Introduction to Fluid Mechanics
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Lecture – 06
Density, Bulk Modulus, Viscosity

Now, we have discussed about one fluid property which is pressure, whenever we are talking about effects of compressibility there are other related fluid properties which come into the picture in a very related manner and those properties, we will look into one by one briefly.

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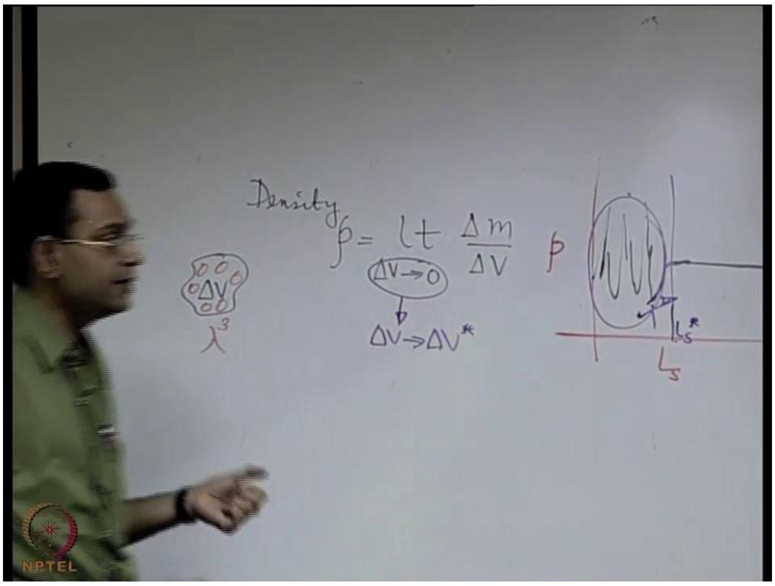

So, one of the important properties will be density. So, loosely what we say that if we have a volume elemental volume say Δ v we have the mass of the molecules, which are there in this ΔV.

$$\rho = \lim_{\Delta v \to 0} \frac{\Delta m}{\Delta v}$$

So we will write $\Delta v \to 0$, we will think that nicely it should give a limiting definition of what is the so-called density. Mathematically very nice, we will see whether it works or not.

So, what it says? It says in the limit as $\Delta v \to 0$; that means, limiting is small volume you find out what is the mass of molecules inside. So, you find that mass per unit volume to get local density at a point that is what this definition is saying. Whether it works or not we have to come back to the continuum hypothesis to adjudge. So, if we remember that in the continuum hypothesis, we disregard the molecular nature and we just consider that it is a continuous medium, does not mean that there are no molecules but; obviously, were obstructed of the molecules and we are just representing the their gross effect, but whenever there are molecules we have to see that what is the number of molecules within this elemental volume?

Again, if the number of molecules, if within this elemental volume is very small then because of the statistical fluctuations even uncertainty in one molecule will give a lot of error and we will give a lot of fluctuation. So, it is critical that what is that elemental volume that you should choose it cannot be too small. What is the smallness? The smallness will come with a length scale; the smallness of the length scale here is the mean free path λ. So, very small volume when we say then that will scale with say λ^3, λ is like a length scale which will correspond to a very small volume.

So, when the volume is of the order of λ^3 elemental volume then it will have lots of uncertainties in the statistical fluctuations of the molecules, because within that length scale you really have uncertainties related to collision. On the other hand, if you take this volume Δv very large then also, we can calculate a density, but it will not be able to capture the local variations it will give a global average.

Therefore, one has to choose a threshold length scale for calculating this density and how it should be sensitive to the length scale, if you make a plot of say the length scale that you choose say we call it Ls and the density that you predict. So, we will see that if you choose a very small length scale, you will get a variation, this type of fluctuation then it will come to a steady one and then if you choose a larger length scale, it will be changing like this. So, what is the significance of such a plot?

