Introduction to Fluid Mechanics
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Lecture – 47
Problems and Solutions

Last time we were discussing about the Linear Momentum Conservation, and we were looking into some examples to illustrate the use of the Reynolds transport theorem for working out problems related to that. We will continue with some more examples.

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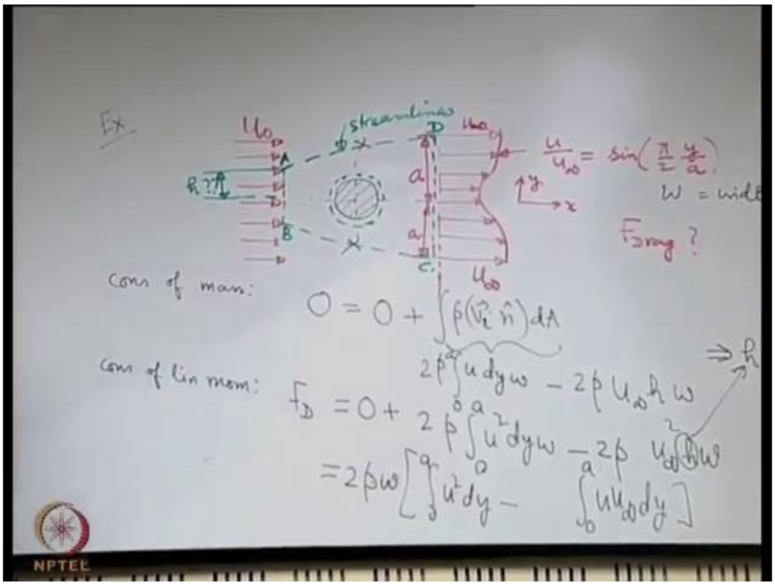

Let us take an example where let us say you have a solid body, say a solid body of whatever shape maybe circular shape if you want it to be so, and fluid is flowing it is coming from a free stream with a velocity uniform velocity say u_{∞} and because of the presence of the solid the velocity is disturbed. And if you go a little bit away from the solid and if you draw the velocity profile say the velocity profile is obtained something like let us make a sketch of how the velocity profile is there. Let us say that the velocity profile varies in this way.

In one of later chapters we will see that what are the factors that will determine that what should be this velocity profile or what should be the nature of variation of this velocity profile, but for the time being let us say that this is a qualitative sketch of how the velocity profile varies. Assume that it is totally symmetric with respect to the center line. And the velocity profile is such that like at the middle, right if you draw it totally in a symmetric manner, at the middle it is like a minimum and then it increases in both sides, comes to almost u infinity at a given height. Let us say that this height at which it comes to almost u_∞ is a.

Let us say this velocity profile is given in terms of the x and y coordinates. Let us say that x is the axial direction and y is the transverse direction, and the velocity profile says given by $\frac{u}{u_\infty} = \sin\left(\frac{\pi}{2}\frac{y}{a}\right)$.

Conservation Of mass: $0 = 0 + \int \rho\left(\vec{V}_c.\hat{n}\right)dA$

$$2\rho\int_0^\infty udyw - 2\rho u_\infty hw$$

$$\Rightarrow h = \frac{\int_0^a udy}{u_\infty}$$

So, the question is that what is the total drag force on the solid body exerted by the water or the fluid. Let us assume that the density of the fluid is rho and we have to find out based on these dimensions. We have worked out and worked out a very similar problem when we were considering flow over a flat plate. This is not flow over a flat plate, this is flow past some body of arbitrary control, but the policy or the philosophy remains the same.

So, we have to basically find out we have to identify a control volume and see what is the net force on the control volume. So, to identify control volume see what part of the control volume will have some inlet and outlet. So, one inlet is this one which is straight forward that the flow is entering, outlet this is straight forward and you can see that outlet is interesting only up to y = a or y = a because beyond that the velocity is uniform. So, if take a control volume say something like this where we consider one inflow boundary one outflow boundary and across other boundaries, we do not want any flow. So, what should be the edges of the other boundaries? There should be streamlines, so that there is no flow across those.

So, let us say that we consider a streamline like this and these are not horizontal lines, these are incline one's. Let us just magnify those a little bit to represent that. Let us say that this is one extreme streamline this is another extreme streamline. So, these are streamlines. Keep in mind that these it is not necessary to choose a control volume which contain streamlines, but only elegance it gives to us is that we do not have to bother about the cross flows. But if we just take say some horizontal lines and the top and bottom and constitutes say a rectangular piece as the control volume then there will flow across that one has to make calculations related to that. It is just a matter of convenience for choosing the control volume, maybe you consider only the water in the control volume, so exclude the solid part.

