**Engineering Drawing and Computer Graphics**
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**Module - 02**
**Lecture – 14**
**Conic Sections – VI**

Hello, everyone. Welcome to our NPTEL online certification courses on Engineering Drawing and Computer Graphics. We are in module number 2, lecture number 14 on Conic Sections.

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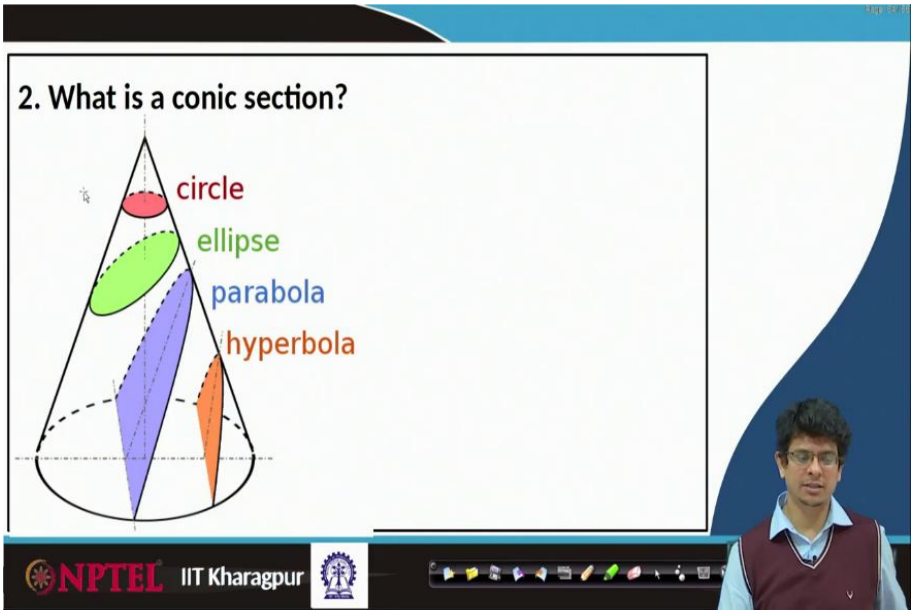


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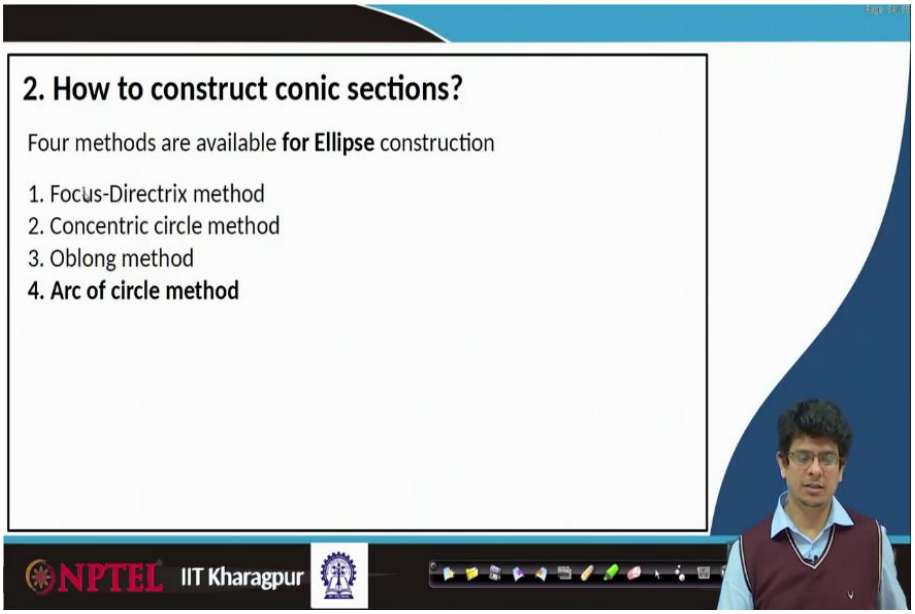

In this lecture, we have covered the focus-directrix method, concentric circle method, rectangle method or oblong method and in today's class, we will look at the arc of the circle method.

