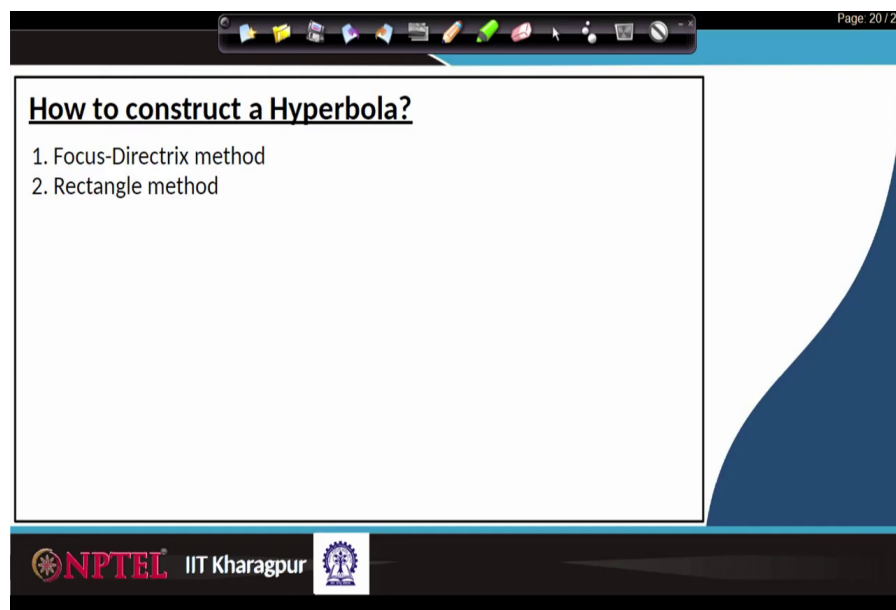


Engineering Drawing and Computer Graphics
Prof. Rajaram Lakkaraju
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Module – 02
Lecture - 17
Conic Sections - IX

Engineering Drawing and Computer Graphics; We are in module number 2, lecture number 17.
We are learning about Conic Sections.

(Refer Slide Time: 00:24)



Page: 20/25

How to construct a Hyperbola?

1. Focus-Directrix method
2. Rectangle method

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In this module, we have already covered how to construct an ellipse and also a parabola. In today's class, we will learn about how to construct a hyperbola. There are two methods quite popular for constructing hyperbola; one is focus-directrix method, the other one is rectangle method. And we are going to learn about the focus-directrix method.

To recall the focus-directrix method, there is a directrix and a focus, away from this directrix at a certain distance. And when eccentricity is greater than 1 for example like 2 eccentricity here, it constructs a hyperbola.

(Refer Slide Time: 00:51)

How to construct a Hyperbola?

When eccentricity
 $< 1 \rightarrow$ **Ellipse**
 $= 1 \rightarrow$ **Parabola**
 $> 1 \rightarrow$ **Hyperbola**

eccentricity = $\frac{\text{distance of point from focus}}{\text{distance from directrix}}$

The diagram shows a coordinate system with a focus and a vertical directrix. Three curves are plotted: an ellipse (eccentricity = 0.5), a parabola (eccentricity = 1), and a hyperbola (eccentricity = 2). A point on the hyperbola is shown with its distance to the focus and its distance to the directrix, illustrating that the ratio is 2.

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The basic definition of this eccentricity and focus from the directrix is, from focus if we are going to measure any point on that curve hyperbola that distance to the distance of that point to the directrix always is equal to the same number. For example, if eccentricity is 2, let us pick this point P from focus to that point and from there to directrix its makes equal ratio 2, whether this point or this or any point. Using this principle, we are going to construct a hyperbola.

So, after the construction of these hyperbola using the focus-directrix method, we will end up with this curve.

(Refer Slide Time: 02:05)

Hyperbola using Focus-Directrix method

The diagram shows a hyperbola with vertices A and B, and foci C and C'. A vertical directrix is shown. Points P1, P2, P3, P4 are marked on the hyperbola. Lines are drawn from each point to the focus and to the directrix, illustrating the focus-directrix method of construction.

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Let us do that step by step how to construct it. The first thing, draw a directrix AB and axis CC'.
First, we have to draw AB and CC' also we have to draw.