These length scales are small enough so that you have really random fluctuations, because of the uncertainties. This length scale is fine beyond this length scale, you have a variation this variation is because over the system length scale the density is varying from one point to the other. So, correct choice of length scale maybe something which can be say in between these two. So, in between these 2 limits therefore, if we say Δv tends to 0 that is not fundamentally correct, because Δv tends to 0 may make you fall on this regime, because Δv tends to 0 means mathematically Δv is as small as possible.

So, as small as possible will obviously, be something smaller than as big as possible so obviously, in that context one has to remember that this Δv tends to 0 should be corrected and how it should be corrected we should change it as not Δv tends to 0, but Δv tends to some Δv^*, which is like if we call this as L_s^* then may maybe it is of the order of $(L_s^*)^3$.

So, it is a threshold length scale beyond which you are not having such uncertainties and fluctuations affecting your density calculation. So, this Δv* therefore, we can say is the smallest elemental volume over which continuum hypothesis is valid. So, it is not tending to 0, but tending to limitingly small volume Δv* over which still continuum hypothesis works, below this limit continuum hypothesis might not work and therefore, this definition will not work because, this definition is on the basis of a continuum description of fluid properties like density. So, we have talked about density, we have talked about pressure.

Next let us talk about bulk modulus.

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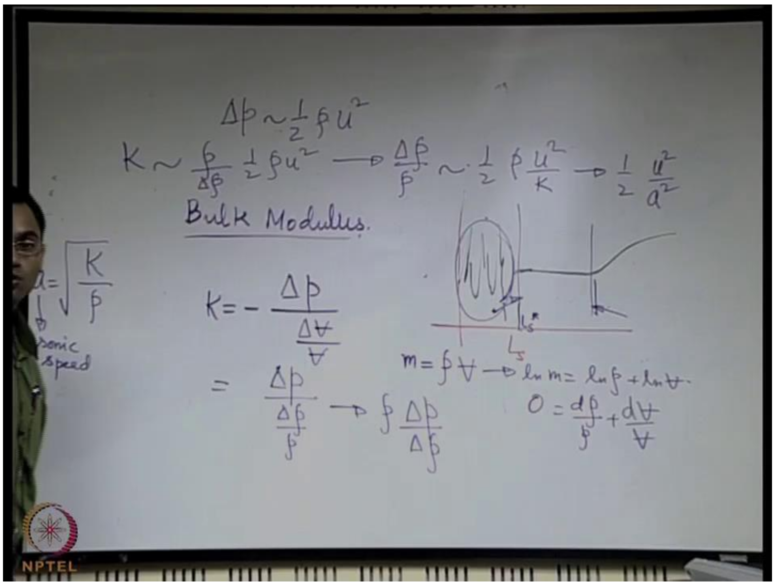

So, all of you are aware of the basic concept of bulk modulus, but let us just see that how you have defined it let us try to define it first in a very loose manner, it is always important to get a qualitative feel and then of course, you can have more sophisticated definitions. We will not go into very detailed sophisticated definition of bulk modulus because, it requires a detailed understanding of thermodynamic processes and therefore, we are not going into that type of definition. So, in a loose sense if you are applying say a pressure differential Δp that is expected to give rise to a change in volume of a fluid element.

Let us say that change in volume is Δv. So, original volume was v. So, $\frac{\Delta p}{\frac{\Delta v}{v}}$ is the rate of or $\frac{\Delta p}{\frac{\Delta v}{v}}$ is the total change in volume per unit volume. So, Δv is the kind of a volumetric strain and Δp is the pressure differential, which is responsible for the volumetric strain and you expect that if Δp is positive Δv is negative because, if you press a fluid element it should compress it is volume should decrease. If you want to give the corresponding fluid property a positive number definition then you should adjust it with a negative sign.

$$K = - \frac{\Delta p}{\frac{\Delta v}{v}}$$

Now you can relate the change in volume with the change in density.

How is it possible to relate the change in volume with the change in density? So, you have said considered the mass of a fluid element. So, that is the density into volume, from now onwards whenever we will be discussing about the volume, we will be using a symbol not v, but $\forall$, because we will be using v for velocity also.

$$K = - \frac{\Delta p}{\frac{\Delta \forall}{\forall}}$$

So, just to avoid that confusion between the symbol of velocity and symbol for volume, we will be just distinguishing those in this way. So, I will not be repeating the symbol many times, but once I will be using this type of symbol, you just take it that we are talking about the volume not the velocity.

