Introduction to Fluid Mechanics
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Lecture – 18
Force on a surface immersed in fluid – Part-III,
Stability of solid bodies in fluid – Part-I

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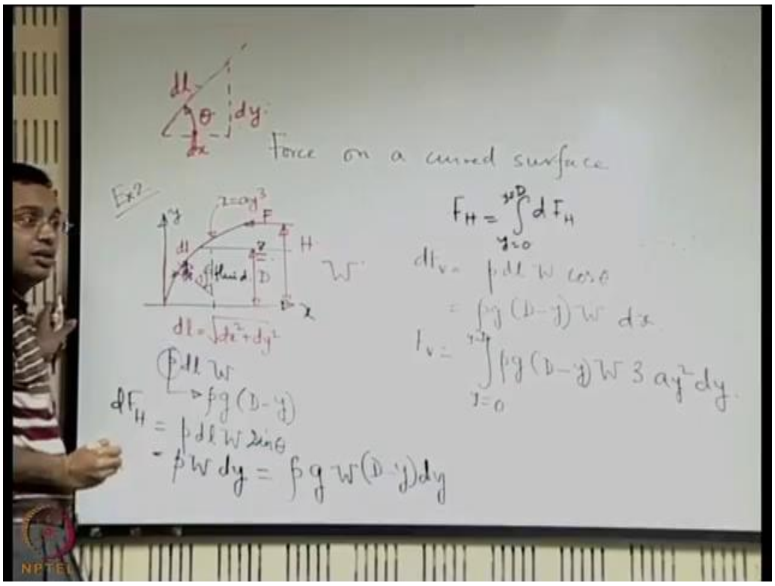

Let us consider a second example. Let us say that you have a curved gate instead of a plain gate, you have a curved gate and there is fluid on one side of it say you have fluid on this side. There is a free surface of the fluid given by this and this fluid tries to exert some force on this curved surface and there is some balancer. So, there is some external force which is applied here to keep it in equilibrium and the location of this external force is given by this H and the depth to which this fluid is filled up is capital D. So, why such problems are practically important let us look into maybe one or two practical cases to see that why such effects are important.

So, we will look into some example applications.

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So, it is a if you clearly see this is like a dam. So, there are forces which are exerted by water and these force components are huge forces which are acting and the structure must be strong enough to sustain it and it is very common and to save us from floods or other calamities, there are many occasions where there are reservoirs in which these water supplies are stored. So, until unless the rainfall is very severe it retains its height, but once it is once the rainfall is so strong, it cannot retain its height.

So, some of the water has to be released and when the water is not released it is under static condition.

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And, it has to be calculated on the basis of fluid statics condition the resultant forces and the situation of equilibrium of course, when it is when the water is very dynamic then you do not consider the fluid statics, but you consider even the dynamic effect of water as it is impinging on the surface. So, you can see the shape of the. So, called dam it is it is not really a plain surface, but it is a curved surface, it is like arch type. There are fluids on different at different ends and it is important to calculate that what is the resultant force that is there.

So, that is the motivation behind solving such a problem that it gives you a clue of how to design maybe dams or switch gates where across which you have different fluid elements which are exerting forces. So, here on one side you have some fluid element as say water on other side say there is atmosphere. So, on the right side there is water in this example and we want to calculate what is the resultant force. So, what we will do we may solve this problem in again two ways; one is by looking into the horizontal and vertical components of forces according to the principle that we have just seen or maybe just by direct integration of the forces on elements.

So, if we do that the second one is little bit may be easy to begin with. So, let us say that we have considered a small element here at a location (x,y). For idealization let us say that this curved surface when its projection is considered it has the equation $x = ay^3$ which is the equation of this curve. So, at the (x,y) if you consider a small element say of size dl then what is the resultant force due to pressure on this dl? First of all how do we specify this dl? Say, at x , y we can consider a dl which is comprising of the resultant some displacement along x say dx and some displacement along y say dy.

So, ā $dl = \sqrt{dx^2 + dy^2}$ So, if you draw a magnified figure if this is dl it is like the sum of a dx along x and dy along y. If this is a small part of the curve, this is approximately the tangent to the curve at that point at that x , y and so, you can consider that this angle θ designates the slope of the curve at that point where it is aligned the dl is aligned with the tangent to the curve at x , y. So, we know what is dl if you know what is the width of the fluid or which is what is the width of the gate say w is the width of the gate then w.dl is the area on which the fluid pressure is acting.

