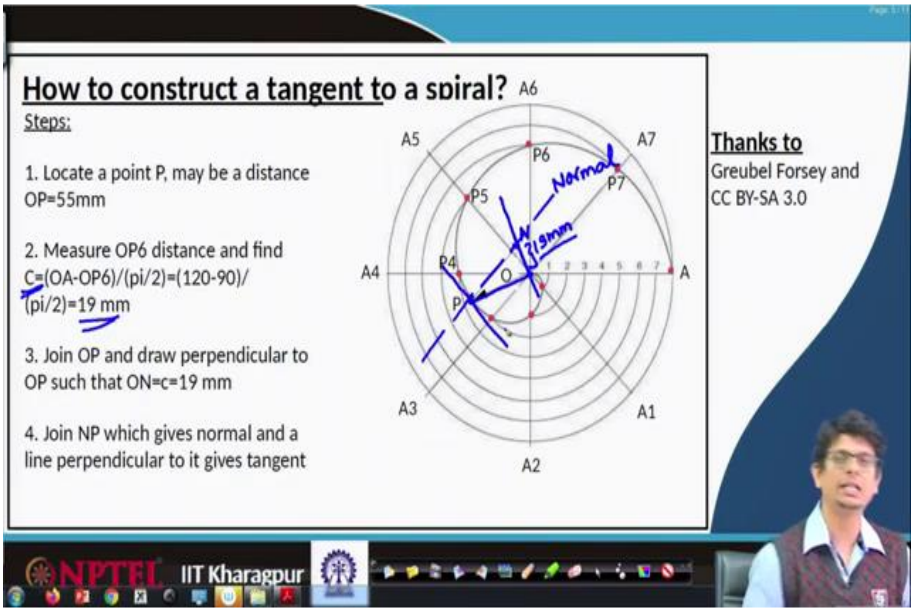

So, first of all, calculate that ratio C of the curve which will turn out to be 19 mm in our case. Now, from centre O to P join a line and draw a perpendicular line to OP in such a way that we are going to measure this distance 19 mm. So, OP and ON are perpendicular with each other, but that is at 19 mm distance.

Once it is done, we will be in a position to locate a point N. Now, join P point and N point. This will be normal to the spiral at point P. To the circle, the normal is always in the radial location, but for this spiral which is continuously moving the line which passes through P and N will be normal.Once we identify this PN PN normal, we will be in a position to make a tangent which will be at 90°. So, let us do that on sheet. On the sheet, we have already drawn a spiral. So, let us measure 19 mm on the sheet. So, 0 is this and 19 is there on the drawing sheet. So, let us look at the drawing sheet; this will be 19 mm. So, from O on the curve at let us write it C for the curve is 19 mm and the point intended point P where we would like to draw a normal tangent is at 55 mm.

So, first of all, locate both 19 mm and 55 mm. So, 55 mm is this one. So, measure this distance. From O make an arc somewhere there. So, call this point as P. So, this one small one is P. Now, join O and P. So, this is the normal to that circle locate a 90° line on that curve that will be here. Join O with that line. On that C constant of the curve is 19 mm. So, make an arc of 19 mm somewhere here. So, call this point as N. Once N is known to join P point and N point. It should be a continuous line, but just for visibility purpose, I am showing it by a dashed line.

So, this is a normal passing through point P. 90° to that we will be in a position to construct a tangent. Join these 2 lines T tangent. Usually, on drawing sheets, we do not represent complete name tangent normal. We show symbols by T T maybe N N to represent normals.

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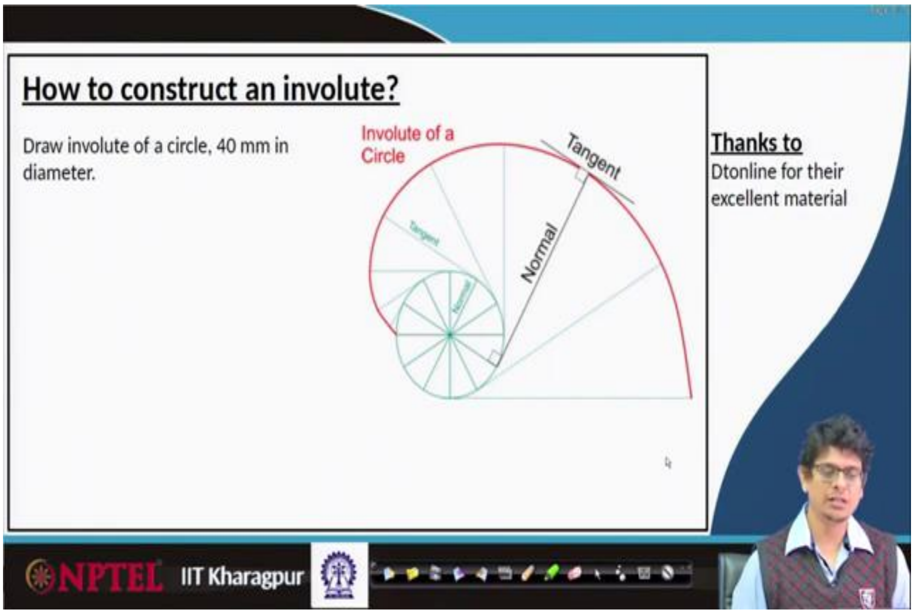

