

**Engineering Drawing and Computer Graphics**  
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**Module – 02**  
**Lecture - 18**  
**Conic Sections - X**

Hello everyone. Welcome to our NPTEL online certification courses on Engineering Drawing. We are in module number 2, lecture number 18. We are covering Conic Sections, especially on special curves.

(Refer Slide Time: 00:31)

The slide, titled "Special curves", is presented in a software window with a toolbar at the top and a page number "Page: 40 / 44" in the top right corner. It features four categories of curves:

- Cycloid**: Shown as a series of circles rolling along a horizontal line, with a red curve tracing the path of a point on the circumference. The parametric equations are given as  $x = a(t - \sin t)$  and  $y = a(1 - \cos t)$ .
- Spiral**: Shown as a red Archimedean spiral with the polar equation  $r = a\theta$ .
- Involute**: Shown as a red curve unwinding from a blue circle. Labels include "circle involute" and "circle".
- Helix**: Shown as a red spiral winding around a blue rectangular prism.

At the bottom of the slide content, it says "In next class, we will learn their construction". On the right side of the slide, there is a note: "Thanks to Wolfram mathworld". The footer of the slide contains the NPTEL logo, "IIT Kharagpur", and the IIT Kharagpur logo.


In the last class, we have introduced the special curves, namely cycloid, a spiral, an involute and a helix and we stopped to learn about these construction in the next class.

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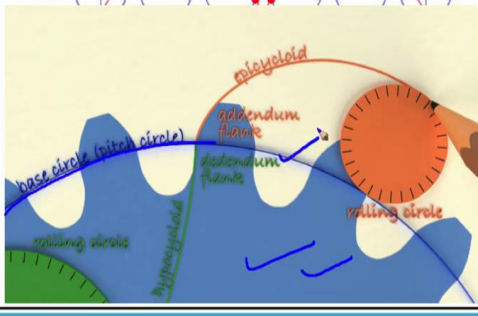
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### Where do we see cycloid curves?

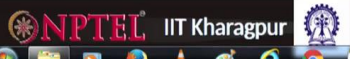
**Cycloid**




$$x = a(t - \sin t)$$

$$y = a(1 - \cos t)$$


**Thanks to**  
Wolfram mathworld





So, in today's class, we are going to learn about a special curve name, cycloid. Where do we see this cycloid curves, first of all, we will ask? For example, if we look at a bicycle, motor wheels where sprocket and gears are located or any automobile engineering machinery, farm machinery anything you open where gears are predominantly located these cycloid curves are quite common.

The cycloid curves give better efficiency in mating the gear so that frictional losses will be a bit less compared to any other kind of mating surfaces; they will be a bit smooth and transmit power in line with the shaft with very less slip on the surfaces. So, if we are looking at carefully, the machine gear where these teeth will be joining with the other teeth let us look at that part.

For example, this blue one is machine gear. This machine gear might be constructed based on a base pitch circle base circle or pitch circle. So, based on a certain radius for example, if this is the gear what we are looking at on the right-hand side if this is the centre let us call this one centre from here to somewhere at a mean level of these gear whatever the radius we are going to use and construct a circle that circle what we call a base circle.

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Slide 41: Where do we see cycloid curves?

$x = a(t - \sin t)$   
 $y = a(1 - \cos t)$

Thanks to Wolfram mathworld

Cycloid

The diagram shows a gear-like base circle (blue) and a smaller rolling circle (orange) moving along it. A cycloid curve (red) is traced by a point on the rolling circle. The curve is divided into sections: 'addendum flanks' (top) and 'dedendum flanks' (bottom). A gear-like shape is shown on the left, and a gear-like shape is shown on the right. The base circle is labeled 'base circle (pitch circle)'. The rolling circle is labeled 'rolling circle'. The cycloid curve is labeled 'cycloid'. The gear-like shape on the left is labeled 'gearing circle'. The gear-like shape on the right is labeled 'gearing circle'. The gear-like shape on the right is also labeled 'hypocycloid'.

So, this is that base circle, above the base circle, we have these flanks. So, these are called flank portions let us use other colours this one is one of the flanks, this is the other flank, these flanks on the top side we will see and also on the bottom side we will see.