(Refer Slide Time: 02:26)

Focus-Directrix method for Hyperbola

Steps to be followed:

1. Draw directrix AB and axis CC'
2. Mark F on CC' such that CF=50 mm
3. Divide CF into 2+3 = 5 equal parts and mark V at the second division from C
 $e = FV/CV = 3/2 = 1.5$
4. At V, draw perpendicular VG=VF and then join GC
5. Mark few points 1, 2, 3, on VC' and draw perpendiculars through them meeting CG at 1', 2', 3',...
6. With F as centre and radius 1-1', cut two arcs on the perpendicular through 1 to locate P1 and P1'
Similarly, with F as a centre and radii=2-2', 3-3' etc, cut arcs on the corresponding perpendiculars to locate P2 and P2', P3 and P3'

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Once done, mark focus point F on CC'.

(Refer Slide Time: 02:55)

Focus-Directrix method for Hyperbola

Steps to be followed:

1. Draw directrix AB and axis CC'
2. Mark F on CC' such that CF=50 mm
3. Divide CF into 2+3 = 5 equal parts and mark V at the second division from C
 $e = FV/CV = 3/2 = 1.5$
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So, somewhere F we have to mark it in such a way that C to F is already given 50 mm, with that 50 mm locate it. Once that is done divide CF into 5 equal parts.

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Focus-Directrix method for Hyperbola

Steps to be followed:

1. Draw directrix AB and axis CC'
2. Mark F on CC' such that CF=50 mm
3. Divide CF into 2+3 = 5 equal parts and mark V at the second division from C
 $e = \frac{VF}{CV} = \frac{3}{2} = 1.5$
4. At V, draw perpendicular VG=VF and then join GC
5. Mark few points 1, 2, 3, on VC' and draw perpendiculars through them meeting CG at 1', 2', 3',...
6. With F as centre and radius 1-1', cut two arcs on the perpendicular through 1 to locate P1 and P1'
Similarly, with F as a centre and radii=2-2', 3-3' etc, cut arcs on the corresponding perpendiculars to locate P2 and P2', P3 and P3'

Thanks to Engineering drawing by Prof. D.A. Johle

$e = \frac{3}{2} = 1.5$

For hyperbola of eccentricity 2, we are going to construct. So, first, we divide this C to F into 5 equal parts in such a way that eccentricity 3 by 2 are 1.5 we are going to construct.

So, once it is divided into 5 equal parts, two parts location we are going to locate V point. So, C to V is 2 units, and V to F is 3 units, in that way we divide this entire C to F part.

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Focus-Directrix method for Hyperbola

Steps to be followed:

1. Draw directrix AB and axis CC'
2. Mark F on CC' such that CF=50 mm
3. Divide CF into 2+3 = 5 equal parts and mark V at the second division from C
 $e = \frac{VF}{CV} = \frac{3}{2} = 1.5$
4. At V, draw perpendicular VG=VF and then join GC
5. Mark few points 1, 2, 3, on VC' and draw perpendiculars through them meeting CG at 1', 2', 3',...
6. With F as centre and radius 1-1', cut two arcs on the perpendicular through 1 to locate P1 and P1'
Similarly, with F as a centre and radii=2-2', 3-3' etc, cut arcs on the corresponding perpendiculars to locate P2 and P2', P3 and P3'

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Once that is done, we draw a perpendicular VG is equal to VF passing through V point. So, from V point draw a perpendicular line in such a way that V to F whatever the distance, V to G is also the same distance.

Then, join G and C in that way then, mark a few points 1, 2, 3 on VC'. So, V point is known, CC' point is known. So, what we are going to do is divide into a number of parts.

(Refer Slide Time: 04:40)

Page: 26 / 31

Focus-Directrix method for Hyperbola

Steps to be followed:

1. Draw directrix AB and axis CC'
2. Mark F on CC' such that CF=50 mm
3. Divide CF into 2+3 = 5 equal parts and mark V at the second division from C
 $e = \frac{FV}{CV} = \frac{3}{2} = 1.5$
4. At V, draw perpendicular VG=VF and then join GC
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Similarly, with F as a centre and radii=2-2', 3-3' etc, cut arcs on the corresponding perpendiculars to locate P2 and P2', P3 and P3'

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These are arbitrary points. And from those points draw vertical lines perpendicular lines in that way. And these lines are going to intersect the extended line CG at 1', 2', 3', 4' and so on points.