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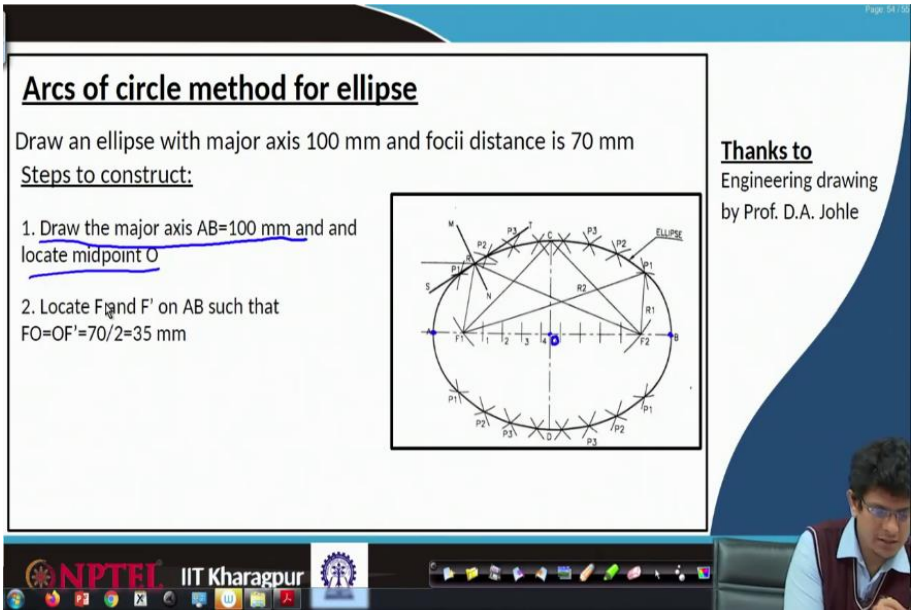


In this arc of the circle method, an ellipse with the major axis is known perhaps let us say in this case 100 mm, and the distance between foci is now something like 70 mm. If that is the case, how to construct an ellipse? So, let us look at this steps, first of all, we have to draw major axis AB 100 mm and locate the midpoint O. Draw major axis AB 100 mm and locate midpoint O. So, point A, point B this is 100 mm, and at 50 mm we are going to locate O.

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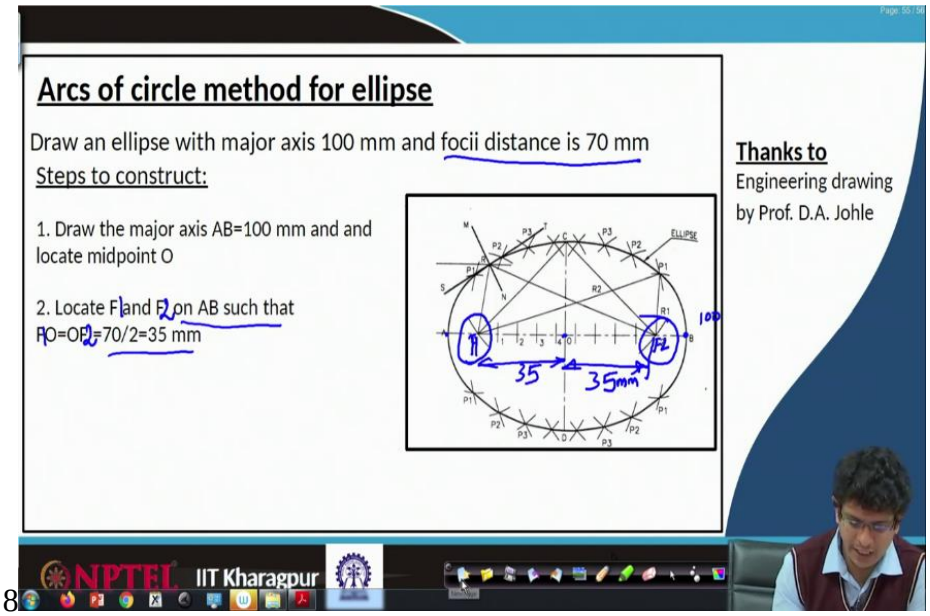

Once it is done, locate foci, locate foci F 1 and F 2. Those are F 1 and F 2 on AB such that F 1 O and O F 2 are 35 mm each. The major axis is 100 mm, and the distance between foci is 70 mm; that means, we have to mark a division from O, here this is 35 units and this one also 35 units. Our standard practice is we do not show these dimensions on the drawing somewhere outside we have to extend those lines and show it. So, once we locate this F 1 foci, F 2 focus, then we will go with the next step.

