Introduction to Fluid Mechanics
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Lecture – 08
Kinematic viscosity, Reynolds number

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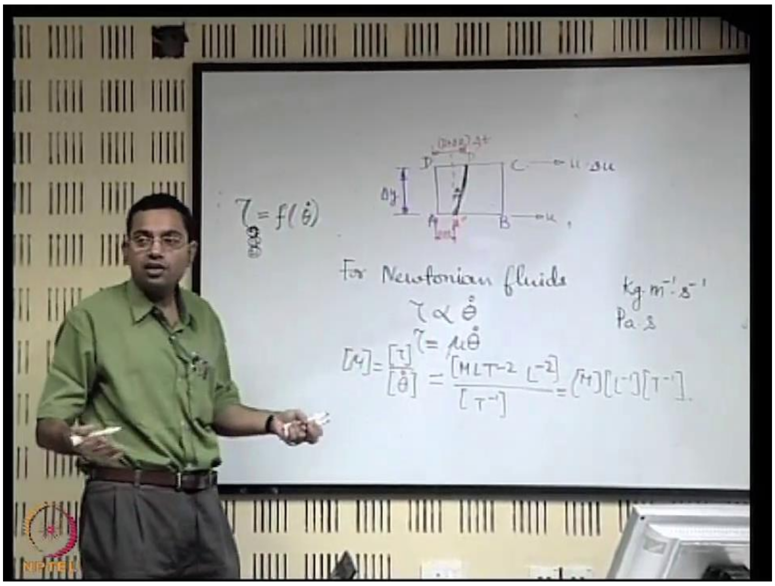

So, if we try to identify the dimensions of the viscosity. So,

$$[\mu] = \frac{[\tau]}{[\dot{\theta}]} = \frac{[MLT^{-2}L^{-2}]}{[T^{-1}]} = [M][L^{-1}][T^{-1}]$$

So, in SI units you can write this as $Kgm^{-1}s^{-1}$, but there are different styles in which this is written also in SI units of course, $Kgm^{-1}s^{-1}$ is one of the styles, but you can also write it in terms of force units and those are typically more common units.

So, this if you write it in terms of force units or units of pressure so, to say, you can also write it as Pa.s. Because you see this is like the stress is like a pressure unit and this is second inverse. So, that goes in the numerator it may also be written as Pa.s. So, these are alternative units for the viscosity, alternative expressions for units of viscosity all in SI units.

So, in our course we will be always following S I units and therefore, it is important that we get conversion to this units in. Of course, in other units there are expressions like; in CGS units this particular unit is given the name as poise. And the reason is like this names have come up to honour the very famous scientist or mathematician who develop the subject of fluid mechanics.

The subject of fluid mechanics as been initially developed mostly by mathematicians and it is important to honour them in various ways. One of the ways is to give units in their names. So, by giving honour to the Poiseuille to the famous scientist Poiseuille the name poise came because Poiseuille had many seminal contributions towards a better understanding of viscosity.

So, this is regarding the units of viscosity and in SI systems. Now, we will learn a concept which is closely related to viscosity and that is known as kinematic viscosity; that is also a property of the fluid.

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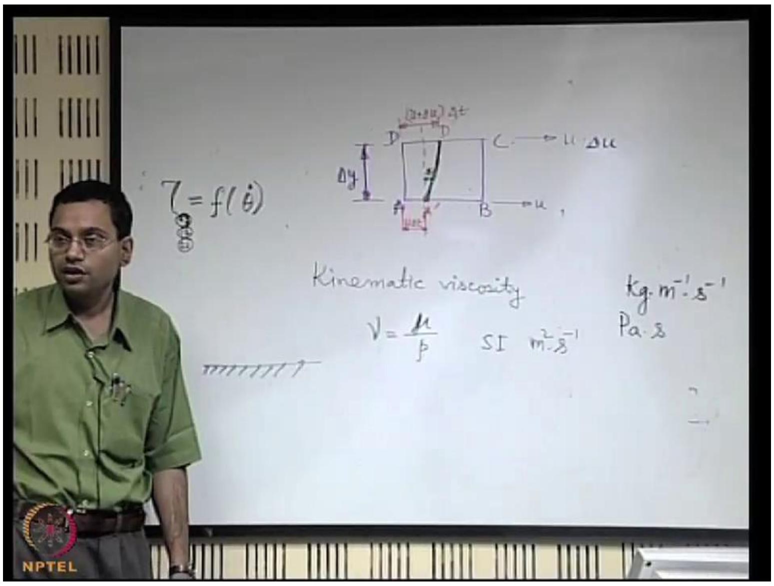

So, when we say kinematic viscosity, we define it with a symbol $\nu = \frac{\mu}{\rho}$. We will see that what is the physical significance of this definition, but first let us look into the units and so, on. So, you can see that if you write its S I units what will be its S I units. So, μ was the viscosity unit and ρ is Kg/m^3. So, it will come to S I unit of $m^2 s^{-1}$. It does not have any maths unit involved with it and that is why the name kinematic. Because when you have a maths unit involved it is as if like you are thinking of a forcing situation where the kinetics also come into the picture.