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Slide 42: Where do we see cycloid curves?

$x = a(t - \sin t)$   
 $y = a(1 - \cos t)$

Thanks to Wolfram mathworld

Cycloid

The diagram is identical to Slide 41, but with blue circles highlighting the 'addendum flanks' and 'dedendum flanks' sections of the cycloid curve.

Furthermore, if we are looking at this top portion that flanks are called addendum flanks and the down ones are called dedendum flanks.

(Refer Slide Time: 03:38)

Slide 43: Where do we see cycloid curves?

$x = a(t - \sin t)$   
 $y = a(1 - \cos t)$

Thanks to Wolfram mathworld

Cycloid

Rollin

Diagram illustrating the construction of a cycloid curve. A base circle (blue) is shown with a rolling circle (orange) moving along its circumference. The path of a point P on the rolling circle is traced as a cycloid curve. Labels include: base circle (blue circle), rolling circle (orange circle), cycloid, and rolling circle. A gear-like image is shown on the left.

Furthermore, this curve usually constructed by cycloid. For example, on this base circle let us consider one more circle this orange one is rolling if this one is rolling in this direction let us pick this point the point P the curve in which direction it forms or tracks that is called we call cycloid.

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Slide 44: Where do we see cycloid curves?

$x = a(t - \sin t)$   
 $y = a(1 - \cos t)$

Thanks to Wolfram mathworld

Cycloid

Diagram illustrating the construction of a cycloid curve. A base circle (blue) is shown with a rolling circle (orange) moving along its circumference. The path of a point P on the rolling circle is traced as a cycloid curve. Labels include: base circle (blue circle), rolling circle (orange circle), cycloid, and rolling circle. A gear-like image is shown on the left.

In that part of the circle above these base circle up to certain length, we call that as epicycloid because it is running over another circle, cycloids for which we usually roll it on a straight line for these epicycloids on top of that point and this circle continuously rolling on another circle.

Similarly, the bottom of the curve constructed by hypocycloid, for example, there is a line on which this green one is rolling then part of the curve in which direction it moves that is what we call this hypocycloid. So, cycloids are quite common for gear construction.

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Slide 47/51: **Where do we see cycloid curves?**

**Cycloid**

$$x = a(t - \sin t)$$

$$y = a(1 - \cos t)$$

Thanks to Wolfram mathworld

Ischronus pendulum

The slide features a series of five circles representing the path of a rolling wheel. A red dot on the bottom of each circle traces a cycloid curve. To the left is a gear icon. To the right is a diagram of an isochronous pendulum with a cycloid-shaped path. The NPTEL IIT Kharagpur logo is at the bottom.

Similarly, if we have pendulums isochronous pendulums here a pendulum connected by a rope and this rope is going up the location of this point with time and the curve along which this point is going up and down.

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Slide 48/52: **Where do we see cycloid curves?**

**Cycloid**

$$x = a(t - \sin t)$$

$$y = a(1 - \cos t)$$

Thanks to Wolfram mathworld

Ischronus pendulum

This slide is identical to slide 47/51, showing the cycloid curve, gear icon, and isochronous pendulum diagram. The NPTEL IIT Kharagpur logo is at the bottom.

If we track these curves makes cycloids, similarly this one also a cycloid.

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### Where do we see cycloid curves?

$x = a(t - \sin t)$   
 $y = a(1 - \cos t)$

**Cycloid**

In Arch-Kimball art museum

Isochronous pendulum

Thanks to Wolfram mathworld

Similarly, if we are looking at arches, for an architectural point of view, the top portions make cycloid curves.

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### How to construct a cycloid curve?

1. Draw the generating circle and the base line equal to the circumference of the generating circle
2. Divide the circle and the base line in to equal number of parts. Draw perpendicular lines from the division of the line

Thanks to Raviteja Kasu for his wonderful post on cycloid

Now how to construct a cycloid curve geometrically? To do that first of all we have to use a generating circle which is rolling on a baseline, our generating circle is this, and this is rolling on this line base, and then we locate a point P, track this motion of this point P; for that what we do is divide this entire circle into 12 equal parts.