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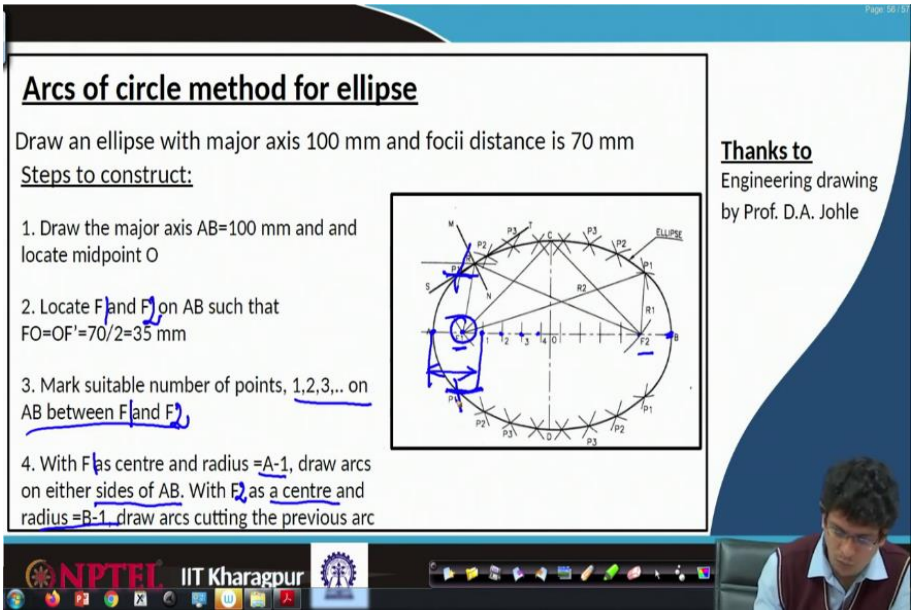


Now, we have to mark the suitable number of points something like 1, 2, 3, on AB between F 1 and F 2. F 1 and F 2 it can be equal or any number of points. It is easy always to go with an equal number of points as a uniform grid first point, second point, third point, fourth point.

Once it is done with F 1 as centre; so, F 1 is centre the distance from A to 1, this distance let us measure it with that distance from F 1 make an arc something like that on both the sides. So, the centre has to be F 1, but the distance is A 1 A to one whatever the distance make an arc on top and bottom on either side of AB.

Now, with focus F 2 from here, as a centre and radius B 1; so, B is here B 1 is that. So, with B to 1 distance, but the centre is F 2 make an arc in this direction; similarly, make it on that side.

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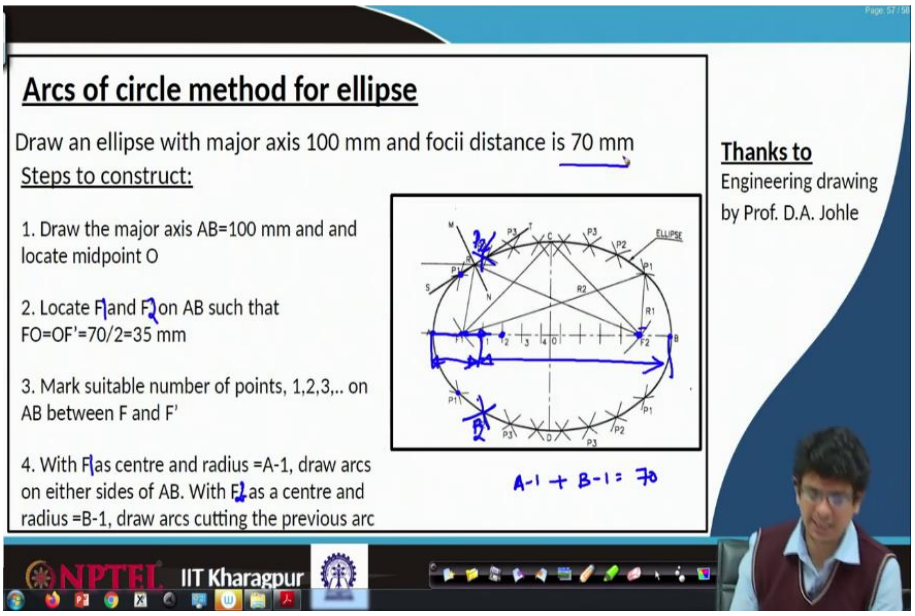


