**Introduction to Fluid Mechanics**
**Prof. Suman Chakraborty**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Kharagpur**

**Lecture – 03**
**Concept of traction vector**

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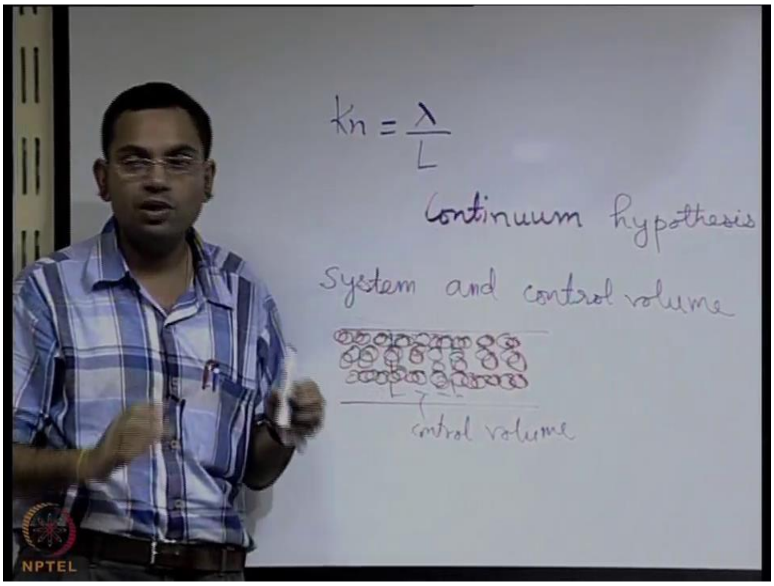


What we left in the last time that is we defined something called as Knudsen number($K_n$)

$$K_n = \frac{\lambda}{L}$$

which is the ratio of the molecular mean free path and the characteristic length scale of the system. So, this L we give it a symbolic note for a pipe it may be the diameter of the pipe for something else it may be some other dimension, but we call it as a characteristic length scale of the system.

So, if this ratio is small, what does it indicate? It indicates that the mean free path is much smaller than the characteristic length scale of the system which implicitly tells that it is a sufficiently densely packed system. On the other hand, if that is not the case that is if the Knudsen number is large; that means, the mean free path is much it may be even larger than the characteristic length scale of the system if it is very rarefied. In such cases what happens? In such cases you have very few number of molecules and then there are lots of uncertainties with respect to presence of molecules in individual elemental volumes.

So, in those cases the macroscopic point of view is not expected to work so efficiently. So, or the macroscopic way of defining the characteristics or properties of the fluid might not work so efficiently. So, whenever the macroscopic way works, we call the fluid as a continuum and the hypothesis concerned is known as continuum hypothesis.

So, what is the continuum hypothesis? Continuum hypothesis tells that we treat the fluid medium as a continuous matter disregarding the discontinuities in the system. There are discontinuities if you look into the molecular level. There are molecules, there are gaps and so on, but if those are sufficiently compact then you may treat it as a continuous matter. Once you can treat as continuous matter then you may use the well-known rules of differential calculus to talk about the changes in properties from one point to the other. So, you can talk about simple gradients, second order derivatives and so on.

So, continuum hypothesis works if there are sufficiently large number of molecules, so that the Knudsen number that we are talking about is very small. If the Knudsen number is greater than 0.1 so to say, then it comes to a state where your mean free path threatens to be 10 percent of the characteristic system length scale and when it goes on larger and larger, there is a stronger and stronger deviation from the continuum hypothesis. So, if there are situations where when we cannot use continuum hypothesis and one need to have a different treatment altogether.

Throughout this course we will be bothered mostly about situations when continuum hypothesis works. So, that means, there are sufficiently large number of molecules in the system. So, that uncertainties with regard to individual molecules do not influence the prediction regarding the fluid flow to a significant extent because we are not looking from a molecular view point, but treating the fluid as a continuous matter. Keeping that in view what we will do is, we will next see that no matter whether we are treating it as in a microscopic viewpoint or in a macroscopic viewpoint how should we describe the fluid as a system.

So, for that we will introduce two important concepts system and control volume. So, we have identified an approach. Now, we have to identify that what should be that fluid over which we apply that concept or approach. So, when we talk about a system by definition system is something of fixed mass and identity. So, something which must have its mass fixed it must have its identity fixed; that means, those are identified. For fluids, sometimes this is not a such a simple concept to implement let us again take an example of flow through a pipe.

