Engineering Drawing and Computer Graphics

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Module – 02
Lecture – 12
Conic Sections – IV

Hello everyone, welcome to our NPTEL online certification courses on the engineering drawing. I am Rajaram Lakkaraju from Mechanical Engineering IIT Kharagpur; we are in module number 2 lecture 12 on Conic Sections. To recap if we are having a right circular cone taking a horizontal slice we will get circle an inclined slice ellipse if this section is going to cut both the slants.

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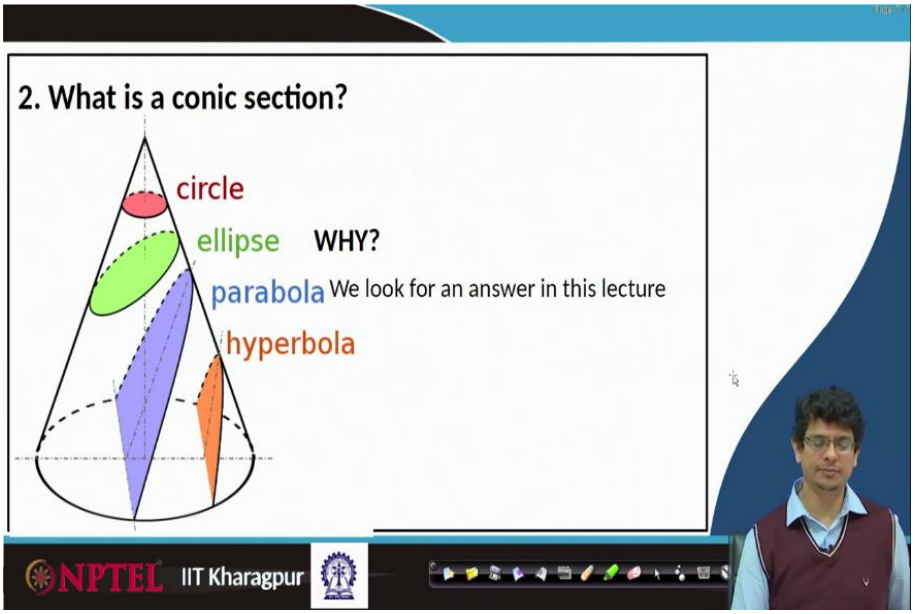

If a parallel slant cut section if we are taking it to construct a hyperbola and any other section it makes hyperbola.

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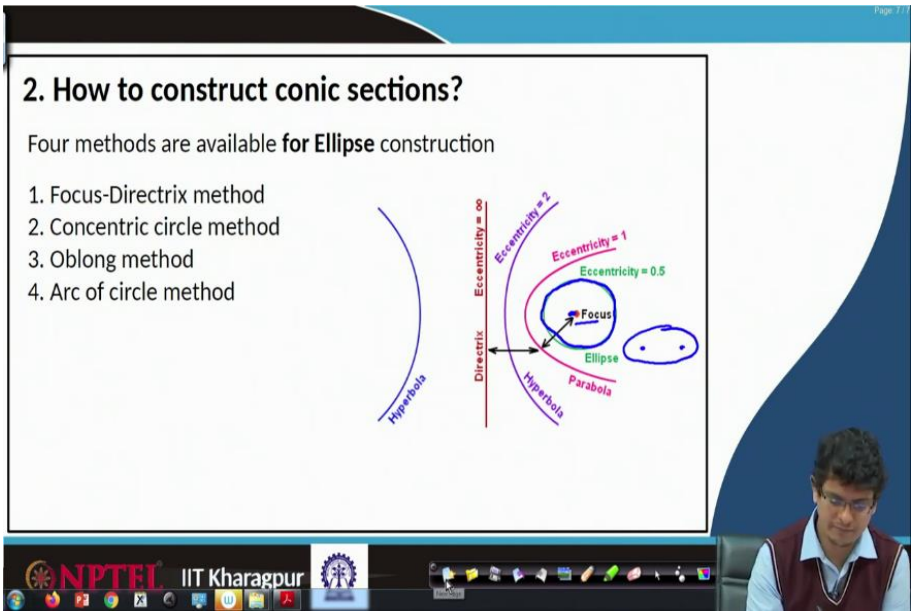

Now in today's lecture, we will see how to construct an ellipse geometrical means. In principle, there are four methods available Focus Directrix method, the second one is Concentric circle method, the third one is Oblong method, and the fourth one is the Arc of the circle method. For the first method focus and directrix method, we have a definition about directrix, eccentricity and focus. Let us here look at this terminology. For example, let us take an ellipse. In this ellipse, there is a focal point which we are calling focus. Usually, for an ellipse, there always be 2 foci. For example, one of the foci here the second foci might be there. For the simple case, we are just showing only one of the foci.

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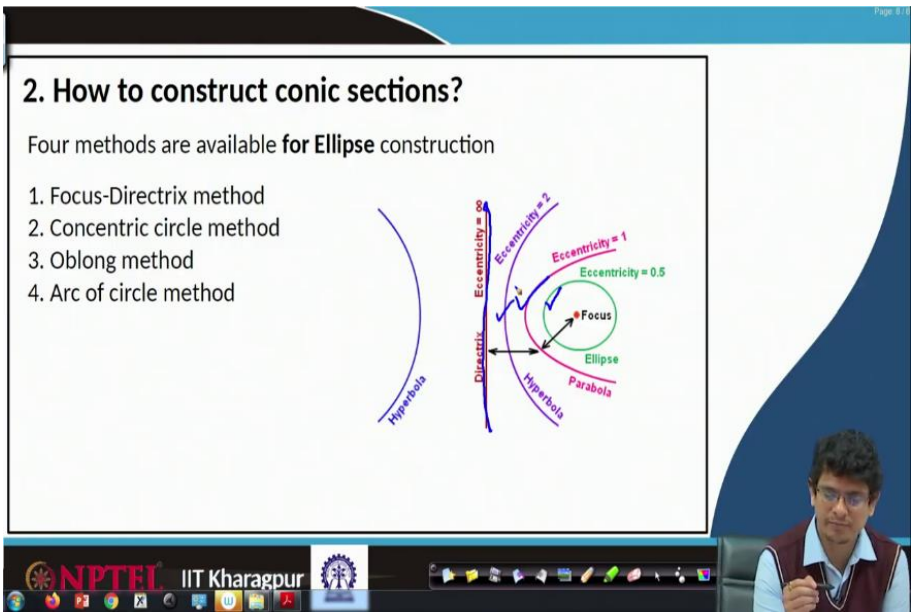

Furthermore, there is a line which we call directrix line, about this directrix line one will be in a position to construct an ellipse parabola or a hyperbola.