(Refer Slide Time: 05:08)

Page: 27 / 32

Focus-Directrix method for Hyperbola

Steps to be followed:

1. Draw directrix AB and axis CC'
2. Mark F on CC' such that CF=50 mm
3. Divide CF into 2+3 = 5 equal parts and mark V at the second division from C
 $e = \frac{FV}{CV} = \frac{3}{2} = 1.5$
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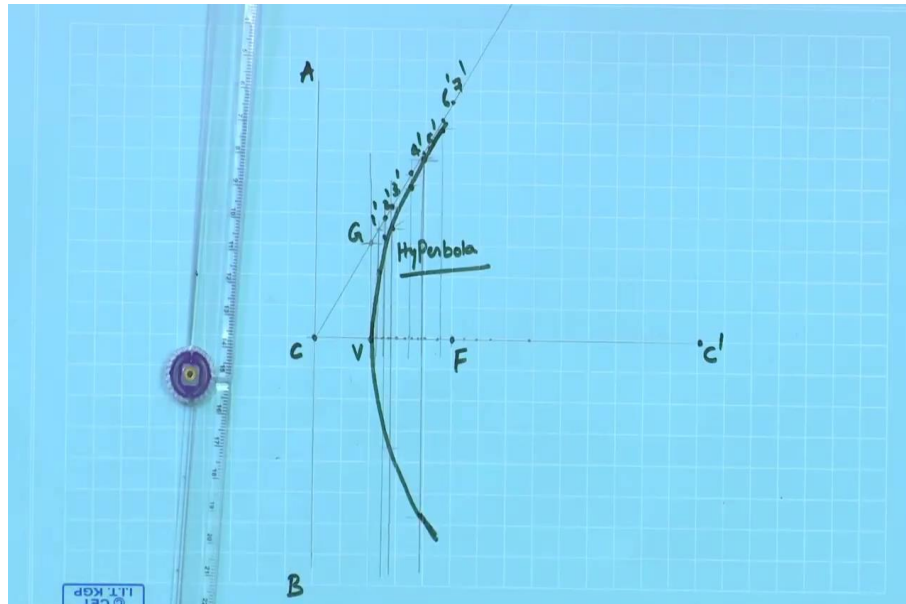
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So, locate this 1. So, where it is intersecting? That is 1-1', 2-2', 3-3', 4-4'. Now, first, measure 1-1', and from focal point use, a distance of 1-1' make an arc.

Similarly, from focal point make an arc of 2-2', similarly from focus point make an arc of 3-3' and so on. Once it is done, we will be in a position to join V all the way passing through this point, so that a hyperbola we will be in a position to construct. Let us do that on our sheet step by step.

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First, we have to draw AB line and then CC'. So, let us construct AB' a directrix. Mark A, somewhere B and in middle line horizontal perpendicular thing, we can construct CC'. So, let us call this CC'. This point is C, somewhere C'. So, two perpendicular lines construction lines we have done.

Now, mark F on CC' such that CF is equal to 50 mm. So, on the sheet locate 50 mm somewhere here. So, mark this one as the focus point. Once it is done, divide that into 5 equal parts. So, 5 equal parts first, second, third, fourth, fifth. So, in that at second point locate V because 1.5 ratio or 3 by 2 ratio we are going to construct, from focus point to the point on the curve V is 3 units and from V to directrix 2 power to 2 units. So, 3 by 2 units we are going to construct it. Once V is done, we can construct a perpendicular line passing through. So, we will be in a position to construct this perpendicular line. Now, transfer VF is equal to VG; this is the one transfer this line. Now, join V and G, a construction line again from C to G extend it, on both sides one can construct it. Once it is done, we have to locate few points 1, 2, 3 on VC'.