So, we have a pipe there are many molecules or even particles whatever are entering the pipe and leaving. So, at a particular instance of time, so, these are the molecules which are present. Now, at a different instant of time you may have different entities different molecules which are present. The reason is quite clear something is entering and something is leaving.

So, it is continuously being replenished, right. In such a situation if you want to track the motion of these particles or individual molecules as something of fixed mass and identity it becomes difficult and tedious because then you have to put a tag on individual entities and follow it as it is moving. So, this kind of approach or the so-called particle tracking approach in mechanics is known as Lagrangian approach.

In fluid mechanics, it is not many times convenient to follow that approach. So, what we do instead is like we focus our attention on a fixed region in space. So, let us say that we have focused attention on this identified region. So, as if we are sitting with a camera focusing the camera on this zone, what we are observing? We are observing whatever is coming into this zone and leaving that, we are only keeping track up to that much, we are ignorant about where from it has come and where it is going. So, rather than focusing attention on individual particles, we are focusing attention on a specified region in space across which matter can flow. So, that region we call as control volume.

So, control volume approach is more convenient for fluid flow because you do not have to track individual particles and fluid is a continuously deforming medium. So, it is very difficult to track individual particles. It is much easier to focus your attention on a specified region and see what happens across that and this particular approach where you use a control volume and analyse what happens across that is also known as Eulerian approach. Just as the Lagrangian approach is following the name of the famous mathematician Lagrange, Eulerian is according to the name of Euler.

So, whatever we will be discussing in this context of fluid flow no matter whether it is a system approach, no matter whether it is microscopic approach or a macroscopic approach we will be mostly using the control volume concept for analysing the flow behaviour. With this basic understanding, we will now go into the concept of a fluid. So, till now we have loosely talked about fluid, we have not seriously defined what is a fluid.

Now, common sense wise if you are ask that what is the fluid; obviously, we will say something what flows is a fluid, right and loosely speaking this is not a bad definition, but there are many things which flow, but those are not so called classical fluids those may be in a borderline between fluids and solid so to say.

A very formal definition of fluids is like this: that fluids are substances which undergo continuous deformation even under the action of very small shear force. So, if you are applying a shear force or there is some shear force acting on the fluid then, even if it is very small it will continuously deform the fluid.

For a solid, obviously, if you are applying a shear force it will not spontaneously deform it till maybe it comes to a threshold limit when it will it will it will appear to be seriously deforming. Obviously, there are substances which are fluids, but which require a threshold shear to be deformed and therefore, the borderline between the fluid and the solid is sometimes not so strict, but for most of the practical cases this definition is something which we will be keeping in mind and we will be classifying fluids or solids according to this behaviour.

So, one of the important consequences is that if there is a fluid which is non-deforming; so, non-deforming fluid may be fluid at rest. So, if you have a container, within the container you have put some water and water is at rest. What is the implication of that? The implication is very straightforward there is no shear which is acting on it.

So, if there is some shear which is acting on a fluid the fluid is deforming converse is also true; that means, if the fluid is deforming there must be some shear which is acting on it. So, when there is a fluid which is there at rest; that means, there is no shear component of force that is acting on it; that means, there is only normal component of force acting on it and that normal component of force per unit area which is acting inverse is called as pressure.

So, obviously, whenever there is a fluid at rest the entire situation of the forcing which is there on a fluid element may be expressed in in terms of pressure. When the fluid is moving it does not mean that there is no pressure; obviously, pressure is very much there which would have been there if the fluid is at rest, but there are additional forcing components which come into the picture which are directly related to the deformation of the fluid and this situation altogether may be tackled in continuum mechanics that is mechanics of a continuous medium through the concept of stress.

So, we will now introduce the concept of stress which we will be introducing in a context of continuum mechanics so, it is valid for both solids and fluids.