$M = \rho\forall$

$\ln(m) = \ln(\rho) + \ln(\forall)$

So, if you want to say see that what is the relationship between the elemental change of density with a elemental change of volume what you can do simply just you can take log of both sides and differentiate. So, if you differentiate keeping in mind that the mass of the fluid element is conserved, so, it's derivative should be 0 therefore, loosely like if you are following this definition, we can relate Δv by v with $\Delta\rho$ by ρ.

$0 = \frac{d\rho}{\rho} + \frac{d\forall}{\forall}$

So, that we will just absorb the minus sign and it will be like K = - $\frac{\Delta p}{\frac{\Delta \rho}{\rho}}$

So, it is like $\rho \frac{\Delta p}{\Delta \rho}$.

So, that we will just absorb the minus sign and it will be like this. So, it is like rho delta p over delta rho like this. Now, we can relate delta p with the velocity of flow in a order of magnitude sense, the delta p and the velocity of flow this is just like if you consider that there is equivalent pressure change, which is brought about by the change in kinetic energy of fluid which is moving with a velocity u then this is not that they are exactly equal it is just to say that one scales with the other in this way. So, you can therefore, write a scale of K as

$$K \sim \frac{p}{\Delta \rho} \times \frac{1}{2} \rho u^2 \rightarrow \frac{\Delta \rho}{p} \sim \frac{1}{2} \rho \frac{u^2}{K}$$

Let us try to see that, what is $\frac{\Delta \rho}{p}$ scale? That means, what is the change in density relative to it is original density. So, if you do that it will be $\frac{1}{2} \rho \frac{u^2}{K}$. Remember one thing, that this K by rho it is something which is the very fundamental quantity, which we have started in physics what is this or square root of K by rho, if it reminds you more.

This is fundamentally sonic velocity; sonic speed. So, to say what is sonic speed? Sonic speed is not just speed of sound, sonic speed is the speed by which a disturbance propagates through a medium and here we are talking about this type of disturbance through the elastic property of the medium. So, it happens to be the speed of sound ok. So, this is the sonic speed a. So, a = $\sqrt{\frac{K}{\rho}}$

This is a very basic high school physics based definition. So, keeping this in view we can write this as $\frac{\Delta \rho}{p} \sim \frac{1}{2} \rho \frac{u^2}{K} \rightarrow \frac{1}{2} \frac{u^2}{a^2}$

So, we can see that the relative change in density is related to a quantity $\frac{u^2}{a^2}$, what is this $\frac{u^2}{a^2}$? This is a non-dimensional quantity that you can see because it is the ratio of two velocities.

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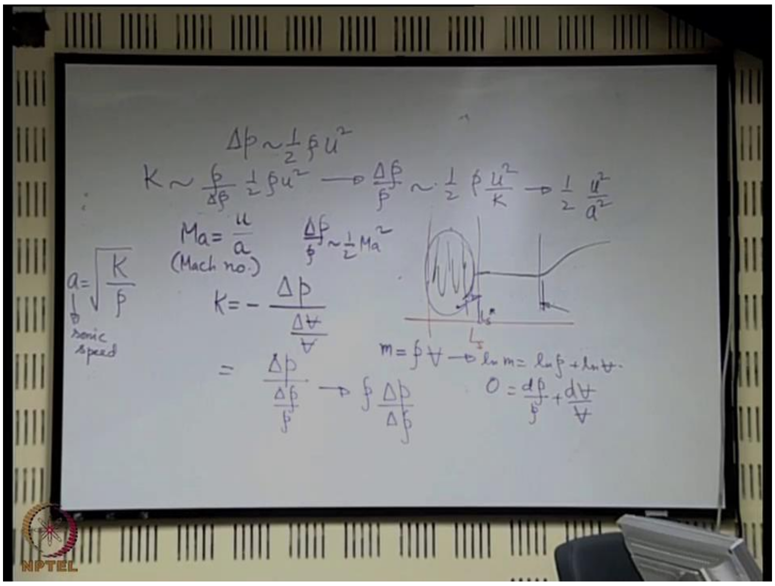

So, in the numerator you have u in the denominator you have a. So, u is the velocity of flow and a is the velocity of a disturbance, which is moving in the medium in which the flow is occurring and these 2 ratios is known as Mach number.