After this spiral is constructed, we will move on to construct an involute. So, involute we get it when a rope or thread is winding on a cylinder. For example, like during these machines if we want to wind a rope or thread on reels thread reels the locus of the endpoint thread the way how it moves that will be constructed or represents in involute.

So, let us construct such in involute. So, here an example a cylinder of particular radius which is winding over this rope or thread which is shown in red colour. For example, let us draw an involute of circle 40 mm in diameter. So, the circle is 40 mm in diameter.

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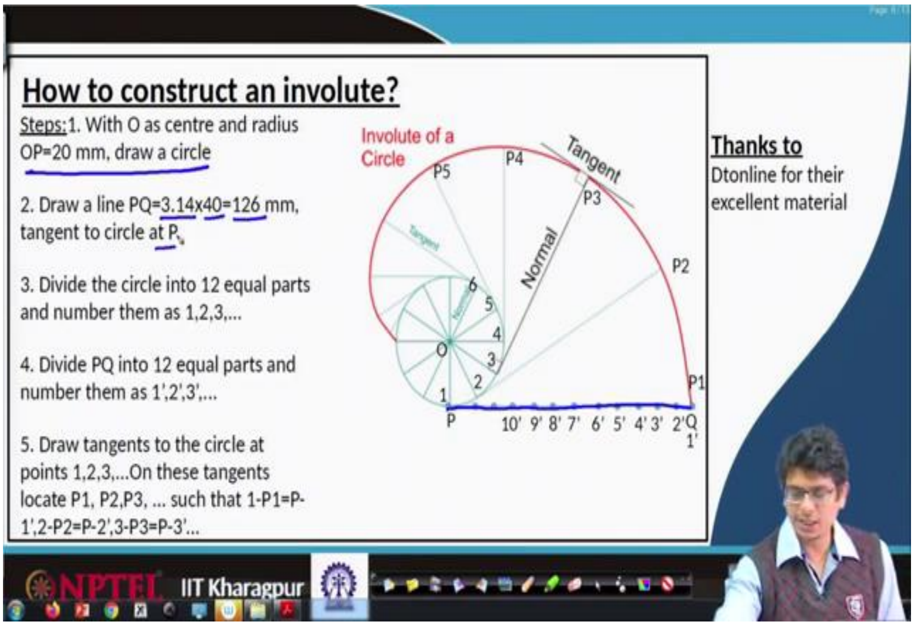

We are going to construct a radius of 20 mm. So, let us locate the points O on the drawing sheet 20 mm as the radius.

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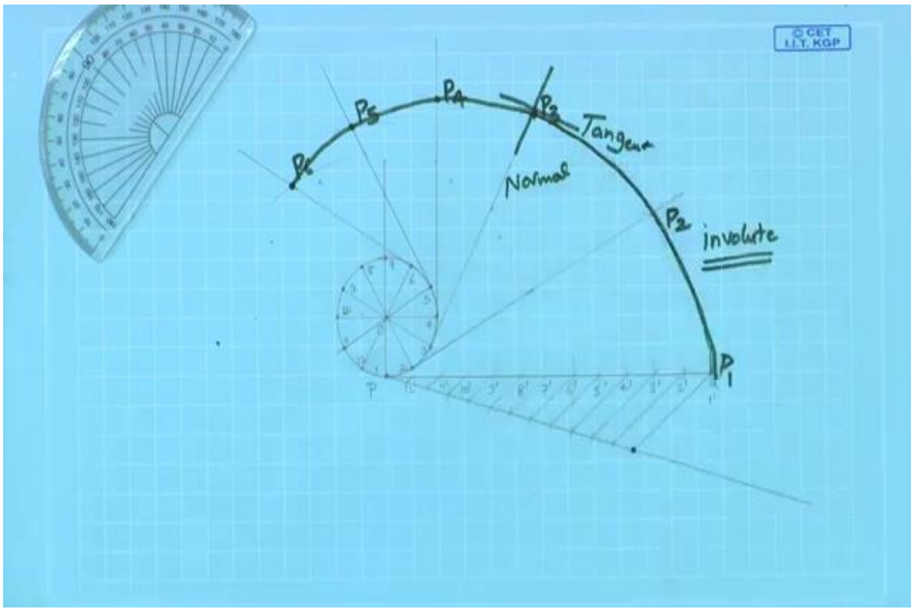

So, the endpoint is this. Draw a circle. The first step is 20 mm radius drawing a circle. Then draw a line PQ which will have a length equal to the periphery of that circle 2 pi r supposed to be the distance which is pi multiplied by d 126 mm. On that the circle if it is rotating as a base point whatever the locus of this thread endpoint moves that is what we call an involute. So, draw the line at point P with 126 mm.