So, let us mark few points somewhere here, 1, 2, 3, 4, 5, 6, 7, 8, 9, maybe 10, 11, 12, 13 and 14. These are the points what we are going to locate it in such a way that our roller scale through that we will be in a position to mark. Draw a few more lines. Make sure that this is always

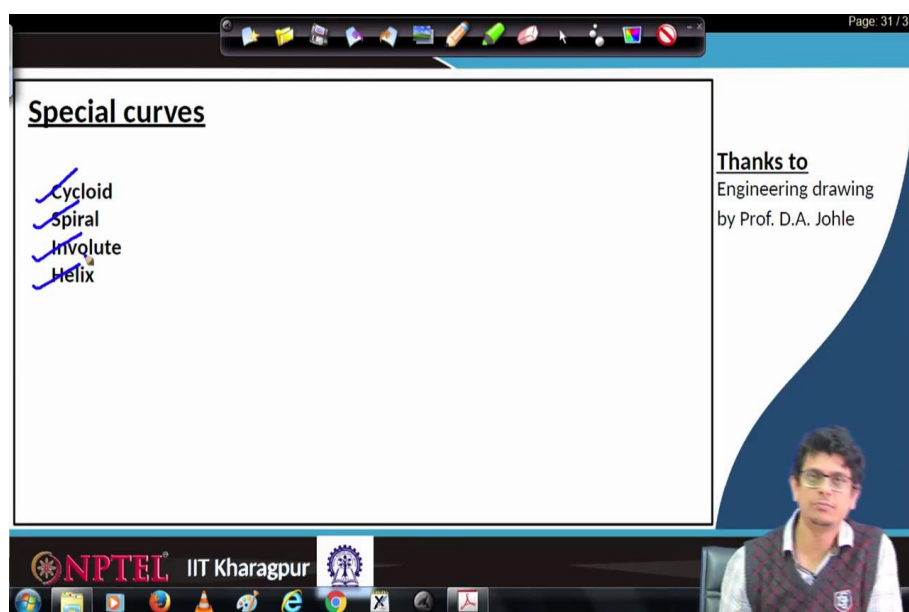
horizontal. So, we will be in a position to draw many more lines. Once that is done we have to construct transfer this lengths, the vertical lengths where they are going to intersect.

Pick this one, let us mark these intersection points 1', 2', 3', 4' and 5' and 6', anyway 7th one is going to construct 7'. Now, transfer these lengths one from the focal point all the way to intersect that, then locate the point intersection 1.

Once done, transfer this point to the second point, the second point here and then the third point and so on. Similarly, from this point 6 intersect that. This is the way we construct all these points. Now, join these points, a hyperbola is ah very fastly changing its curvature and also the slope from here. So, having many more points, we will be going to have a very smooth curve there. So, let us join these points through a smooth curve. This is the way we construct part of that curve. Let us call hyperbola.

The same lengths extending these lines down one will be in a position to construct. For example, let us pick this one and similarly pick this line. Transfer these lengths on the vertical axis because of symmetry we will be in a position to transfer these lengths, otherwise from the focal point again we have to make such kind of lengths and transfer it. Similarly, extend these lines also from 1; oh this not anymore. Use a roller scalar, construct such kind of lengths. Once it is done, we will be in a position to join in that way. This is the way we construct a hyperbola.

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So, till now we have looked at very interesting curves like ellipse, parabola and hyperbola. These are the curves generated from a cone, and we usually call them as conic sections. And

now we are going to look at special curves used in mechanical engineering. These are cycloids, spirals, involutes and helix.

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Special curves

Cycloid $x = a(t - \sin t)$
 $y = a(1 - \cos t)$

Spiral

Involute

Helix

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The first one is a cycloid. So, if we are taking a circle, for example, let us pick a circle, locate the first point something like P which is going to touch the bottom. And if the circle is rolling on a flat horizontal base, if it is rolling on these perfectly horizontal base and this P point never slips, then the motion of this P point as circle rolls whatever the curve tracked by that we call it as cycloid. For example, this P point, next instant of time reaches to somewhere there. And this entire circle rolls. So, if we are tracking or keeping a pencil or pen on that point P whatever the track we are going to get that is what we call cycloid.