I mean ratios of these 2 numbers is known as mach number. So, you have heard about the mach number like a jet moving with a mach of these. So, higher the mach number higher is the velocity of flow relative to the velocity of the disturbance with which the disturbance propagates within the medium and therefore, we say that it is having a more and more compressible effect, the reason is if you have if you just write this $\frac{\Delta\rho}{p} \sim \frac{1}{2} \mathrm{M_a}^2$

Therefore higher the mach number higher is the effect of the change of density relative to its original density. So, mach number therefore, is a very important indicator of something which is called as compressibility of a fluid. So, what is the signature of compressibility of a fluid? We will say that a fluid is compressible when it has a change in density, because of change in pressure.

So in that way, all fluids are compressible right because all fluids will have some change in density, because of change in pressure, but when we say that a fluid is incompressible what we mean is that, that effect is negligibly small. So, compressible fluid and an incompressible fluid these are just conceptual paradigms, there is no fluid as such which is in compressible.

But when we say that a fluid is incompressible we mean that it's compressibility effect is very very small, again how small or how large that is something which may be debated. So, let us say that we are talking about a change; this relative change say 5 percent. So, let us say that if we say that this change is less than 5 percent, we say that it is almost incompressible. So, if we want to see that what will lead to that 5 percent.

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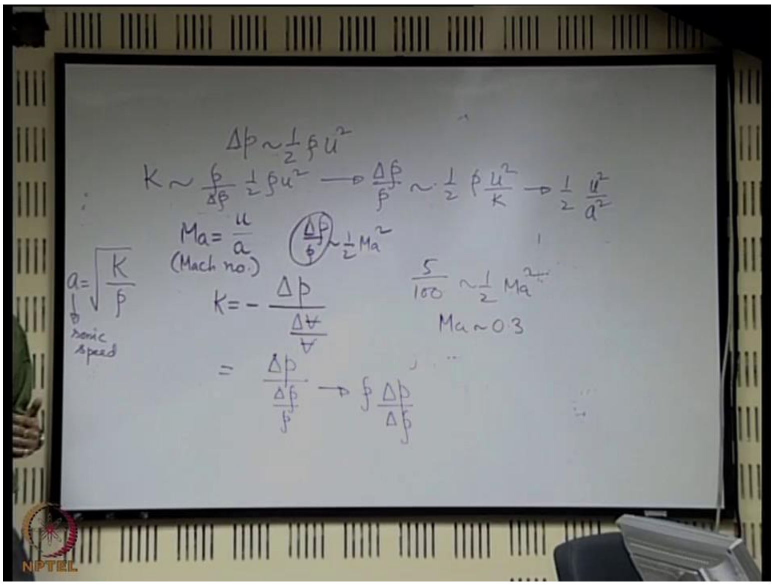

So, one may work it out with say 5 percent means 5 divided by 100. So, what would be the threshold mach number for this roughly 0.3 right. So, 0.33 or whatever, but roughly 0.3; that means, if we say that relative density change less than 5 percent is something, which we do not consider as the compressibility effect, that implicitly means that a mach number less than 0.3 something which is not going to give us any serious compressibility effect.

So, this is the important because whenever you are analyzing an engineering flow, nobody will tell you that whether the flow is compressible or incompressible as an analyzer, it is your responsibility to make a judgment of whether you are going to use the concept of compressible flow or incompressible flow for the analysis of your problem and then you have to be confident that whether a particular analysis methodology is going to work or not.