So, what will be the direction of action of the fluid pressure? It is acting in this way normal to the surface. So, that will have two components; one is the horizontal component, another is a vertical component. So, how do you calculate the horizontal component and the vertical component? The force because of this is $p.dl.W$ where p is the pressure at (x , y). So, how can you write express p in terms of the depth capital D?

$$p = \rho g(d - y)$$

From the free surface, this is the resultant force, but if we break it up into components that is what is our objective and we should break it up into components because we cannot just integrate it like this because the direction of such force on each element is changing. So, we cannot just algebraically or scalarly add. We should take out extract individual components and then add up the components. So, when we take the horizontal component, what is the horizontal component because of this?

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$p.dl.W$ is the resultant force. So, the dl makes theta with the horizontal. So, normal to the dl should make an angle theta with the vertical.

$$F_H = p.dl.W.\sin\theta = pWdy = \rho gW(D-y)dy$$

$$F_H = \int_{y=0}^{y=D} dF_H$$

Something very interesting it shows as if this is independent of the function of the graph $x = ay^3$, it does not matter what is x and intuitively it is supposed to be that way because when you take the projection of the surface in the side plane does not matter how it is curved because the projection will always be straight. So, you just require to know the extent of the depth and you can verify that this will be nothing, but the projection on the surface which is basically a surface projected on a vertical plane.

$$dF_V = p.dl.W\cos\theta = \rho g(D-y)W.dx$$

$$F_V = \int_{y=0}^{y=D} \rho g(D-y)W3ay^2dy$$

We will not go into the integration in details because it is a very simple integration and it is not worth to waste time just on that, but we will focus on something which is bit more important, that is, let us say that we want to find out the same vertical component of force, but using the method of the weight of the fluid that is contained within that projected volume. One way to see that whether it is correct or not is that, if we find out that weight and we come up with an expression it should be same as this one, so that you can of course, find out the weight and check that the expression at then is same as what you get out of this integral, but even before that how you qualitatively assess it.

So, what was our method you project from the corners; vertical lines which meet the free surface or it is extended form. So, one line projected here and another it becomes a point and it should be the shaded volume of fluid.

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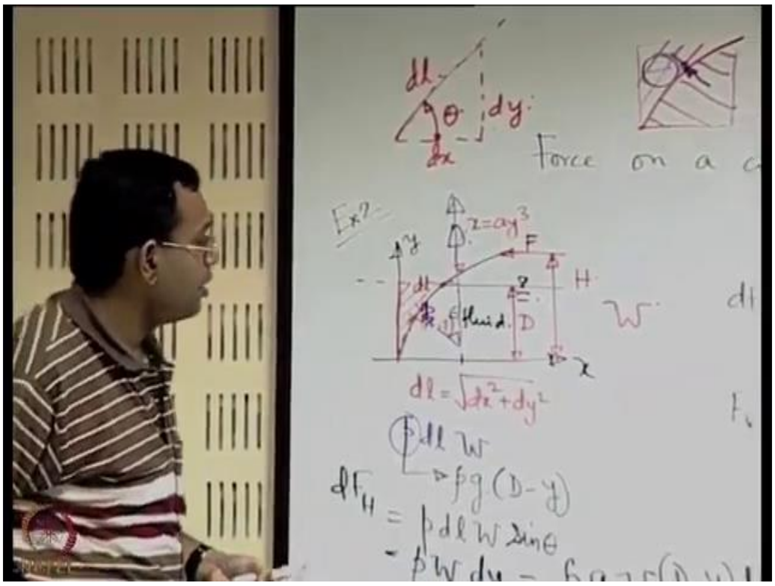

So, just think in this way let us say that this is a curved surface and let us consider that there is volume of fluid one on this side another on this side. If that was the case then you can see that in a static condition it is naturally in equilibrium because whatever is the force due to pressure or from one side the same is the force due to pressure on the other side. So, the two sides are keeping it in equilibrium.

