Introduction to Fluid Mechanics

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Lecture – 36
Problems and Solution

Let us try to work out maybe one or two simple problems to illustrate these equations that we have developed. We will deliberately try to consider examples where we can use the stream wise and cross stream wise components or maybe the polar coordinate systems like that.

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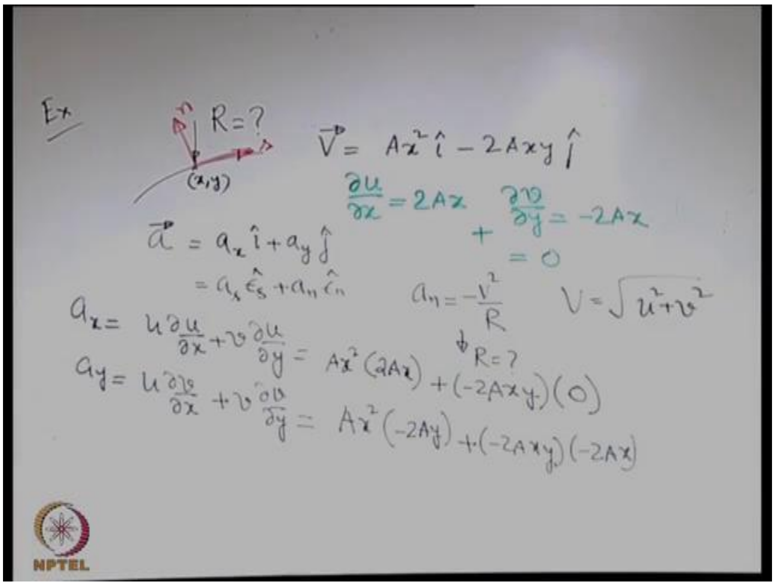

Let us take an example where, let us say that you have a stream line. And the velocity vector is given not in the streamline coordinate system, but in the x y coordinate system. So, at a given point (x , y) located on the streamline, the velocity vector is given $\vec{V} = Ax^2\hat{i} - 2Axy\hat{j}$ where again A is a dimensional constant and x and y are the coordinates. We are interested to find out, what is the radius of curvature of the streamline at this point.

$$\frac{\partial u}{\partial x} = 2Ax, \frac{\partial v}{\partial y} = -2Ax$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\vec{a} - = a_x\hat{i} + a_y\hat{j}$$

This is also resolved in two components, but the components that we are very much looking for in the streamline coordinate system are s and n. So, this may be resolved say along s. s is virtually like the tangent, but again i fundamentally told that it is not a tangent.

It is it is basically located oriented along the streamline. $\vec{a} = a_x \hat{\varepsilon}_s + a_n \hat{\varepsilon}_n$

$$a_n = -\frac{V^2}{R}, V = \sqrt{u^2 + v^2}$$

Acceleration along x, $a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = Ax^2(2Ax) + (-2Axy).0$

Acceleration along y, $a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = Ax^2(-2Ay) + (-2Axy).(-2Ax)$

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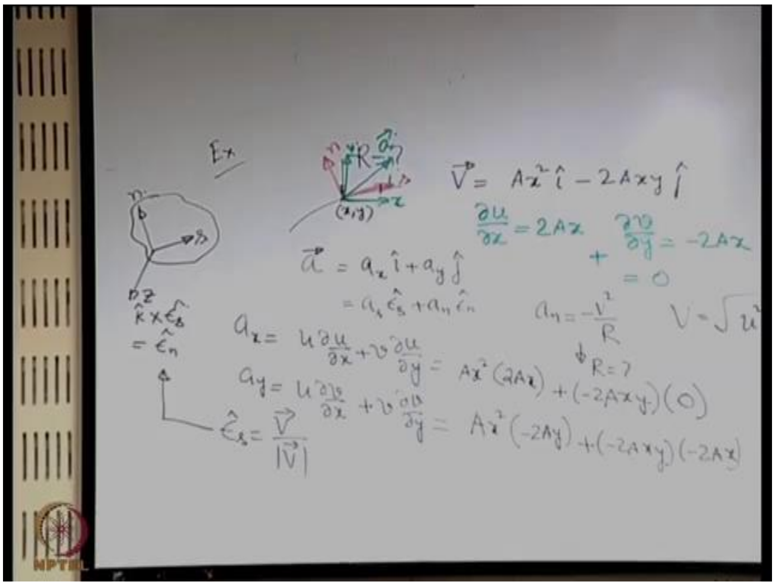

$$\hat{\varepsilon}_s = \frac{\vec{V}}{|\vec{V}|}$$

So, unit vector along z is $\hat{k}$.

