Engineering Drawing and Computer Graphics
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Module – 02
Lecture - 17
Conic Sections - IX

Engineering Drawing and Computer Graphics; We are in module number 2, lecture number 17. We are learning about Conic Sections.

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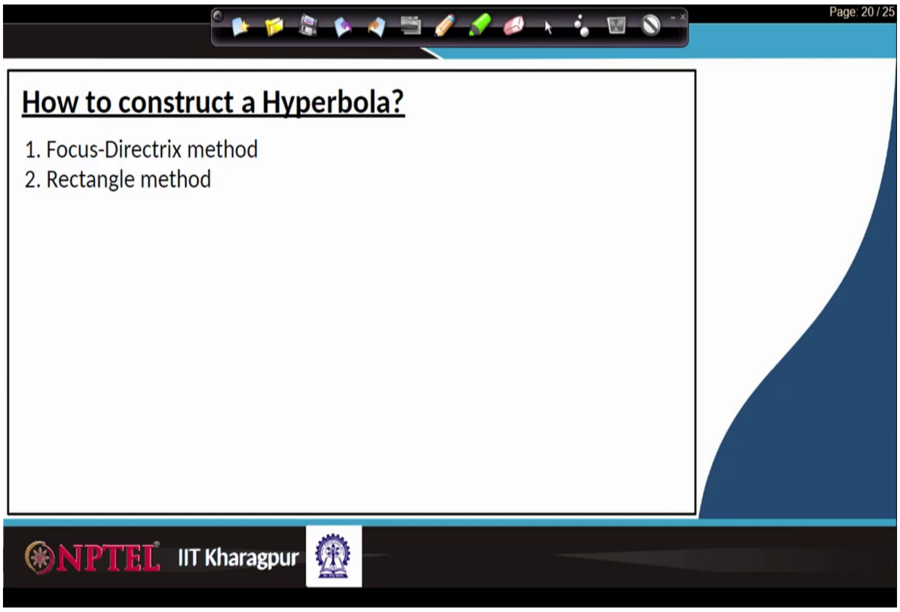

In this module, we have already covered how to construct an ellipse and also a parabola. In today's class, we will learn about how to construct a hyperbola. There are two methods quite popular for constructing hyperbola; one is focus-directrix method, the other one is rectangle method. And we are going to learn about the focus-directrix method.

To recall the focus-directrix method, there is a directrix and a focus, away from this directrix at a certain distance. And when eccentricity is greater than 1 for example like 2 eccentricity here, it constructs a hyperbola.

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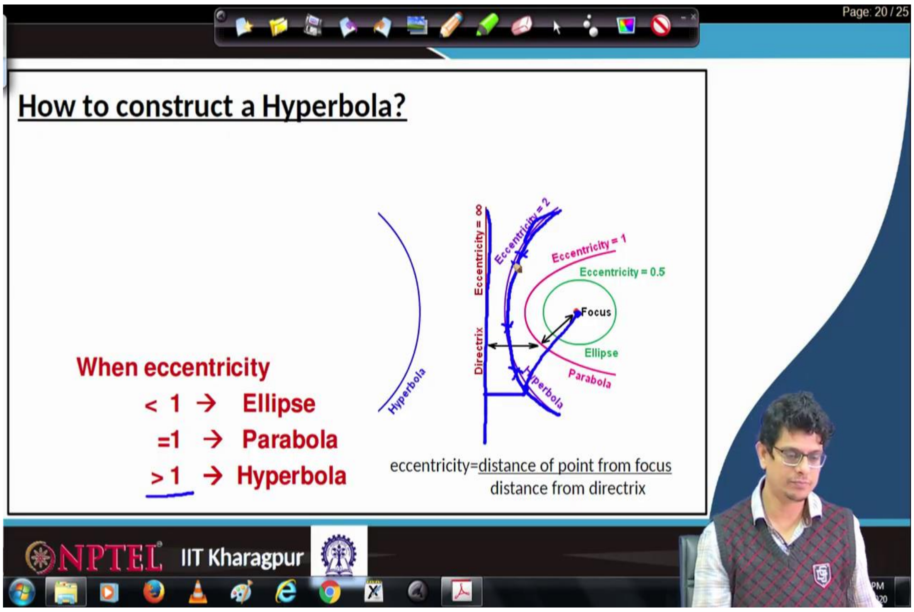

The basic definition of this eccentricity and focus from the directrix is, from focus if we are going to measure any point on that curve hyperbola that distance to the distance of that point to the directrix always is equal to the same number. For example, if eccentricity is 2, let us pick this point P from focus to that point and from there to directrix its makes equal ratio 2, whether this point or this or any point. Using this principle, we are going to construct a hyperbola.

So, after the construction of these hyperbola using the focus-directrix method, we will end up with this curve.