Now, when you do not have a fluid here; that means, that this part is like minus it is subtracted. So, it is a condition equivalent to a deviation from equilibrium because of a lack of presence of these rather than the real presence of this. So, the deviation from equilibrium is because of this equivalent volume of element which is which is in the upper part. So, if you calculate that volume of fluid and find the corresponding weight of that you will see that you will get it exactly the same as this one, and you can clearly tell that what should be the direction of force acting on vertical component of force acting on this upwards or downwards.

Upwards; because the vertical component of this one is like upwards, I mean directed in this way. So, it is not apparently the vertical component of this shaded volume of fluid because it will appear that if there is a volume of fluid here it is weight should act downwards. You have to remember it is an imaginary volume of fluid and it is just giving you the equivalent volume that needs to be employed for calculation of the vertical component of force. The exact sense of the force has to be determined from the physical meaning of this type that is; so, if you subtract a kind of volume like this it is as good as extra upward force because something is missing some weight is missing from the top otherwise also from the direction of pressure itself it will follow.

So, either way whenever you are calculating either by the fundamental method of finding force components or individual elements summing the vector summing them up; that means, summing up scalar forms in terms of the x and the y or the horizontal and the vertical components or finding out the vertical and horizontal components by the alternative method that we have seen, whatever is it you must ascertain the correct sense from the physical condition and that will not always be dictated by the rule base. It will come from the consideration of where is the volume of fluid that is present is exerting the force and what is the sense of that force when it is exerting a pressure on the element. So, that consideration should give you the proper sense of the vertical component of the force.

Now, we have considered forces on our force components on plane and curved surfaces. We have seen that there are some simple ways by which we can evaluate these force components. Now, what we have assumed is that when the surface is put in the fluid the surface is in equilibrium and that equilibrium is not disturbed, but if there is a slight tilt because of whatever reason then that equilibrium may be disturbed and if that equilibrium is disturbed what will happen we have to understand.

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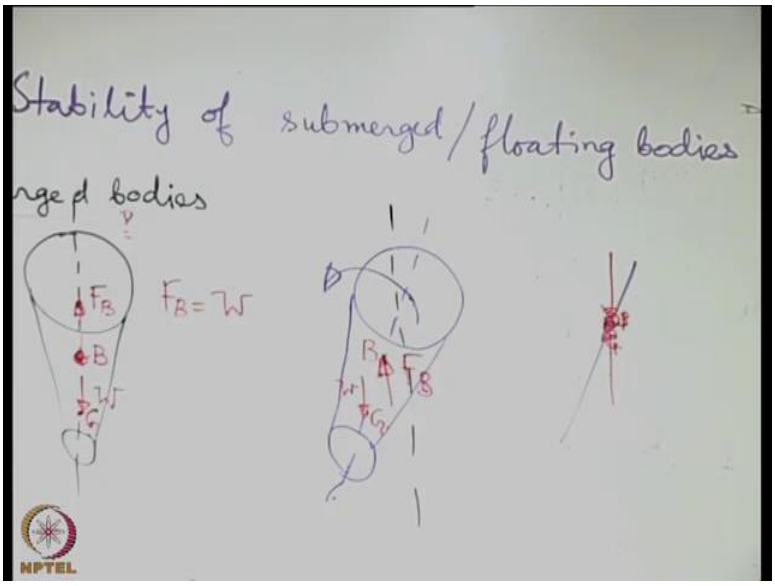

So, we have to now go through the concept of stability of floating and submerged bodies. Let us say that first we consider submerged bodies. So, submerged body is something which is completely immersed in the fluid and floating means a part is at the top I mean above the free surface and the part is below the free surface.

So, let us take an example; let us say that we have this kind of a body, looks like a parachute type the top is light and the bottom is heavy because of some added mass. Initially it is completely submerged and the resultant forces whatever are acting on this you may write in terms of the buoyancy force and the weight. So, the buoyancy force will be acting on acting through some point and the weight will be acting through some point. So, the buoyancy force is based on the volume which is submerged not the mass.

So, when you consider the volume that is submerged the greater portion of the volume is at the top. So, may be the resultant buoyancy force acts through this, but the mass is more concentrated towards the bottom. So, the weight maybe is more concentrated or the resultant force due to the weight distribution is passing through the point which is G or the centre of gravity that is located somewhat below and the resultant buoyancy force is now acting through some point B. So, what is the point B? So, B is the so called centre of buoyancy; that means, whatever is the location of the centroid of the displaced volume. So, it is fully a geometrical concept.