So, here it is solely like a dictated by units determining the motion. So, that is why the name kinematic viscosity or that is something more superficial, but the concept is more subtle that, how we can utilise the concept of this to get a physical feel of what is happening within the fluid. Again take that example where you have like flow over a plate or any solid boundary. Now what is the viscosity that tries to do? It tries to defuse the disturbance in momentum.

The solid boundary creates a disturbance in momentum viscosity is a fluid property that tries to defuse that disturbance in momentum to the outer fluid. So, that is there in the numerator, in the denominator what is there? In the denominator there is a fluid property which tries to maintain its momentum; because it is density. So, it is directly related to the mass and mass is a measure of inertia.

So, what it tries to do? That denominator represents a physical property by virtue of which the fluid tries to maintain its momentum and the numerator is a property by which it tries to disturb its momentum or propagates the disturbance. So, it is an indicator of the relative tendency of the fluid, to create a disturbance in momentum as compared to its ability to transmit a momentum. A rather to maintain its momentum not to transmit to maintain its momentum through its inertia right.

So, that is the physical significance of the kinematic viscosity. So, it is not just only the viscosity that is important, the density is also important; because when you are thinking of the characteristic of a fluid in transmitting momentum, you must also try to compare it with a situation where it is maintaining its own momentum and not responding to a change in momentum. So, that is that is why this is a very critical and important ratio.

The next question that we will ask again qualitatively that, what is the origin of viscosity. That where from such physical property originates is; it just by magic or where from it occurs.

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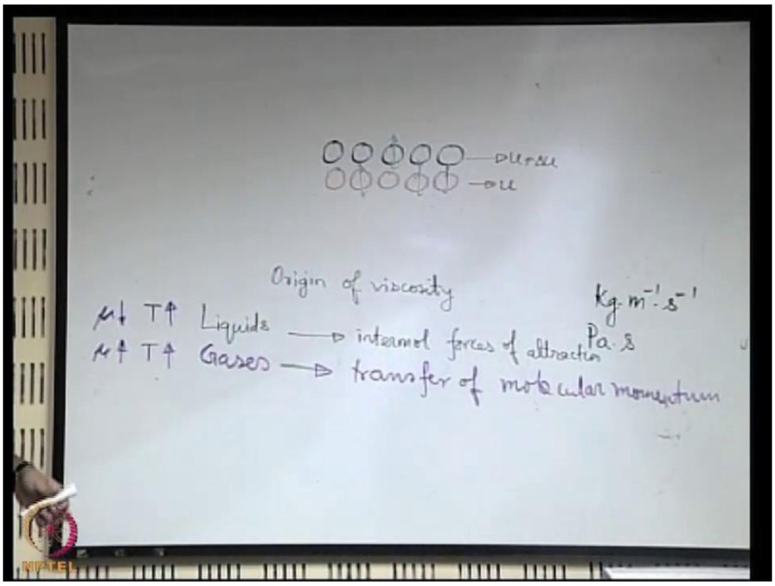

So, we will see what is the origin of viscosity in a very qualitative way. Let us first concentrate on liquids. Why we separately concentrate on liquids and gases is, a very straight forward reason that the molecular nature of gases and liquids are different and therefore, the viscosity or the viscous behaviour is going to be different.

And it is important to appreciate that at the end because through a continuum description like viscosity is a continuum description of a fluid property, we are usually abstracted of the molecular nature. But we have to keep in mind that a molecular nature of the fluid, that has eventually given rise to this property. So, there is a direct relationship it is like an up-scaled version of the molecular behaviour. So, that you are abstracted from it, but you have to keep in mind at that it is the molecular behaviour that has given rise to these properties.

So, when we talk about liquids. So, when we talk about liquids, the main origin of viscosity in liquids is intermolecular forces of attraction. So, there are different types of forces of attraction like, if you have like molecules and if you have unlike molecules you might have accordingly different types of forces like cohesion and adhesion and so on.