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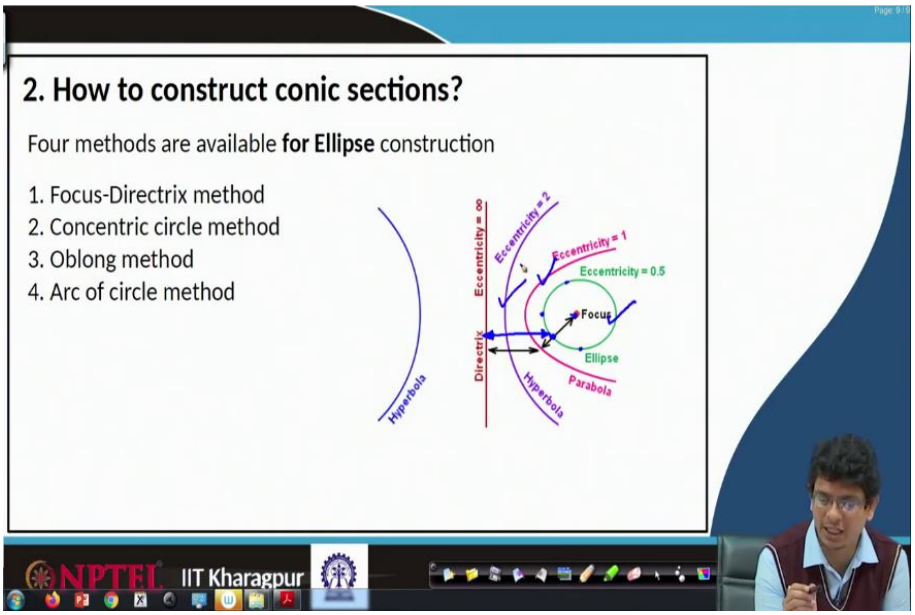

Usually for any curve, whether it is ellipse parabola or directrix, from focus to a point on the curve. What is the distance from the focus to that point, and what is the distance from that point to directrix? This ratio makes one unique thing, based on this ratio focus to the point of the curve and point of the curve to directrix horizontal thing. If it makes one particular unit, for example, less than 1 we get one kind of curve, if it is equal to 1, we get one kind of curve, if it is greater than 1 we get another kind of curve.

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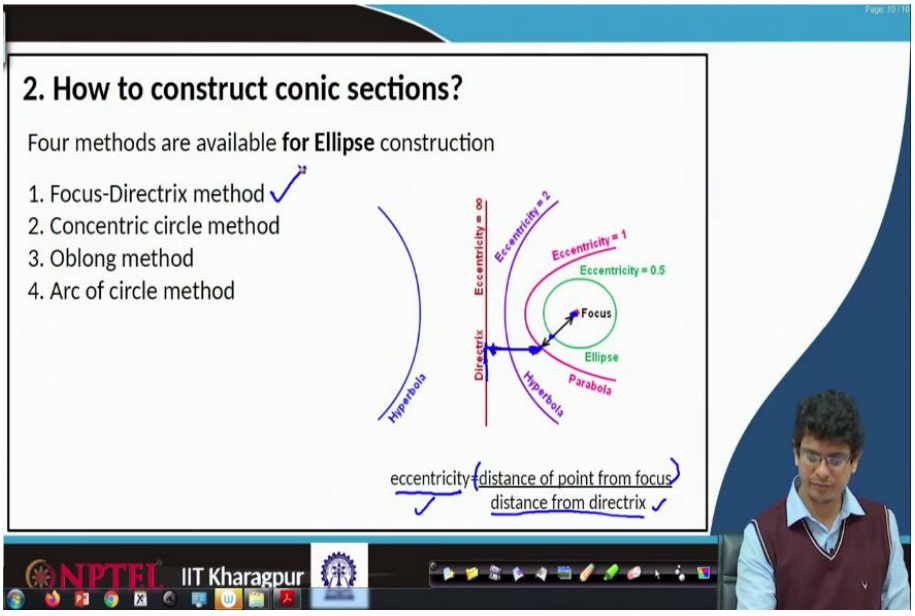

Furthermore, we call that ratio as eccentricity. So, eccentricity is defined as the distance of a point from the focus. For example, if it is an ellipse this point to that point. If it is a parabola from this point to that point distance of that point from there to focus by distance from directrix for that point.

So, if this is the directrix vertical line from that to that particular point what we are calling this denominator, that forms a unique ratio eccentricity. This eccentricity and directrix are the main things to construct an ellipse or parabola or hyperbola using focus directrix method.

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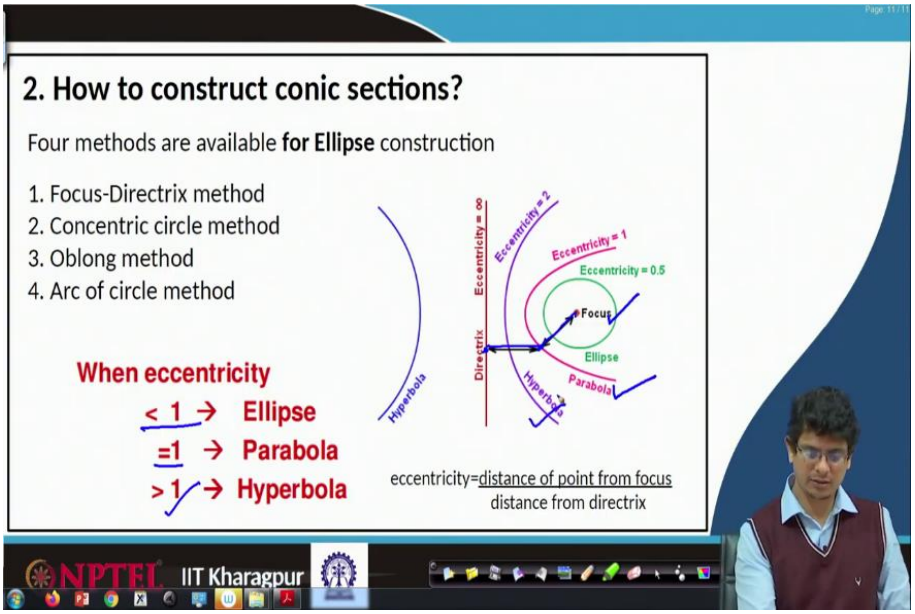

If the eccentricity is less than 1 we get an ellipse, if it is equal to 1, we get a parabola, that means from focus to point on this parabola and distance from this point to this directrix they are the same. Because the ratio is 1, such kind of curve what we call parabola, if that eccentricity is greater than 1, we get a hyperbola.

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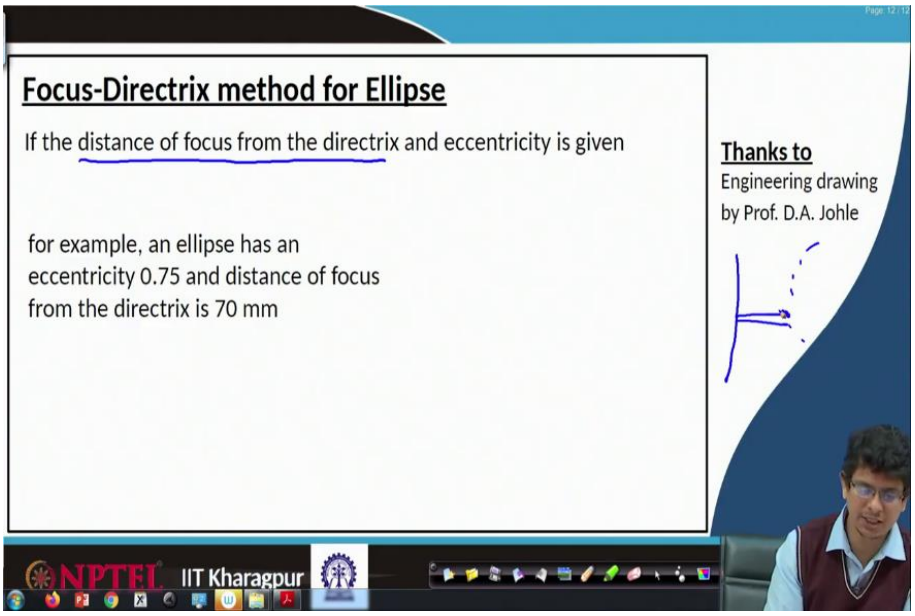

First of all, let us focus on this focus directrix method. In this case, the distance of focus from the directrix will be given by the distance of focus from the directrix.