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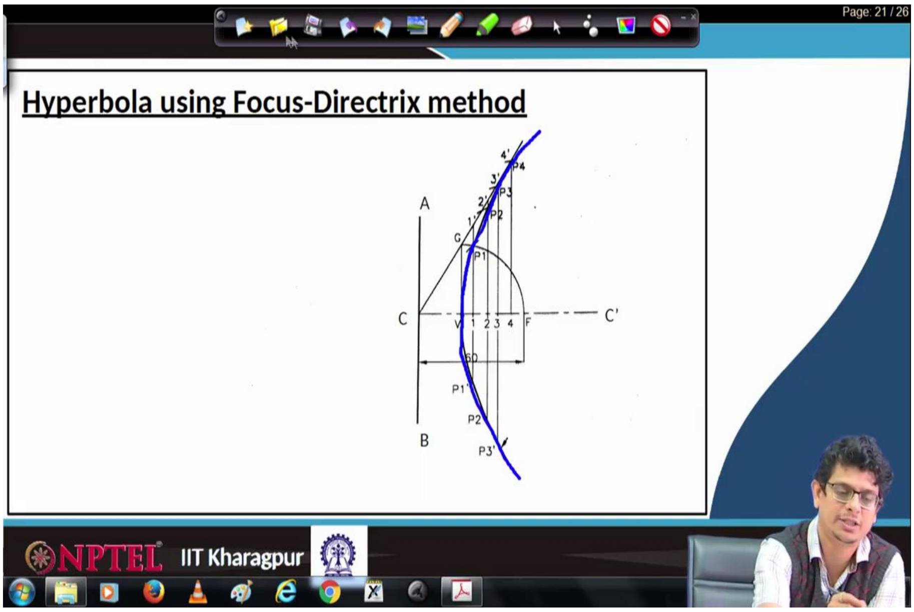

Let us do that step by step how to construct it. The first thing, draw a directrix AB and axis CC’. First, we have to draw AB and CC’ also we have to draw.

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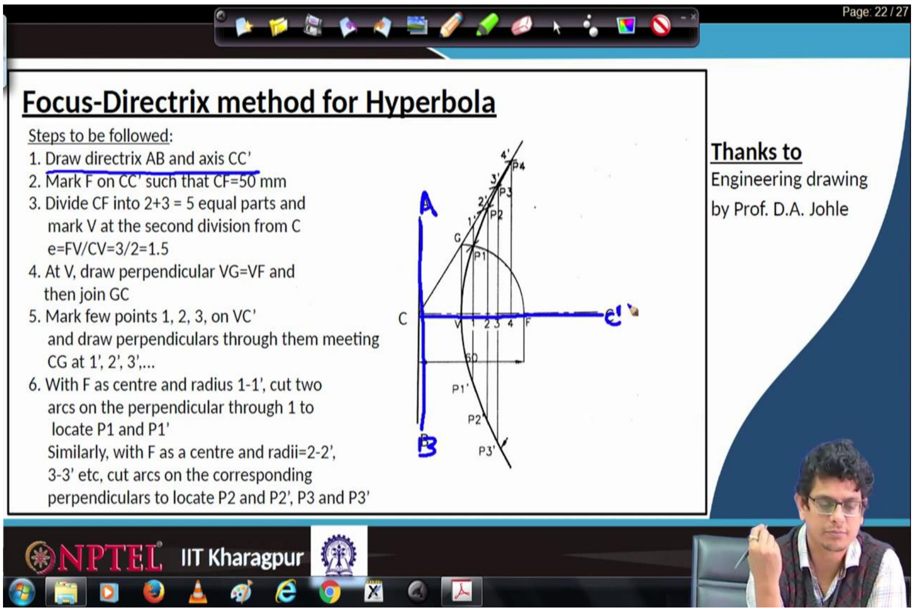

Once done, mark focus point F on CC’.

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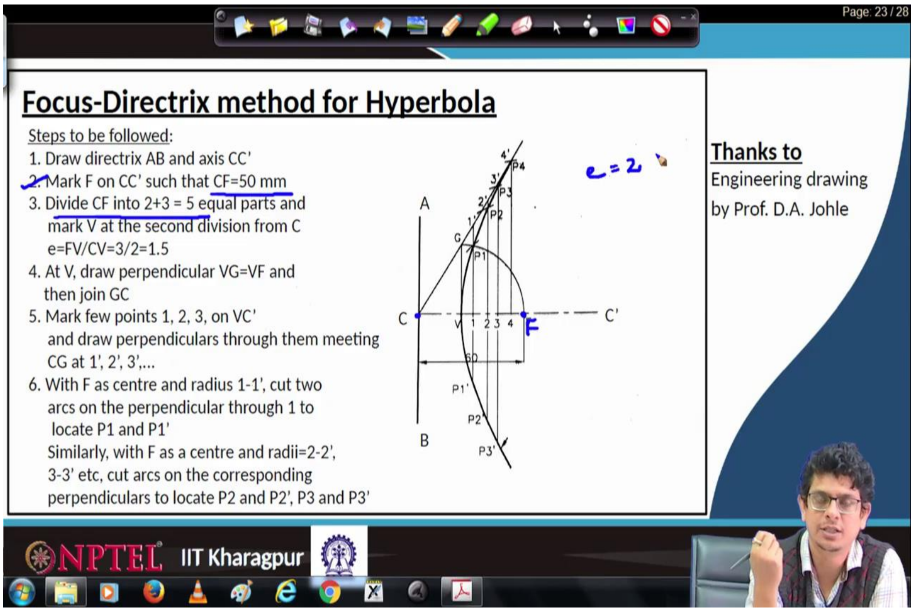

So, somewhere F we have to mark it in such a way that C to F is already given 50 mm, with that 50 mm locate it. Once that is done divide CF into 5 equal parts.