Once we are done we will have this point P 1 and P 1', these are F 1, and this is F 2, and this is also F 2, and this is F 1.

Now, if we are moving to the second point to construct P 2 what we have to do is from A to 2 whatever the distance use that distance, but centre as F 1 makes an arc on either side. Similarly, from B to 2 distance whatever the distance F 2 as centre make an arc on both sides. Whatever the intersection point let us call P 2, P 2' or P 2' in such a way that A to 1 distance plus B to 1 distance always gives us major axis 70 mm.

Let us look at this. This is A 1; A 1 and B 1 is this. So, if we are going to add these two distances, it is supposed to give us major axis 70 mm.

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Similarly, if we are using A to 2 distance and B to 2 distance that also consists of A 2 plus B 2 distance that is also 70 mm; in that way, we will be going to construct P 2 point P 3 point and so on things.

So, let us begin our example construct it step by step. Major axis 100 mm. So, let us look at on this drawing sheet. So, let us mark 100 mm on this drawing sheet.

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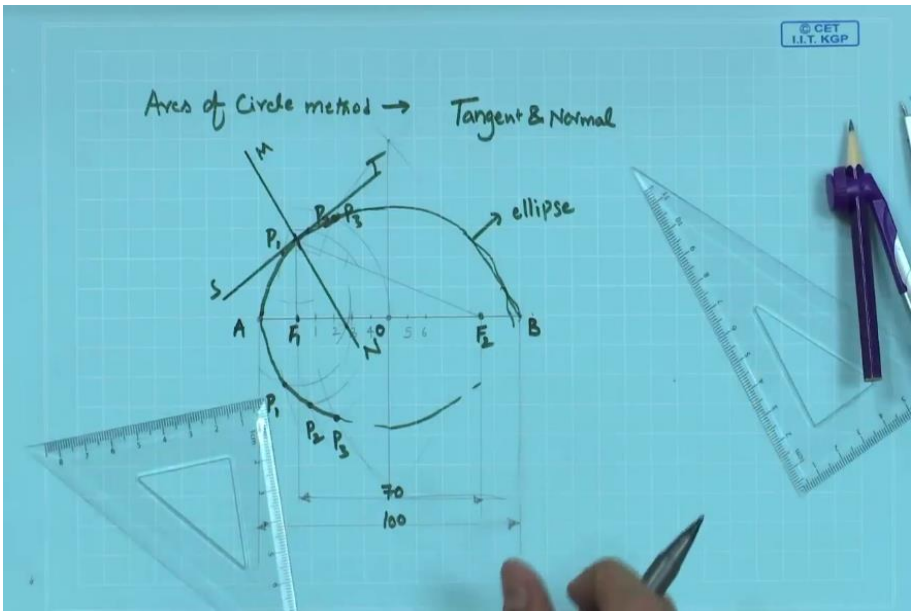