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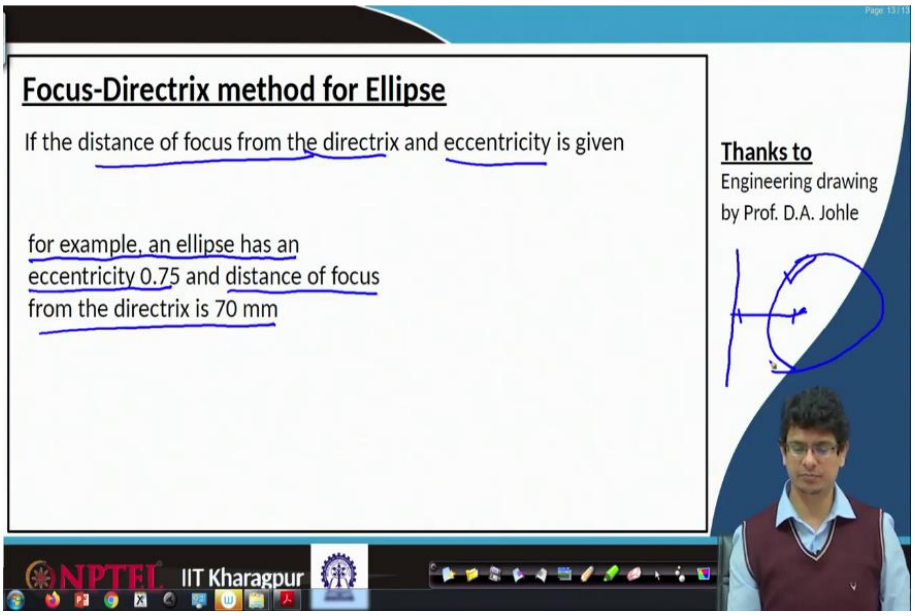

So, a directrix and perhaps an arbitrary curve it has a focus So, distance from directrix will be given to the focus and what kind of eccentricity, whether it is lower than 1 greater than 1 will be given if that is the case construct this arbitrary curve. For example, here an ellipse has an eccentricity 0.75, any eccentricity less than 1 we will be getting this ellipse and distance of focus from the directrix is 70 mm, if that is the case construct an ellipse.

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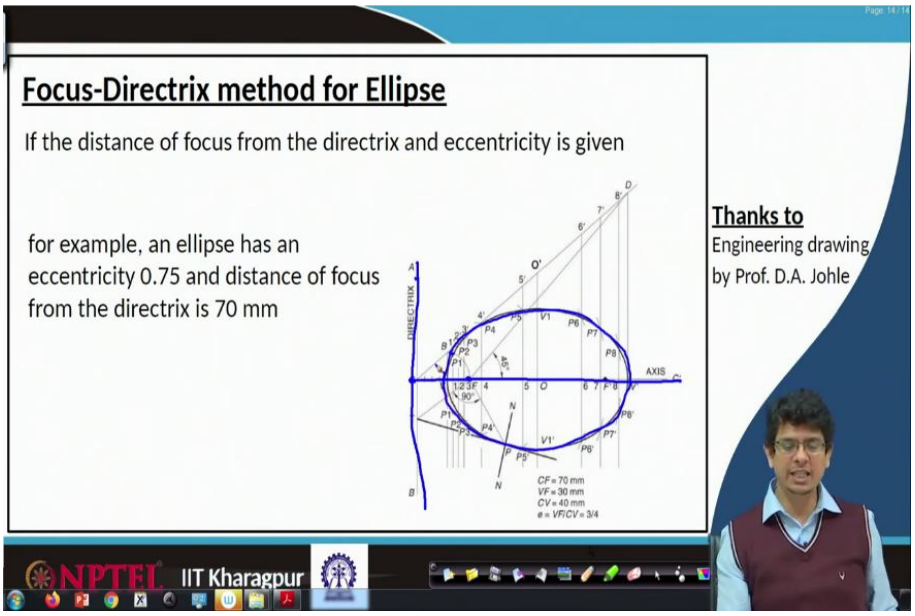

So, using this directrix focus method, first of all, we will be drawing this directrix. We will be going to construct something like a line C from directrix to that focal point we will locate it something like 70 mm, do geometric construction then draw tangent lines and vertical lines horizontal lines construct intersecting points, then follow the procedure to fill this gap like an ellipse. We will see that step by step.

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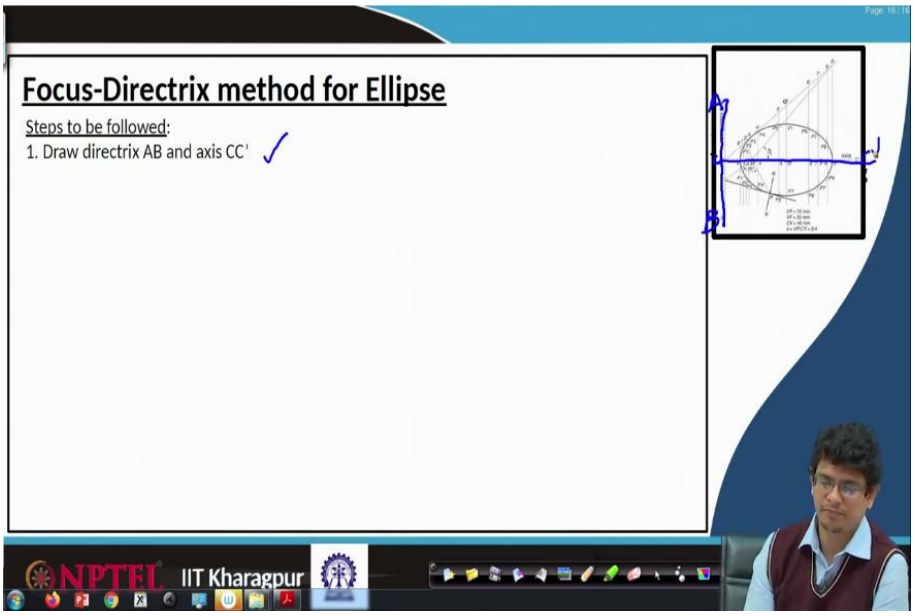

Steps to be followed, so on the right-hand side I am showing the complete picture to indicate the points what we have to follow, on the left-hand side we will show the steps.

The first one is: draw directrix A B, so that means, first of all, a vertical line we are going to draw name it A and B and also axis CC'.

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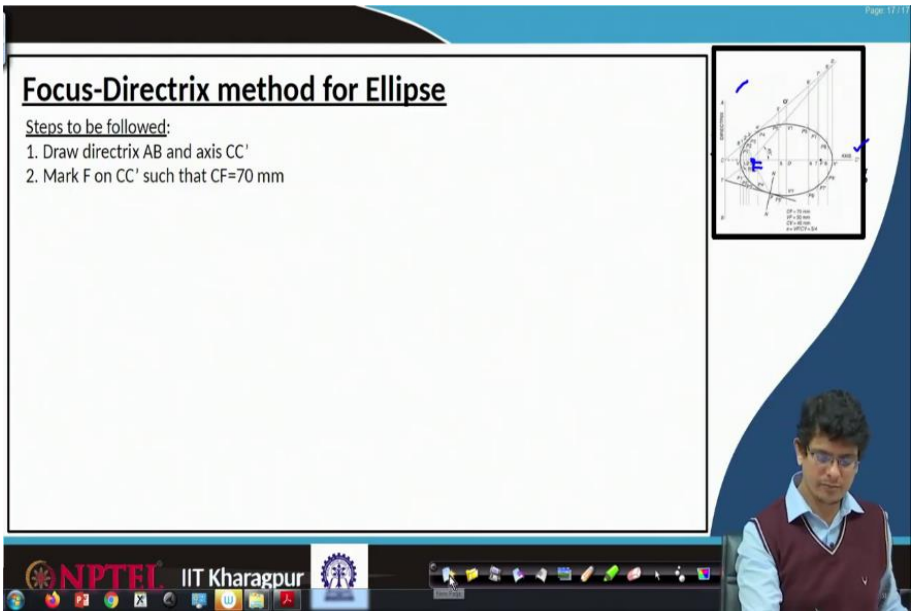

So, somewhere at point C draw a line name it CC'. Once it is done mark a point F on CC'. So, we have already constructed AB and CC'. So, now locate a focus F, which will be at 70 mm.