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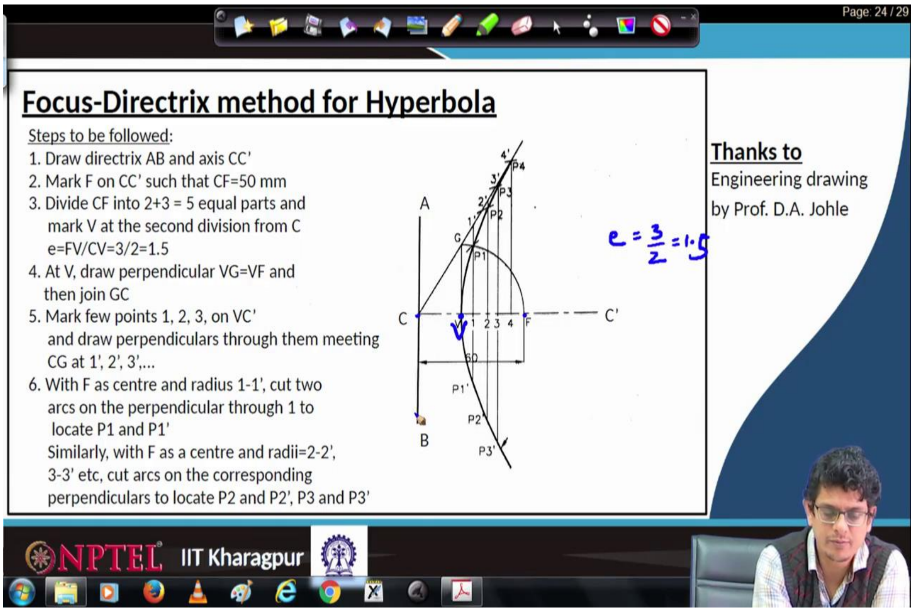

For hyperbola of eccentricity 2, we are going to construct. So, first, we divide this C to F into 5 equal parts in such a way that eccentricity 3 by 2 are 1.5 we are going to construct.

So, once it is divided into 5 equal parts, two parts location we are going to locate V point. So, C to V is 2 units, and V to F is 3 units, in that way we divide this entire C to F part.

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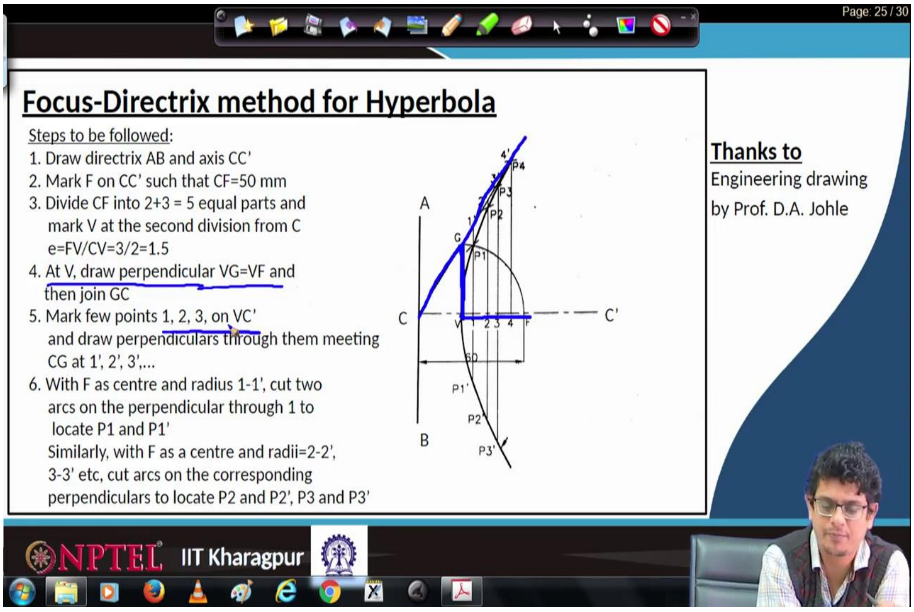

Once that is done, we draw a perpendicular VG is equal to VF passing through V point. So, from V point draw a perpendicular line in such a way that V to F whatever the distance, V to G is also the same distance.

Then, join G and C in that way then, mark a few points 1, 2, 3 on VC'. So, V point is known, CC' point is known. So, what we are going to do is divide into a number of parts.