Whereas, when you have G this depends on the distribution of mass over the body. So, this is something where for equilibrium you have $F_B = W$ it is in equilibrium. Now, let us say that you have slightly tilted it. So, when we have slightly tilted it; it has a deformed not deformed, but deflected configuration like this. So, its axis has got tilted from the original vertical one maybe because of some disturbance. Now, the entire body is within the fluid. Therefore, the location of the center of buoyancy and center of gravity relative to the body does not change because the entire body is within the fluid itself.

So, if you have say this as G, this still remains as G if you have this as B this still remains as B because it is already totally within the fluid, so, the it is volume distribution within the fluid it is mass distribution everything it does not change. So, you have F_B acting like this, you have W acting like this only thing what has now changed is that no more F_B and W are collinear. So, when they are not collinear they will still be equal and opposite forces, but not passing along the same line. So, it will create a couple moment.

So, what will that couple moment try to do? So, if you see the sense of this couple moment what it will try to do? So, it will try to create a rotation like this which is shown in the figure if you look into the senses of the forces and this rotation what it will try to do it will try to bring it back to its original position or configuration. So, we call it at a restoring moment. On the other hand if G was above B then this would have been downwards and this would have been upwards and that would have tried to increase the angular displacement even further.

So, what is the hallmark of a stable equilibrium? That is, if you have a slight displacement it will try to come back or be restored to its original configuration. So, this type of situation ensures that it tends to come back to it is original configuration. However, if G was above B, it would have tried to increase the angular displacement even further not restoring, but helping the disturbance. So, in that way it will be unstable equilibrium. What would be the situation if B and G are coincident somehow?

So, wherever it is there still it will be a collinear say you have somehow an arrangement where you have this as B, this as the same point is G. So, this is B this is as good as G. So, whatever is the weight and the buoyancy they are always acting along the same line, no matter whether it is tilted or not. So, wherever it is tilted it will locally attain equilibrium and that equilibrium is known as neutral equilibrium. So, the stability of submerged bodies the equilibrium condition depends on the relative location of B with respect to G. So, what we can summarize from this?

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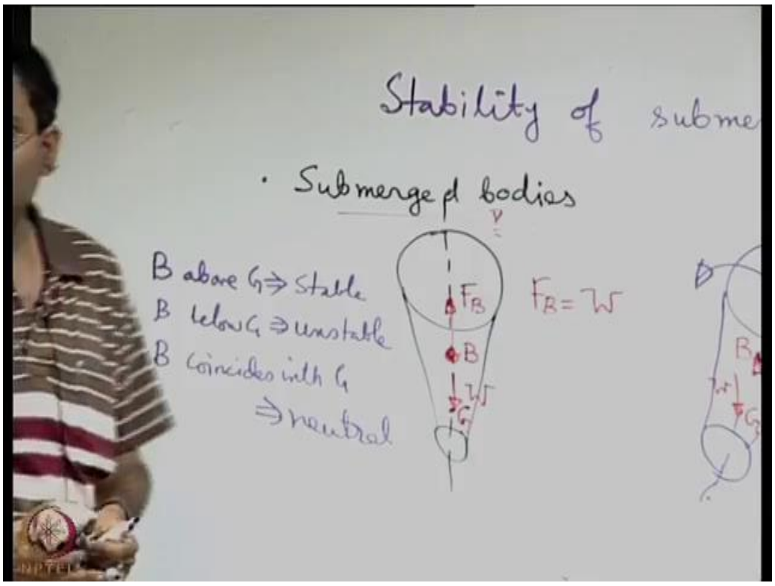

If B is above G what it will imply? It will imply stable equilibrium. If B is below G, it is unstable equilibrium and if B coincides with G that is neutral equilibrium. So far so good, but we have to remember that submerged bodies are not the only types of bodies that we need to consider. Many practical examples are cases of floating bodies like ships. So, a part is within the fluid and the part is outside the fluid. So, what will be the situation for that, let us take an example.