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Special curves

Cycloid

Spiral

Involute

Helix

$x = a(t - \sin t)$
 $y = a(1 - \cos t)$

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And at the mathematical level the x coordinates and y coordinates, one will be represented in a parametric form x is equal to a multiplied by t minus sin t, where t is the time or parametric variation. Similarly, y is equal to a multiplied by 1 minus cos t.

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Special curves

Cycloid

Spiral

Involute

Helix

$x = a(t - \sin t)$
 $y = a(1 - \cos t)$

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So, what about this x coordinate, y coordinate? We are going to get from this parametric equations, that is what we call cycloids.

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Special curves

Cycloid
 $x = a(t - \sin t)$
 $y = a(1 - \cos t)$

Spiral
 $r = a\theta$

Involute
Helix

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Spirals. So, they begin somewhere at the centre; Archimedean spirals are quite popular in that sense. They begin as the angle increases the radius also increases. So, as we are going beginning from 0° increases to 10, 20, 30 and so on 360° , gradually the radius also increases. Radius can increase, decrease based on how we are going to move this curve if we are moving outside radius increases, as we go inside the radius decreases. These kinds of things are called spiral. And the parametric form for the spiral is r is equal to A theta. As theta increases, r also increases, for a positive definite a value and for a negative definite a value r decreases as theta increases.

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Special curves

Cycloid
 $x = a(t - \sin t)$
 $y = a(1 - \cos t)$

Involute

Helix

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The other form of the special curve is called involute. If we are winding a thread or rope around a cylinder, so, let us take a rope of fixed length l , tie it on one side of the cylinder, now spin the cylinder. The rope length continuously it tries to own the circle or cylinder and the length whatever the apparent length we are going to see that continuously decreases. And the curved track by one of the ends of this rope is called involutes.

For example, this is the cylinder. Let us consider this is the cylinder circle in two dimensions and this is the rope which perhaps we might have tied it there. Now, spin this cylinder, so the point P is perhaps winded this rope so that point P it moves on a specialized path. Gradually, the distance, the apparent distance what we are going to see from the rope point to that end of the cylinder decreases and it follows a curve named involutes.

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The slide is titled "Special curves" and contains the following content:

- Cycloid:** A diagram showing a series of circles of radius a rolling along a horizontal line. A red curve passes through the top of each circle. The parametric equations are given as $x = a(t - \sin t)$ and $y = a(1 - \cos t)$.
- Spiral:** A diagram showing a spiral starting from a center point and winding outwards. The equation $r = a\theta$ is shown next to it.
- Involute:** A diagram showing a circle of radius a with a red curve (the involute) starting from a point on the circle and unwinding around it. The labels "circle involute" and "circle" are present.
- Helix:** A 3D diagram of a cylinder with a blue helix curve wrapped around it. A blue letter 'A' is written next to the helix.

On the right side of the slide, there is a "Thanks to Wolfram mathworld" note. The slide is part of an NPTEL presentation from IIT Kharagpur, as indicated by the logos at the bottom.

The other ones are helix. So, for example, here a point which goes in a circular format, but it has axial motion also so that it rises upward direction and so on. Typically, small size air bubbles in coke bottles or perhaps in water beakers when they are rising they make this kind of spirals. So, it is a circle which is trying to move in the axial direction; such kind of things are called a helix.

For example, if we open a pen, we find a spring in it. These springs always have a constant radius, and they are continuously increasing in the axial direction. They move in that axial direction—such kind of things what we call helix.

In spirals, the radius is continuously increasing or decreasing. For helix, this radius is nearly constant, but axial directions z -axis increases.

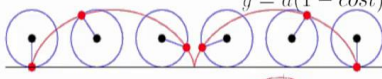
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Page: 40 / 44

Special curves

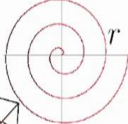
$x = a(t - \sin t)$
 $y = a(1 - \cos t)$

Cycloid

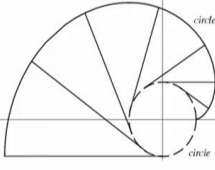


Spiral


$r = a\theta$



Involute


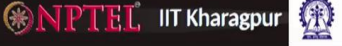


Helix



In next class, we will learn their construction

Thanks to
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In the next class, we will learn about how to construct special curves like cycloids, spirals, involutes and helix.

See you in the next class.

Thank you.