Let us call A and B; then we have to locate the centre which bisectors we can use it otherwise 50 mm also we can use in this case. Join these lines. So, this is the centre of that circle call O. Now, we have to locate foci which is at 35 mm on either side. So, let us mark 35 mm scale. So, this is one of the foci; the other one is at 70 mm. So, call these points F 1 and this point F 2 after that we have to take distance A to 1; for that, we have to arbitrarily or equal divisions we can take it to let us take equal divisions. So, it is 35 mm. So, we would like to mark after every 7 points. So, 35, 7 5s are 35 so, 6, 7, 14, 21, 28. Similarly, here one can make 35 next 42, 49 and so on. Once this division is done, we have to measure distance A to 1, let us name them as 1, 2, 3, 4, 5, 6 and so on. A to 1 whatever the distance use your compass A 1 use F 1 as centre mark an arc on both sides from B to 1 also we have to find the distance. Use foci as F 2 make an arc, call that intersection point as P 1 P 1, P 1', P 1'. Now, A to 2; pick that point; this is A to 2 centred around the focus on both sides then B to 2 whatever the distance we get use F 2 intersect this curve which is A 2. So, mark this point P 2. Similarly, use A 3 distance centre around F 1 B 3 distance centred around F 2 Now, we will be in a position to join a smooth curve using French curve it will be better and so on. Similarly, we will be in a position to construct a line which passes through P 3. In that way, we will be in a position to construct all that curve which pass through B point and also at this point, so that we will be in a position to construct ellipse. This is by arcs of the circle method.

In this case, we require a major axis. So, if it is construction lines we have to show 100 mm; foci distance also we know. So, smaller dimensions are always inside. There is this standard

convention what we are going to use. The construction procedure can be much lighter also instead of darkening it. So, this one is 70, and this one is 100, the distance between foci.

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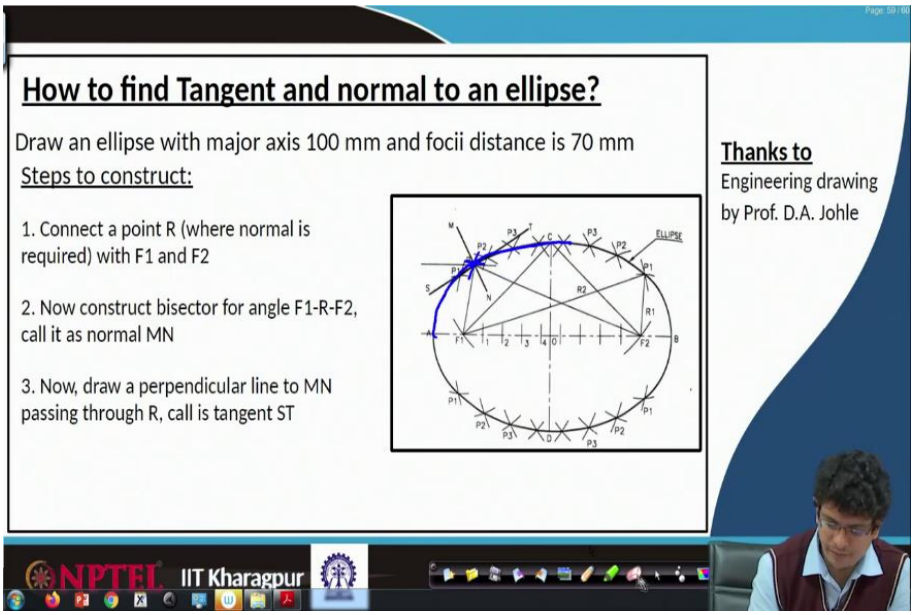


Now, we will ask a question: how to draw a tangent and normal to an ellipse. For example, part of the curve we have constructed as an ellipse. Now, pick any point where we would like to construct a normal or tangent. In this case, we would like to construct, for example, at point R. So, this is the ellipse which we have joined, and we intend to construct something like normal to that. For that purpose, what we have to do?