Of course, for all flows compressible flow and analysis we will work because, all flows are compressible, but it is like if you have a mosquito you will not like to kill it with a cannon. So, if you are ready or if you are having the possibility of doing a relatively simple analysis one should not go for a complex analysis that is what all of us have learnt in engineering that do not go for unnecessary complication until and unless it is absolutely required. So, whenever compressible flow analysis is not required, we should not go for it and this Mach number of flow will give us a guideline of whether we should go for compressible analysis or not.

Couple of other important points or remarks are there regarding this definition of bulk modulus one is see in this definition, we have talked about a change in volume because of a change in pressure or equivalently a change in density because of a change in pressure, but pressure effect of change of density it depends on the type of process all of you have heard of certain thermodynamic processes like adiabatic process, isothermal process and so on.

So, given a particular system how the density will change with pressure will depend on the nature of the thermodynamic process. So, this definition as such fundamental is not incorrect, but incomplete, because it does not talk about the thermodynamic process by which you are trying to have this change of state. So, there are more fundamental or correct definitions of these in terms of specifying it as say either a reversible isothermal process, reversible adiabatic process and so on.

But we are not going into those details here because thermodynamics is not the scope of this particular course, but we should keep with that in mind because, whenever you will be studying thermodynamics again, this type of definition will come into the picture and there more detailing will be done in terms of whether it is a reversible adiabatic process, reversible isothermal process and so on.

So, that is one of the important concepts the second important concept is as follows. Say, you are interested to identify whether a flow is incompressible or not and in that respect there is a subtle difference between the concept of incompressible fluid and incompressible flow these are very very subtle concepts. So, when you talk about an incompressible flow, what you mean is that if you have a volume element of a fluid that volume does not change. So, incompressible flow means that there is no volumetric strain of the fluid element there is no change in volume, but you cannot directly always relate it with this definition, because the change in volume may not always be due to change in pressure directly.

It may be because of something else also I mean there are reasons for which you might have change in volume of a fluid element not because of the change in density due to change in pressure, but maybe because of change in density due to change in temperature not directly due to pressure. So, whenever we are talking about incompressible fluid we are talking about that we are asking ourselves a question that is there a change in density, because of a change in pressure. If that answer is that yes, it is significant we call it a compressible fluid, but not compressible flow definition is something more general compressible flow means fluid element which if it is going to have a volumetric strain or change in volume per unit volume by whatever reasons.

It need not be just due to pressure or it may be because of anything then we say that is a compressible flow. So, compressible fluid and compressible flow are related because of course, one of the reasons of being a fluid compressible or being a flow compressible is because of the fluid itself is compressible. So, the density change due to change in pressure is significant, but there could be other effects that are creating the change in the volume. So, this is the concept that we should remember. Next what we will do is we will try to learn about very important property of fluid, which is called as viscosity.

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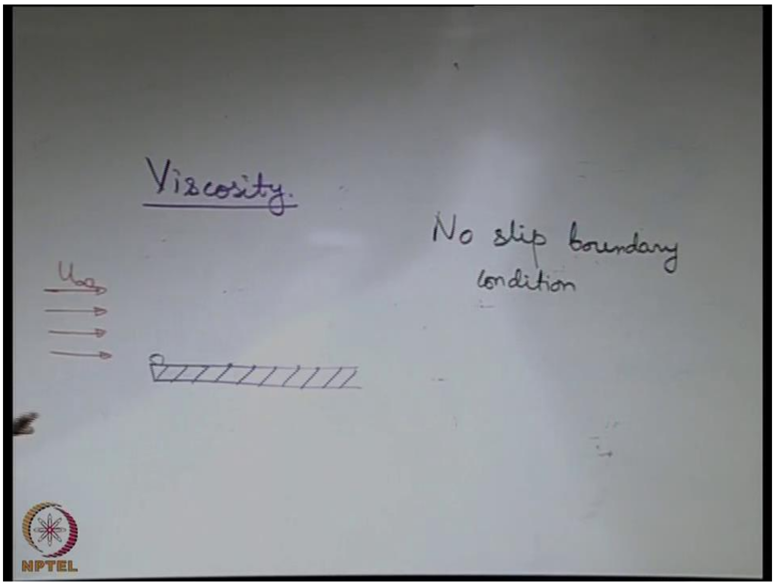

So, when we talk about viscosity we will not try to just learnt it in abstraction, but we will start with an example. Let us say that you have a flat plate just like the top of a table flat plate and fluid is coming from far stream just like say fluid is being blown from that side it is coming on the top of the table and going away.