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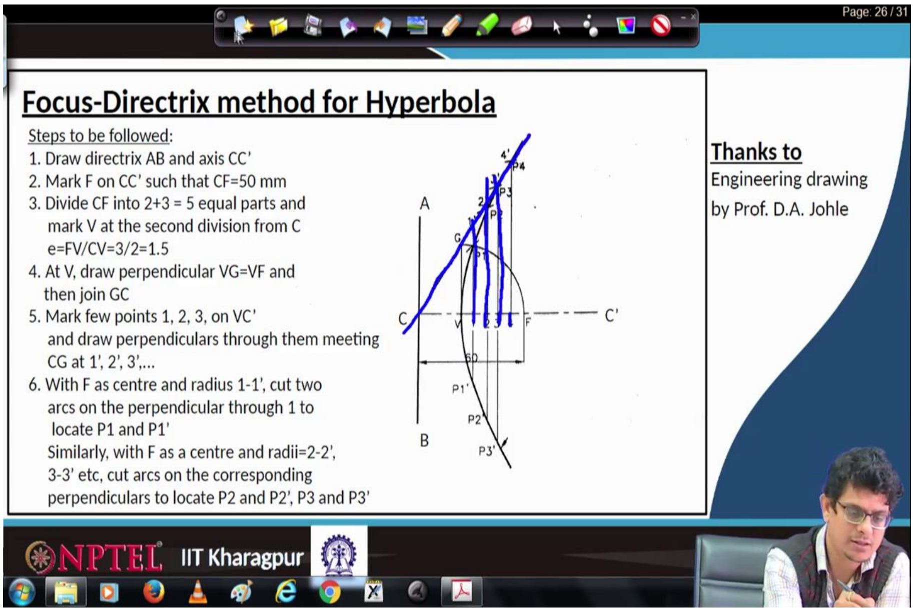

These are arbitrary points. And from those points draw vertical lines perpendicular lines in that way. And these lines are going to intersect the extended line CG at 1', 2', 3', 4' and so on points.

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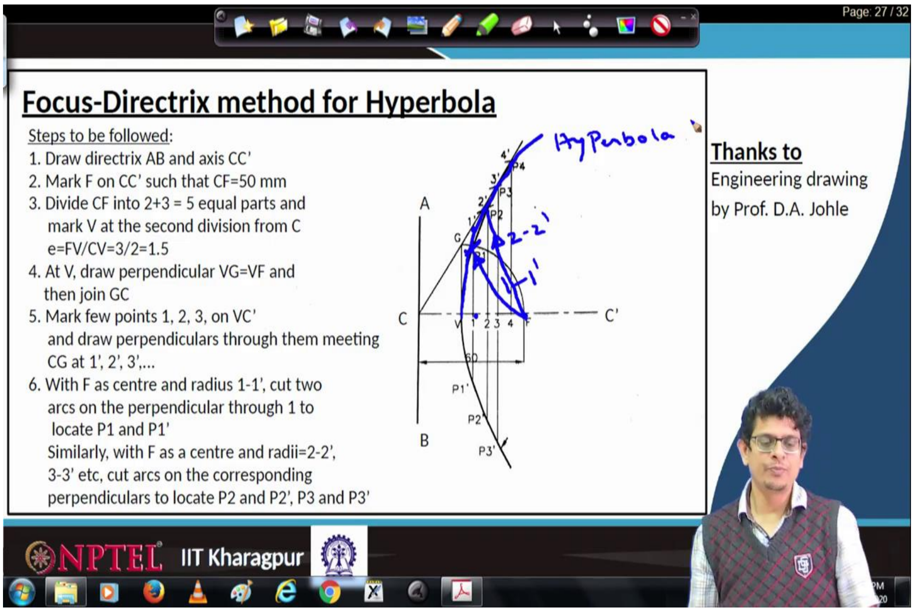

So, locate this 1. So, where it is intersecting? That is 1-1', 2-2', 3-3', 4-4'. Now, first, measure 1-1', and from focal point use, a distance of 1-1' make an arc.

Similarly, from focal point make an arc of 2-2’, similarly from focus point make an arc of 3-3’ and so on. Once it is done, we will be in a position to join V all the way passing through this point, so that a hyperbola we will be in a position to construct. Let us do that on our sheet step by step.

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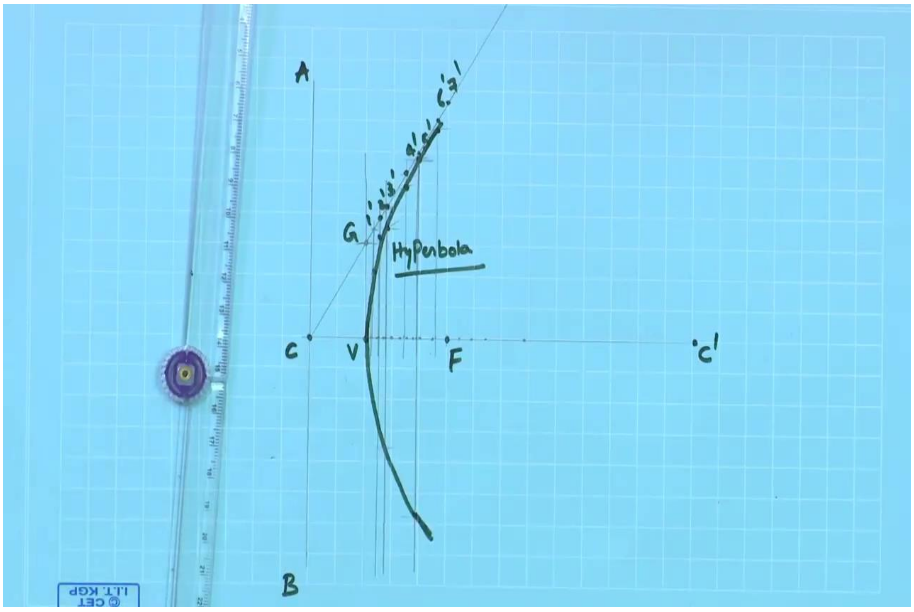

First, we have to draw AB line and then CC’. So, let us construct AB’ a directrix. Mark A, somewhere B and in middle line horizontal perpendicular thing, we can construct CC’. So, let us call this CC’. This point is C, somewhere C’. So, two perpendicular lines construction lines we have done.