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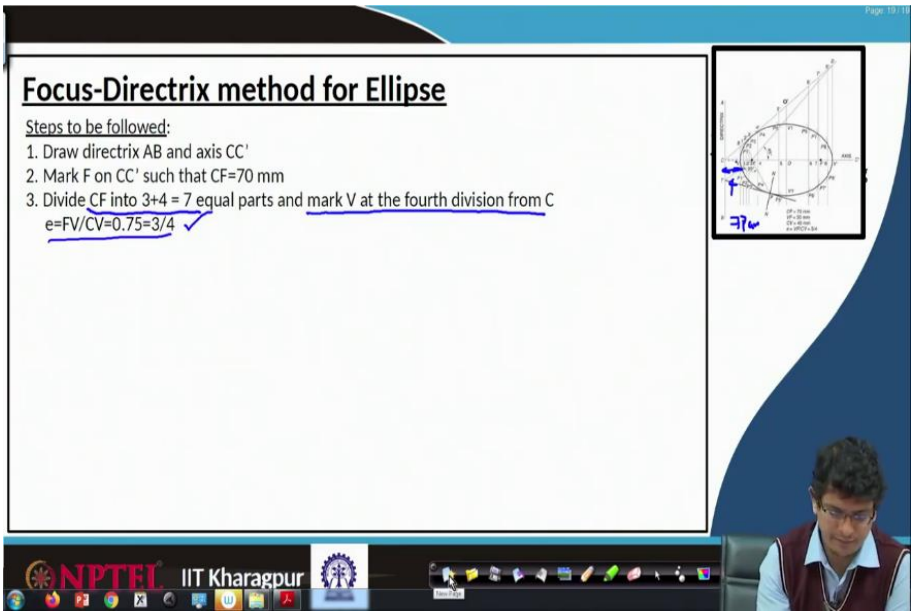

Once it is done, divide CF. So, C point we know already and F point also we know divide this C to F distance into 3 plus 4 7 equal parts so this 3 plus 4 comes because of eccentricity.

So, focal point to any point on the curve, because if this curve passes through this point B somewhere here the focal point V to F will be 3 units and C to V will be 7 unit ah 4 units. So, total distance C to F will be 7 part. So, 3 by 4 units we are going to construct.

So, divide the complete CF into 7 equal parts 7 parts we have to divide and once we divide that into 7 parts mark V at the fourth division from centre C. So, V will be 4 unit distance. So that eccentricity will be three fourth after that at V draw perpendicular VB is equal to VF.

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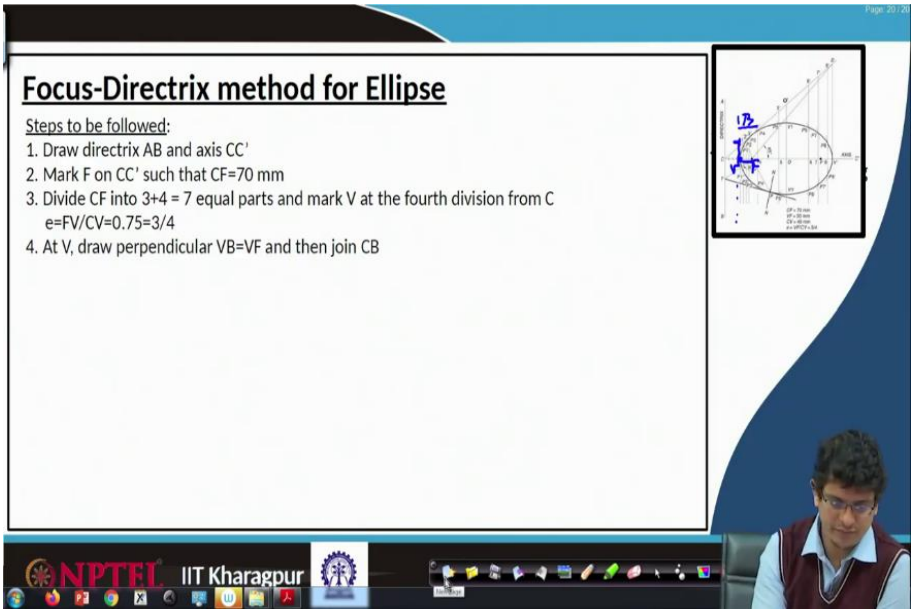

So, we have already located point V on that curve. So, there using a mini drafter draw a perpendicular line this is the one. So this point is V somewhere point B, such that V B is equal to V F. So, V we know F we know on that perpendicular line what is this distance V F equal distance move it name it as B.

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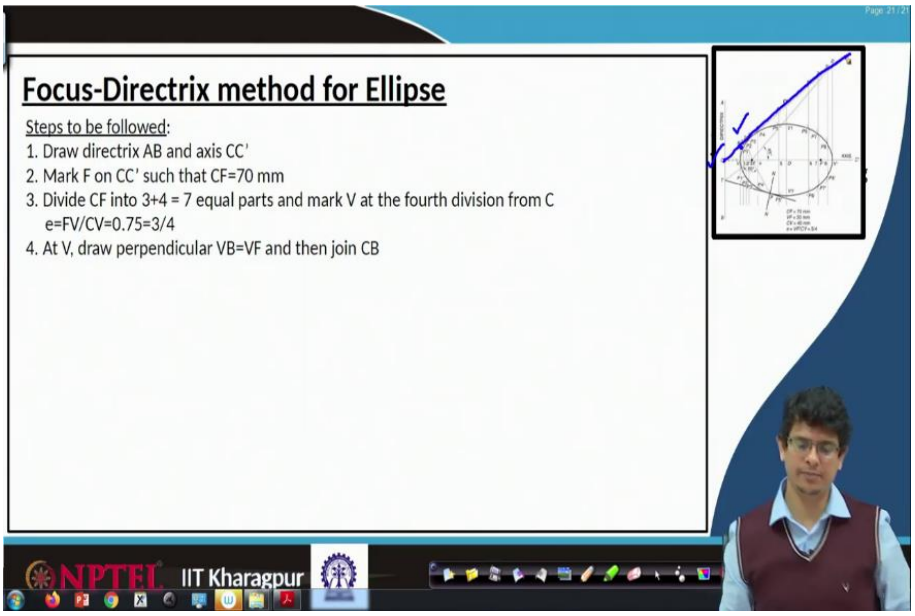

Then join B point to C point. So, once we locate that point join it. We have already this F point focus draw the line at 45°s. So, through F point, draw the line at 45°s to meet CB.