So, the top of the table may be like a flat plate. So, let us say that the fluid is coming with a uniform velocity from a free stream in fluid mechanics usually, we give such a symbol infinity with a subscript to indicate that it is a free stream condition. So, infinity subscript is like a free stream velocity. So, it is coming with a uniform free stream velocity, now that freeness will be disturbed because of the presence of the plate and let us see that what is going to happen. So, when this fluid first comes in contact with the plate what happens?

Let us first try to understand that say, there is a fluid molecule which comes in contact with the plate. So, what will the plate like to do with the fluid molecule? Let us consider 2 different examples, one is for a gas and another is for a liquid, usually whenever we discuss about fluids, we are either talking about gases or liquids, but sometimes their physical behavior it is better to discuss distinctly or differently. So, let us say that there is a gas molecule as a first example, which is coming falling on this plate.

So, what will happen there will be first a tendency that the gas molecule is absorbed on the surface. So, once it is absorbed on the surface then what will happen that it will exchange some of this momentum with the surface. So, it will try to have it slow down and then again, it will try to be get getting ejected from the surface. So, it is like a molecule falling on the surface adsorbed on the surface, getting ejected from the surface like this. So, in this process many molecules are colliding with this and their exchanging their momentum with a wall. So, if there are very large number of collisions.

So, to see theoretically infinitely large number of collisions then this kind of momentum exchange will bring on an average the molecules in equilibrium with the surface. So, if the surface is at rest the molecules will also be at rest so that will imply that there is 0 relative velocity between the fluid and the solid at the point of contact and this is something which is known as no slip boundary condition. So, fundamentally what is a no slip boundary condition? It is 0 relative tangential component of velocity to be more accurate 0 relative tangential component of velocity between the fluid and the solid at their points of contacts.

We are not talking about the normal component because, still the molecule may be colliding like it may have a sort of elastic collision. So, it may bounce back. So, it may have a normal component. Now; obviously, regarding the normal component there are issues like if the molecules are sufficiently large in number and they are at the wall they cannot penetrate and go through the wall walls wall is not having holes. So, that is called as a no penetration boundary condition then there even the normal component of velocity will become 0, but no slip boundary condition does not talk about that is a separate consideration.

No slip boundary condition talks only about the tangential component of velocity. So, 0 relative tangential component of velocity between the fluid and the solid at the point of contact. Now as I am telling this to you, you are tending to believe that this is always the correct picture and this has happened really for a long time. So, for a long time this no slip boundary condition was taken as something, which is like a ritual which we should not change and the reason was that for many of for most engineering flows it is still valid or it has been experimentally found to be very very accurate, but whenever we are understanding this concept we should ask our self a question: Are there conditions in which the no slip boundary condition maybe violated?

It is important because in many of the modern-day applications of fluid mechanics especially fluid mechanics in small length scales, this boundary condition is something which is put under serious question. So; obviously, we need to see that or we need to appreciate that this is just a conceptual paradigm it is not something which is a ritual and which is expected to work always. Let us see let us try to look into an example within the context of gas flow that no slip boundary condition does not work. So, let us say that you have gas molecules, but not very large number of gas molecules. So, then what will happen? The molecules will be exchanging momentum with the wall, but there will not be very large number of collisions.

Because there will not be very large number of collisions the momentum exchange will not be complete. So, there will be some velocity of the fluid relative to the solid boundary even if otherwise, we tend to believe that there should not be any slip. So, that is just because of the rarified nature of the medium that there is not sufficiently large number of molecules to have a theoretically large number of or infinite number of collisions. On the top of that, there may be local strong gradients in density and temperature and that might itself induce motion of molecules of gases over the solid surface.