Fundamentally if you have two different types of molecules or two molecules of the same type, there will always be some attractive and repulsive potentials which are acting amongst themselves. So, net effect is an intermolecular force of attraction that binds the molecular configuration together, otherwise molecules will just escape. And for liquids it is the intermolecular forces of attraction, that gives rise to the viscosity mainly; so, intermolecular forces of attraction.

However, if we want to give this logic for gases, it sounds to be weak why it sounds to be weak? Because we know that gases are much densely populated systems much less densely populated system and therefore, intermolecular forces of attraction are not that strong. So, what give rise to a strong viscosity in gases many times? So, let us look into an example.

So, let us say that you have a system of gas molecules. So, one layer of gas molecules which is at the top of another layer which is like this: what is the difference between these two layers? The bottom layer is moving at a slower velocity and the top layer is moving at a faster velocity and we have seen that how such velocity profiles occur in a system of fluid.

Now, for gases these molecules have strong random motion with respect to their mean position. So, these are vibrating with respect to their mean positions. So, in this way what is happening? It is very likely that a molecule from the slower moving layer joins the group of a faster moving layer and what it will try to do? It will try to reduce the velocity of the faster moving layer.

On the other hand, it is very likely that because of this random thermal motion, this motion is because of a thermal energy of the system. So, if the temperature is absolute zero it will go to a stop, but in any other case this thermal motion will be there. So, from the upper layer there will be molecules which are joining the bottom layer, and this will now try to enforce the bottom layer to move faster.

So, that is how there is an exchange or transfer of molecular momentum and these gives rise to the viscous properties of gases. So, for gases it is mostly because of transfer of molecular momentum. If we say now that there are certain factors which are affecting the viscosity for gases and liquids, then we should be able to explain how those factors influence the viscosity through this basic understanding.

Let us concentrate on one of the factors as examples let us say temperature. So, viscosity of fluids is generally a strong function of temperature. Now let us ask ourselves a question if you increase the temperature of gas and a liquid, intuitively would you expect the viscosity to increase or decrease?

It is expected to decrease why? If you increase the temperature the intermolecular forces of attraction would be overcome by the thermal agitation and therefore, the basic origin of viscosity will be disturbed. So, it is intuitively accepted that for liquids, the viscosity should decrease with increase in temperature.

However for gases what will happen?

If you increase the temperature there will be more vigorous exchange of molecular momentum and therefore, for gases it is expected that, the viscosity will increase with an increase in temperature. Of course, there may be exceptions, but we should not discuss exceptions these are more rules than exceptions. So, what is the basic understanding? See whenever we learn a particular concept it is important to learn the basic science that leads to the concept.

So, if we just learn it like a magic rule, that viscosity of a liquid decreases with increase in temperature, it serves no purposes only in your examination you may answer it well, but the next day you forget and many times in the examination also you may confuse between these two. But if you recall what is the correct physical reason reasoning that goes behind that, it is very easy to appreciate that why this should be more intuitive or not.

Now, just as an example for gases, let us try to see that whether we may quantify the viscosity may be for ideal gases through the concept of exchange of molecular momentum. So, let us say that you have n as the number density of the gas molecules.

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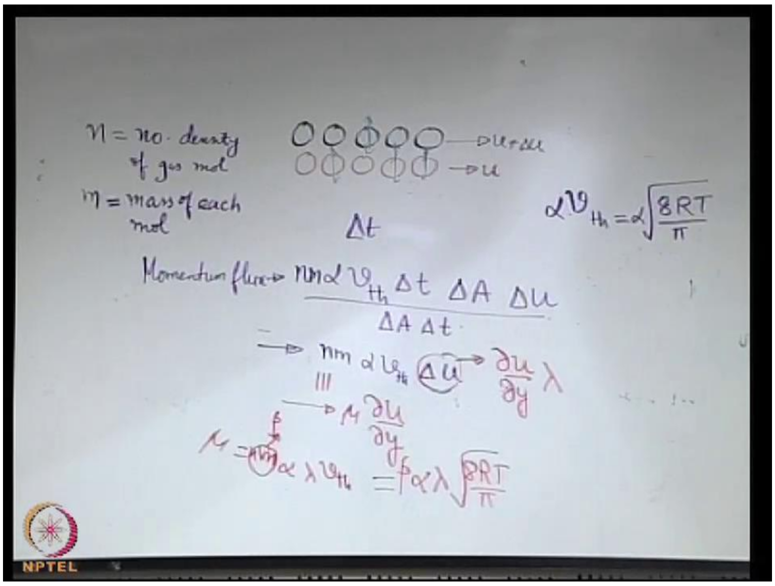

So, let us say n is the number density. Number density means, basically the number of gas molecules per unit volume. So, with this number density and with this type of interaction let us see that: what is the extent of exchange of molecular momentum. Let us say that m is the mass of each molecule. Now, let us say that these molecules are having a random thermal motion and the random thermal motion is being characterized by a velocity which if you follow the kinetic theory of gases it may be expressed as $v_u = \sqrt{\frac{8RT}{\pi}}$ for ideal gases.