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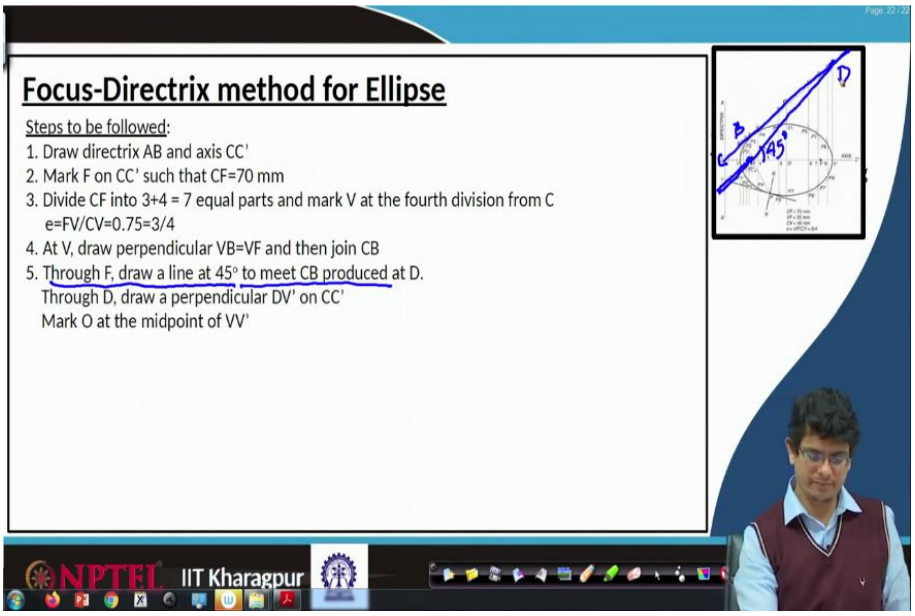

So, already we have noted C point and B point this we have extended, then from F a 45° line we have to construct it extends down to meet tangent, so this angle is 45°s. So, that we have an intersection point let us call that point as D.

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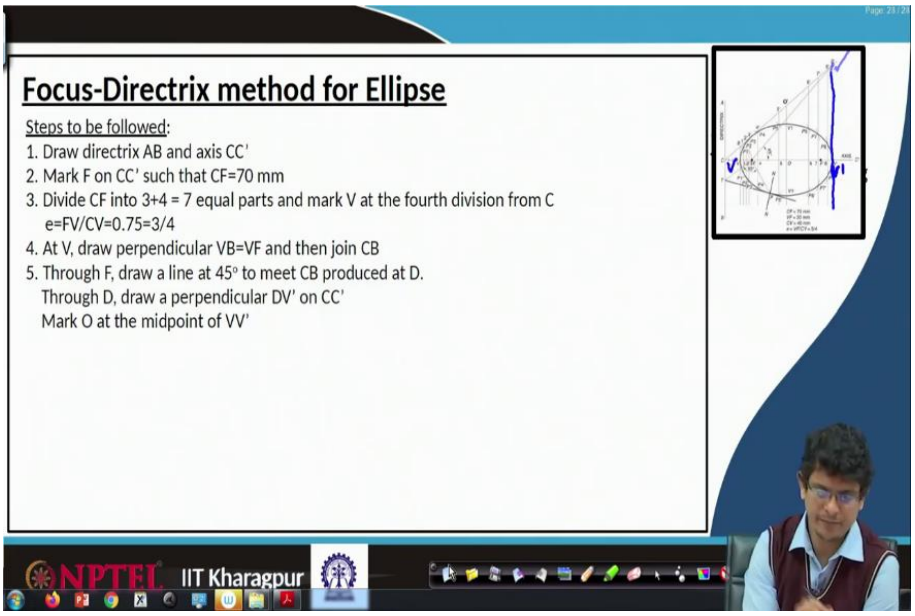

Now, through D, draw perpendicular lines D V’ on C. So, this is the intersection point So, use your mini drafter construct a vertical line perpendicular thing thus name it V’, on one side we have V, on another side we have V’.

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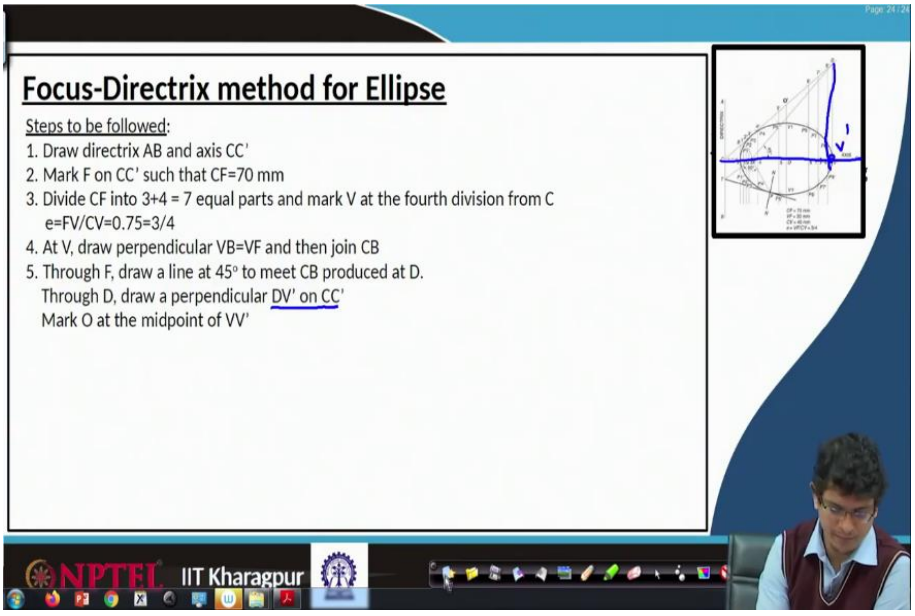

So, DV' on CC' is this line DV' line is this. So, we have identified V'. Now we have to mark O at the midpoint of VV'.

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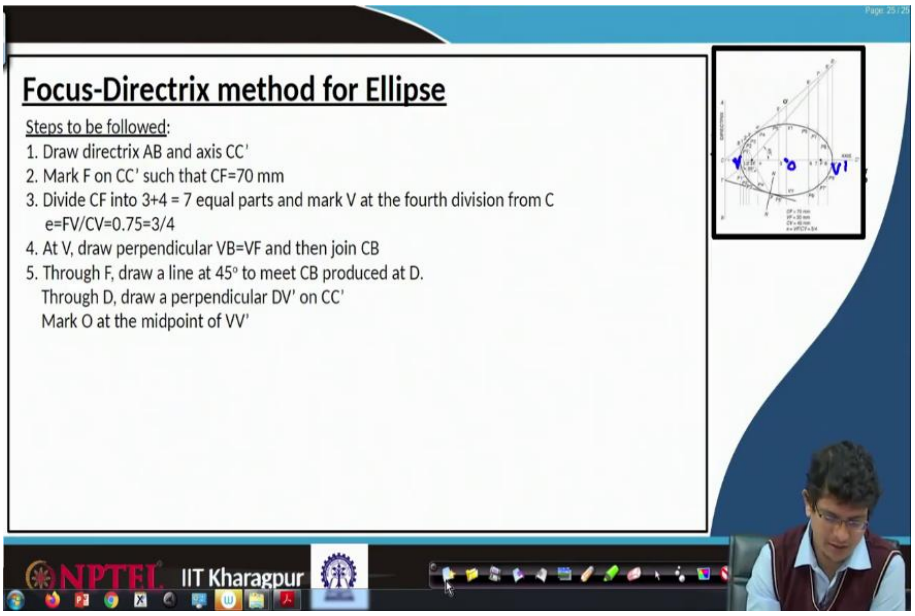

So, V is one end another end for this ellipse is VV'. We know perpendicular bisector how to draw that. So, using VV' use arcs construct then drop it. So, that perpendicular bisector O we will be in a portion to know. Then mark a few points 1, 2, 3 on VB'.