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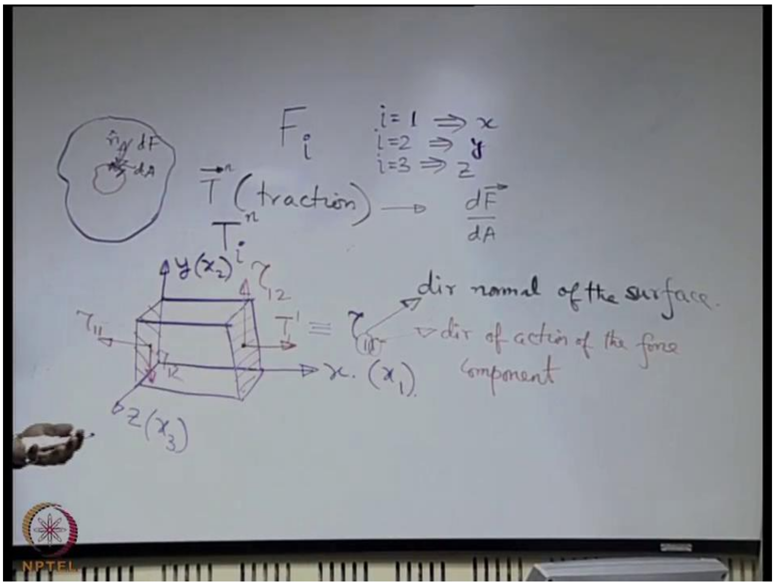


Now, let us say that we have an element. When we say we have an element it may be an element of a solid fluid whatever we are not very particular about it and we are identifying a small chunk from that. On that small chunk we are taking a small area say dA. What we are interested is to see that whenever we are taking out the small chunk the other part of the body will exert some force on this, just by Newton's third law.

So, we are interested to identify that force and let us say that that force is directed like this, it is absolutely arbitrary. So, it depends on many situations let us say that the force is dF. So, if we want to define some force per unit area then we are just giving it a name we are calling it T or traction, we will call it a vector because it is having the nature of a force. It will be implicitly determined by $\frac{d\vec{F}}{dA}$

where F is a force, but we have to remember that it is not unique until and unless we specify the area.

What it means, let us say that centred around the same point we take a different elemental area same dA, but differently oriented. So, if we take that differently oriented area now, if we find out the resultant force on that it is likely to be different; that means, given the point around which we take the differentially small area as fixed that is the location fixed given the magnitude of the so-called elemental area as fixed still this ratio is going to be different.

So, this strongly depends on not just that dA, but how that dA is oriented. So, it is important to give a kind of superscript or subscript to this. So, let us give us superscript n. So, this n denotes that we are talking about an area which is having its outward normal in the direction of the n vector. So, whenever we denote the orientation of area it is customary to denote it by the unit vector in the outward normal direction. Let us say that n is such a vector, even if it is not the unit vector there is no problem because we can always normalise it in the form of a unit vector, the direction is what is important.

If n is changed; that means, the orientation of the area is changed, obviously, this traction vector will change. Now, this traction vector therefore, is not denoting something which is ordinarily like any other vector. So, if you are talking about say a force; so, whenever you are writing the component of a force say F is the force you are using an index i to denote the component of the force. So,

$$i = 1 \Rightarrow \text{x}$$

$$i = 2 \Rightarrow \text{y}$$

$$i = 3 \Rightarrow \text{z}$$

Therefore, by using one index i and varying it from 1 to 3 we may denote the components of a vector , but when we are trying to denote this traction vector yes it has a component because it is like a force per unit area. So, based on the direction of the force it has its own direction, it has its own components, but it’s specification depends on also another sort of index n which denotes the choice of the area orientation that has been employed to calculate this T. So, it is something more general than an ordinary vector.

How general it is to understand that we will take some special examples. What are the special examples? Let us say that we take an element of a fluid which is of a rectangular shape like this. We may orient axis like x, y, z in terms of the index we call this as x1, x2, x3. Index using the index is a very convenient way because just by varying the index you can vary the directions.

Now, let us try to see that what are the special cases of traction vectors on the phases of this cuboid. So, this has six phases and these phases are special surfaces. Why these are special surfaces? These have their normal directions either along x or y or z. This is absolutely an arbitrary area and what we are interested to do is to figure out what happens for an arbitrary area by referring to special areas which are either oriented along x, y or z. So, that is the motivation of taking such an element.

So, once we have taken this element what we are going to see is we are going to write such expressions for different phases. So, when we come along this phase, we are interested to write the components of the traction vector. So, let us write that. So, components of the traction vector there is a component along x. In terms of this notation what should be the subscript?