Now, mark F on CC’ such that CF is equal to 50 mm. So, on the sheet locate 50 mm somewhere here. So, mark this one as the focus point. Once it is done, divide that into 5 equal parts. So, 5 equal parts first, second, third, fourth, fifth. So, in that at second point locate V because 1.5 ratio or 3 by 2 ratio we are going to construct, from focus point to the point on the curve V is 3 units and from V to directrix 2 power to 2 units. So, 3 by 2 units we are going to construct it.Once V is done, we can construct a perpendicular line passing through. So, we will be in a position to construct this perpendicular line. Now, transfer VF is equal to VG; this is the one transfer this line. Now, join V and G, a construction line again from C to G extend it, on both sides one can construct it. Once it is done, we have to locate few points 1, 2, 3 on VC’.

So, let us mark few points somewhere here, 1, 2, 3, 4, 5, 6, 7, 8, 9, maybe 10, 11, 12, 13 and 14. These are the points what we are going to locate it in such a way that our roller scale through that we will be in a position to mark. Draw a few more lines. Make sure that this is always horizontal. So, we will be in a position to draw many more lines. Once that is done we have to construct transfer this lengths, the vertical lengths where they are going to intersect.

Pick this one, let us mark these intersection points 1’, 2’, 3’, 4’ and 5’ and 6’, anyway 7th one is going to construct 7’. Now, transfer these lengths one from the focal point all the way to intersect that, then locate the point intersection 1.

Once done, transfer this point to the second point, the second point here and then the third point and so on. Similarly, from this point 6 intersect that. This is the way we construct all these points.Now, join these points, a hyperbola is ah very fastly changing its curvature and also the slope from here. So, having many more points, we will be going to have a very smooth curve there. So, let us join these points through a smooth curve. This is the way we construct part of that curve. Let us call hyperbola.

The same lengths extending these lines down one will be in a position to construct. For example, let us pick this one and similarly pick this line. Transfer these lengths on the vertical axis because of symmetry we will be in a position to transfer these lengths, otherwise from the focal point again we have to make such kind of lengths and transfer it.Similarly, extend these lines also from 1; oh this not anymore. Use a roller scalar, construct such kind of lengths. Once it is done, we will be in a position to join in that way. This is the way we construct a hyperbola.

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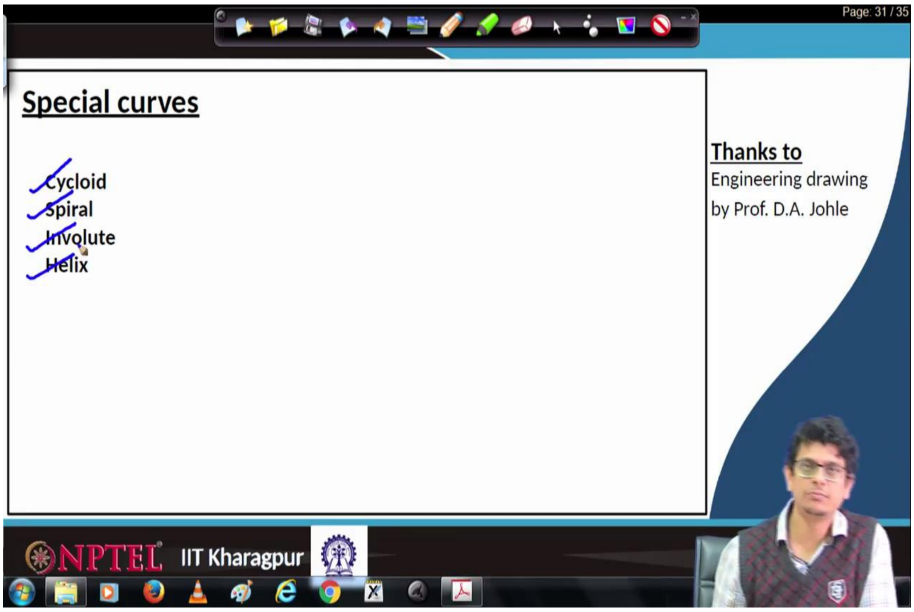

So, till now we have looked at very interesting curves like ellipse, parabola and hyperbola. These are the curves generated from a cone, and we usually call them as conic sections. And now we are going to look at special curves used in mechanical engineering. These are cycloids, spirals, involutes and helix.