So, these are called as phoretic motions if these are induced by temperature these are called as thermophoresis and this may be induced by any other effect, but temperature is one of the common effects by which by introducing a very high gradient of temperature, you can introduce local flow of molecules of gases over the solid boundary. So, we can see that there may be situations and there are likely to be such situations when the no slip boundary condition is not valid, but well in most of the engineering systems that we are talking about the no slip boundary condition will work for gases, except for rarified gases or maybe gases, which are not having sufficiently large number of molecules or gases being subjected to very high local gradients of density or temperature. For liquids it is difficult to believe that the no slip boundary condition will not work, because liquid liquids are very compact systems.

So, liquid molecules will not be insufficient in number to have in inadequate collisions with the wall, but for liquid molecules there may be slip because of certain reasons.

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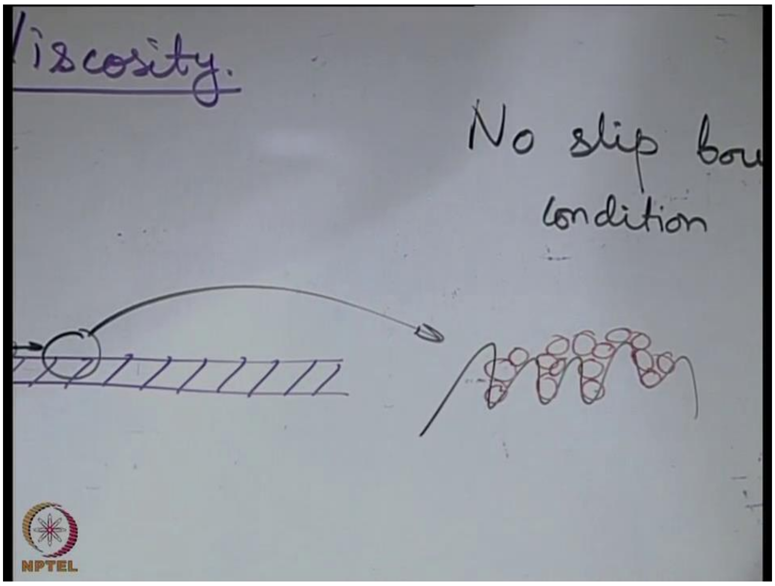

So, to understand the picture of the what happens in the liquid molecules let us consider a small element of surface like this, the surface may look very very smooth on the top, but if you look it into a very powerful microscope it will be much much worse than what I have drawn here. So, it will have lots of peaks and valleys and what will happen is that the molecules will nicely see it on these peaks and valleys. So, some will be entrapped like this and because of a compact nature, what will happen whatever is entrapped is not easily being escaped and that will make us believe that yes it will be no slip boundary condition.

At the same time, if you are having a very high shear rate which is being introduced on the liquid say a very high rate of shear strain, what will that try to do that will try to forcefully bid this out from or take this out from these locations. So, then in that kind of contexts the liquid molecules may also slip on the surfaces. Otherwise, if you have very smooth surfaces say you must have heard about carbon nanotubes, these days those are very sophisticated and fascinated technologies to produce carbon nanotubes. So, those are very smooth tubes and if you are having liquids in contact with them. Now; obviously, there will be the Van Der Waals' forces of interaction between the surface and the liquids.

But in such case water flowing through those nanotubes will have very ordered hydrogen bonding and then the motion of that water will be such that it can overcome the Van der Waal's forces of interactions those are relatively weak in comparison to this strong bonding in the water to overcome the wall attraction and flow on the top of such surface. So, it may actually slip and these are called as highly slipping surfaces therefore, we have to keep in mind that no slip boundary condition is a paradigm, which will work for most of the engineering problems that we are going to consider, but at the same time we should not take it as a ritual.

We should keep in mind that there are situations in which it might be violated, but for practical purposes for almost all the problems that we are going to solve in this particular course no slip boundary condition will work ok. So, let us stop here today and in the next class we will take this up and introduce the concept of viscosity through this no slip boundary condition that we have discussed today.

Thank you.