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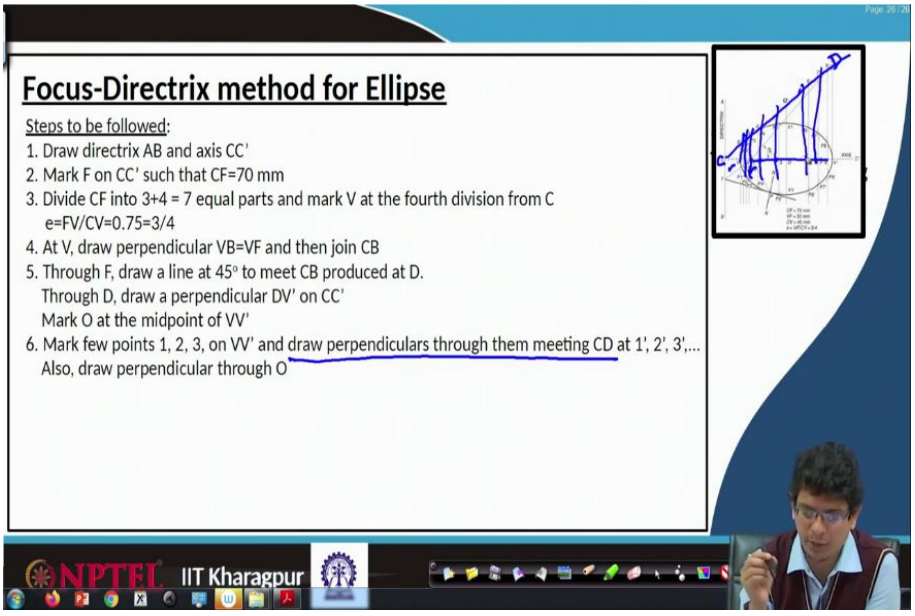

So, these can be at different kind of lengths already we divided this CV and VF. So, already we have made some division like CV and VF. So 1, 2, 3 points already we know similarly locate some other points on that V B' line somewhere here 4 somewhere here 5 somewhere here 6 and 7, so these are arbitrary points.

Once we know that what we can do is draw perpendicular through them meeting C D. So, through this 1 point, 2 point 3 point, 4, 5, 6 and so on, draw the perpendicular so, that they will intersect the curves at first 1' 2' 3' and so on 6'. Similarly, draw one perpendicular line through O also.

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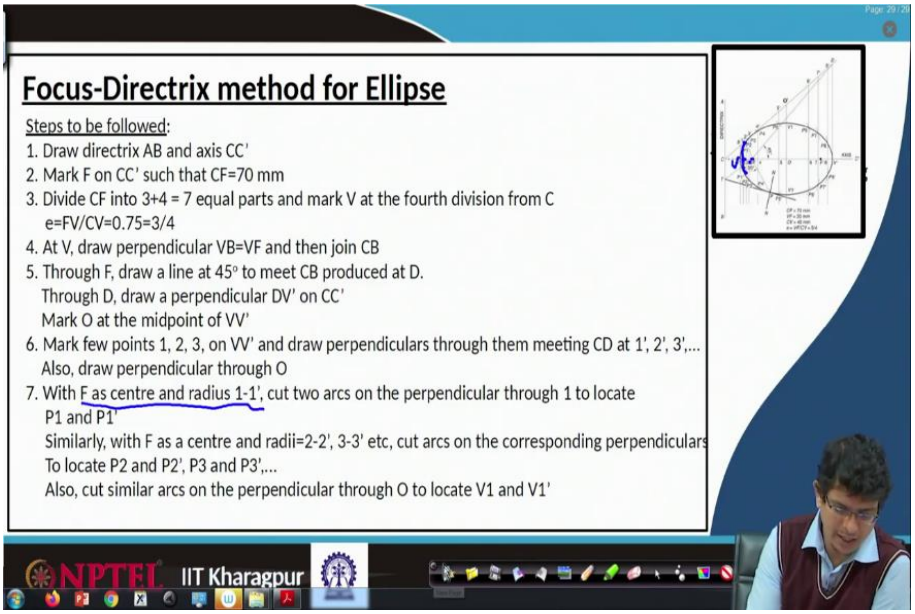

Now with F as the centre and radii 2 2' and with F as centre and radius 1 1'.

So 1 to 1' whatever that radius pick that length from F as centre cut two arcs on the perpendicular. So, the perpendicular line we already know on 1 1' So, make an arc from the centre this F to locate point P 1 somewhere here and somewhere here. So, use F as centre whatever the distance 1 1' cut this one, so that where it is going to intersect this perpendicular line that mark as P 1 point similarly mark other points as P 1'.

So, once we know, these are the points P1 and P 1' through which ellipse is passing already V point we already know focus F, P 1, P 1' join that curve so that an ellipse begin to construct. Similarly what we can do is with F as centre and radius 2 2'. So, once we construct 2 2' distance from F cut an arc so that P 2 point P 2' arc we will be in a position to construct.

Similarly, 3 3' from F make an arc, so that is going to intersect these perpendicular lines and from there we will be in a position to find out how this ellipse is going to move.

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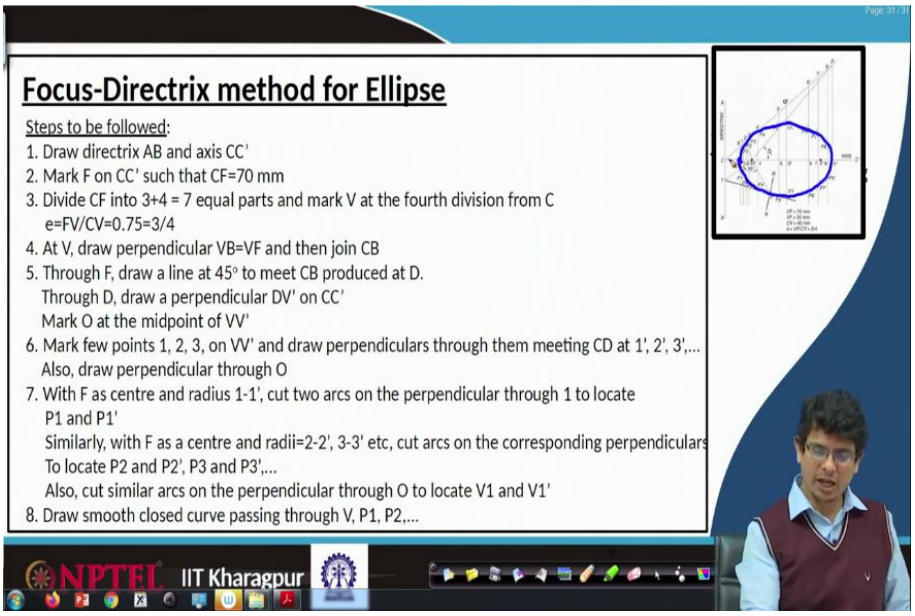

So, once we are done with this entire procedure, a smooth curve passing through this V point P 1 P 2 P 4 and so on, we will be in a position to construct an ellipse.