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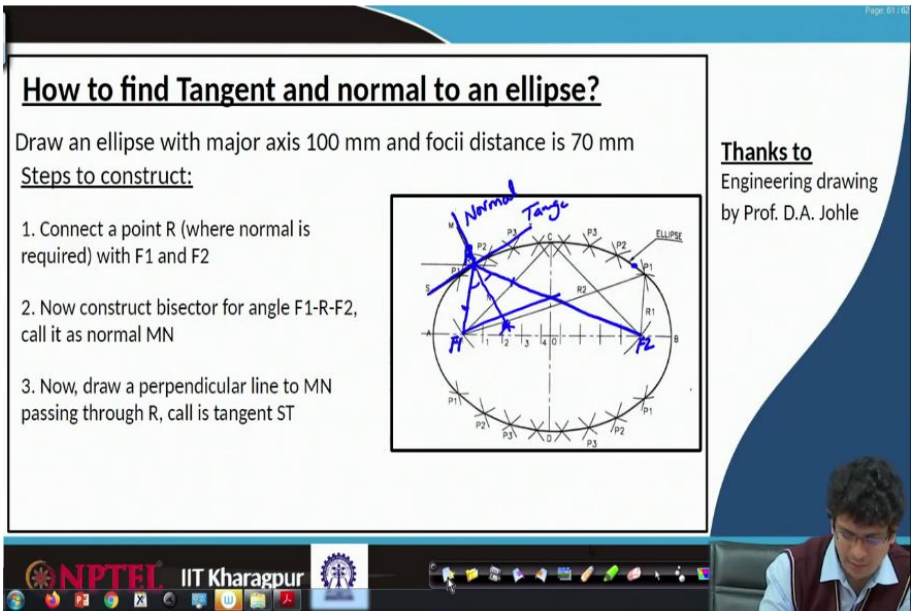

After deciding where you would like to draw the normal or tangent connect that point R with focus F 1. Similarly, join this R with focus F 2. Now, we have an angle. We have this angle F1-R-F2. Now, based on this centre somewhere distance we just make an arc we have to bisect it. So, from there again with the same radius make an arc on both sides. So, we will have some point and extend that. So, it makes equal angles on both sides, and this point extended. If we can do that, that is what we call normal. So, to an ellipse, this is the way we constructed normal; tangent always be 90° and touch at only one point passing through R, and we will be in a position to construct tangent.

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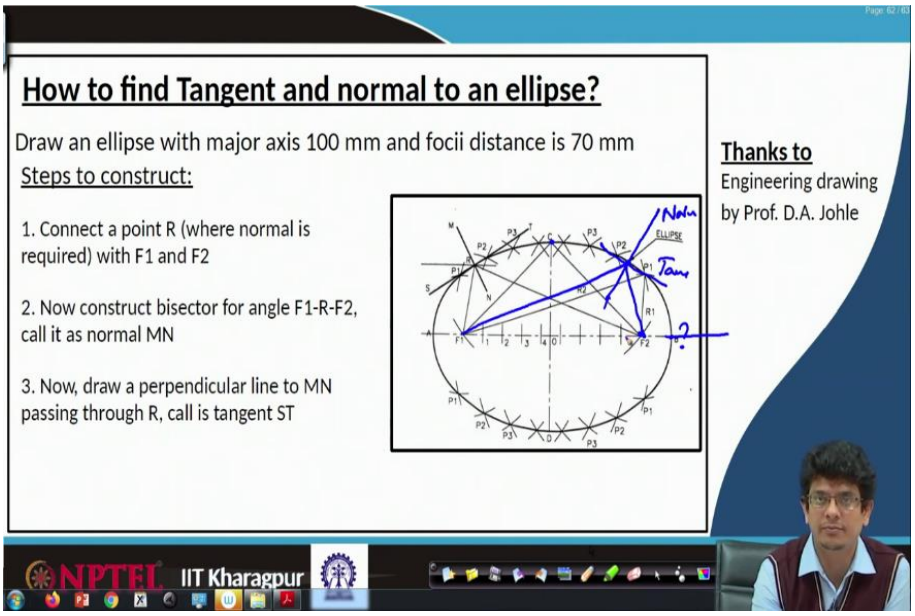


If some other point here then we will join F 1 point, for example, let us look at we would like to construct a normal there. What we do is connect this F 1 and this point similarly connect this F 2. Now, make a bisector join it, construct normal, this is normal, and that will be tangent.

If one would like to construct normal here, what to be done, think about it. Precisely on the major axis or minor axis, you would like to construct normal, will that be straightforwardly this one or do I have to join the lines from F 1 F 2 and bisect it?