(Refer Slide Time: 07:00)

**How to construct a cycloid curve?**

1. Draw the generating circle and the base line equal to the circumference of the generating circle
2. Divide the circle and the base line in to equal number of parts. Draw perpendicular lines from the division of the line

**Thanks to**  
Raviteja Kasu for  
his wonderful post  
on cycloid

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In that way, we make 12 equal parts, and we will name them, 1 2 3 4 and so on all the way.

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**How to construct a cycloid curve?**

1. Draw the generating circle and the base line equal to the circumference of the generating circle
2. Divide the circle and the base line in to equal number of parts. Draw perpendicular lines from the division of the line
3. With compass and circle radius make arcs on the lines with centers as C1, C2, C3,....

**Thanks to**  
Raviteja Kasu for  
his wonderful post  
on cycloid

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Once that is done we draw parallel lines to these points 1 2 3 4 5 6. So, 1 line from there we draw a parallel line similarly from point 2 we are going to draw a parallel line from 3 4 5 6 again from 7 8 and so on we repeat to this point P.

After that with using a compass, we are going to locate the intersection points of this curve on these horizontal lines. So, after the division of this circle, we make 12 equal parts from 1 2 3 4 6 we draw horizontal lines then the line which is passing through the centre we divide that into an

equal number of parts from extending this 1 2 3 4 5 in the perpendicular direction pick each one as centre mark an arc on the first line.

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**How to construct a cycloid curve?**

1. Draw the generating circle and the base line equal to the circumference of the generating circle
2. Divide the circle and the base line in to equal number of parts. Draw perpendicular lines from the division of the line
3. With compass and circle radius make arcs on the lines with centers as C1, C2, C3,....

**Thanks to**  
Raviteja Kasu for his wonderful post on cycloid

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For example, pick this one mark an arc on that first line pick C1 mark an arc, pick C2 to mark an arc, C3 mark an arc and so on joining these things so, that it forms a cycloid. Let us look at that procedure step by step using our graph sheet.

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**How to construct a cycloid curve?**

1. Draw the generating circle and the base line equal to the circumference of the generating circle
2. Divide the circle and the base line in to equal number of parts. Draw perpendicular lines from the division of the line
3. With compass and circle radius make arcs on the lines with centers as C1, C2, C3,....
4. locate the points which are produced by cutting arcs and joining by a smooth curve.

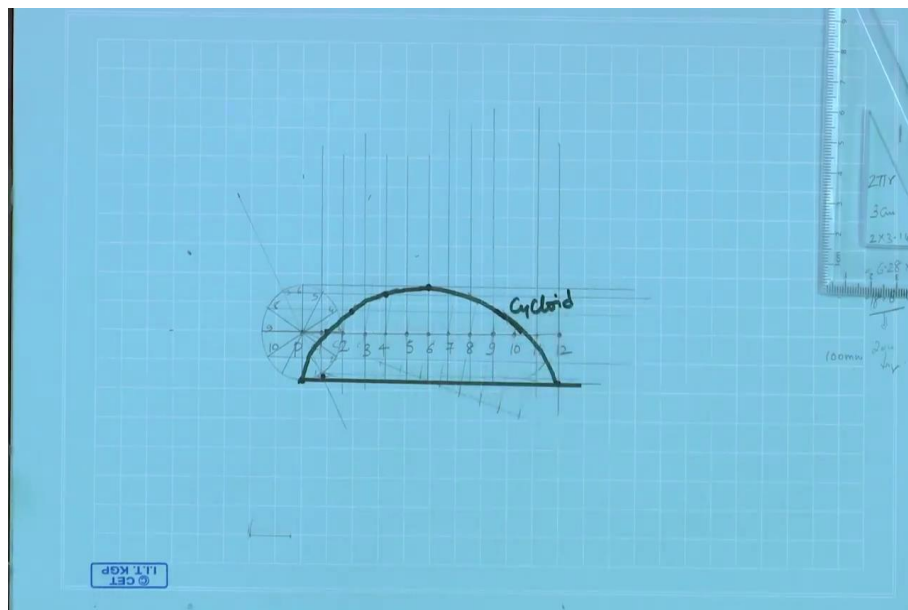
**Thanks to**  
Raviteja Kasu for his wonderful post on cycloid

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First, we have to draw a circle of chosen radius and know that  $2\pi r$  distance draw a horizontal line.