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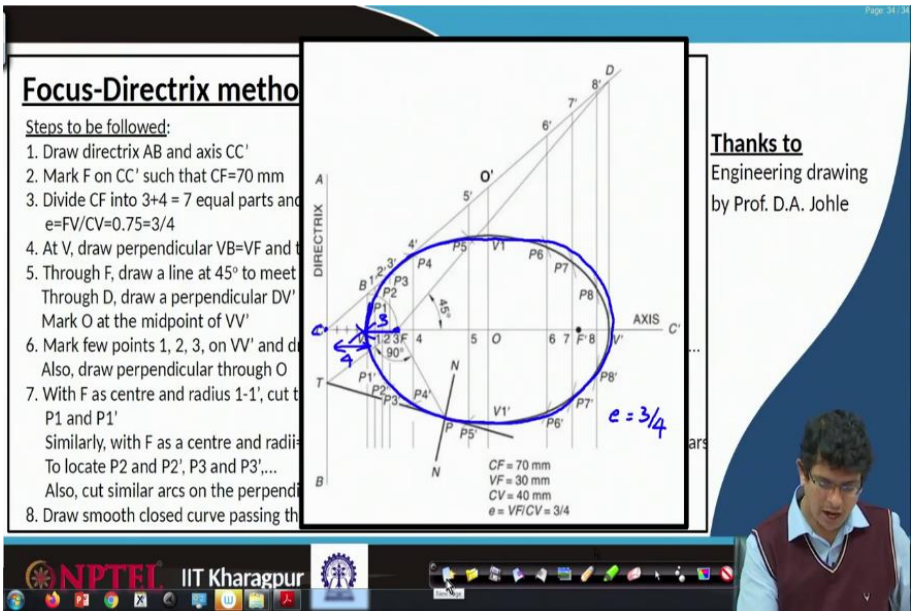

After doing that we will end up with this ellipse.

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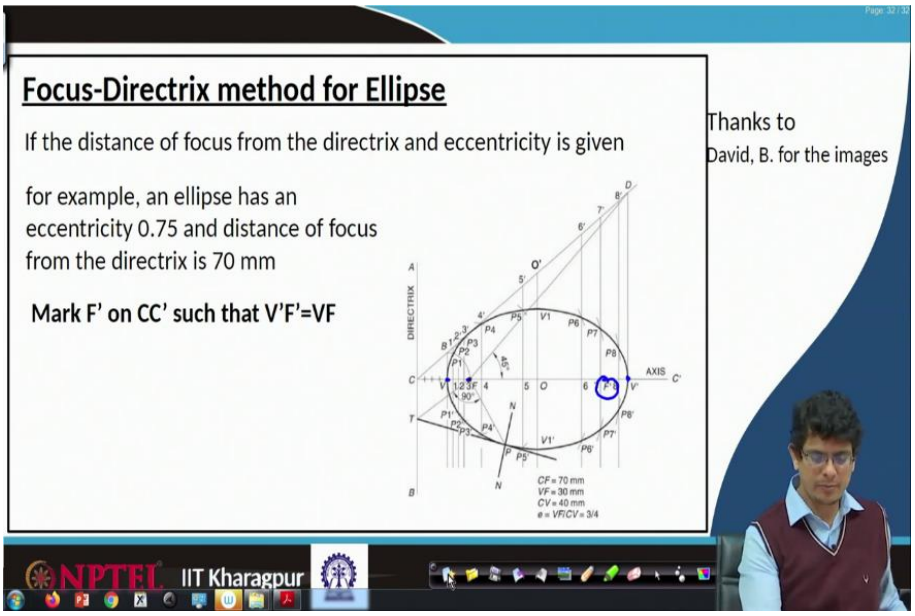

If one is interested in constructing tangent normal, we will see in the next class how to construct this normal and tangent. Let us do that step by step on our drawing sheet.

So, first of all, draw a directrix AB, and the dimensions what we are going to follow is focus from this C point supposed to be 70 and eccentricity ratio is 3 by 4. So, let us note down that eccentricity is three fourth and from the directrix is supposed to be at 70 mm for the focus. So, first of all draw a directrix line a vertical directrix line we are going to construct.

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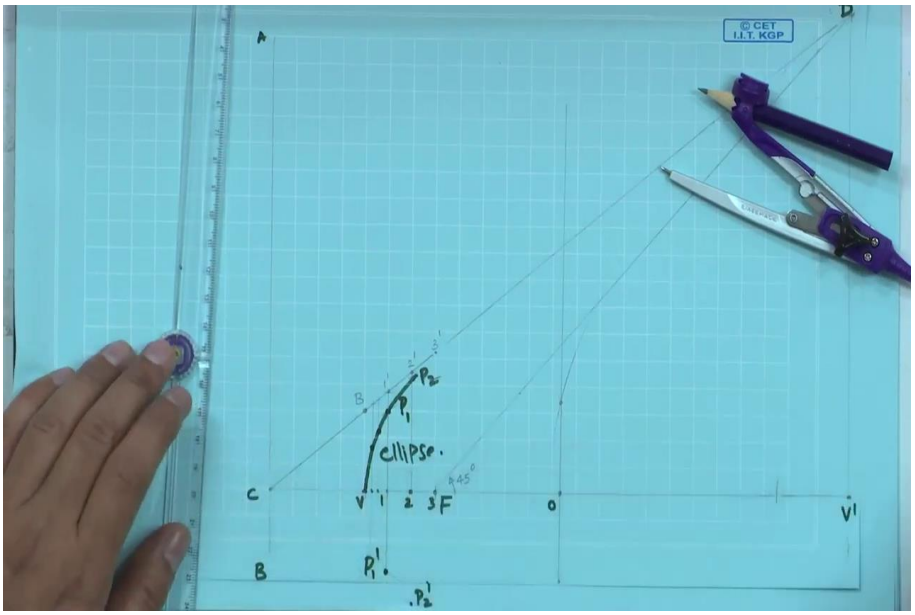

Let us name that as AB directrix something like A something as B, then a CC' line we have to construct. Let us use CC' for this a horizontal CC' we are going to use, a horizontal line name it as C somewhere it goes C' axis. So, it is not visible somewhere here C'.

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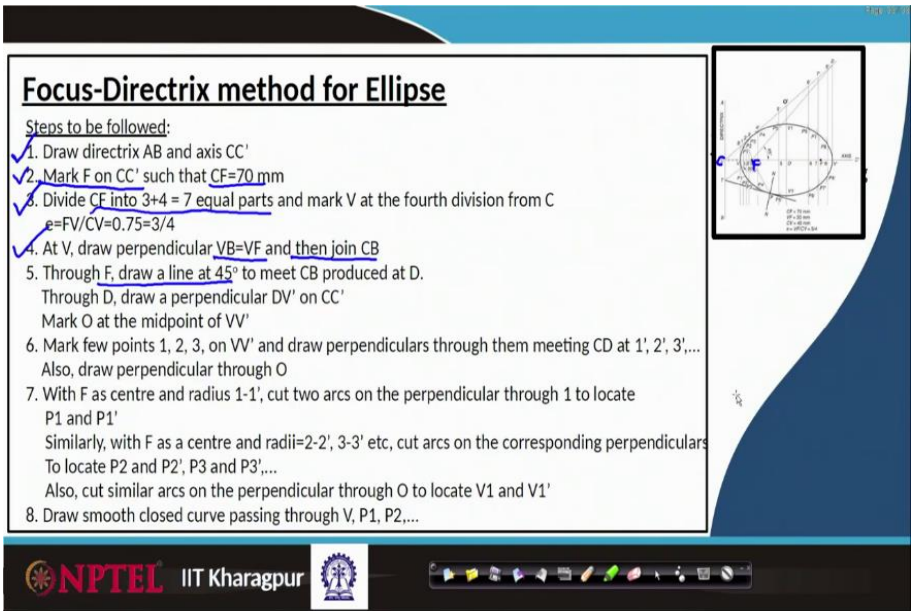

So, the 1st point draw directrix AB and CC' we have noted down, then mark F on CC'. Second step is we have to mark CC' such that C F is equal to 70 mm C is this point F is that point.