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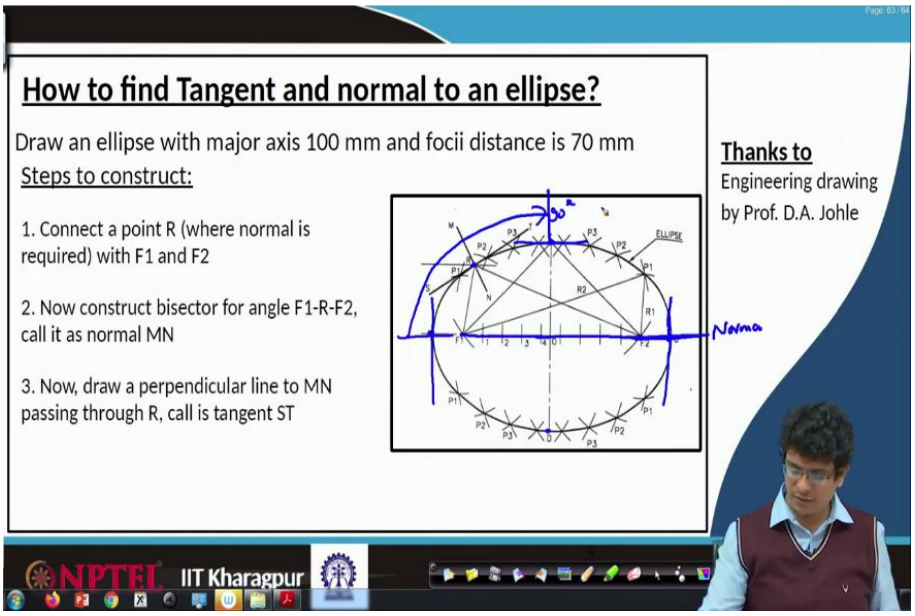


So, let us look at it carefully. If the point is straight away at the major or major axis, the procedure is to connect any point on that curve with two foci. So, if I am going to connect point B with F 1, the line will be this one. Similarly, this point we are going to connect with F 2; that means, for this point B again connect that. Both are still making that 0° or 180° kind of thing the equal division which comprises of the same line.

So, this is the one which makes normal to that circle at this point, and tangent will be perpendicular to that this will be. The same thing happens here normal will be in that direction tangent will be in that direction. If we are picking C point normal will be in that direction, tangent will be in another direction. Except for these four points A, B, C, D at remaining points, these normal angles continuously increases as you go in that direction from 0 angles to 90° it increases.

So, let us construct this normal on a tangent on sheet. So, we have a part of the curve here on the drawing sheet. Now, let us identify a point here.

Let us consider this is the point where I would like to construct normal on the drawing sheet here. Now, the procedure is to join this F 1 with that point it is a construction line; similarly, construct join this F 2 with that particular point. Now, this is the point what we are identifying.

Now, use angle bisector division for that purpose what we have to do is pick anything like arbitrary radius. Now, use this intersection point with the same radius, intersect this curve locate this point and join this point with this one. This will be the normal M N and the tangent always be perpendicular to that.

So, we can use a set of squares these set squares we can use it, align that or perhaps your drafter is the best way to construct these perpendicular lines and extend this line. So, we have this tangent S T, the arc of the circle method, and we have learnt how to construct both tangent and normal.

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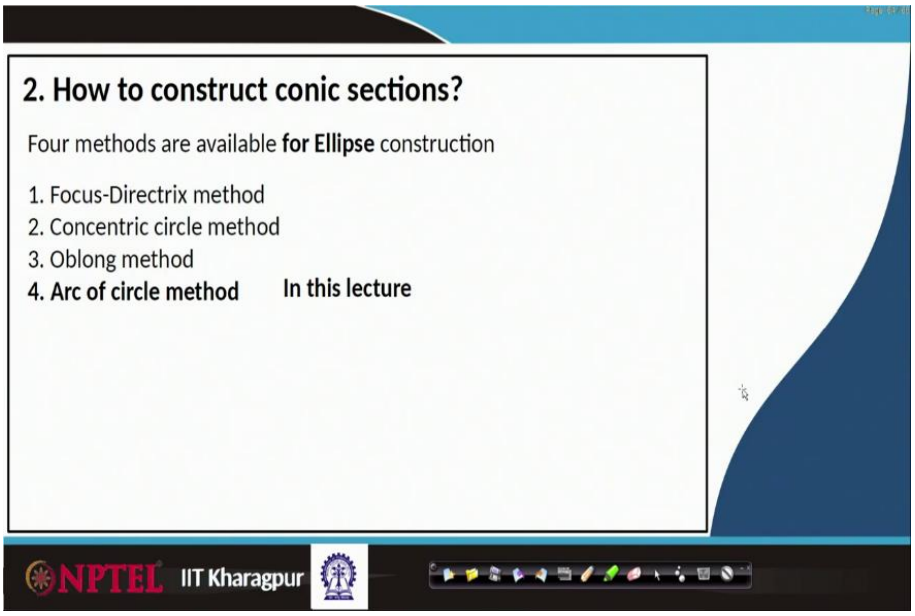