Now, let us write the expression for the mass and the linear momentum conservation for the control volume. So, one thing that remains unknown is that what is this height h because we have constructed streamline from the right, from the edge of this layer where u become u infinity this is not a boundary layer. We will discuss later that what is the difference between this and the boundary layer as such. So, when we have the streamline starting from the edge the streamline will end up here at some arbitrary point which is not known to us. So, we have to find that out. So, let us say that we mark the edges as A,B,C,D and try to identify that what are the expression for the conservation of mass.

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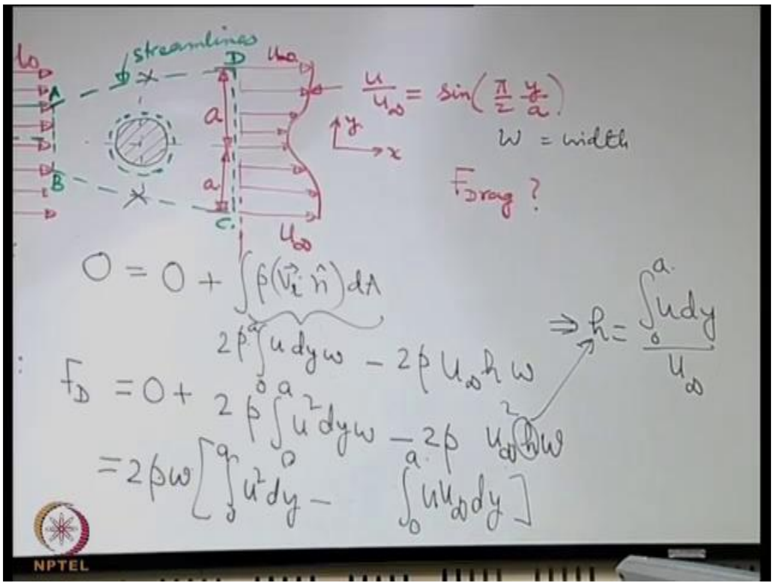

Conservation of linear momentum

$$F_D = 0 + 2\rho\int_0^a u^2 dyw - 2\rho u u_\infty hw = 2\rho w\left[\int_0^a u^2 dy - \int_0^a u u_\infty dy\right]$$

There are two important effects which are important; which prevalent here, one is the viscous effect, another is the contour of the body. As the fluid flowing over the contour of body there is a change in pressure. So, one is the geometrical effect another is the viscous effect and this velocity profile is a combined consequence of what has taken place.

There are certain interesting things that we can observe from this problem one interesting thing is, it appears as if this force does not depend on the shape of this object, but that is an illusion.

The velocity profile will very much depend on what is the shape of the object. So, we have assumed a velocity profile, but this is like it is does it is not that it is it comes just arbitrarily. This whatever is the velocity profile that velocity profile should come from the shape of the body and therefore, that is where the shape of the body becomes critical.

Now, the other thing is that it will also appear as if the force does not depend on the viscosity of the fluid, it appears so. So, it is a kind of like not an intuitive thing that you expect the force due to viscosity because if there is no viscous effect perhaps would be no drag, but there is no viscosity here, there is no presence of the parameter viscosity here.

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Once you get a velocity profile see that is what the integral balance is giving, integral balance the net effect it is not microscopic looking into what has happened individual points, but it has a gross consequence. The consequence is some velocity profile at the outlet.

And this kind of gross consequence is important because you can measure it experimentally. Experimentally point to point measurement is difficult, it is not impossible, but it is it is always more expensive to do that. But experimentally you can at least find out velocity profile on a given section. You can have the different probes all set may not be as simple as a pitot tube probe, but you may have any velocity measurement along this section that is not difficult and uniform velocity which is the free stream velocity that you know.

So, from the experimental understanding of what is the velocity profile at the inlet and the outlet you may be in a position to experimentally calculate or rather to calculate from the experimental data what is the drag force. And the limitation of that is it does not pin point that how the flow field vary from one point to another point to give rise to the drag force, but it gives the total effect in an integral sense. Now, how it varies from one point to another point? For that we have to look in to the corresponding differential equation for viscous flow. That, we will do in our next chapter.