$$\hat{k} \times \hat{\varepsilon}_s = \hat{\varepsilon}_n$$

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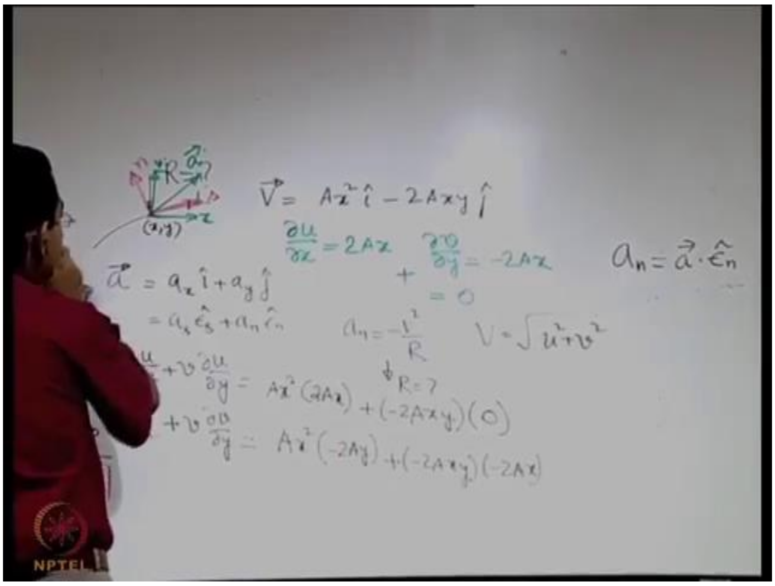

$$a_n = \vec{a}.\hat{\varepsilon}_n$$

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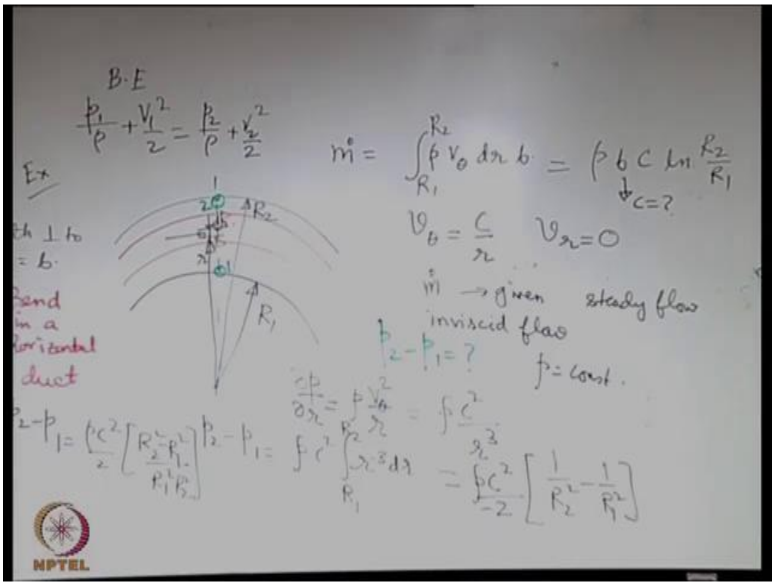

Let us say that you have a curve, or a bend that we were discussing. Say there is a bend in a pipe line. So, that the streamlines which we are originally a parallel to each other. They may remain parallel to each other, but they are now curved in instead of being straight. So, this is like a bend in a horizontal duct. So, if these are parts of circles, you have let us say the inner radius of the bend as R_1 and outer radius of the bend as say R_2. Circular curves are special cases of general curves where the radius of curvature is a constant.