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So, let us use a generating line  $2\pi r$  we have to do. Maybe let us pick something like 3 centimetres. So,  $2\pi r$  length 2 into 3.14 into 3 centimetres. So, 8.26 into 3.2467818 so, 18.84 number we have to use. Now a geometry this is always difficult in terms of choosing it.

So, usually, we draw something like the nearest number something like 20 centimetres kind of line then construct what might be the equivalent radius use that equivalent radius construct a circle. So, we will do it in this same way. First of all, let us construct a baseline of 10 units. So, the baseline begins here ends here, which is 100 millimetres is the baseline.

Let us call that point 1 here, and 12 divide this 100 mm into 12 equal divisions. So, draw an inclined line divide that into 12 equal divisions 1 2 3 4 5 6 7 8 9 10 11 and 12. Now join this point to 12, we have to use go parallelly in this direction. So, that we can make we can mark these points ok, go carefully 12 divisions we have to mark.

So, these are the points, mark them as 1 2 3 4 5 6 7 8 9 10 11, I think we made this is 12 let us use an equal number of divisions. So, this length, we are going to measure it.

So, this length whatever that we are going to make it into 12 equal parts 1 2 3 4 5 6 7 8 9 10 11 ok we are right into 12 points the notation is this begins with P point to 12. So, let us call this point P 1 2 3 4 5 6 7 8 9 10 11 and 12 points that is 100 mm. So, use your calculator to calculate what is the equivalent radius  $2\pi r$  is equal to 100 mm. So, 100 by 2 by 3.1416 that gives you 15.9 mm this is the approximate circle what we can use.

So, let us first of all mark 15 to 16 mm. So, the least count what we can have is 16 mm in this case, otherwise what we can use is take the bigger circle. So, that we will be in a position to increase that least count and the length also increases. So, use this length ok draw a circle from P point divide this circle into 12 equal parts; that means,  $30^\circ$  angle is the one what we can use it.

So, mark 30 60 90 120 150 180, use our scale to join these lines similarly join this  $30^\circ$  line, join this one also. So, we make 12 divisions; we can have 24 divisions and so on so that a better curve can be tracked. Now name them carefully, always point P somewhere here.

So, this line on this, we can now draw this line parallel to that because the circle is rolling on a line. So, this is the line on which it is going to roll, and these are the centres. Now draw perpendiculars through this line this is the line and through that passing through these points construct 1 2 3 4 these lines go all the way to base one has to be careful with these construction lines, and it goes all the way to 6 7 8 9 11 and then 12 lines.

Let us darken the base this is the base on which our circle is rolling these are the centre lines so, C1 C2 C3 and so on and draw horizontal lines going through these points now this is the curve; let us mark these points as 1 2 3 4 5 6 and so on.

So, let us draw few horizontal lines first one, the second one, the third one goes there, the fourth one also goes in that way 5 and 7 goes in that way, and 6 also goes in that way. So, on these lines, we have to make arcs. So, first of all, what we have to do is with compass and circle radius make arcs on the lines with the centre as C1 C2 C3 and so on. So, first of all, this is the radius what we can locate.

Now, from C1 make an arc on 1 somewhere there locate that point, so the first point is that, the second point is that from C2 locate it on line 2. So, one locate it there, from 2nd point locate a curve here and 3rd point locates it, and 4th point locates it, 5th point makes an arc, now 6th one this. So, let us join these points 1st one, 2nd one, 3rd one to one somewhere we have lost.

So, let us join these points which are going through it. Let us extend that for the 7th one again goes there 8th one on 8th one 10th one on to the 10th. So, 7 8 9 10 line is this. So, from 10 make an arc, and 11th one on that and 12th again comes to that point, this is the way we construct a cycloid. In the next class, we will learn about how to construct a spiral and involute.

Thank you very much.