The unit vector outward normal of the surface chosen. So, here the surface chosen has unit vector along $x_1$. Alternatively, we use a notation equivalent to this is $\tau_{11}$. So, two indices are there. What are these indices representing? The first index is representing the direction normal of the surface and the second index is representing the direction of action of the force component. So, in general it is like $\tau_{ij}$ where i represents the direction normal of the surface which you have chosen and j represents the direction of action of the force component itself.

Interestingly, let us look into the opposite phase of this one. So, for this phase the outward normal is along negative x, right. So, we will develop a sign convention that if the outward normal of the surface is along negative i we will have the sign conventions such that positive $\tau_{ij}$is along negative j; that means, here the. So, this one we will call as positive $\tau_{11}$ for the surface. Why? Because, the first one is actually along negative x, the first index. Therefore, the positive sense of the j which is the second one is along negative x.

So, on this surface if you want to draw positive sense of $\tau_{12}$, so, that should be downwards. These are sign conventions. So, if it actually is the other way it will come as minus of this number just like in free body diagram you draw force. The force might have come in the negative; that means, it is actually in the opposite sense than what you have drawn in the figure. So, these are also like that. So, as if you are drawing the free body diagram of a chunk or an element. So, we are establishing sign conventions for that.

So, what we have learned as the sign conventions is if the direction i is along the negative of one of the coordinate axis, then the $\tau_{ij}$ positive sign will be oriented along the negative j direction and if it is positive it is the other way.

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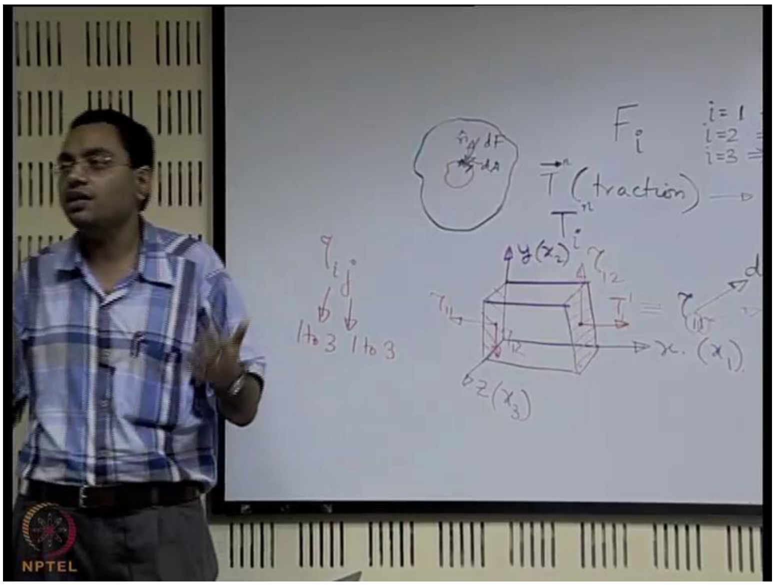


So, this $\tau_{ij}$ , so, when you write $\tau_{ij}$, this i may vary from 1 to 3 and j may vary from 1 to 3. So, these are certain quantities in general you may have 3 into 3, 9 $\tau_{ij}$components.

We will later on see that actually out of these nine you have six which are independent and utilising those independent six components which are called as components of a stress tensor we can actually find out the state of stress for any arbitrary plane which is neither oriented along x nor y nor z. So, this particular quantity is which are like called as components of a stress tensor we may understand that these are not like vectors.

So, what are the differences between these and the vectors? So, very logically you can see a vector requires a single index for its specification, i. This requires two indices for its specification. What are the special indices? One index is just like making it act like a vector, but the other index is specifying the direction normal chosen to calculate that quantity. So, it is something more general than a vector. This actually is called as a second order tensor.

We will not be defining in general what is the tensor because it is an involved mathematical concept and there is not enough scope here that we discuss about that. But, at least from common sense you can appreciate that the order of tensor in this Cartesian notation is like the number of indices that you are requiring to specify it. So, vector is also a tensor. It is the tensor of order 1, scalar does not require any index for its specification. So, it is like a tensor of order 0.

So, we have very easily come across three different orders of tensors tensor of order 0 which is a scalar, tensor of order 1 which is a vector and tensor of order 2 example is a stress. And, we will see examples where we will be having tensor of order 4 as of course, there may be n-th order tensor in general, but we will see that there are certain important tensors in the context of mechanics in a continuum or fluid. So, fourth order tensor is one such example which we will come across later on in this course.