We will write the velocity components in the polar coordinate system. $v_\theta = \frac{C}{R}, v_r = 0$. Vortex is something which will have only the cross radial component of velocity. Not the radial component of velocity. And that cross radial component of velocity is a function of the radial coordinate.

$\dot{m}$ is the rate of mass flow per unit time say Kg/s. Assumptions: Steady flow and Inviscid flow. If the flow is originally irrotational, it is likely to remain irrotational.

Our objective is to find out the difference in pressure between two points 1 and 2. One is in the just close to the or adjacent to the inner radius within the fluid. And two is just at the outer radius. So, we are interested to find out P_2-P_1.

$$\frac{\partial p}{\partial n} = \rho \frac{V^2}{r} = \rho \frac{C^2}{r^3}$$

$$P_2 - P_1 = \rho C^2 \int_{R_1}^{R_2} r^{-3} dr = \frac{\rho C^2}{-2}\left[\frac{1}{R_2^{\,2}} - \frac{1}{R_1^{\,2}}\right] = \frac{\rho C^2}{2}\left[\frac{1}{R_2^{\,2} - R_1^{\,2}}\right]$$

Let us say that we consider a small element at a, at a radius r we consider a small element of. So, this is local r. So, at a local r we consider a small element of width dr there is a flow across this element.

Let us say that in the other direction it has a uniform width. So, let us say that width perpendicular to the plane of the figure, is equal to say b. Area of the strip is b.dr .

$$\dot{m} = \int_{R_1}^{R_2} \rho V_\theta dr.b = \rho b C \ln \frac{R_2}{R_1}$$

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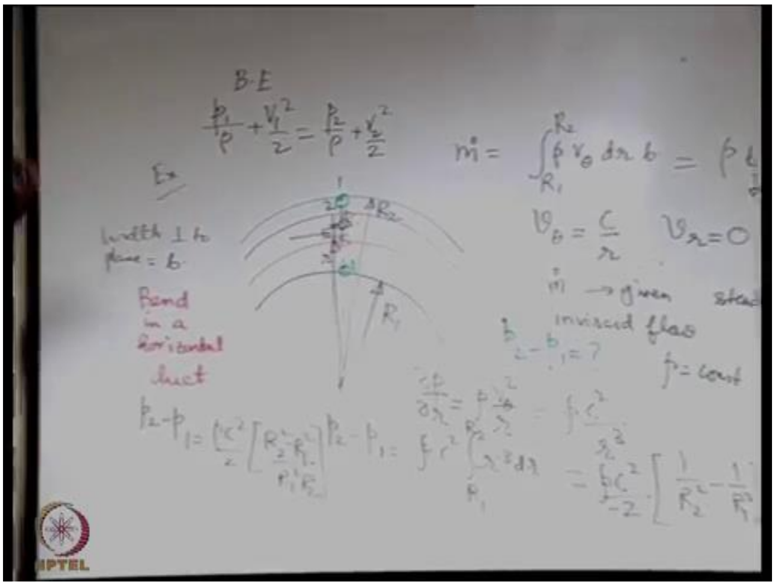

Bernoulli Equation: $\frac{P_1}{\rho}+\frac{V_1^2}{2}=\frac{P_2}{\rho}+\frac{V_2^2}{2}$

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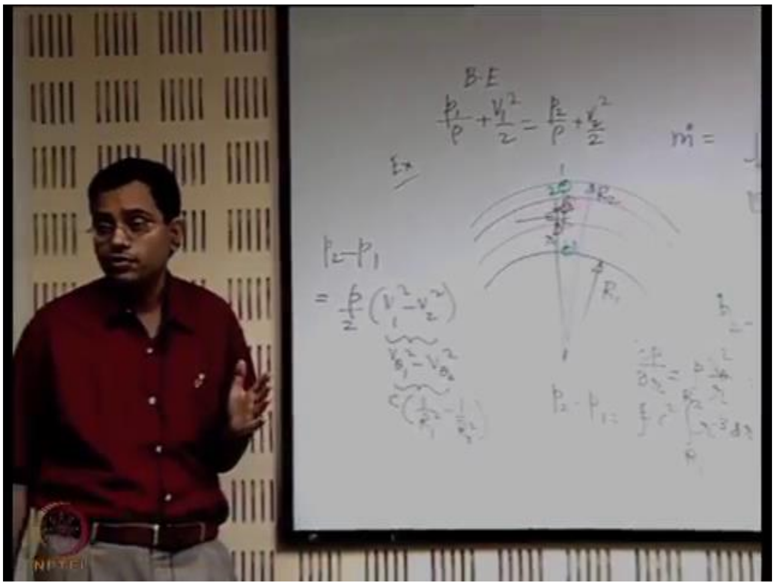