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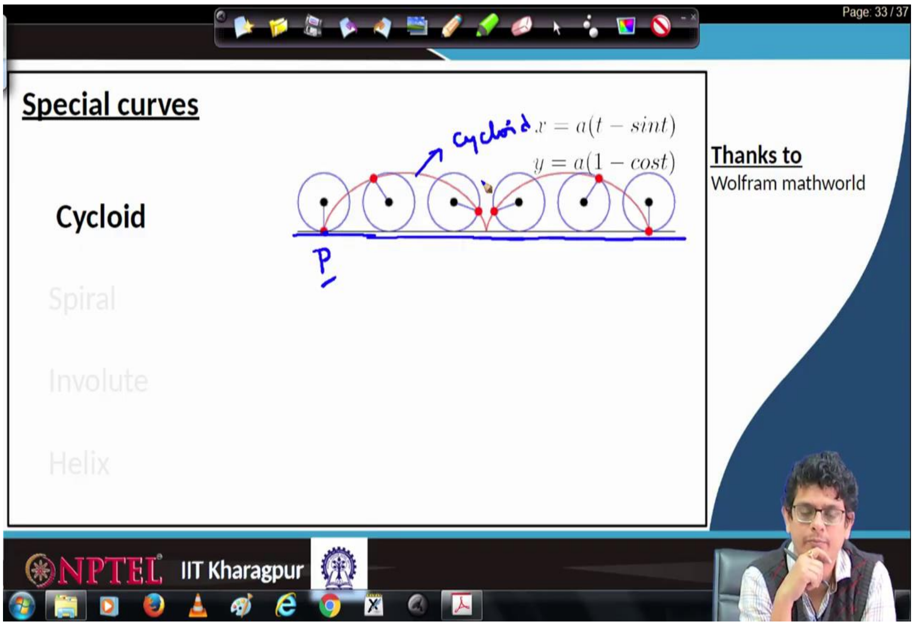

The first one is a cycloid. So, if we are taking a circle, for example, let us pick a circle, locate the first point something like P which is going to touch the bottom. And if the circle is rolling on a flat horizontal base, if it is rolling on these perfectly horizontal base and this P point never slips, then the motion of this P point as circle rolls whatever the curve tracked by that we call it as cycloid. For example, this P point, next instant of time reaches to somewhere there. And this entire circle rolls. So, if we are tracking or keeping a pencil or pen on that point P whatever the track we are going to get that is what we call cycloid.

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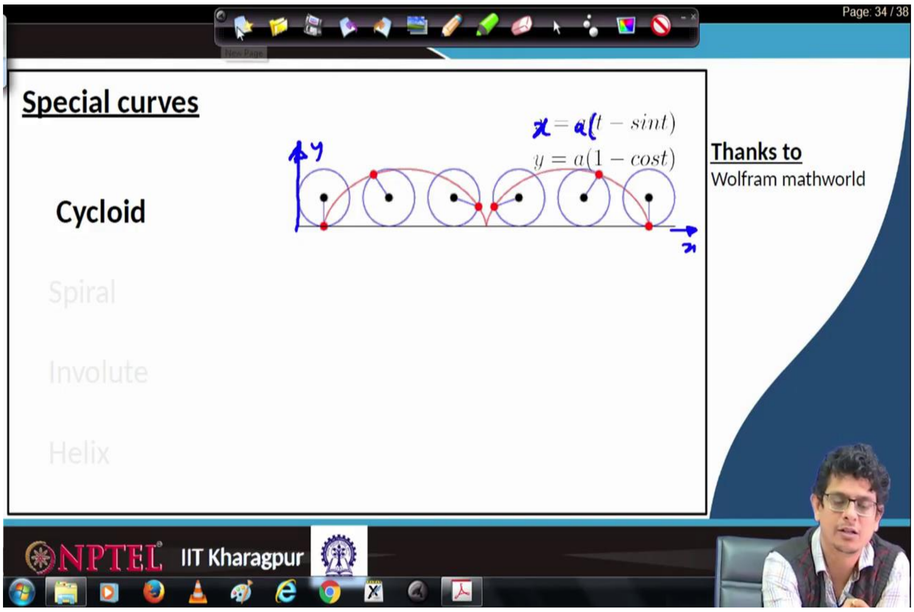

And at the mathematical level the x coordinates and y coordinates, one will be represented in a parametric form x is equal to a multiplied by t minus sin t, where t is the time or parametric variation. Similarly, y is equal to a multiplied by 1 minus cos t.