So, on the drawing sheet at the bottom point, this is the point P. Let us name it a 126 mm locate it 126 somewhere here. So, draw a line. Now, divide this entire line which we call PQ line into 12 equal parts.

So, to construct 12 equal parts what we have to do is make an inclined line, use our compass to make 12 sections something 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. So, the last point we are going to join using a scale parallel to that we use our roller looks like. Construct parallel lines which are passing through these points.

Name these points as 1’ 2’ 3’ 4’ 5, 6, 7, 8, 9, 10, 11 and 12 parts. So, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 equal parts we have. Now, the circle also supposed to be divided into 12 equal parts.

So, we can use our protractor to make 12 threes a. So, 30° angle we should be in a position to construct every 30, 60, 90, 120, 150, 180. Let us join these lines and the last line this once constructed name it 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 points. These are the points through which this rope length is a winding rope.

Now, draw tangents to these points. For example, here we have to draw a tangent at every point of this. So, 8 points 2 draw a tangent join this line. Similarly, 8 points 3 draw a tangent which is at 90°. Similarly, at 4, 5 and 6. Let us extend this tangent which is passing through point 2 up. Now, point Q might be getting winded up on the cylinder.

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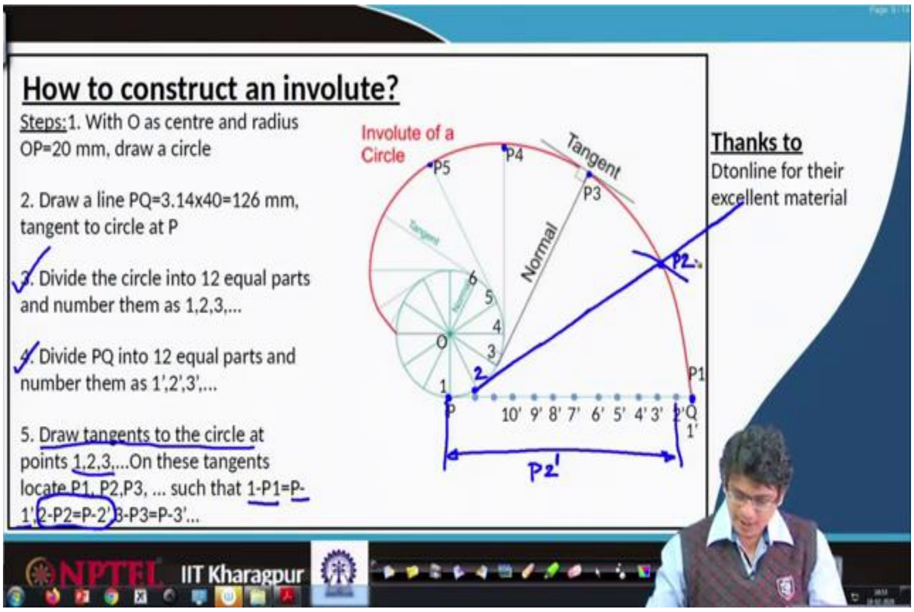

After dividing PQ line into 12 parts circle into 12 parts after drawing these tangents to that circle at point 1, 2, 3 we have to locate points P1, P2, P3, P4 and so on in such a way that 1 to P1 distance let us pick the first point 1 to P1 point equal to P2 1'. So, P point is that, and 1' is that. Similarly, we have to locate point P2 in such a way that 2 to P2 distance always be equal to P to 2' distance. So, P to 2' distance whatever it is there use that distance using compass from P try to intersect the line tangent which is passing through 2 point. So, tangent through 2 point goes in that direction. Using this P2' that distance we are going to make an arc based on centre 2 such that we will get P 2 point.

Let us do that on the sheet. Already we know this point P1 locate it on the sheet. Now, use distance P to the second point whatever the distance locate it on that point. This is P2 point.

Similarly, from P use 3' distance locate there. This will be a P3 point. Similarly, here to 4' whatever distance is there make an arc such that we will have P4. From 4 to 5, locate a distance P5. From there to 6', locate a distance P6. If we continue, it goes and makes a point somewhere here.

So, let us join by freehand sketch. We should begin from this P1 point, but just for simplicity, we are going. Let us draw that line again. If we are interested in constructing tangent and normal to that anyway, this is tangent which is going. So, this will be the normal line for us which is going via that and tangent always be perpendicular to that that will be tangent. In the next class, we will learn about helix how to construct that.

Thank you very much.