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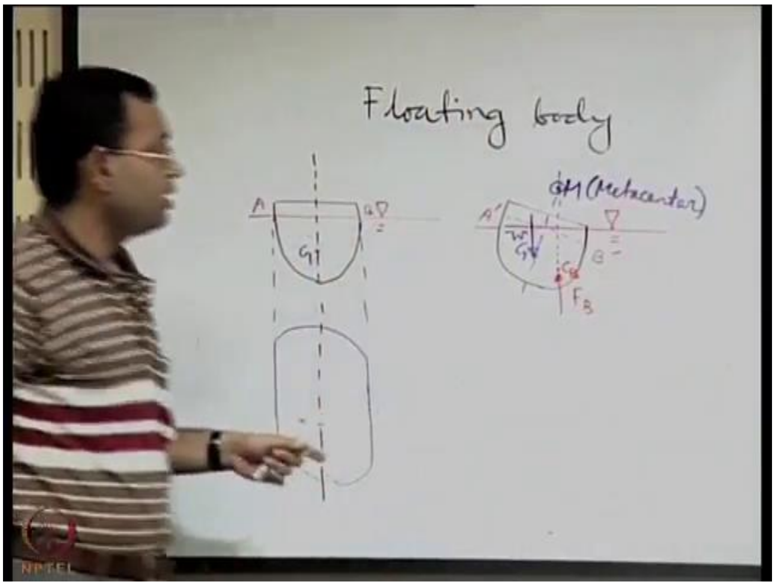

Let us consider a floating body. So, when you consider a floating body let us see that how is it different first from a submerged body. So, let us say that this is the free surface of the fluid, this is the body which is floating something like a boat shape or similar to that. If you consider its intersection with the free surface of the fluid that is the sectional view of the intersection with the fluid, let us say that it is something like this.

So, this part that we have drawn is like this one whatever we have drawn that plane is like this. Now, this we are assuming that this is having an axis of symmetry; say, this is the axis of symmetry. Now, let us say that this is tilted. Let us see what happens when this is tilted. We will assume that it is tilted very slightly because when we test the stability we just give a small displacement and see how it responds to the small disturbance. So, we just tilt it like this and it comes to this configuration.

So, whatever was the line of interface before, now say that line of interfaces with relative to the body whatever was the line say AB, now say it becomes A’ B’. If it is symmetrical with respect to the axis at over which it tilts you will see that one interesting thing has happened. Some new part has gone down into the fluid, some new part has come up and if it is very symmetric these two parts are of same volume.

So, the volume that was earlier immersed is still the same, but the distribution of the volume has changed.

The centre of buoyancy has changed.

So, there is no sanctity with respect to the location of the centre of buoyancy, that is very important. So, for the submerged body when you have the location of the centre of buoyancy relative to the body it does not change whereas, when you have a floating body depending on it is tilted configuration there will be an extent of dominance of one side of the body relative to its immersed conditions with respect to the other and accordingly there will be a bias that there will be a preferred side across which the centre of buoyancy will be moving.

So, the centre of buoyancy cannot be one of the fixed parameters with respect to which we may decide whether it will be stable or not that is the first thing. So, the stability criteria for submerged body will not work. So, whenever we are trying to learn something new we have to understand that why are we trying to learn it afresh; I mean if the same criteria for submerged body would have worked we would have not gone into this exercise. So, first we are getting that motivation that how or where is the difference. The difference; again, I sum up is like this, the centre of buoyancy location is now not fixed with respect to the body, but it goes on evolving as the body is tilting.

So, let us say that the centre of buoyancy now comes to this position say C_B, centre of buoyancy. We expect that it will be coming towards this direction because it is now more tilted towards the right, so, more part of the body is now into the fluid towards the right. So, you have the centre of buoyancy in this way. So, you have the resultant buoyancy force like this. The centre of gravity is something which is fixed with reference to the body that does not change.

So, if the centre of gravity earlier was say relative to the body here, let us say that the centre of gravity still here. So, the weight of the body is like this. So, now, again you can see that there is a couple moment and whether it is restoring or helping it depends on that whether it is if it is extended where it will meet the axis. If it meets if it meets down of this one then it is one way if it meets above G it is the other way. So, where it meets the axis that point is known as metacentre.

So, in our next class we will see that what is the consequence of this metacentre, and how the location of the metacentre will dictate the stability under this condition. So, it is not the centre of buoyancy that is important here, but the location of the metacentre relative to the body is what is going to decide whether the body should be stable or not for a floating body that we will take up in the next class.

Thank you.