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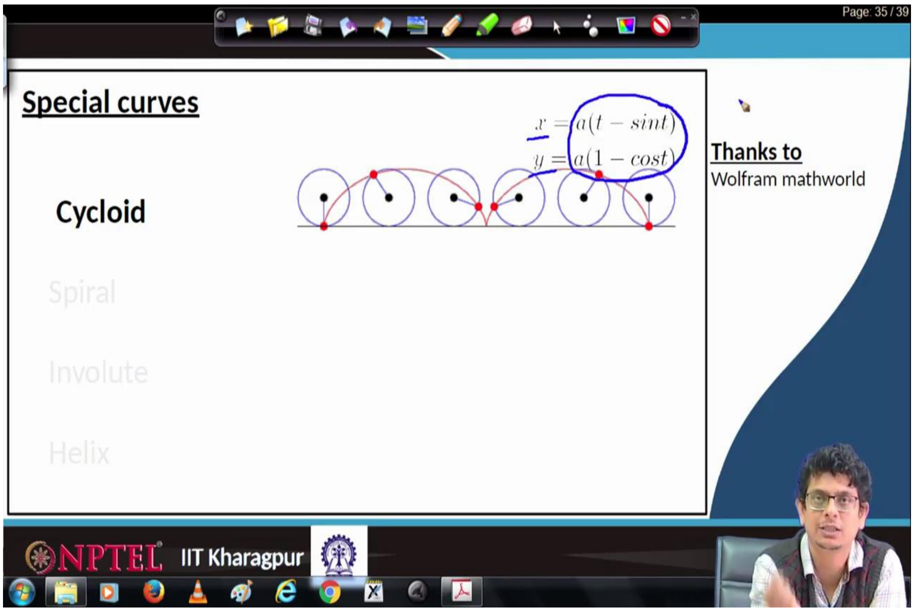

So, what about this x coordinate, y coordinate? We are going to get from this parametric equations, that is what we call cycloids.

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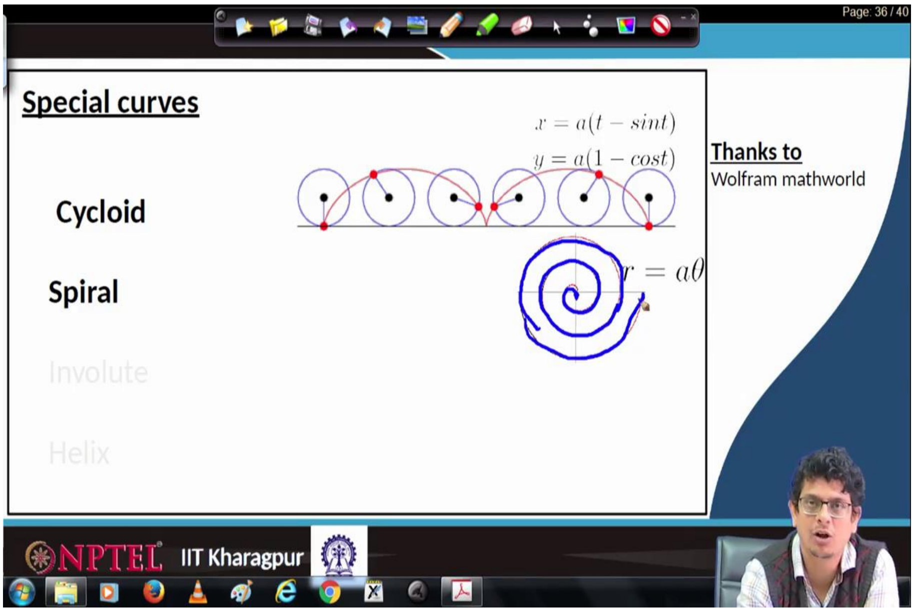

Spirals. So, they begin somewhere at the centre; Archimedean spirals are quite popular in that sense. They begin as the angle increases the radius also increases. So, as we are going beginning from 0° increases to 10, 20, 30 and so on 360°, gradually the radius also increases. Radius can increase, decrease based on how we are going to move this curve if we are moving outside radius increases, as we go inside the radius decreases. These kinds of things are called spiral. And the parametric form for the spiral is r is equal to A theta. As theta increases, r also increases, for a positive definite a value and for a negative definite a value r decreases as theta increases.

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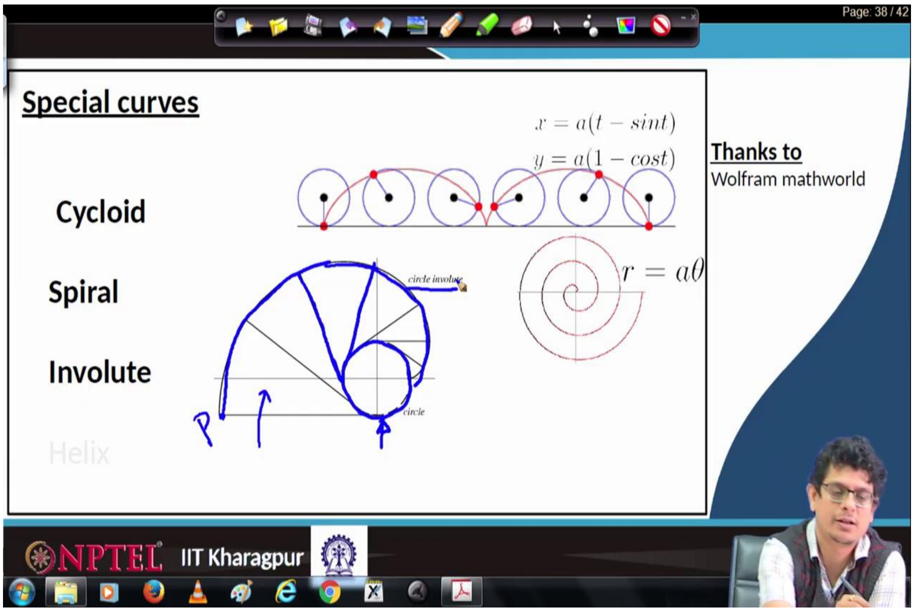

The other form of the special curve is called involute. If we are winding a thread or rope around a cylinder, so, let us take a rope of fixed length l, tie it on one side of the cylinder, now spin the cylinder. The rope length continuously it tries to own the circle or cylinder and the length whatever the apparent length we are going to see that continuously decreases. And the curved track by one of the ends of this rope is called involutes.