So, we have to mark that 70 mm on our drawing sheet. So, locate 70 mm point here so this is the focus point. So, 2nd step done so C to F is 70 mm. Now divide CF into 7 equal parts, because it is 70 mm is quite easy for us to divide this line. If it is fraction something like 69 mm 68 mm we have already seen how to divide into number of equal parts by using this inclined line and constructing it.

So, because it is 70 mm is easy for us to locate point 1, 2, 3, 4, 5, 6, 7 mark V at the fourth division from C 1st 2nd 3rd 4th, so that distance is 1 unit 2 units 3 units 4 unit 40 mm and from focus to point of the curve. Because curve is passing through this point this will be three units that will be 4 units, so 3 by 4 units we are going to get.

Let us mark this point as V this is the point V. So, that F V to C V will be 3 by 4 units which are 0.75. So, 3rd part 3rd point is done. If we draw perpendicular V B is equal to V F. So, V B is equal to V F line we have to draw, for that purpose first of all V B is equal to V F.

So, first of all measure this distance transfer this V B to this point. So 1, 2, 3 units, let us call that by name B 1, 2, 3 units and then join CB, we have to join this CB line. Now, through F, draw the line at 45°s.

So, focus point we locate something like this point oh oh ok. From F to that point draw these two lines are going to intersect somewhere here. Now, in this case, it is outside of the box, let us pull it down. So, it is going to intersect somewhere there. So, one has to be careful with these drawing sheets. So, it will be going to intersect somewhere at this point.

Once we know that points we will locate D somewhere here, drop a vertical thing so that we will be in a position to construct where is that V', so, we are at point number 5, where we have drawn a 45° line from point F, and already we have located V F equal to V B. So, that a line is passing from C all the way B we extended it and a line at 45°s from focus point F which is going to intersect at D.

Once we know this D point intersection point drop a perpendicular from D down to a horizontal line where C O V' meets. So, V is one end of this ellipse V' is another end of an ellipse.

If we are drawing an ellipse, it goes via this points, once we know V' and V we can make intersection point so that a perpendicular bisector meets at O. So, this is the centre of that ellipse 2 foci we always are going to have from V to F and O is centre. So, FO again equal to this new point F' where another focus one can locate it.

The other way is from V whatever the distance of F we have same another focus we are going to have it at this point. So, the focus point again will be at that location, once we defined these lines, so let us look at only half part of this curve once we know that locate points 1, 2, 3 and 4 and 5 kinds of points on this curve.

Use a distance of 1 to 1' whatever this one as the distance from focal point F try to make an arc which is going to cut this 1 1' let us call this one. Similarly, on the other side, make a cut, so name these points as P 1 and P 1'. Similarly, as distance 2 to 2' and F as the centre, make an arc. So, call that one as P 2. Similarly, make another arc from here 1 to 2 line.

So, if we are extending these lines, it is going to intersect at this point. So, this is P 2' and this one P 2. Based on how many points we are going to have if we are going to divide

many more points drawing these vertical lines. We can have many more points, only thing is from this point we have to mark what is this distance from focus point make an arc.

Similarly, from this point, what is this vertical distance, use that vertical distance make an arc so that we will be in a position to mark these points. So, if we are going to have a free hand curve for this purpose, one can use French curves also. If we are making a freehand curve, it forms an ellipse which tracks all the way there and again pass through V 1 point and comes back. This is the way one supposed to construct this ellipse.

So, we have constructed part of the ellipse here. If we go with this procedure, finally we will construct this ellipse. For any ellipse, we pick a point from focus what is the distance, let us call that in this case 3 and from point to directrix C that is 4 in this case. So, eccentricity here 3/4.

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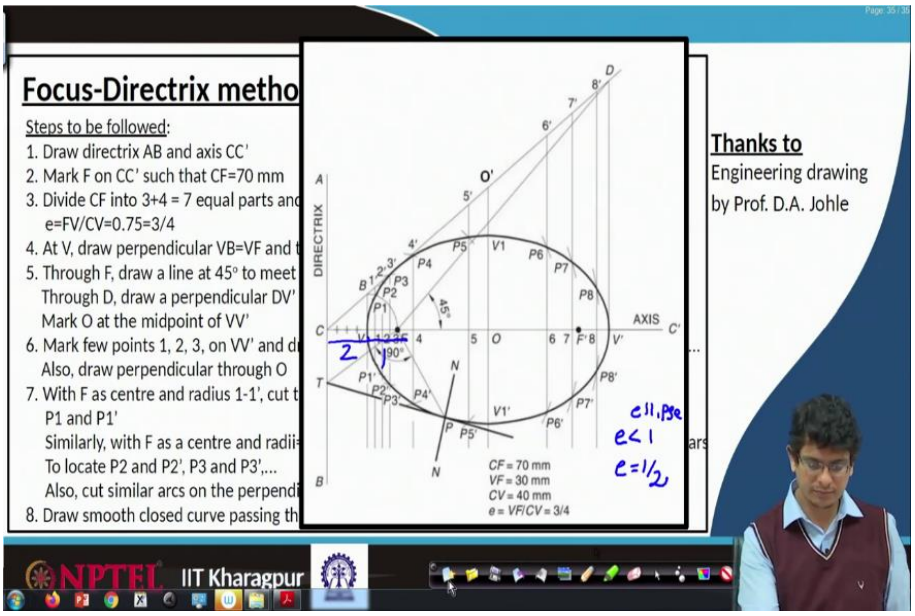

If it is something like another factor any e less than 1 we get an ellipse, in this case, we have used three fourth; one can also use something like half. If e is equal to half three fourth ratio 0.75, it makes. So, 1 2 ratio means that we have to make it into 3 parts first of all, then locate 2 parts in that direction 1 part then go with the same procedure we will get another ellipse. This is the way any general ellipse one will be in a portion to construct it.

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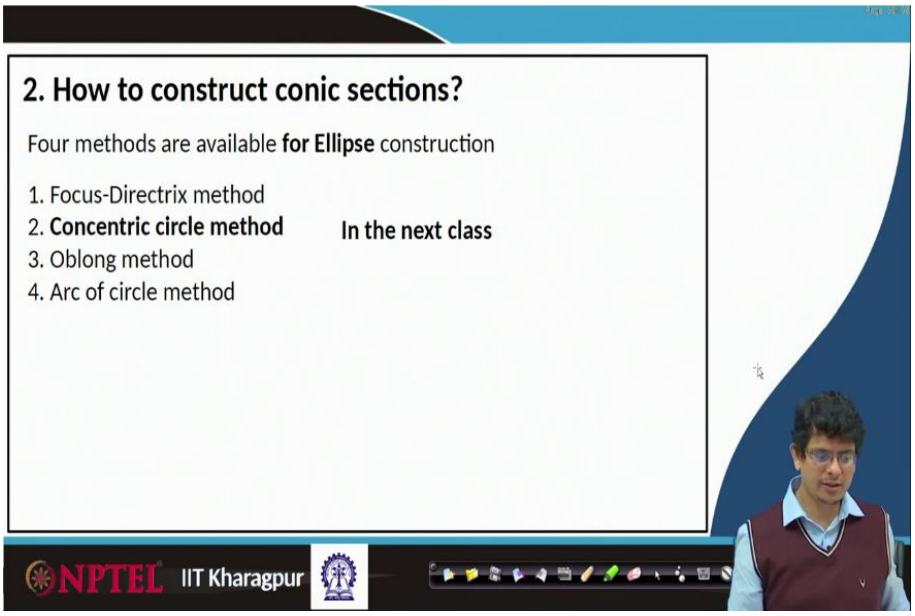

In the next class, we will learn about the concentric circle method and after that oblong method.

Thank you very much.