Now, of course, this is a random thermal motion; so, not that this full motion is utilized for just the transverse velocity component because, the molecules have degrees of freedom thermal degrees of freedom in all direction. So, let us say that a fraction of that alpha is what is utilized for the exchange of transverse molecular momentum in the y direction.

So, if we consider a time of Δt, then with the time of Δt what is the distance that we swept? In the vertical direction that is $\alpha v_{th}\Delta t$ that is the distance swept in the y direction. And what is the number of molecules which is taking part in that interaction? So, this is the distance in the y direction, if you multiply $\alpha v_{th}\Delta t$ with the cross sectional area say ΔA of the phase that we are considering across which this molecules are moving along y. So, this represents the total volume and what is the mass within the total volume?

So, first n is the number density. So, n into this is the number of molecules in the total volume that if you multiple with m that is the mass of each molecule then $nm\alpha v_{th}\Delta t\Delta A$ is what is the total mass in this volume. So, with this total mass, there is a molecular momentum of the upper layer $nm\alpha v_{th}\Delta t\Delta A\times(u+\Delta u)$ of the lower layer is $nm\alpha v_{th}\Delta t\Delta A\times(u)$. So, the net exchange of molecular momentum is the difference between that two and that is $nm\alpha v_{th}\Delta t\Delta A\Delta u$

Now, if you want to find out that: what is the rate of exchange of this molecular momentum, say per unit time per unit area that is known as flux. So, any quantity which you are estimating as a rate in terms of per unit time and per unit area normal to the direction of propagation of that, then that is known as a flux. So, we call it as a momentum flux. So,

$$\text{momentum flux} = \frac{nm\alpha v_{th}\Delta t\Delta A\Delta u}{\Delta A\Delta t}$$

So, the momentum flux is nothing, but $nm\alpha v_{th}\Delta u$. So, this Δu we may express in terms of a gradient let us think that we are having interactions between two layer of molecules. When the interactions is possible what is the characteristic length that should separate them which should make them interact that is the mean free path. Because that is the distance over which characteristically one molecule should transverse before colliding.

So, this may be expressed in terms of a gradient like this. So, this is like the rate of change that multiplied the distance over which it is traversing. So, that is like the characteristic Δu. So, λ is the molecular mean free path now if you. So, we have seen that the molecular momentum flux that is nothing, but the shear stress in a fluid. So, this if it is a Newtonian fluid, this should also be expressible as equivalent to $\mu \frac{du}{dy}$ by the Newton's law of viscosity.

So, this is identically equal to shear stress. Shear stress is nothing, but like in this case the molecular momentum flux. So, if you equate these two, then what you get out of this? $\mu = nm\alpha\lambda v_{th}$. $n \times m$ is what? It is basically the density the mass the density in terms of mass of the fluid. So, it is $\rho\alpha\lambda v_{th}$.

$\mu = \rho\alpha\lambda v_{th} = \rho\alpha\lambda\sqrt{\frac{8RT}{\pi}}$. So, this is like roughly an expression that talks about, the viscosity behaviour of ideal gases and you can clearly see that as the temperature increases the viscosity increases. Of course, for real gases it is not so, simple for real gases you also have to consider other interactions, but this just gives the qualitative picture of the entire scenario.

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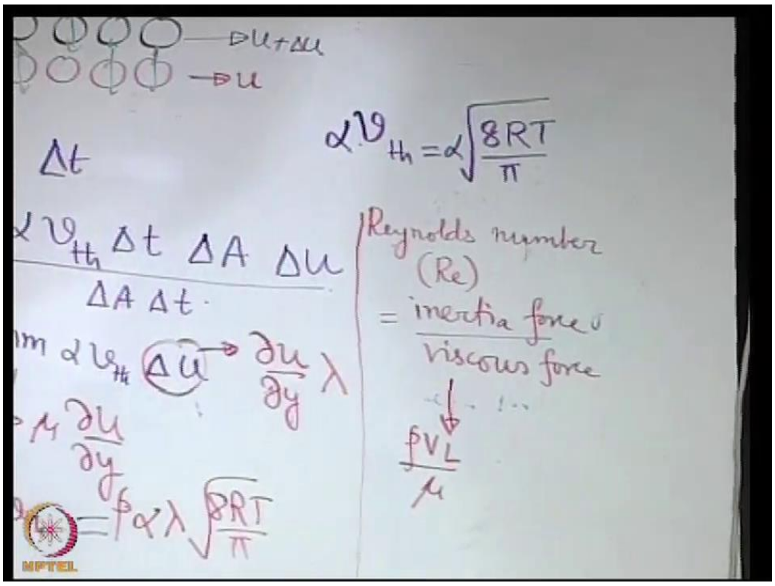

Keeping this in mind, what we will just do? We will briefly define a few non dimensional numbers with which we will try to correlate this behaviour. The first non dimensional number that we will be defining is something which you have heard many times is the Reynolds number.