For example, this is the cylinder. Let us consider this is the cylinder circle in two dimensions and this is the rope which perhaps we might have tied it there. Now, spin this cylinder, so the point P is perhaps winded this rope so that point P it moves on a specialized path. Gradually, the distance, the apparent distance what we are going to see from the rope point to that end of the cylinder decreases and it follows a curve named involutes.

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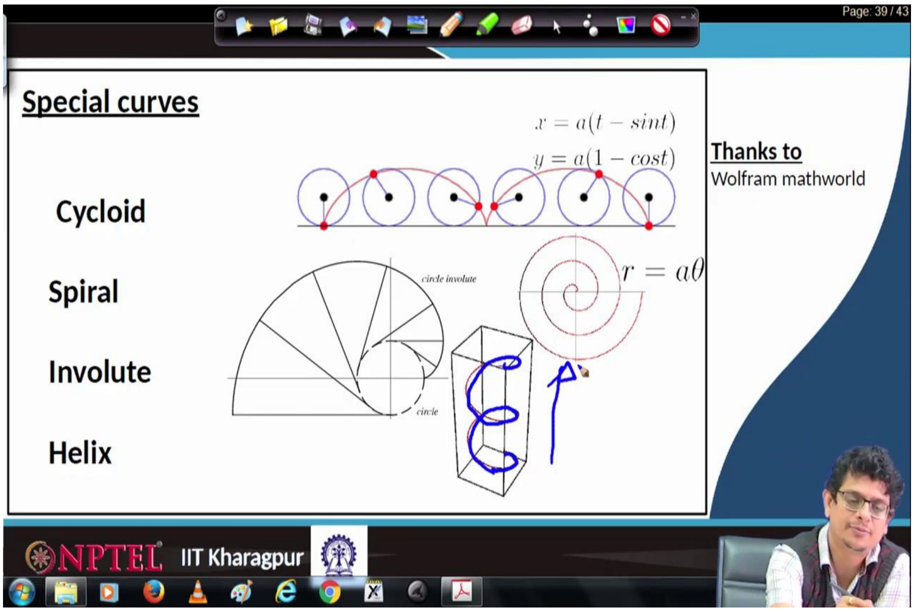

The other ones are helix. So, for example, here a point which goes in a circular format, but it has axial motion also so that it rises upward direction and so on. Typically, small size air bubbles in coke bottles or perhaps in water beakers when they are rising they make this kind of spirals. So, it is a circle which is trying to move in the axial direction; such kind of things are called a helix.

For example, if we open a pen, we find a spring in it. These springs always have a constant radius, and they are continuously increasing in the axial direction. They move in that axial direction—such kind of things what we call helix.

In spirals, the radius is continuously increasing or decreasing. For helix, this radius is nearly constant, but axial directions z-axis increases.

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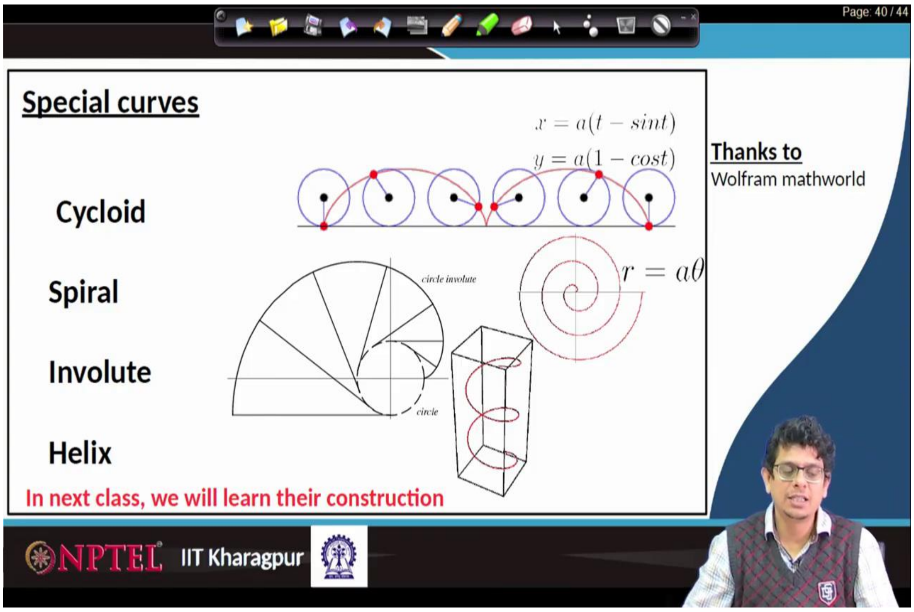

In the next class, we will learn about how to construct special curves like cycloids, spirals, involutes and helix.

See you in the next class.

Thank you.