$$P_2 - P_1 = \frac{\rho}{2}(V_1^2 - V_2^2) = \frac{\rho}{2}\left(V_{\theta 1}^2 - V_{\theta 2}^2\right) = C\left(\frac{1}{R_1^2 - R_2^2}\right)$$

Here you get rid of that complication simply because it is an irrotational flow. So, you can apply Bernoulli's Equation between any two points in the flow field. No matter whether they are located along the same streamline or not provided other assumptions of the Bernoulli's

Equation they are valid. Now, we will work out some additional examples to illustrate the unsteady Bernoulli's Equations. So, till now we have discussed about the steady version of these equations. But we have made a remark that it is possible that you have an unsteady version.

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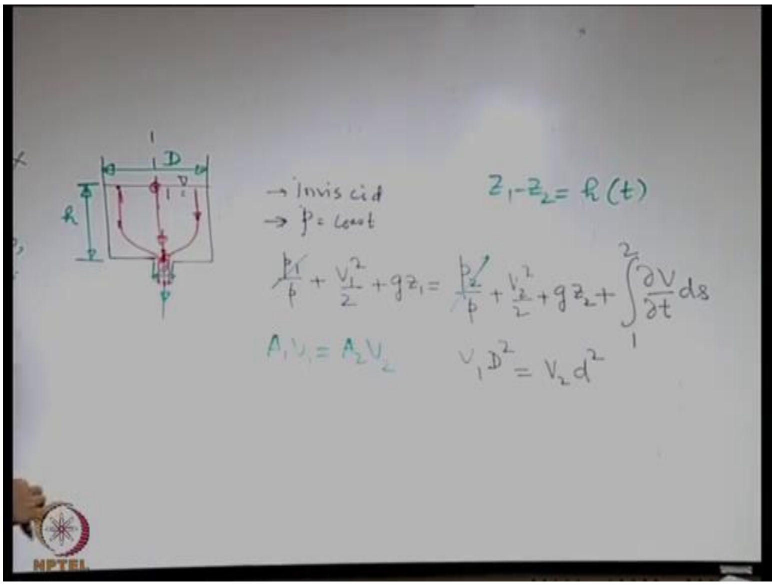

So, to look into such an example we will again consider a case, which you have encountered many times in your earlier experiences with solving fundamental, very simple flow problems. So, you have a tank which has water to some depth. And there is a small opening at the bottom of the tank through which water is being drained out. So, you are interested to find out how the height of the tank or the depth of the tank is changing with time, because of the water that is being drained out.

Let us say that the original height,h. At t=0, $h=h_0$. So, our objective is to find out the time may be required to empty the tank, as a special interest. But here the general interest will be to write the equation of motion for this case.

Originally the streamline will be parallel but when they come close to the constriction, they will try to come close to each other because the streamlines then have to be confined through this small hole.

Assumptions: 1) Inviscid; 2) $\rho = const$.

$$\frac{P_1}{\rho}+\frac{V_1^2}{2}+gZ_1=\frac{P_2}{\rho}+\frac{V_2^{\,2}}{2}+gZ_2+\int_1^2\frac{\partial V}{\partial t}\,ds$$

Now let us see when you are considering the points 1 and 2. So, 1 is exposed to atmosphere and 2 is also exposed to atmosphere. So, at both locations the pressure is the atmospheric pressure.

$$V_1D^2=V_2d^2$$

$$V_1=-\frac{dh}{dt}$$

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