So, when the fluid is flowing, the fluid is being subjected to different forces one of the forcing mechanisms is the inertia force. So, the fluid has an inertia, because of that inertia it tries to maintain its motion. On the top of that there is a resistance in terms of viscous force which tries to inhibit the motion. So, we may try to get a qualitative picture of what is the ratio of this inertia force and viscous force, which will give us an indication of the extent of the effect of viscous force in terms of influencing the fluid motion when it is subjected to an inertia force. So, the inertia force is like mass into acceleration.

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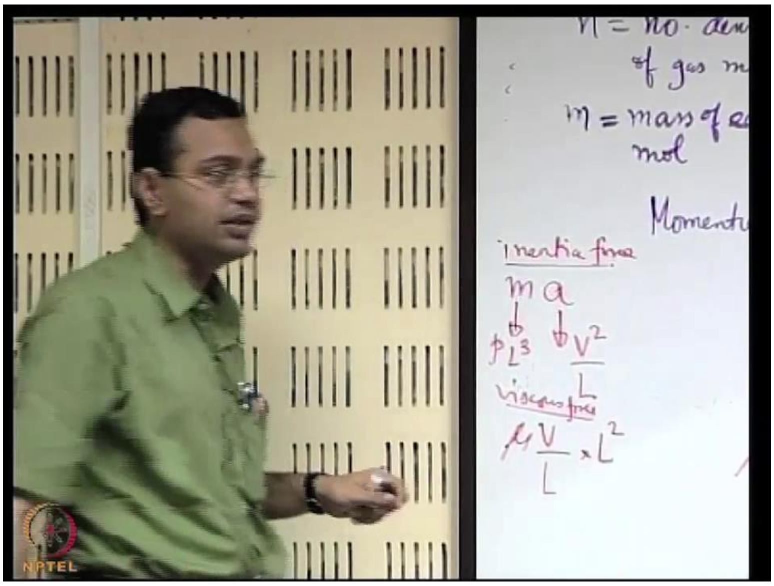

So, if you write m$\times a$ this is the inertia force. And the acceleration in terms of velocity can be written as $\frac{v^2}{L}$. This is also an unit of acceleration just expressed in terms of velocity. So, mass into acceleration this is the inertia force and the viscous force you can express like using the Newton's law of viscosity.

So, let us try to express the viscous force, the viscous force is the shear stress times the area. Shear stress is $\mu\times\frac{V}{L}\times L^2$. So, we are just trying to write it dimensionally. m$\times a$ this is the inertia force. $\mu\times\frac{V}{L}\times L^2$ is viscous force.

So, if you find out the ratio of these two what you will get? Now when you write the mass into acceleration, you have to keep in mind that the mass is the density times the volume. So, the mass is $\rho\times L^3$ in terms of dimension. So, if you find out the ratio of these two forces what you will get is, $\frac{\rho VL}{\mu}$. So, this L is a characteristic length scale of the system.

We have earlier discussed that: what is the characteristic length scale of the system. So, in a system with particular characteristic length scale, the Reynolds number expressed in terms of the length scale and we will see what is this velocity, this is also a characteristic velocity a system may have different velocity at different points. So, this is a kind of characteristic velocity taken and with these characterizations, it may be possible to have a quantification of the ratio of inertia and viscous force.

Even when the inertia force is absent, this tells as a way by which we can have the concept of Reynolds number utilizable. That not the inertia force by viscous force, but if it was possible to have an acceleration by having a driving force, what would have been that equivalent inertia force? The ratio of that equivalent inertia force by viscous force may be interpreted as the Reynolds number in cases when the fluid is not accelerating.

So, when the entire energy of the fluid was utilized to accelerate it, what would have been that equivalent inertia force that by viscous force would still then we interpreted as an equivalent Reynolds number. So, we will try to stop here today and in the next class we will try to utilize the concept of this non-dimensional number and relate it with the viscous behaviour for gases ok.

Thank you.