Introduction to Fluid Mechanics
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Lecture –43
Problems and Solutions

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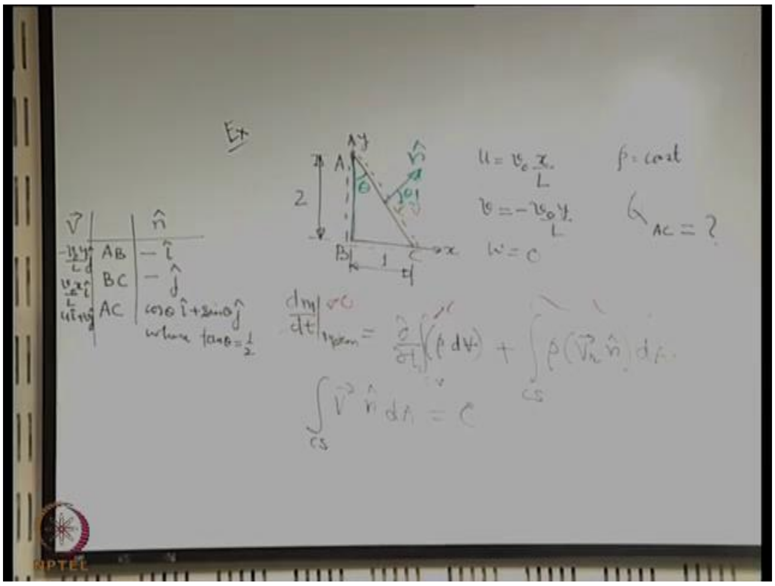

Last time we were discussing about the integral forms of the conservation equations. And as an example we looked into the integral form of the mass conservation and its corresponding differential form also we revisited. And we found out that it is possible to convert one form to the other.

Now, let us look into some more examples of the use of the integral form of the mass conservation equation. Let us say that you have a wedge shaped element like this with the axis oriented along x and y. And the velocity field is a two-dimensional velocity field is given by $u = v_0 \frac{x}{L}, v = -v_0 \frac{y}{L}$, w=0.

So, the net rate of transport should be given by the integral form of the mass conservation

$$\left.\frac{dm}{dt}\right|_{system} = \frac{\partial}{\partial t}\int_{CV} \rho dV + \int_{cs} \rho\left(\vec{V}_c.\hat{n}\right) dA$$

What is our control volume? Let us say that this triangular shaped element is our control volume. So, whenever we are making a control volume analysis, it is important to identify what is the control volume that we are taking.

So, this triangular shaped thing is a fixed control volume, the volume of that is not changing with time. So, it is not a deformable control volume neither the density is changing with time.

$$\int_{CS} \vec{V_c}.\hat{n}dA = C$$

	$\hat{n}$
AB	$-\hat{i}$
BC	$\hat{j}$
AC	$\cos\theta\hat{i} + \sin\theta\hat{j}$

Where $\tan\theta = \dfrac{1}{2}$

For AB it is $-\dfrac{v_0 y}{L}\hat{j}$, for BC it is $\dfrac{v_0 x}{L}\hat{i}$, for AC it is $u\hat{i} + v\hat{j}$.

There is no net flux or influx or out flux of flow across AB because there is no normal component.

Now, let us look into some other problem which is not as trivial as this one.

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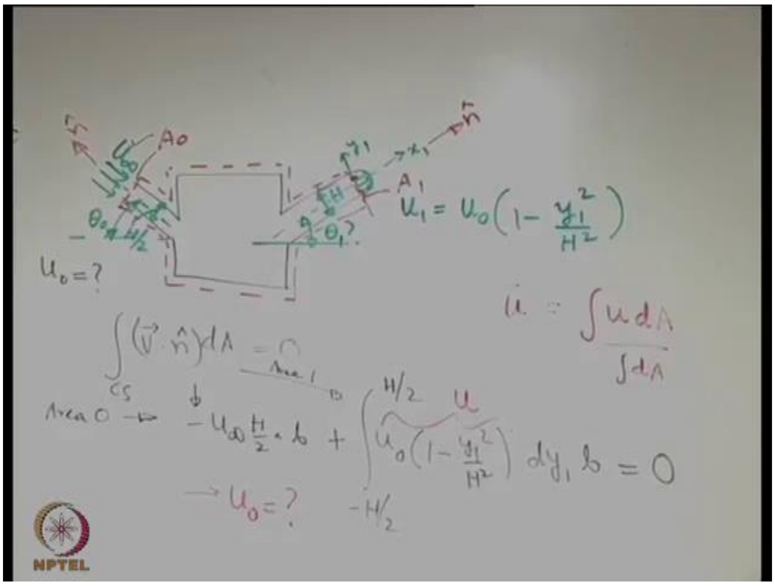

let us say there is a tank like this. The velocity profile at the exit of the tank is through this pipe is given by this one. And in terms of a local coordinate system, this let us say that the local coordinate system is x_1, y_1 it is given as $u_1 = u_0\left(1-\frac{y_1^2}{H^2}\right)$. The fluid entering here it is given that it is a uniform velocity profile here with a velocity u_∞ which is uniform, and let us say this is $\frac{H}{2}$.

The directions of the axis are not given that means it is not given that what is this angle theta. it is given that the density is a constant, $\rho = const$

$$\int_{CS}\left(\vec{V}.\hat{n}\right)dA = 0$$

$$\vec{V}.\hat{n} = u_0\left(1-\frac{y_1^2}{H^2}\right),\; dA = dy_1.b$$

$$\text{Area } O \rightarrow -u_\infty \frac{H}{2}.b \;+\; \int_{-\frac{H}{2}}^{\frac{H}{2}} u_0\left(1-\frac{y_1^2}{H^2}\right)dy_1.b = 0$$

Clearly from the minus sign of the first term you can make that it is inflow and the plus sign of the second term means that it is outflow.

$$\text{Average velocity } \bar{u} = \frac{\int u dA}{\int dA}$$

If it was uniform, then that uniform velocity times the area will give you the flow rate that is what the first term has told us. And if it is not uniform, obviously, you have to integrate it to get the flow. So, if the velocity profile was uniform in a hypothetical case, but the flow rate becoming still the same as it is in the real case. Then if you equate those two flow rates, then that equivalent hypothetical uniform velocity this is called as average velocity.

So, it is like a equivalent uniform velocity that would have prevailed across the section satisfying the same volume flow rate as it is there in the real case. So, it is it is just like $A_1V_1 = A_2V_2$.

Those we wrote for points 1 and 2, now we are writing for sections 1 and 2. When it is uniform, it does not matter it is as good as writing for a point because velocity does not vary from one point to the other, but you do not have anything called as area at a point.

So, it is basically when you write A_1, no matter in the context of what we write what we have written earlier for a two points when you use the Bernoulli's equation. We should keep in mind that there also A_1 was for the section that contained the point 1. So, it is not that area of a point 1 or something like that, but there we use velocity at the point 1.

The reason is that in the Bernoulli's equation we use velocity at a point. So, we had to link it with velocity at a point. Now, it is like we are linking things through velocity over an area. So, if you complete this problem, you will get what is u naught because all other things are known. Now, let us look into some other examples where maybe we look into different case maybe unsteady case.