So, in today's lecture, we have covered this arc of circles method. So, in principle to construct ellipse, the popular methods are four - focus-directrix method, a concentric circle method, oblong method and arc of circle method. In focus-directrix method we know something like a directrix, eccentricity we know, equally divide the distance between these any point on the curve by an equal number of divisions, draw 45° line and intersect it and construct this ellipse.

In the concentric circle method, we have a major axis, minor axis and two circles we draw, two concentric circles by horizontal and vertical projections on an equal number of divisions we will construct these ellipse. In rectangle method or oblong method, what we have is major axis, minor axis, a rectangle; in some cases parallelogram also we will use to construct an equal number of divisions on the major axis, minor axis. Connect minor axis elements with major axis elements and get the intersection lines from there construct this ellipse.

In the arc of the circle method, we have seen major axis is given, the distance between foci is given. So, using a principle any point where we would like to construct these ellipse A 1 plus B 1 always be major axis length using that principle and radius is F 1 F 2 the foci centres, make many arcs intersect them and construct an ellipse this is. These are the ways we construct ellipse.

And especially focus-directrix method is quite popular: one because from directrix we know the distance and eccentricity we know; that means, in the last classes we have seen an ellipse has eccentricity less than 1, is for parabola eccentricity is equal to 1 and for hyperbola, eccentricity is greater than 1. That means if we know the directrix distance and eccentricity, in principle, we will be in a position to construct it can be ellipse, parabola, hyperbola.

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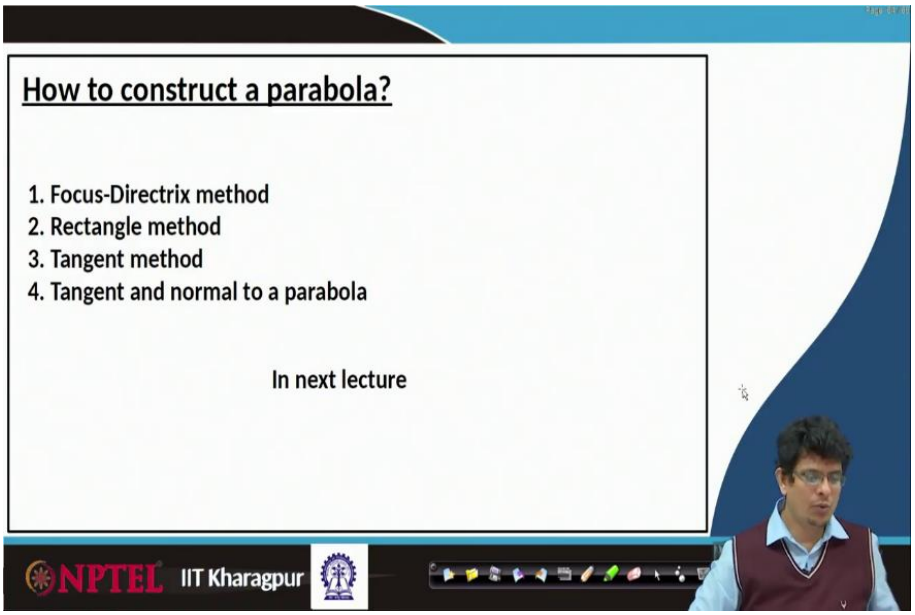


And, in next class, we will look at these four methods especially focus-directrix method, rectangle method and there is a particular case like the tangent method to construct parabolas. We will also see how to draw tangent and normal to parabolas.

Thank you